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Article

Study of the dynamic vertical interaction of multi-axle rolling stock with railway track

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Abstract: The continuous growth in the demand for increased capacity of the existing railway network of the Republic of Kazakhstan, particularly in regions bordering the Russian Federation and China, makes the issue of improving the operational safety of heavy-haul freight trains with increased mass and length especially relevant. The introduction of heavy trains represents a complex engineering and operational task that involves the use of more powerful locomotives, increased axle loads, reconstruction of track infrastructure and power supply systems, and the improvement of transportation technologies. In addition, higher train speeds combined with increasing freight traffic intensity require improvements in the strength and stability of the railway track structure. The technical policy of Kazakhstan Temir Zholy is aimed at a full transition to continuously welded track constructed on reinforced-concrete foundations. Edge stresses arising from rail bending and torsion under vertical and horizontal loads from rolling stock are among the key parameters determining rail strength. It is well known that the half-sum of the edge stresses characterizes the vertical impact on the track, whereas their half-difference represents the horizontal impact (lateral force). These parameters also reflect the influence of force moments generated by lateral loads and by the displacement of the conventional center of the wheel-rail contact patch relative to the rail head.

Keywords: railway track; vertical and lateral forces; mechanical modelling; high-speed operation; dynamic characteristics

1. Introduction

Early locomotives, similar in design to the steam locomotives that preceded them, were built with a rigid frame structure. In such locomotives, all driving wheelsets were mounted within a single main frame that carried the vertical and horizontal forces arising from wheel–rail interaction, as well as the longitudinal forces associated with traction and braking.

These frame-type locomotives were equipped with leading or guiding bogies that transmitted part of the vehicle weight to the track and helped reduce lateral forces between the locomotive and the track when passing through curved sections of railway lines [1,2].

With the subsequent increase in operating speeds and locomotive power, as well as the need to improve their ability to negotiate curves, a new structural element of the mechanical system was introduced—the bogie. In this configuration, the locomotive body frame is supported by bogies that are capable of rotating relative to the body, thereby improving running stability and curving performance.

The introduction of bogies into the mechanical structure of locomotives required the solution of several related engineering problems. These included supporting the locomotive body on bogies capable of rotating about the vertical axis, transmitting longitudinal and transverse forces while the locomotive body and bogies move relative to each other, and connecting bogies to improve running performance in curved track sections [3,4].

As a result, the mechanical structure of railway rolling stock was supplemented with a number of additional components, including body-to-bogie support units, pivot mechanisms, and articulation devices between bogies. To reduce the probability of wheel slip, load equalization mechanisms between the axles of the driving wheelsets were also introduced. These devices help maintain a sufficient vertical load on the leading axles of each bogie, especially when the locomotive develops maximum tractive effort during starting [5]. In such conditions, maintaining adequate wheel–rail contact forces is essential to ensure effective traction and prevent loss of adhesion.

The study of the dynamic characteristics of rolling stock is typically based on representing the train as a mechanical system consisting of rigid or deformable bodies interconnected by structural elements. This representation reflects the geometric and structural properties of the vehicle. Using the principles of mechanics, the behavior of such a system can be described mathematically through a set of differential equations of motion. The resulting mechanical–mathematical representation is referred to as a dynamic model. A properly developed dynamic model should reproduce the main characteristics of the real system with sufficient accuracy, allowing the dynamic behavior of the rolling stock to be evaluated with the required level of precision.

The scientific novelty of the research lies in the analysis and refinement of the dynamic characteristics of a locomotive with a bogie structure of a mechanical part based on mechanical and mathematical modeling. The paper proposes an approach to describing the dynamics of rolling stock using a system of differential equations of motion, which makes it possible to more accurately take into account the geometric and power parameters of the structure.

The practical significance of the research lies in the possibility of applying the results obtained in the design and improvement of the mechanical part of locomotives. The use of the proposed dynamic model makes it possible to more accurately assess the load distribution between the wheelsets, the conditions for fitting the locomotive into the curved sections of the track, and the likelihood of wheel boxing. This helps to increase the reliability of locomotives, improve their traction properties and reduce dynamic loads on structural elements and the railway track.

The purpose of this work is to develop and analyze a mathematical model of the dynamic vertical interaction of multi-axle rolling stock of railways and tracks based on a continuous elastic foundation approach. The model is designed to estimate rail deflections at various operating speeds, axial loads, bogie configurations and seasonal variations in track stiffness. Particular attention is paid to the identification of critical operating conditions associated with maximum rail deflection and increased dynamic load on the track.

To achieve this goal, the paper examines the design features of the mechanical part of locomotives and the evolution of the transition from a frame scheme to a bogie. The conditions of transmission of vertical, longitudinal and transverse forces between the body and bogies are analyzed, as well as the influence of support devices and load balancing between wheelsets on the stability of movement.

