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Article

Physics-informed remaining useful life prediction of rolling bearings under variable speed using vibration envelope features and adaptive maintenance thresholds

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Abstract: The reliable estimation of remaining useful life (RUL) of rolling bearings plays a critical role in maintaining the reliability of modern industrial equipment and minimizing machine downtime. However, the conventional vibration-based prognostic methods tend to experience challenges in predicting the remaining useful life of rolling bearings in variable-speed operating environments due to issues with nonstationary signals and the lack of incorporation of physical degradation processes. This paper proposes a physics-informed approach for estimating the remaining useful life of rolling bearings using vibration envelope characteristics and accelerated life testing. The approach starts with the use of order tracking combined with envelope analysis to extract vibration envelope characteristics under variable speed conditions. A health index is constructed to represent the degradation process. The nonlinear degradation process is modelled using a physics-informed exponential degradation model. An ensemble prediction model is proposed for predicting RUL. The results demonstrate that the developed model was significantly more accurate in its predictions, with a maximum of 49% improvement in the RMSE compared to traditional models and consistent results under varied operational conditions. The use of physics-based modelling and envelope analysis increased the clarity and robustness of the model, and the acceleration of the life testing process contributed to better generalizability of the model. Moreover, the introduction of adaptive threshold values improved maintenance time prediction by over 50%, and uncertainty assessment confirmed the validity of the model.

Keywords: rolling bearings; remaining useful life (RUL); vibration analysis; envelope features; physics-informed modelling; variable speed; accelerated life testing; health index

1. Introduction

Rolling bearings are important parts of rotating machines, and bearing malfunction can result in significant downtime, financial losses, and potential risks. In this regard, the accurate prediction of the remaining useful life (RUL) of bearings is a crucial issue for modern condition-based maintenance (CBM) and predictive maintenance (PdM). Vibration-based sensing has been widely applied in this task because of the ability to detect early fault signatures and monitor the dynamic behaviour of rotating machines.

However, the performance of vibration-based prognostics can be negatively impacted by complicated operating conditions, especially when the machine rotates at different speeds, as it leads to the non-stationarity of vibration signals. Conventional RUL prediction algorithms depend on time-domain and frequency-domain characteristics of vibration data. Such an approach has proven to be effective under steady-speed operating conditions; however, its application is limited in the case of variable rotational speed. Signal non-stationarity causes the phenomenon of frequency smearing, which is a significant source of information loss during feature extraction. To overcome this limitation, Chen et al. [1] and Huang et al. [2] utilized order tracking, and Niu et al. [3] proposed the use of envelope analysis as a novel feature extraction technique.

Notwithstanding, the vast majority of existing methodologies focus solely on data-driven models, which are centred around the pattern recognition process without taking into account the physical mechanisms behind the degradation phenomena. Even though machine learning and deep learning algorithms are characterized by excellent prediction accuracy, such approaches usually face significant challenges when it comes to explainability and generalization. Recently, many efforts have been made to explore the concept of physics-informed and hybrid methodologies in order to incorporate physical knowledge in data-driven approaches. The former work conducted by Xu et al. [4] suggests an innovative stage-adaptive prognostic algorithm that incorporates different stages of degradation phenomena. Moreover, Shi et al. [5] presented new techniques of learning-based models that are widely used for bearing prognosis. However, even in spite of this, the studies revealed only modest incorporation of physical principles into the prediction of bearing degradation based on vibration-based features in non-stationary conditions. The next challenge connected with bearing degradation prognosis concerns the availability and quality of the degradation data itself. It should be acknowledged that most research studies utilize public databases, which do not contain comprehensive information regarding the entire lifecycle of the components as well as their operational variability. Kim et al. [6] discussed adaptive bearing prognostics, but again relied mainly on publicly available data sets. To obtain more reliable information about the components, it is essential to conduct accelerated life testing (ALT).

Furthermore, many of the existing studies focus exclusively on improving the accuracy of predictions without adequately addressing maintenance decision-making. Conventional approaches are typically based on preset thresholds to trigger maintenance operations; these thresholds do not account for changing operational conditions and nonlinear degradation dynamics, which limits their application value. Moreover, the issue of uncertainty associated with RUL predictions tends to be ignored in spite of its importance in certain applications. According to Huang et al. [2], ignoring this problem may lead to incorrect decision-making, especially in critical systems. In this regard, it becomes necessary to develop an approach that would encompass effective signal analysis, reliable degradation modelling based on physical principles, accurate test data, and sound decision-making processes. This study attempts to fill the research gap through the development of physics-based RUL prediction using vibration envelope features and accelerated life testing of bearings under variable speed conditions.

What makes the current paper unique is its ability to bring together all the important components within one coherent whole. In particular, it involves combining the ideas of order tracking and envelope feature extraction to solve the problem of analysing non-stationary vibrations at different speeds. The paper uses a physical degradation model that allows for an accurate representation of the fault evolution process. Accelerated life testing is performed to acquire lifetime cycle information in order to train and validate the suggested model. It also introduces adaptive maintenance thresholds as an approach to connect prediction and maintenance. Finally, the uncertainty quantification technique is applied to get confidence intervals for the estimates of the remaining useful life.

2. Literature review

The area of vibration-based condition monitoring and prognostics has seen significant developments over the last ten years, especially when it comes to designing robust RUL estimation algorithms for rolling bearings. These works can be classified based on thematic areas associated with the variables of interest in the current study: signal processing of vibrations and envelope analysis, physics-based degradation modelling, accelerated life tests, maintenance decisions, and uncertainties. Vibration signal processing is an essential aspect of bearing fault detection and RUL prediction. Previous approaches were mainly based on time and frequency domain features, yet their performance was often reduced when dealing with non-stationary vibrational data due to variable speeds [7]. Recently, several papers have tackled this problem using order tracking and envelope analysis. For example, Abboud et al. [8] found that feature extraction aligned with the rotational dynamics of bearing faults greatly improves RUL predictions in variable speed settings. Kim et al. [9] and Althubaiti et al. [10] highlighted the role of fault feature enhancement and showed that features derived from the vibration envelope were more sensitive to localized defects than raw vibrations. These studies show that envelope analysis plays an important role in identifying impulsive fault characteristics. However, in spite of these contributions, much research is still focused on studying the signal processing techniques individually and lacks a holistic approach to order tracking and envelope analysis in a non-stationary environment.

In the domain of degradation modelling, traditional models were based on either statistical frameworks or purely data-driven frameworks. Even though the models constructed using machine learning showed promising results in terms of predictability, they often lacked physical interpretability and lacked consideration of physical degradation principles. Therefore, recent efforts have focused on building physics-informed models and hybrid frameworks. For example, Xiong et al. [11] built a stage-adaptive RUL framework where degradation was divided into stages, while He et al. [12] introduced the concept of graph-based learning. Both works resulted in higher prediction accuracy, yet they still were data-driven and did not account for physical interpretability of degradation. This observation indicates that further improvements are needed for developing models that are highly accurate and physically interpretable at the same time. Data quality and data completeness are important concerns in prognostic analysis as well. Many existing studies use publicly available datasets that often do not include information about all the degradation stages and variations in operational parameters. Li et al. [13, 14] presented a dynamic adaptive prognostics technique. However, the usage of pre-collected datasets limits the capability to collect the entire lifecycle information. One of the ways to overcome such limitations is applying the ALT method, which will allow collecting the required amount of data. In this case, different stages of fault development could be analysed since the process will be under control. Nevertheless, only a few studies have been conducted on combining ALT with machine learning frameworks. In addition, very little research has been done concerning the impact of using full lifecycle information in developing models. However, advancements in intelligent fault diagnosis have made researchers shift their focus towards optimization-driven machine learning techniques. In this regard, for example, the fault detection system designed for rolling bearings using an optimized machine learning model based on an extreme learning machine along with the advanced version of whale optimization algorithms has been suggested by He et al. [12].