2. Materials and methods

During the operation of railway transport, various external and internal disturbances arise that affect the dynamic interaction of railway rolling stock and track. These irregularities include rail surface irregularities, changes in track stiffness, wheel defects, axle load fluctuations, and changes in operating speed. These factors create dynamic forces that affect both the suspension system of the vehicle and the structure of the railway track.

When analyzing the vertical interaction of the vehicle and the track, it is necessary to take into account the dynamic characteristics of the rail support system, including track stiffness, damping properties and inertial effects. Deviations of these parameters significantly affect rail deflections and the resulting wheel-rail contact forces.

In the study, the railroad track is considered as a dynamic system subject to loads on moving axles. The response of the track structure is estimated by analyzing the change in rail stiffness along the track and the resulting vertical displacements under different operating conditions [6, 7].

Parametric disturbances affect the variation of track stiffness along its length. As the rail joint is approached, the vertical stiffness of the rail–track system decreases. However, in the central part of the joint, the presence of joint bars causes the load to be distributed between two rail sections, which results in a local increase in stiffness (point *d* in **Figure 1**). On average, the length of a joint-related track irregularity is about 2.55 m. During winter conditions, the stiffness of the track, including the joint zone, may increase by approximately 2.5–3 times [8, 9].

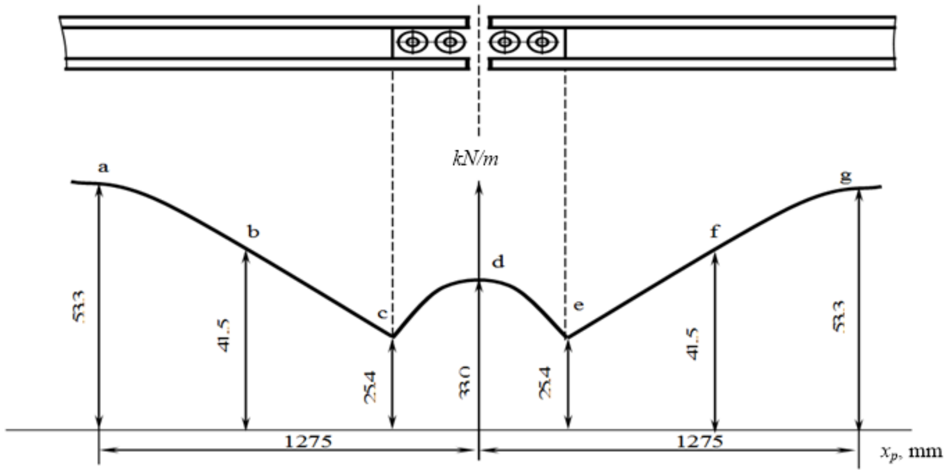

Figure 1. Variation of railway track stiffness in the vicinity of a rail joint.

The issue of track stiffness and its dissipative characteristics under real operating conditions is closely related to the spatial coordinate of the track. These parameters are non-uniform and stochastic, varying under the influence of several factors, including track design, structural system, maintenance conditions, and environmental influences. In practice, the variation of stiffness and dissipation properties along the track should therefore be considered random.

In the central part of a rail segment, the vertical stiffness of the track typically varies randomly within the range of 30–130 kN/mm. Accordingly, the change in vertical stiffness and the dissipative characteristics of the track along its length can be represented as a random function of the track coordinate [1,10].

2.1. Mathematical model of vehicle–track interaction

The railway track was modelled as an Euler–Bernoulli beam resting on a continuous viscoelastic Winkler foundation. The rail is characterised by bending stiffness EI, distributed mass m, foundation stiffness k, and damping coefficient c.

The governing differential equation of vertical rail vibrations under a moving wheel load is expressed as follows [11]:

$$EI\partial^4 w(x,\,t)/\partial x^4 + m\partial^2 w(x,\,t)/\partial t^2 + c\partial w(x,\,t)/\partial t + kw(x,\,t) \;=\; P\delta(x - vt) \quad (1)$$

where: *w(x, t)* is the vertical rail deflection;

EI is the rail bending stiffness;

m is the distributed rail mass;

c is the damping coefficient of the track structure;

k is the equivalent stiffness of the rail foundation;

P is the wheel load;

v is the vehicle speed;

δ(x − vt) is the Dirac function representing the moving load.

The formula makes it possible to estimate the dynamic response of the rail under different operating conditions and provides a basis for assessing the effect of vehicle speed, axle arrangement and track stiffness on rail deflection.

The Winkler foundation model was chosen because it provides an acceptable balance between computational efficiency and modeling accuracy for analyzing vertical deflections of rails in medium-speed freight traffic. More advanced formulations such as the Pasternak foundation model or elastoplastic track models can account for shear interactions in the ballast layer, previous studies have shown that Winkler's approach provides reasonably accurate predictions of rail deflections and wheel-rail forces for engineering applications involving operating speeds up to approximately 120 km/h [12].

The main limitation of the Winkler model is the neglect of shear stress transmission between adjacent support elements. Therefore, the findings should be interpreted as engineering estimates rather than accurate predictions of path behavior under all operating conditions.

2.2. Track stiffness variability and external disturbances

The dynamic response of a railway track is strongly influenced by spatial fluctuations in track stiffness and external disturbances that occur during train operation. In practical railway systems, the rigidity of the rail support structure cannot be considered constant, since it depends on numerous factors, including the characteristics of the rail bond, the condition of the ballast, the type of sleeper, the quality of track maintenance, environmental conditions and seasonal temperature fluctuations [13, 14].

One of the most significant sources of local stiffness variation is the rail junction area. As the wheel approaches the rail junction, the effective stiffness of the rail-track system decreases due to the rupture of the rail structure. However, directly in the connection area, the presence of linings and additional fasteners can temporarily increase local stiffness by redistributing the applied load between adjacent rail sections [15]. The resulting stiffness distribution along the track is schematically shown in **Figure 1**.

In addition to structural irregularities, railway tracks are subject to external dynamic disturbances caused by defects in the rail surface, geometric irregularities, wheel ovality, axial load fluctuations and changes in operating speed [16]. These disturbances create dynamic forces of interaction between the wheel and the rail, which affect the deflections of the rails and the vibrations of the vehicle.

Seasonal climatic conditions also affect travel behavior. During winter operation, the freezing of the ballast and roadbed layers can significantly increase the effective rigidity of the rail support system. According to published studies, the equivalent rigidity of the track structure can increase by about 2.5–3 times at low temperatures, which will lead to a change in the vibration characteristics and dynamic forces of interaction of the wheel with the rail.

Since the mechanical properties of a railway track vary along its length and depend on operational and environmental factors, the stiffness and damping characteristics of the track are considered as spatially variable parameters [17]. This assumption allows for a more realistic representation of the interaction of the vehicle and the track and allows us to investigate the effect of local stiffness changes on the behavior of the rail during deflection.

2.3. Numerical simulation conditions

Numerical simulations were performed to estimate the dynamic vertical interaction between the multiaxial rolling stock and the railroad track under different operating conditions. Calculations were performed using the continuous track model described in Section 2.1. The principal parameters adopted in the simulations are summarized in **Table 1**.

The railway track was presented in the form of a rail with continuous support, supported on an elastic-viscous base. The model included basic dynamic parameters affecting rail response, including rail bending stiffness, equivalent track stiffness, damping characteristics, axle load, bogie configuration, and train speed [18].

The simulation was carried out for locomotive speeds in the range of 60–120 km/h, corresponding to typical operating conditions on freight railway lines in Kazakhstan. Three typical vehicle configurations with different bogie arrangements and axle loads were considered to assess their effects on rail deflection behavior.

Table 1. Parameters used in numerical simulations.

Parameter	Symbol	Value
Train speed	v	60–120 km/h
Axle load	P	215–230 kN
Bogie base	L	2,950–4,600 mm
Rail support stiffness (summer)	k	24–56 MPa
Rail support stiffness (winter)	k	92–100 MPa
Track model	-	Winkler foundation
Analysis type	-	Dynamic vertical interaction

To study the effect of seasonal conditions, two ranges of rail support stiffness were adopted. The first range corresponds to summer operating conditions, when the equivalent modulus of elasticity of the base of the rail support ranges from 24 to 56 MPa. The second range corresponds to winter conditions, when the support stiffness increases to about 92–100 MPa due to ballast freezing and increased foundation stiffness.

The dynamic performance of the track was evaluated in terms of maximum rail deflection, the shape of the rail deflection curve, and the sensitivity of the track system to changes in train speed, axle load, and support stiffness [10, 19].

The adopted modeling conditions make it possible to analyze the effect of the main operating parameters on the interaction of the vehicle with the rail track and determine the operating modes associated with an increased dynamic load on the railway infrastructure.

2.4. Model validation

To assess the robustness of the proposed modelling approach, calculated rail deflections were compared with results reported in previously published experimental and numerical studies of vehicle-rail interaction.

According to literature data, the maximum vertical deflection of a rail under a freight rolling stock is usually 1.5–3.0 mm, depending on the axle load, vehicle speed, rail support rigidity and the condition of the track structure. Similar ranges were specified by literature [1, 4, 9] for heavy-duty and conventional rail services. Comparative indicators of modeling results with published studies are shown in **Table 2**.

Table 2. Comparison of model results with published studies.