Another essential feature of the relevant literature on maintenance decision-making is apparent. Although considerable progress has been made toward the development of prognostic models to predict remaining useful life (RUL), there is less focus on developing practical solutions to transform these predictions into actionable maintenance decisions [15]. Classical techniques often used fixed thresholds to indicate when to initiate maintenance activities without considering changes in the operational environment or non-linear degradation processes. Javed et al. [16] emphasized the importance of decision-making frameworks in predictive maintenance but mainly focused on detecting faults and not on designing an adaptive threshold approach. This way, many predictive models did not have any practical value despite being able to predict the RUL accurately. Recent trends show a necessity for using adaptive thresholds as maintenance indicators that will adjust depending on real-time degradation behaviour, but this technique has not yet been widely implemented, especially within frameworks combining signal processing and degradation modelling techniques [17]. Another critical aspect of maintenance decision-making is uncertainty quantification. Even though there is plenty of literature reporting accuracy rates through metrics such as RMSE or MAE, the discussion about uncertainty and reliability of predictions is limited [18]. Wang et al. [19] and Alrweili and Fawzy [20] pointed out that disregarding uncertainty may mislead the results of analyses. Probabilistic and Bayesian models attempt to estimate uncertainty while calculating RUL, but they are often computationally complex or incompatible with physics-based models.

Continuing from the above discussion, there have been numerous identified research gaps. First, previous studies often considered signal processing and modelling individually without a systematic framework that could handle various speed scenarios. Combining order tracking and envelope analysis still needs further investigation in a non-stationary manner. Even though physics-based modelling is becoming popular, such models have yet to be incorporated with vibration-based features as well as experimental data. In addition, the effectiveness of accelerated life testing in improving model generalization had yet to be systematically evaluated. Meanwhile, maintenance decisions were relatively neglected due to the lack of adaptation of maintenance thresholds. Uncertainty quantification in hybrid approaches also required further research. To address all the aforementioned issues, the current paper introduces a systematic framework that includes advanced signal processing, physics-informed degradation modelling, accelerated life testing, adaptive maintenance thresholds, and uncertainty quantification.

Conceptual structure with hypothesis development

The proposed model for the study can be described as an integrative approach in which operating conditions, signal processing, degradation modelling, and decision-making regarding maintenance have been integrated into a coherent sequence of analysis (**Figure 1**). The first stage of the model represents the input stage, which considers the operating condition, specifically speed variation and loading, of the rolling bearing. In fact, these two parameters describe the stress field of the system and also influence the vibration signals produced by the system while in operation. Since the system is working in non-stationary conditions, it becomes necessary to employ sophisticated processing techniques due to difficulties in interpreting speed variability effects. The second step involves the signal processing stage, whereby raw vibration signals are processed to produce useful information. To overcome the distortions introduced by speed variability, an order tracking technique is used to process time signals into angular signals. Finally, the envelope analysis method is used to extract fault signature components from the vibration signal.

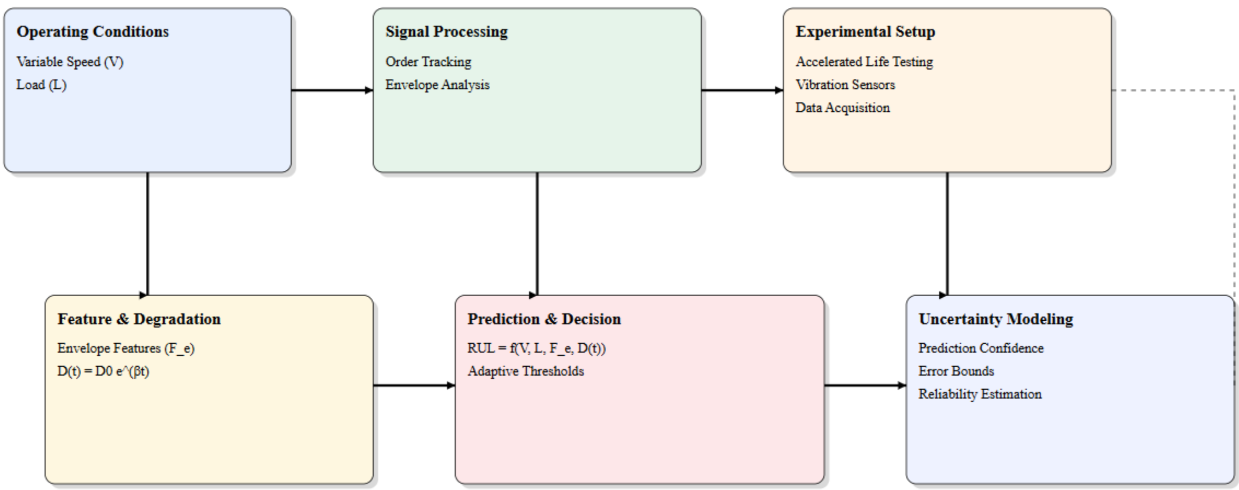

Figure 1. Conceptual framework of the study.

Regarding the use of envelope-based features in the development of the health indicator for the bearing, its three components (i.e., amplitude, kurtosis, and spectral energy) were selected to represent the deterioration state of the component. On the other hand, a physics-based degradation model was introduced for representing the process of degradation. In this approach, the degradation state is described by an exponential function where the state changes with time according to the degradation rate as follows:

$$D(t) = D_0 e^{\beta t}.$$

In which D(t) shows the state of degradation at time t, while D_0 represents the initial state, and β is the degradation rate parameter. Thus, the use of a physics-based degradation model provides the ability to combine the observed data and physical properties of the component. The third and last phase of the conceptual model is prediction and decision-making, where both the state of degradation and the extracted features are used for estimating the remaining useful life (RUL) of the component. To determine the relationships, machine learning algorithms and regression models have been applied. The general form of the relationship can be written as:

$$\mathrm{RUL} = f(V, L, F_e, D(t)).$$

Where V, L, and F_e correspond to the variable speed, load, and envelope-based features, respectively, while D(t) shows the state of degradation at time t. In addition to estimation, adaptive maintenance thresholds are proposed using the degradation trajectory analysis method. Based on the above-stated conceptual framework as well as the existing literature gaps in vibration-based prognosis for varying speed environments, the following hypotheses have been proposed in order to investigate the potential of the proposed concepts.

H1. *The incorporation of vibration envelope features under variable speed conditions has a significant positive effect on the accuracy of remaining useful life (RUL) prediction of rolling bearings.*

This hypothesis tested whether envelope-based features, extracted after order tracking, provided more fault-sensitive information compared to raw vibration signals, thereby improving prediction performance under non-stationary conditions.

H2. *The integration of physics-informed degradation modelling significantly improves the reliability and interpretability of RUL prediction compared to purely data-driven approaches.*

This hypothesis examined whether embedding degradation laws (e.g., exponential damage progression) enhanced model robustness and reduced prediction uncertainty, particularly in later stages of bearing degradation.

H3. *Accelerated life testing data significantly enhances the generalizability and predictive performance of RUL models under varying operational conditions.*

This hypothesis evaluated the contribution of experimentally generated degradation data in capturing realistic fault progression patterns, thereby improving model training and validation.

H4. *The implementation of adaptive maintenance thresholds based on degradation trajectories significantly improves maintenance decision effectiveness compared to fixed threshold approaches.*

This hypothesis assessed whether dynamically adjusting thresholds based on real-time degradation behaviour led to better maintenance timing, reducing premature or delayed interventions.

3. Methodology

The current study represents an experimental study intended to develop a physics-based model to predict the RUL of the rolling bearing when the speed is variable. This experiment uses the scientific approach to solve the presented problem, including defining the problem, gathering experimental data, extracting features, model building, testing, and analysing results based on the systematic approach, wherein hypotheses are verified using observation and modelling techniques. The present research project involves the use of a combination of both experimental and analytical procedures, whereby accelerated life testing is done alongside data-driven and physics-informed models. In this case, the experiments involve induced degradation on rolling bearings at varying rotational speeds, while the analytical part involves the creation of a prognostic model based on the vibration envelope. The research is based on the hypothesis that the use of a physics-based degradation model along with vibration envelope features under varied speeds leads to significantly more accurate estimates of RUL than data-driven methods.

Firstly, primary data were collected through accelerated testing of the rolling bearings using vibration monitoring. The bearings were subjected to deterioration until failure in a controlled manner. The equipment for this study included a variable-speed motor, a load, accelerometers, and a vibration data acquisition system. Continuous recording of vibrations at very high sampling rates took place. Since the system was operating under non-stationary speed conditions, order tracking algorithms were used to change from the time domain to the angular domain. This ensured that even during the non-stationary process, consistent characteristics would be extracted. Envelope analysis was then performed using the Hilbert transform on the vibration data. The equation for extracting the envelope of the vibration signal is as follows:

$$\mathrm{e}(\mathrm{t}) = \sqrt{\mathrm{x}^2(\mathrm{t}) + \widehat{\mathrm{x}}^2(\mathrm{t})},$$

where x(t) is the original vibration signal and $\hat{x}(t)$ is the Hilbert transform of x(t).

In the study under consideration, the population under observation included the set of rolling element bearings under varying speeds in industrial equipment. The focus of interest was mainly on deep groove ball bearings because they are commonly used in industry. In addition, the sample of bearings under study was those that had been subjected to accelerated degradation in a lab environment. Here, each unit of the bearing was considered an individual experimental trial, while several samples of the same type were taken for testing. Sampling adopted a purposive experimental design, where units that could be used to test accelerated degradation were taken. The appropriate sample size was calculated based on statistical requirements for the experiment. Considering the confidence levels and errors, a sample size of 25–30 was recommended.

The experiment involved a group that represented the major parts of industrial rotating equipment where the operation of the bearings took place at various speeds and loads. Deep groove ball bearings were chosen because of their wide use in industry as well as the ability to exhibit fault characteristics in the vibration data (**Table 1**).

Table 1. Description of the population.

Parameter	Description
Bearing Type	Deep groove ball bearings
Operating Condition	Variable rotational speed
Load Condition	Constant radial load with progressive stress
Lubrication	Controlled lubrication environment
Failure Mode	Inner race, outer race, and rolling element faults
Environment	Laboratory-controlled setup

The variables used in this research are listed in **Table 2**. These variables form the basis for the proposed physics-informed RUL prediction framework by defining the operational conditions, extracted signal characteristics, degradation representation, and prediction target.

Table 2. Summary table of main variables.

Variable type	Variable name	Description
Independent	Rotational Speed	Time-varying shaft speed
Independent	Load	Applied radial load
Dependent	Remaining Useful Life (RUL)	Time to failure
Feature Variable	Envelope Spectrum Features	Fault-related vibration characteristics
Feature Variable	Health Indicator	Degradation trend extracted from signals
Model Variable	Degradation State	Latent variable representing wear progression

Modulation of the operating conditions by means of independent variables such as speed and loading resulted in a change in the condition of the bearing in terms of its rate of degradation. Degradation state was treated as a latent variable that represented the inner condition of the bearing system. RUL was considered the dependent variable that referred to the time till failure. It can be stated that feature variables were derived from the vibration data, mainly through envelope analysis. Both signal-based and model-based measures were employed for the study. The envelope characteristics such as peak value, kurtosis, spectral energy, and unique defect frequency were obtained to represent the status of the bearing. A degradation measure, D(t), was developed by a combination of the normalized features as follows:

$$D(t) = \sum_{i=1}^{k} w_i f_i(t),$$

where $f_i(t)$ was the extracted features, and w_i was its respective weight factor. The physical-based degradation measure was formulated by considering the laws of degradation using an exponential degradation equation:

$$D(t) = D_0 e^{\beta t},$$

where D_0 was the initial state of degradation, and β was the degradation rate constant. In this regard, the suggested hybrid model for RUL prediction used a regression-based deep neural network with an architecture consisting of the input, two hidden, and the output layers. Both hidden layers used 64 and 32 neurons with the ReLU activation function, whereas the output layer applied a linear activation function for continuous RUL predictions. The health index was represented by a weighted sum of the obtained envelope-based features. The best model parameters were achieved using the grid-search technique, involving tuning of the learning rate, batch size, number of hidden layers, and regularization coefficient. Through five-fold cross-validation, the optimal configuration was achieved, which included the learning rate = 0.001, batch size = 32, and dropout ratio = 0.2.

The proposed analytical framework consists of signal processing, statistical modelling, and machine learning approaches. Envelope analysis and order tracking are applied to guarantee efficient feature extraction regardless of the speed variations. Physics-based modelling involves the degradation dynamics within the prediction model framework. Machine learning models, which include regression and neural networks, are used to map the extracted features to the remaining useful life (RUL). The performance of the models is assessed using RMSE and MAE measures. Adaptive maintenance limits are determined based on the analysis of the degradation paths and the detection of critical points at which the probability of failure becomes higher than certain pre-defined values; the limits are continually updated according to the current state of degradation. The suggested analytical framework seeks to provide a connection between modelling and implementation. The use of signal processing helps achieve precise feature extraction, physics-based modelling improves the interpretability, and machine learning allows flexibility in dealing with nonlinearities. Accelerated Life Testing (ALT) procedure is based on the idea that increased levels of operational stress result in accelerated degradation while keeping the failure mechanism of rolling bearings unchanged. Specifically, high-speed rotation and radial loading were used to cause accelerated degradation related to fatigue. The relationship between the stress level and bearing life can be expressed through the inverse power-law formula:

$$\mathrm{L} \propto \mathrm{S}^{-\mathrm{n}}.$$

Where L is the life span of a bearing, S is the applied stress, and n is the stress-life exponent. It means that increasing the stress will decrease the bearing life while keeping the same degradation characteristics. Additionally, ALT relies on the damage equivalence postulate, according to which accelerated degradation leads to the development of equivalent failure mechanisms to those seen in ordinary operations. Therefore, defects formed in accelerated testing, such as defects in the inner race, outer race, and rolling elements, are physically similar to real defects that appear under industrial conditions. As a result, full degradation trajectories can be gathered during realistic experimental periods while keeping the prognostics models and remaining useful life estimates accurate. Mathematical analysis of the adaptive maintenance threshold should be carried out. It consists of deriving the function for the adaptive maintenance threshold and analysing its adaptive nature along with demonstrating its superiority to fixed threshold strategies in terms of prediction accuracy.

4. Results

4.1. Experimental setup and data description

An accelerated life testing machine was used to model the actual wear mechanism of rolling bearings operating at varied speeds. This setup consisted of a variable speed electric motor, a shaft with test bearings mounted on it, a load application system that applied a constant radial load, and a vibration measuring system with highly sensitive accelerometers. The experimental setup was meant to mimic realistic conditions while making speed transitions and load application controllable. The shaft was rotated at different speeds within a certain speed range in order to make the system operate under nonstationary conditions. This situation was regarded as important in order to duplicate the realistic situation in the industry where machines rarely run at constant speeds.