Parameter	Literature	Present study
Rail deflection	1.5–3.0 mm	1.8–2.6 mm
Critical speed range	1.5–3.0 mm	60–80 km/h
Influence of stiffness	Significant	Significant

The results obtained in the present study show comparable trends. In particular, an increase in train speed and axle load leads to an increase in the dynamic sensitivity of the rails, while changes in track stiffness significantly affect the amount of deflection of the rails. The predicted maximum deflections remain within the ranges specified in previous studies.

In addition, the model reproduces several well-known physical phenomena described in the literature, including:

(i) increased dynamic load under conditions close to resonance;
(ii) sensitivity of the rails to changes in support stiffness;
(iii) changing the shape of the rail deflection curves with increasing train speed.

Thus, although no direct field measurements were performed in the present study, the consistency between the results obtained and previously published experimental observations confirms the applicability of the adopted modeling approach for the engineering assessment of vehicle-track interaction.

3. Results

Experimental studies have shown that the vertical stiffness of the railway track is not constant but varies depending on several factors, including the magnitude of the applied load, the selected clearances in the rail fastening system, and the compression of the rail pads. At the initial stage, the stiffness is relatively low; however, as the load increases, it rises significantly until reaching a certain limiting value. Beyond this point, the stiffness remains nearly constant, indicating an approximately linear behavior of the system within the normal operating range.

The results of the study therefore demonstrate that the vertical structural response of the railway track changes depending on load conditions, fastening clearances, and the compression characteristics of the rail pads [10,11]. Although the initial stiffness is relatively small, it increases substantially under load and subsequently stabilizes, which leads to a quasi-linear dynamic response of the track structure under typical service conditions.

In other words, when rolling stock moves along the track, the disturbances acting on the system cause each wheelset to experience a displacement at the wheel–rail contact point that varies with time. Thus, the displacement of each wheelset can be described as a time-dependent function.

In addition to random disturbances caused by rail irregularities, the forced vibrations of the rail and vehicle system may also be generated by deterministic (regular) excitations. These periodic disturbances can arise from repeating structural features of the track or from the dynamic characteristics of the vehicle–track interaction system [9]:

$$\eta(t) = \sum\nolimits_k \eta_{0k} \cos k\omega t = \sum\nolimits_k \eta_{0k} \cos 2\pi k x / L_0 \tag{2}$$

where η_{0k} is the amplitude of the k-th harmonic component of wheelset irregularities;

x—wheelset coordinates.

$$x = vt \tag{3}$$

The circumference of the wheel pairs in the riding circle:

$$L_0 = 2\pi r_K = 2\pi v / \omega \tag{4}$$

where $r_K = v/\omega$ is the radius of the wheelsets in the riding circle.

The study of this problem is generally carried out using two principal approaches to the analysis of coupled oscillations in the railway track–vehicle system. The first approach considers the railway track as a system possessing elastic, dissipative, and inertial properties [4,5,7,10,20]. Within this framework, track irregularities are treated as external disturbances, while the motion of the wheelsets is regarded as an internal dynamic response of the vehicle–track system. A key difficulty in this approach is the proper selection and representation of the parameters describing track irregularities.

The second approach considers a simplified system in which the vertical displacements of the wheelsets act as the excitation for the vehicle. In this case, it is assumed that the two approaches are essentially equivalent, differing mainly by a time shift in the dynamic response. Such vertical displacements are usually determined experimentally; therefore, the applicability of these disturbances to vehicles of other types or operating at different speeds remains uncertain, although several proposals addressing this issue have been presented in the literature [8].

For the analysis of vertical vibrations of railway vehicles, three main types of track models are typically used: discrete models [11,21–23], continuous models [4,10,15], and combined (mixed) models [6, 9, 12]. The track model illustrated in **Figure 2**, commonly referred to as the continuous model, is widely applied in the analysis of steady-state oscillations. In this representation, the railway track is treated as a system with parameters continuously distributed along its length. The most widely used formulation is the beam on an elastic foundation proposed in the Winkler foundation model, where the bending response of the beam is considered at the point of application of the external load.

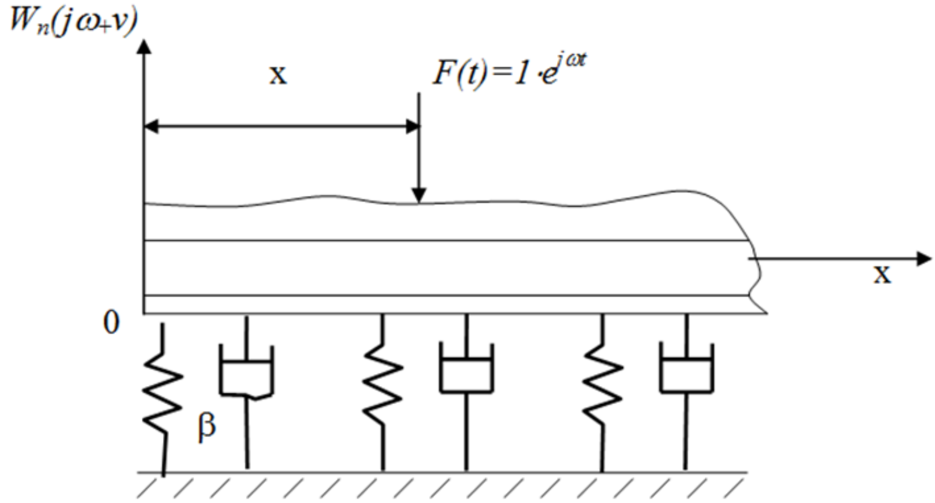

Figure 2. Continuous model of the railway rail.

The continuous model of the railway track (**Figure 2**), commonly used for the analysis of steady-state vibrations, represents the track as a system whose mechanical properties are continuously distributed along its length. In this approach, the rail track is treated as a continuous structural element rather than a set of discrete components.

A frequently applied representation is the beam resting on a viscoelastic foundation of the Winkler foundation model type. In this model, the foundation exhibits both elastic and viscous properties. This means that the supporting medium can deform under the applied load and subsequently return to its original state, while also providing damping of vibrations.

In such a representation, the deformation of the foundation occurs primarily in the vicinity of the point where the external load is applied [16,17].

For the analysis of vertical vibrations of the railway vehicle, a continuous track model was selected, as it most adequately satisfies the requirements of the present study. The main characteristic of the adopted rail model is the overall dynamic stiffness of the track system, which is determined using an approximate solution of the differential equation describing the vibrations of a beam subjected to a harmonic load [18].

In this work, the rail is represented by a continuous scheme that sufficiently reflects the conditions governing the vertical oscillations of the vehicle. The principal parameter of the selected track model is the equivalent dynamic stiffness (or dynamic modulus) of the rail–track system [16,24].

Within the vehicle–track system, all dynamic properties can be expressed through various parameters characterizing the considered processes. In general, two main approaches are used in the development of computational models for studying the oscillatory behavior of railway vehicles.

The first approach involves constructing a detailed mechanical–mathematical model that represents the real system with a large number of degrees of freedom. This method makes it possible to incorporate a wide range of system properties into a single model. In some cases, such a comprehensive system may also include subsystems that have only weak interactions with the main structure. However, solving specific problems related to the dynamic properties of individual subsystems may require analyzing the entire system as a whole, which significantly complicates the programming process and increases computational costs and machine time.

The second approach involves the development of specialized computational programs designed to solve specific problems in which individual components or subsystems must be analyzed separately. Such an approach allows researchers to focus on particular elements of the vehicle–track system without constructing an excessively complex general model.

At the same time, the increase in computational time required for analyzing such systems on modern high-performance computing systems is no longer considered critical, especially when the implemented algorithms are optimized to reduce computational costs and machine time [6,25].

An increase in train operating speeds leads to the formation of dynamic waves in the rail ahead of the first wheel of the bogie. The maximum rail deflection in front of the wheel can reach values of up to 0.7 mm, while the total (absolute) rail deflection may reach approximately 2.6 mm. Frequent changes in the sign of rail bending ahead of the wheel increase the stress state of the rail and may negatively affect train safety.

These dynamic waves may also contribute to the development of corrugation-type wear of the rail [11]. When wheels pass over such areas, sections of the rail with positive deflection are subjected to more intensive abrasion, which accelerates the formation of wave-like wear patterns (**Figure 3**).