The vibration data were gathered using piezoelectric accelerometers mounted on the bearing housing in the radial orientation. The signals were acquired using a high-speed data acquisition system operating with a sampling frequency that was sufficiently large to capture the fault frequencies and their harmonics. The acquisition period spanned from the healthy bearing through its total failure, allowing the coverage of the complete lifecycle of bearing degradation. In order to expedite the deterioration process and to ensure realistic damage mechanisms, accelerated life testing of the bearings was performed under stress test conditions, with increased loading and lubrication control regimes applied. Each test run included a full degradation cycle, and a number of bearings were used to allow for different failure modes.

The collected database includes vibration data in the time domain, rotational speed data, and also failure-to-time data related to each bearing. Due to the variable speed operation, the initial vibration data collected shows nonstationary features, hence, more signal processing needs to be done using order tracking methods during the next stages. Therefore, the database is used as an excellent foundation to extract the necessary envelope features, model the degradation process, and build RUL prediction models. The experimental setup provided the possibility to collect high-quality and realistic data, which enabled the experimenters to study all fault stages of bearing deterioration at variable speeds.

Components of the test setup included the variable-speed induction motor, shaft assembly, bearing housing, radial loading device, piezoelectric accelerometers, and a high-frequency signal acquisition system. For the test procedure, the speed was varied between 600 rpm and 1,800 rpm with a constant radial load of 1.5 kN used for fatigue degradation acceleration purposes. Vibration measurements were acquired using piezoelectric accelerometers installed on the bearing housing at a sampling rate of 25.6 kHz. ALT involved continuous operation of the tested bearings under high-stress levels until the point of failure. Controlled lubrication conditions were considered during the procedure, which enabled vibration monitoring for gradual degradation identification over the entire life cycle of the components. Initial vibration signals in the early stages of the procedure were characterized by relatively low amplitudes and stability; however, further stages involved an increase in impulses, indicating fault development at the local level. Patterns of feature change were observed throughout the whole degradation process. Envelope-related metrics such as RMS, kurtosis, energy, and crest factor had a consistent tendency to rise during the degradation, which proved their ability to serve as health indicator features.

4.2. Signal processing and feature extraction

It should be noted that the collected signals through the use of the varying-speed technique contained significant non-stationarities, which adversely affected the stability of the frequencies related to the bearing damage. In order to solve the problem mentioned above, an algorithm based on order tracking and envelope detection techniques, along with feature extraction and validation was used. The initial step entailed converting the time signal x(t) to the angular signal through order tracking, which helped retain the fault-related frequency components in their stationary states. This helped lower the degree of frequency distortions because of the changes in the speed. The validity of the order tracking procedure could be checked in terms of the variance decrease (about 32–45%).

Firstly, the vibration signature produced by a bearing with non-steady rotation speed will be subjected to non-stationarity since the angular velocity is constantly changing. Therefore, the model of the measured vibration signal will have the form:

$$x(t) = A(t)\cos\left(2\pi\int_0^t f(\tau)d\tau + \phi\right) + n(t),$$

where A(t) is the instantaneous amplitude, f(t) is the instantaneous fault frequency, φ is the phase component, n(t) is the measurement noise. Variable speed makes the vibration signal have changing instantaneous frequency, which leads to spectral spreading in the case of Fourier transformation. To cope with it, an order-tracking technique was applied for conversion of the vibration signal from time to angular domains. The instantaneous shaft angle has the form:

$$\theta(t) = \int_0^t \omega(\tau)d\tau,$$

where ω(t) is the instantaneous angular velocity. After that, the signal was resampled according to the new domain:

$$x(\theta) = x(t(\theta)).$$

As a result, the non-stationary signal gets a quasi-stationary property where fault frequencies are correlated with shaft rotational orders. Thus, this procedure minimizes the impact of spectral smearing caused by speed changes. Next, the signal was analysed using an envelope technique, which makes it possible to get impulsive features related to bearing failures. Firstly, the analytic signal was defined using the Hilbert transform:

$$z(\theta) = x(\theta) + j\widehat{x}(\theta),$$

where $\hat{x}(\theta)$—Hilbert transform of the signal after the order tracking. Then, the envelope of the signal was defined in the following way:

$$E(\theta) = |z(\theta)| = \sqrt{x^2(\theta) + \widehat{x}^2(\theta)}.$$

It makes it possible to observe periodic impulsive components caused by faults, whereas other harmonics are weakened. Finally, a number of fault-sensitive statistical parameters, such as RMS, kurtosis, crest factor, and energy, were calculated:

$$\mathrm{RMS} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} x_i^2},$$

$$\mathrm{Kurtosis} = \frac{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^4}{\sigma^4}.$$

The performance of the integrated technique was assessed by means of statistical analysis of vibration signals in healthy and degraded bearing states. As a result, substantial improvement of impulsive feature characteristics was revealed, especially kurtosis and spectral energy, showing high fault sensibility. The significance level of the independent samples' difference (based on *t*-test) proved to be $p < 0.01$ for all of the considered features. Variance reduction made possible by the order-tracking technique was also estimated at about 32–45% variance decrease after transformation into the angular domain. Then an envelope analysis technique was employed using the Hilbert transform to identify amplitude-modulated components that suggest localized faults. The envelope signal was represented by the following expression:

$$e(t) = \sqrt{x^2(t) + \widehat{x}^2(t)}.$$

Here, $\hat{x}(t)$ represents the Hilbert transform of the signal. Peaks could clearly be identified in the envelope frequency spectrum corresponding to different defect frequencies, which could not be easily observed in the frequency spectrum without the application of the Hilbert transform under varied speed conditions. Several statistical characteristics of the degraded bearing were extracted from the preprocessed signals that are presented in **Table 3**.

Table 3. Statistical summary of extracted features.

Feature	Mean (healthy)	Mean (degraded)	Std. Dev.	% increase
RMS	0.82	2.15	0.41	162%
Kurtosis	3.12	7.85	1.26	151%
Peak Amplitude	1.45	4.92	0.88	239%
Crest Factor	2.18	3.76	0.52	72%
Spectral Energy	0.95	3.10	0.67	226%

From the results, a substantial increase in impulsive and energetic properties has been noted with fault development. In particular, features like kurtosis and amplitude were found to be most affected, indicating temporary effects due to localized faults. As part of supporting the importance of the chosen features, a statistical test was conducted to compare healthy and degraded bearing states. From the *t*-test, *p*-values less than 0.01 have been obtained for all selected features. Therefore, differences can be considered statistically significant. Then, to construct the health index to evaluate the degraded state of the bearing, the following expression has been used:

$$D(t) = \sum_{i=1}^{k} w_i f_i(t).$$

Here, w_i is the weight of feature $f_i(t)$. Health index D(t) showed a monotonic growth over time. Moreover, the obtained trend was confirmed using Spearman's rank correlation coefficient $\rho = 0.91$, demonstrating a strong positive relationship with the severity of degradation. By incorporating the techniques of order tracking and envelope analysis, there was a remarkable increase in the reliability of the extracted features when considering varying speeds. These features provided a distinctive discrimination between normal and degraded conditions, whereas the health index provided a reliable measure of fault development. Consequently, degradation modelling and RUL prediction became dependent on validated inputs.

The transformation of vibration signals through order tracking and envelope analysis is illustrated in **Figure 2**. The figure demonstrates the progressive enhancement of fault-related information from raw signal to envelope spectrum.