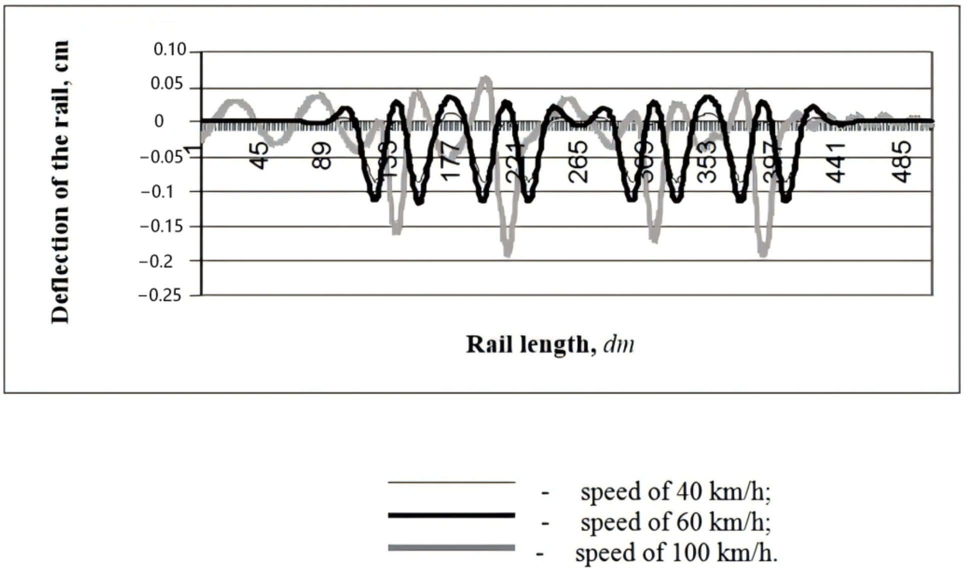

Figure 3. Rail deflection curves under locomotive loading.

Seasonal conditions also influence the magnitude of rail deflection. During the summer period, rail deflections are generally greater than in winter, regardless of train speed. However, at a speed of 100 km/h, the rail deflection decreases compared with the value observed at 60 km/h. At approximately 60 km/h in summer conditions, a resonance phenomenon may occur, which intensifies the oscillatory response of the rail under the locomotive.

Processing the calculated results of rail deflections caused by the locomotive made it possible to obtain numerical values of the maximum rail deflection (**Figure 2**). The analysis of **Figure 2** shows that two peaks of rail deflection are observed depending on seasonal conditions.

The first peak occurs at a speed of 60 km/h, which corresponds to the movement of the vehicle during the summer period, when the modulus of elasticity of the rail support base was assumed to be within 24–56 MPa. The second peak appears at a speed of 80 km/h, corresponding to winter operating conditions, when the modulus of elasticity of the rail support base increases to approximately 92–100 MPa.

These results indicate that at speeds of 60 km/h and 80 km/h, the impact of rolling stock on the railway track becomes most unfavorable. This effect is associated with the coincidence of rail bending diagrams generated by the group of wheel loads acting on the rail structure [3, 8].

However, there are still reserves for increasing train operating speeds. As demonstrated by several theoretical and experimental studies [4, 26], an increase in train speed may lead to a reduction in the vertical impact of rolling stock on the railway track (**Figure 4**).

This phenomenon can be explained by the appearance of aerodynamic forces, a reduction in the inertial forces acting on the rolling stock, and the influence of gyroscopic effects.

The observed unloading of the wheelsets that occurs with increasing train speeds is determined by the specific features of the interaction between the track and the rolling stock. Studies conducted by the All-Russian Scientific Research Institute of Railway Transport have shown that, for freight locomotives and wagons, a reduction in wheel loads begins to be observed at speeds of approximately 80 km/h when operating on tracks with reinforced-concrete sleepers.

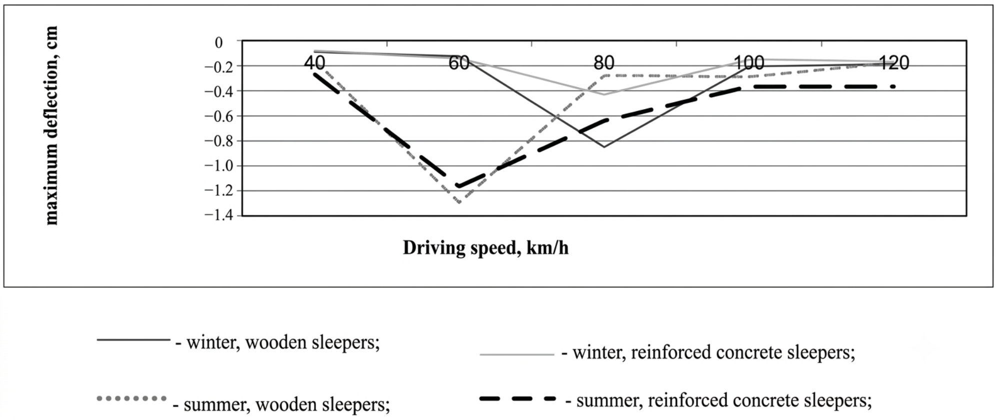

Figure 4. Maximum rail deflection under locomotive loading.

As the speed of an electric locomotive increases from 80 to 100 km/h, the rate of increase in rail deflection becomes smaller. The largest values of vertical rail deflection were recorded at a speed of 80 km/h.

Using the methodology described in the literature [2,3], the influence of the main characteristics of the vehicle running gear on the dynamic rail deflection was also analyzed. **Figure 5** presents the structural schemes of three bogie designs, while **Table 3** provides the parameters of the rolling stock considered in the study.

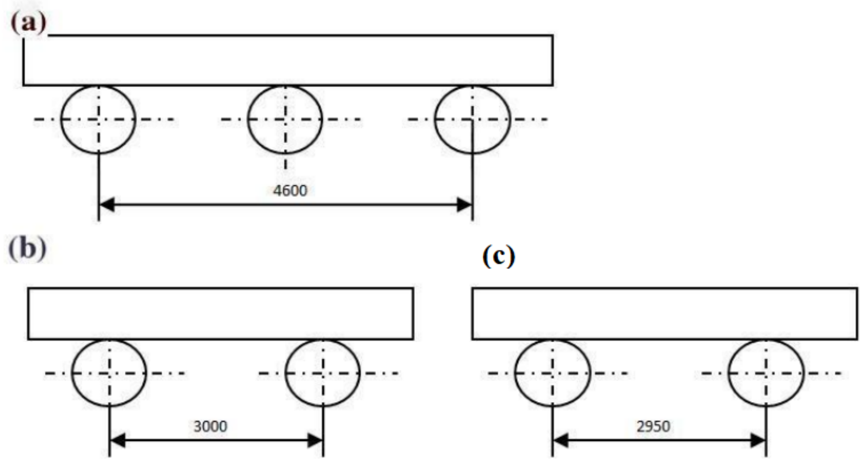

Figure 5. Trolley diagrams.

Table 3. Main parameters of the rolling stock considered in the study.

Parameters	Name of the crew Figure 5a	Figure 5b	Figure 5c
Trolley base, mm	4,600	2,950	3,000
The axial formula	3_o–3_o	$2(2_o$–$2_o)$	$2(2_o$–$2_o)$
Axial load, kN	230	215	230

Figure 6 presents the results of calculations of the dynamic impact of the adopted

bogie models on the rail during high-speed operation (100 and 120 km/h).

The analysis of the data shown in **Figure 6** indicates that, at different train speeds, the considered vehicle configurations produce approximately the same level of influence on the railway track. The main difference lies in the length of the track section involved in the oscillatory process [9, 10].

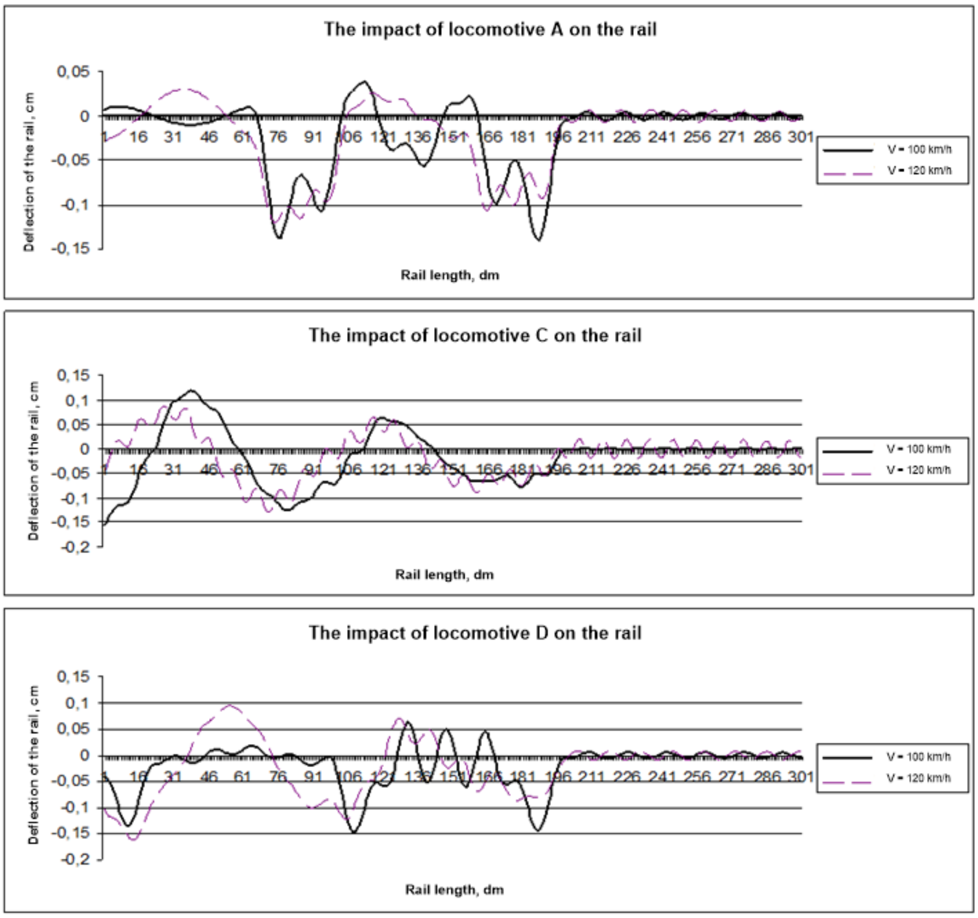

Figure 6. Graph of rail deflection.