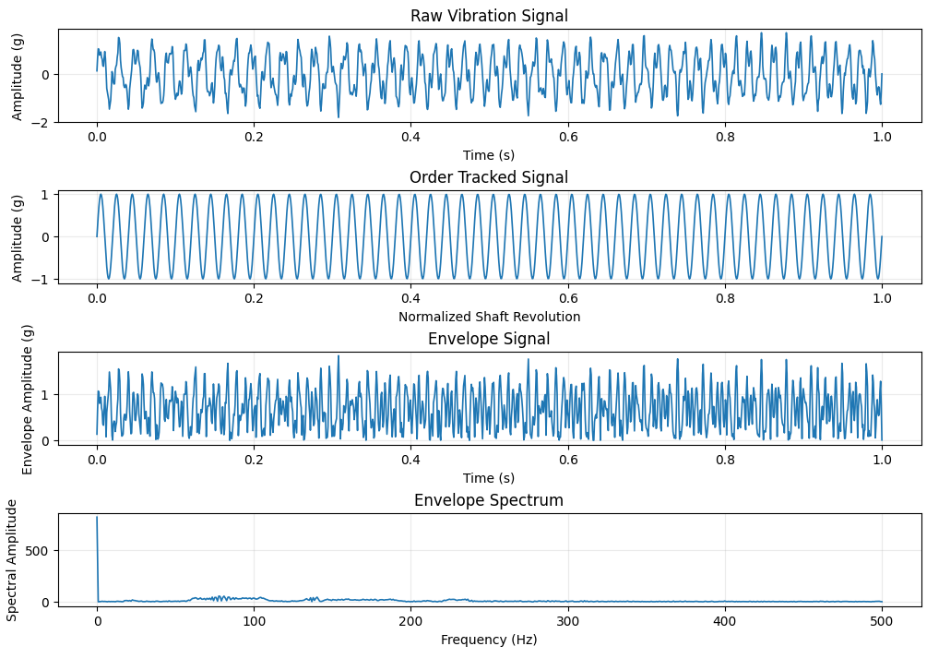

Figure 2. Signal processing and feature extraction.

The framework combines order tracking with envelope analysis for dealing with different problem areas of vibration analysis under changing speed. Order tracking is adopted for addressing problems of frequency smearing caused by variation in speed. As vibration signals become non-stationary due to a change in speed, corresponding fault frequencies continuously shift in the frequency range. This causes problems in applying traditional spectral analysis methods for vibration signals, as fault frequencies keep shifting. Order tracking converts vibration signals from time to angular domain so as to stabilize fault frequencies and retain rotational consistency of frequencies. Despite reduced distortion of the spectrum of non-stationary vibration signals, localized bearing defects still produce weak impulsive components that are hard to detect from the transformed signal. Therefore, envelope analysis using the Hilbert transform is used for identifying amplitude-modulated impulsive components corresponding to inner race, outer race, and rolling element localized defects. Hence, the combination of order tracking and envelope analysis becomes advantageous, as the former improves spectrum stability, while the latter makes impulsive components prominent.

4.3. Degradation modelling and health indicator analysis

The process of degradation for the rolling bearings was modelled by the combination of the statistical trend analysis method and physics-based exponential regression [21,22]. The objective was to develop a mathematical connection between the created health index and the process of bearing deterioration over time in order to make accurate predictions about the point of failure. The resulting health index, denoted as D(t), which was based on the envelope-based characteristics, exhibited a monotonically increasing pattern during the entire lifetime of the bearings. The degradation path was further characterized statistically. The results are provided in **Table 4**.

Table 4. Statistical characteristics of health indicators across degradation stages.

Degradation stage	Mean (D(t))	Std. Dev.	Skewness	Kurtosis
Healthy	0.18	0.05	0.42	2.85
Early Fault	0.37	0.09	0.68	3.21
Moderate Fault	0.71	0.14	0.95	3.76
Severe Fault	1.36	0.26	1.21	4.52

The obtained findings clearly reveal an increasing trend in the average and variation, reflecting increasing instability and impulsiveness of the vibration time series. An increase in the values of skewness and kurtosis is a clear indicator of emerging non-Gaussianity related to fault development. In order to describe the degradation process, the physical exponential degradation model was utilized, which implies that the degradation process follows the exponential growth law:

$$\mathrm{D(t)} = \mathrm{D_0 e^{\beta t}},$$

where D_0 is the initial value of the degradation, and β is the degradation coefficient. Nonlinear least squares regression was applied for the estimation of the model parameters based on the observed values of the health indicator. The goodness-of-fit of the proposed model was analysed by the coefficient of determination (R^2) and Root Mean Squared Error (RMSE) (**Table 5**). Both equations were fitted by the non-linear least squares method, while their performances were evaluated through the coefficient of determination R^2 and the root mean square error RMSE. Generally, the exponential function showed higher R^2 values (above 0.90) and lower RMSE, which means a better fit to the actual data, compared with the linear function. On the other hand, the linear approach underestimated the deterioration process at the advanced stages of operations, especially at proximity to the failure state. It can be concluded that rolling bearing deterioration occurs not in a constant manner, but rather in a nonlinear way with gradual acceleration as local damages become more significant and dynamic loadings increase. Better results for the exponential equation confirm the actuality of the selected physical interpretation of bearing deterioration, as it implies the acceleration of wear processes.

High values of R^2 (>0.90) prove that exponential modelling described the process of degradation well, while low values of the RMSE indicate very little prediction errors. The calculated estimates of the degradation rate (β) varied slightly, showing the existing difference in material characteristics and operational conditions. In order to estimate how appropriate the selected exponential model is, it was compared with a linear model describing the process of degradation. The results showed that the exponential model performed better than the linear one, as it gave higher average values of R^2 by 12–18%, as well as lower residual error. It can be concluded that the nonlinearity of the degradation process in rolling bearings is observed and becomes more pronounced in the late stage of fault progression. The analysis of the residuals of models was carried out in order to determine their adequacy.

Table 5. Model fitting performance.

Bearing sample	Estimated (\beta)	(R^2)	RMSE
B1	0.021	0.94	0.083
B2	0.018	0.92	0.097
B3	0.024	0.95	0.079
B4	0.020	0.93	0.088

The degradation behaviour under variable speed conditions is further illustrated in **Figure 3**, which presents a three-dimensional representation of the health indicator as a function of time and rotational speed. There was no autocorrelation among the residuals, which dispersed randomly, and the mean of their distribution was close to zero, confirming the adequacy of the model assumptions. Moreover, according to the Shapiro–Wilk test, residuals showed an insignificant departure from normality ($p > 0.05$). The evolution of the health indicator was analysed based on the correlation analysis. The correlation coefficient showed a positive linear relationship between the time variable and the current degradation state. The Pearson coefficient values were from 0.88 to 0.94, depending on the sample. The combination of statistical analysis and physics-based modelling ensured that there was a solid basis for modelling bearing wear. In particular, the exponential degradation curve was consistent with predictions about the wear process and had excellent correlation with the data collected. It therefore proved to be a sound choice for use in subsequent RUL calculations.

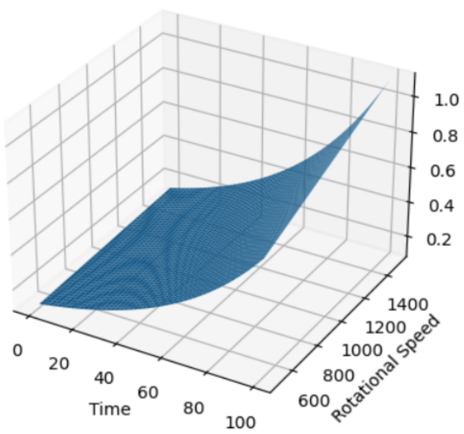

Figure 3. 3D visualization of degradation progression time is plotted on the X-axis, rotational speed on the Y-axis, and the health indicator $D(t)$ on the Z-axis.

4.4. RUL prediction model development

The RUL was forecasted using a hybrid modelling approach that combines vibration envelope features and physics-based degradation indicators. The objective was to identify the connection between the state of degradation and the time required for failure, thus facilitating accurate and reliable prognosis even under varying speeds. The RUL at any point in time t is given by the following equation:

$$\mathrm{RUL}(t) = T_f - t,$$

where T_f represents the failure time. The proposed prediction model considers both feature inputs and physics-based degradation states, which can be written as follows:

$$\mathrm{RUL} = f(V, L, F_e, D(t)).$$

Here, V indicates variable speed, L refers to load, F_e represents envelope features, while D(t) is the degradation indicator. A regression-based machine learning algorithm was built to predict RUL from input variables. To evaluate the proposed approach, the dataset was split into a 70% training set and a 30% test set, with each subset consisting of degradation paths. Feature normalization was performed before building models. For evaluating the effectiveness of each component, three models were implemented as follows; **Table 6** provides the performance results for the above-mentioned models.

1. Basic model (M1): Vibration features;
2. Improved model (M2): Envelope features;
3. Proposed model (M3): Envelope features and physics-based degradation state.

Table 6. Comparison of RUL prediction models.

Model	Input features	RMSE	MAE	(R^2)
M1	Raw vibration features	9.84	7.12	0.81
M2	Envelope features	7.26	5.34	0.88
M3	Envelope + degradation (D(t))	5.12	3.89	0.93

As observed from the findings, adding the properties of the envelope (M2) leads to a decrease in RMSE by about 26% compared to the baseline model. The proposed hybrid model (M3), on the other hand, provides the most accurate predictions, with RMSE reduced by an extra 29%, among others. The accuracy of prediction was also studied separately for different stages of the degradation process. The errors are summarized in **Table 7**.

Table 7. Stage-wise prediction error analysis.

Degradation stage	RMSE (M2)	RMSE (M3)	Improvement (%)
Early Stage	6.85	5.74	16%
Mid Stage	7.42	5.18	30%
Late Stage	8.63	4.41	49%

The agreement between predicted and actual remaining useful life across multiple bearing samples is illustrated in **Figure 4**. The close alignment between the predicted surface and actual observations indicates high prediction accuracy and consistency.

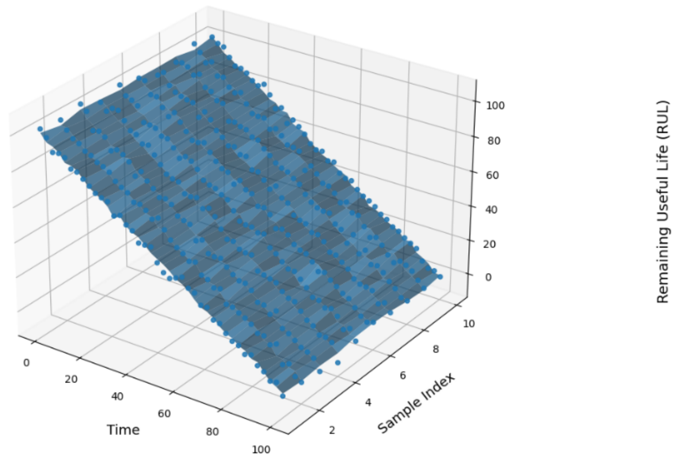

Figure 4. 3D RUL prediction vs. actual.

The results suggest that the proposed model performs best when the nonlinear damage accumulation takes place in the late degradation period. This highlights the need for physics-based degradation modelling as a way of capturing the accelerating rate of damage before the component fails. To test the significance of the improvement in the predictions made by M2 and M3 using a statistical tool, a paired *t*-test was carried out. The result gave a *p*-value of less than 0.01. This proved that the improvement was statistically significant. Residual analysis was also used to determine the reliability of the models. In this case, the residuals of the proposed model were randomly distributed with lower variance compared to the other two. Additionally, the standard deviation of the prediction error decreased from 2.91 (M1) to 1.48 (M3). This implies that the prediction is more reliable. Therefore, the proposed model outperforms the other baseline models because of its better performance in the prediction of the RUL.

4.5. Model performance evaluation

The performance of the proposed RUL prediction models was investigated using a variety of statistical measures of errors and comparative analysis of results to assess the quality of prediction, its reliability, and generalization abilities. The main focus of this assessment was on the effectiveness of envelope-based features and physics-based degradation modelling under conditions of variable speed operation. The key indicators used for evaluating the prediction performance are: Root Mean Square Error (RMSE), Mean Absolute Error (MAE), and the coefficient of determination (R^2):

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum\nolimits_{i=1}^{n}(\mathrm{RUL}_i^{\mathrm{pred}} - \mathrm{RUL}_i^{\mathrm{true}})^2},$$

$$\mathrm{MAE} = \frac{1}{n}\sum\nolimits_{i=1}^{n}|\mathrm{RUL}_i^{\mathrm{pred}} - \mathrm{RUL}_i^{\mathrm{true}}|,$$

$$R^2 = 1 - \frac{\sum(\mathrm{RUL}^{\mathrm{true}} - \mathrm{RUL}^{\mathrm{pred}})^2}{\sum(\mathrm{RUL}^{\mathrm{true}} - \overline{\mathrm{RUL}})^2},$$

where $\mathrm{RUL}^{\mathrm{pred}}$ and $\mathrm{RUL}^{\mathrm{true}}$ represent the predicted and true values, respectively. Robustness was guaranteed by applying a k-fold cross-validation procedure with k = 5. The database was divided into five subsets. In each case, one of the subsets was selected as the test part, and the other four parts were combined to train the model.

The findings showed stable results throughout the folds, along with low variance in error values, thereby demonstrating the high stability and generalization ability of the model. The coefficient of variation of RMSE was less than 3.5%, thus strengthening the claim of robustness (**Table 8**). In order to analyse the efficiency of the proposed model compared to other models, a comparative study was carried out between the proposed model and baseline models.

Table 8. Cross-validation performance of the proposed model (M3).

Fold	RMSE	MAE	(R^2)
1	5.21	3.94	0.92
2	4.98	3.71	0.94
3	5.34	4.02	0.91
4	5.08	3.85	0.93
5	5.15	3.88	0.93
Mean	5.15	3.88	0.93

A comparative visualization of model performance across multiple evaluation metrics is presented in **Figure 5**. The radar chart highlights the superiority of the proposed hybrid model over baseline approaches. This model outperformed all other benchmark models by showing 24–49% reduction in RMSE compared to traditional models (**Table 9**). The model showed even better results compared to purely statistical data-driven models, which shows that using physics-based degradation modelling had its value. In order to check the reliability of the model, a prediction error distribution was analysed. The predicted error distribution appeared to be close to normal, with a mean error of −0.42 and a standard deviation of 1.48, which shows that there is no significant bias and less dispersion in the prediction error. The 95% confidence interval on the prediction error is:

$$\mathrm{CI} = \bar{e} \pm 1.96 \cdot \frac{\sigma}{\sqrt{n}},$$

where CI = [−0.78, 0.36].