Axle load has a significant effect on the stress–strain state of the railway track. An increase in axle load leads to stronger interaction forces between the track and the rolling stock. At the same time, increasing the axle load is one of the indicators of the intensification of railway operation and improves the adhesion between wheelsets and rails, thereby enhancing the traction performance of the locomotive.

The magnitude of rail deflection is also strongly influenced by the number of locomotive axles and the train speed. The use of two-axle bogies improves the interaction between the track and the rolling stock, which is reflected in reduced values of rail deflection.

The obtained diagrams of rail deflection indicate that with an increase in train speed the deflection curve becomes asymmetric. In this case, a lag of the maximum rail deflection relative to the position of the moving wheel is observed. In addition, a damping wave appears ahead of the vehicle, the length of which depends on the speed of the rolling stock.

The data presented in **Figure 6** correspond to the results obtained for **Figure 4**, which illustrates the curves of maximum rail deflection under locomotive loading. The figure shows the effects of different locomotive configurations on the rail, designated as locomotives A, C, and D.

The final assessment of the dynamic characteristics of the investigated rolling stock is carried out using extended schemes of vertical oscillations [27]. This model takes into account the specific features of the oscillatory processes occurring in the traction drive system, as well as the bouncing, pitching, and lateral rolling of the bogies and the locomotive body. In addition, the model considers the bouncing and lateral oscillations of the wheelsets.

At high operating speeds and when heavier types of track superstructure are used, it becomes necessary to consider these factors when solving problems related to the interaction between the railway track and the rolling stock. The non-elastic vertical resistance forces that arise under loading manifest themselves partly as friction forces that do not depend on the vertical load, and partly as forces related to the vertical components of vibration velocities.

4. Discussion

The results obtained confirm that the transition from the frame scheme of locomotives to the bogie design has become one of the key stages in the development of the mechanical part of rolling stock. The use of trolleys has significantly improved the conditions for the interaction of wheelsets with the track, especially when driving in curved sections. The presence of pivoting devices and body support units ensures a redistribution of loads between structural elements, reduces lateral forces on the track and increases the stability of the locomotive's movement with increasing speeds and traction forces [28].

The analysis also revealed that the dynamic characteristics of a locomotive are largely determined by the transmission of longitudinal, transverse and vertical forces between the body and the bogies. Structural elements that ensure load balancing between the axles of wheelsets play an important role in preventing boxing and in maintaining the required level of wheel-rail adhesion [29]. This is especially evident when the locomotive starts moving, when the tractive effort reaches its maximum values and there is a possibility of a redistribution of vertical loads between the axles of the bogies.

The use of mechanical and mathematical modeling makes it possible to study in more detail the processes of dynamic interaction of rolling stock elements. The representation of a locomotive as a system of interconnected solids makes it possible to take into account the geometric parameters of the structure, the characteristics of suspension and the features of force transmission. The construction of a system of differential equations of motion provides a quantitative assessment of dynamic parameters and allows predicting the behavior of a locomotive under various operating conditions with the required accuracy.

The data obtained can serve as a basis for further improving the efficiency of bogie structures, optimizing load distribution between wheelsets, and improving the performance of locomotives in the context of modern requirements for speed, reliability, and safety of railway transport.

5. Conclusion

A mathematical model of the dynamic vertical interaction of a multiaxial rolling stock with a railway track was developed on the basis of an Euler-Bernoulli beam resting on a continuous elastic base. The model allows you to estimate rail deflections at different train speeds, axle loads and track stiffness conditions.

The simulation results showed that the maximum dynamic impact on the railway track occurs in the speed range of 60–80 km/h. In summer operating conditions, the greatest deviations of the rails were observed at about 60 km/h, while in winter conditions the maximum response speed shifted towards 80 km/h due to increased track stiffness.

The calculated maximum rail deflections reached approximately 2.6 mm, while dynamic waves before wheel loading caused local rail displacements of up to 0.7 mm. These values correspond to the ranges specified in previous vehicle-track interaction studies.

Seasonal fluctuations significantly affect the dynamic behavior of the railway track. Increasing the equivalent stiffness of the rail support system from 24–56 MPa in summer to 92–100 MPa in winter changes the vibration characteristics of the track and the critical operating speed associated with the maximum deflection of the rail.

The analysis showed that axle load, bogie configuration and train speed are the main parameters affecting rail deflection. The use of biaxial bogies improves the interaction of the vehicle with the track and reduces rail deflections compared to heavier multi-axle configurations.

The proposed approach to modeling can be applied for engineering evaluation of railway track characteristics, detection of critical operating conditions and optimization of rolling stock parameters aimed at reducing dynamic loads and improving reliability of railway infrastructure.

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