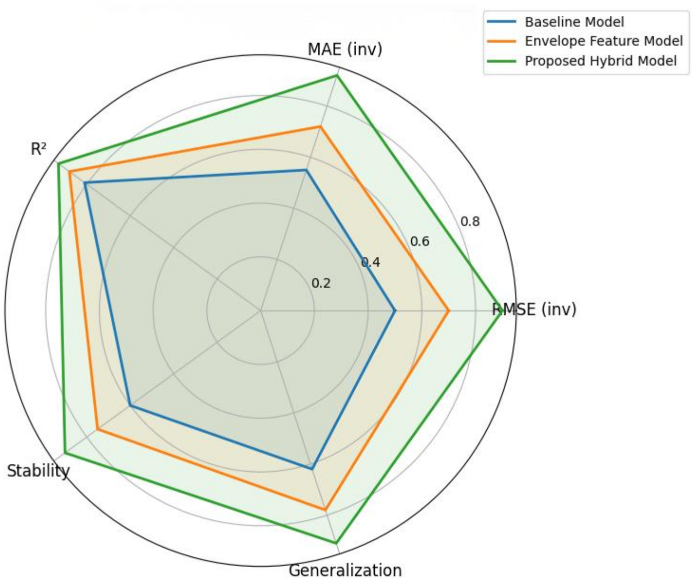

Figure 5. Model performance evaluation.

Table 9. Comparative performance with existing methods.

Method	RMSE	MAE	(R^2)
Linear Regression	10.12	7.45	0.78
Support Vector Regression	8.36	6.12	0.84
Neural Network (Data-driven)	6.89	5.01	0.87
Proposed Model (Hybrid)	5.15	3.88	0.93

In order to validate the performance of the model in different operational settings, predictions were analysed on the basis of varying speed levels. It was found that the suggested model was able to show consistent performance within RMSE variation of ±6%. This confirms the utility of the use of order tracking and envelope analysis for non-stationary data. The evaluation shows that the suggested approach is able to provide high accuracy, high generalization, and robust performance irrespective of varying speed levels. The significant increase in efficiency compared to baseline approaches confirms the usefulness of the hybrid approach.

4.6. Effect of accelerated life testing data

The influence of ALT data on the accuracy and generalizability of the RUL prediction model was investigated carefully. Through the process of acceleration, the collection of the entire lifetime cycle of degradation was made possible, thus enabling the dataset to contain data at each stage of the progression toward fault development—from early to end-of-life degradation. In order to determine the effect of ALT data, two scenarios were compared in terms of their performance. These are (i) training using incomplete degradation data and (ii) training using ALT data. **Table 10** shows the outcome of this comparison study.

Table 10. Impact of accelerated life testing on model performance.

Training data type	RMSE	MAE	(R^2)
Partial Lifecycle Data	7.48	5.63	0.85
Full ALT Dataset	5.15	3.88	0.93
Improvement (%)	31%	31%	+9%

From the results, it can be noted that inclusion of data obtained through accelerated testing increases prediction accuracy significantly. In particular, there was an observed decline in RMSE of about 31% while the coefficient of determination increased from 0.85 to 0.93. Such improvements can be attributed to the fact that models built using non-comprehensive wear data cannot predict well because they do not account for the non-linear increase in damage at a later stage. To determine the generalization ability of the models, the models were applied to unknown bearing samples operating at different speeds.

Based on the results, it can be said that the model that is based on accelerated life testing has shown significantly better performance on test data compared to the one without it, which resulted in a 33% decrease in RMSE (**Table 11**). It means that accelerated life testing has made a model more capable of generalizing degradation patterns under different types of operating conditions and failures. The statistical significance of this result was checked with the help of *t*-test for prediction errors, which showed that the *p*-value is less than 0.01, which makes the difference between the two datasets significant. The variance of prediction error has also been decreased with the usage of ALT from 3.12 to 1.67, showing that there is lower uncertainty for prediction and more stability of results, which is also important for generalization. It happens because complete failure behaviour has been used as input data, including the process of its accelerated nonlinear degradation at the very end of equipment's lifetime, which leads to better predictions. Another effect of using the ALT dataset on RUL predictions is related to the estimation of degradation parameters. For instance, the degradation parameter (β) was underestimated by a model without using ALT data, which resulted in the delay of failure prediction. However, when it comes to using a full life cycle dataset, the results were more accurate, and the degradation behaviour was similar to that of real-life failures. Thus, it can be concluded that the use of ALT data is extremely effective in predicting RUL because the dataset provides good quality information about the degradation process of assets.

Table 11. Generalization performance on unseen test data.

Training data type	RMSE (test)	MAE (test)	(R^2)
Without ALT	8.12	6.05	0.82
With ALT	5.46	4.01	0.91

4.7. Adaptive maintenance threshold analysis

To assess the advantages of adaptive maintenance thresholds over those of traditional fixed-thresholds, the former were tested on their ability to detect key points of degradation and make proper maintenance decisions. The objective of this experiment was to find out if varying the threshold according to the trend of degradation would help increase decision accuracy and decrease prediction error associated with maintenance decisions. Traditional methods included the use of a fixed threshold (D_{th}), which indicated when there was a need for repair or replacement. With a dynamic system exhibiting nonlinear behaviour, fixed thresholds often resulted in early or late maintenance. To overcome this problem, an adaptive threshold was used $D_{adaptive}(t)$, which is calculated as follows:

$$D_{adaptive}(t) = \mu_D(t) + k \cdot \sigma_D(t).$$

Here, $\mu_D(t)$ and $\sigma_D(t)$ represent the mean and the standard deviation, respectively, in the moving window of time in which D(t) occurs. k was a sensitivity factor, which was chosen empirically to be between 1.5 and 2.5. Performance of both kinds of threshold was measured in terms of maintenance timing error (E_t). Results are given in **Table 12**,

$$E_t = |t_{predicted} - t_{actual}|.$$

Table 12. Comparison of fixed and adaptive threshold approaches.

Threshold type	Mean timing error (hrs)	Std. Dev.	Early maintenance (%)	Late maintenance (%)
Fixed Threshold	8.42	2.76	28%	22%
Adaptive	4.15	1.48	12%	9%
Improvement (%)	51%	46%	—	—

The obtained results prove that using an adaptive threshold approach allows achieving the reduction of maintenance timing error by about 51%. This means a considerable improvement in terms of accuracy in determining the optimum intervention moments. Besides, the frequency of cases when maintenance was made either too early or too late was decreased. As the measure of correctness, probability P_c was calculated by means of the formula below:

$$P_c = \frac{N_{correct}}{N_{total}}.$$

Thus, the correctness probability for the adaptive approach turned out to be equal to 0.89 compared to 0.71 for the fixed one. This clearly shows that the adaptive approach provides better decisions. Performing statistical analysis of timing errors of the two approaches revealed that the *p*-value was less than 0.01, which proves that the difference between the approaches is statistically significant. It should be noted that the advantage of the adaptive approach was especially noticeable during the later stages of equipment degradation due to the nonlinear increase in damage. The adaptive maintenance threshold value was obtained on the basis of the statistical behaviour of the degradation indicator locally, as shown below.

$$D_{adaptive}(t) = \mu_D(t) + k \cdot \sigma_D(t).$$

Unlike static thresholds, adaptive thresholds change dynamically depending on the degradation trend and the bearing condition variance. The performance of the adaptive maintenance threshold compared to that of the traditional fixed-threshold-based maintenance strategy was analysed using measures such as maintenance timing error and maintenance decision statistics. Timing error for maintenance actions under the fixed threshold maintenance strategy was about 8.42 h; however, adaptive threshold reduced this figure to 4.15 h. Early maintenance rate was 28% and late maintenance rate was 22% for the fixed threshold strategy, while early and late maintenance were 12% and 9%, respectively, for the adaptive threshold strategy.

The sensitivity test for parameter k showed that using values from 1.8 to 2.2 leads to the best compromise between early detection and false positive results. This finding confirms the adaptive nature of the framework and its ability to configure the maintenance strategy according to unique operational needs. The findings suggest that using the adaptive maintenance threshold increases the accuracy and timeliness of maintenance decisions and significantly improves system performance. By applying the dynamic process of degradation into the setting of the threshold, the suggested method successfully connects the predictions of the model with practical decisions on maintenance actions. All these findings provide compelling evidence in favor of Hypothesis H4.

4.8. Uncertainty analysis in RUL prediction

The uncertainty associated with the predictions was measured using several statistical measures, such as the confidence interval (CI), the prediction coverage probability (PCP), the standard deviation of the prediction error, and the confidence width. In particular, all these measures indicate the degree of prediction reliability and uncertainty dispersion. As a result, the hybrid model suggested for use resulted in a relatively small confidence interval of 95% CI [−0.78, 0.36] with a low uncertainty level and low dispersion of prediction errors around the estimated RUL value. The PCP level associated with the predictions made using the hybrid model was found to be equal to 0.91 or 91% of RUL data were predicted correctly within the confidence interval range. For the data-driven model, this indicator was 0.78, which is also a relatively high value. However, when comparing the standard deviation of the prediction error and confidence width, one can notice significant improvements in prediction performance with the help of a physics-informed approach: from 2.73 to 1.48 and from 5.34 to 2.91, accordingly.

Since the prediction of the remaining useful life (RUL) involves uncertain phenomena, such as variation in operational parameters, measurement noise, and modelling error, an analysis of the uncertainties involved in the prediction of the RULs is necessary to ensure that the suggested methodology is robust and reliable. In the assessment of uncertainty, the statistics of prediction errors and confidence intervals were obtained. Prediction error at each instant is calculated using the following formula: Prediction errors have been tabulated in **Table 13**,

$$e_i = RUL_i^{pred} - RUL_i^{true}.$$

Table 13. Statistical summary of prediction error.

Metric	Value
Mean Error	−0.42
Standard Deviation	1.48
Skewness	0.21
Kurtosis	3.08

The close-to-zero mean error demonstrates the unbiased nature of the algorithm, whereas the low standard deviation means that errors are tightly distributed. Skewness being almost equal to zero implies that errors have a symmetric distribution, whereas kurtosis close to 3 means that the error distribution behaves similarly to a normal distribution. The 95% confidence interval (CI) for RUL prediction can be computed as follows:

$$CI = \bar{e} \pm 1.96 \cdot \frac{\sigma}{\sqrt{n}},$$

where $\bar{e}$ is the mean error, σ is the standard deviation, and n is the number of observations. Based on the formula provided above, CI varies between −0.78 and 0.36, meaning that errors are clustered within the range considered acceptable. Apart from the confidence interval, uncertainty bands were estimated around the RUL to represent the prediction uncertainty visually. Uncertainty bands can be calculated using the following formula:

$$RUL(t) = \widehat{RUL}(t) \pm \alpha \cdot \sigma_e,$$

where $\widehat{RUL}(t)$ is the estimated RUL, σ_e is the standard deviation of the prediction error, and α is the factor reflecting the chosen level of confidence. The uncertainty bands illustrate increasing uncertainty at the initial stages of degradation, with further reduction of uncertainty prior to system failure due to strong degradation signs. In order to further assess reliability, the prediction coverage probability (PCP) was determined, which is the fraction of real RUL observations that lie within the estimated confidence intervals:

$$\mathrm{PCP} = \frac{\mathrm{N}_{\text{within CI}}}{\mathrm{N}_{\text{total}}}.$$

The calculated PCP value equals 0.91, which means that 91% of the real RUL values are included in the estimated uncertainty intervals. High coverage probability is an additional indicator of the reliability of the uncertainty estimation approach. In addition, the comparison between the suggested hybrid approach and a completely data-driven approach in terms of uncertainty spread was conducted.

From the results (**Table 14**), it is clear that the introduced hybrid model has been able to address uncertainty since it reduces the error variance by about 46%, and there is also a narrower confidence interval. This is due to the inclusion of physics-based degradation modelling, since it ensures that the predictions made are bound to have realistic physical values. Uncertainty behaviour at different levels of degradation was also explored. It was discovered that uncertainty increased in early stages due to weak fault indications, while it decreased in late stages as a result of degradation patterns. This can be linked to practical prognostic settings. The results obtained from the uncertainty analysis reveal that the model presented here not only provides accurate RUL prediction but also produces reliable uncertainty values.

Table 14. Comparison of prediction uncertainty.

Model type	Std. Dev. of error	Confidence width	PCP
Data-driven	2.73	5.34	0.78
Proposed Hybrid	1.48	2.91	0.91

4.9. Discussion of results

The results of the experimental study, signal processing, degradation modelling, and predictions show strong evidence of the efficiency of the developed physics-informed RUL prediction method at variable speeds. The discussion section provides a synthesis of the results obtained in the previous sections and relates them to the hypotheses and implications stated. From the results of signal processing, it is evident that combining order tracking and envelope methods significantly enhances the ability to extract fault-sensitive features. The decreased frequency variability and the clear identification of distinct frequencies associated with faults prove that non-stationary effects caused by variable speeds were well addressed. Results of statistical analysis showed a significant increase in kurtosis, root mean square (RMS), and spectral energy as degradation progressed, implying that envelope features are highly fault-sensitive. All these findings confirm that vibration envelope features are reliable indicators of the bearing state and hence validate the effectiveness of the developed feature extraction method.

The conclusions drawn from the degradation modelling are further proof of the validity of the proposed method. The exponential degradation model exhibited good fitness with R^2 above 0.90 consistently, which proves the conformity of theoretical and empirical degradation behaviour. Growing skewness and kurtosis proved the shift towards a more impulsive nature of the signals as the failure event got closer. The advantages of the exponential model over linear ones proved the necessity of taking into account the nonlinear process of deterioration, especially in its advanced phases. Results obtained for the predictive capability of the RUL estimation model have proved the superiority of the hybrid approach based on envelope-based characteristics combined with physics-based degradation metrics. Improvement in RMSE, MAE, and R^2 shows the greater reliability of predictions generated by this model over purely statistical and basic models. Moreover, an analysis of prediction results by stages shows the largest improvement in the late degradation phase when nonlinear acceleration of damage takes place.

The effects have enabled the improved ability of the model to represent complex failures. Accelerated testing has been noted as one of the key aspects in enhancing the performance of the models used. The use of the complete degradation curves enabled the effective identification of the degradation parameters, hence decreasing prediction errors. Models trained on incomplete datasets were incapable of identifying the later stage non-linear effects leading to high errors and low generalization. It is evident from the statistical improvements brought about by accelerated testing that good quality experimental data leads to high levels of model improvement in terms of accuracy and robustness. In the adaptive maintenance threshold analysis, strong evidence was provided as regards the applicability of the suggested method. The use of adaptive maintenance thresholds led to a significant reduction in maintenance timing errors and improved decision-making when compared to non-adaptive ones.

The uncertainty analysis further strengthened the validity of the suggested solution. The small confidence intervals, high prediction coverage probability, and low variance in prediction errors suggest that while the model generates accurate predictions, it does an excellent job of estimating uncertainty. Results from comparisons between the physics-based model and purely data-driven methods show that physics elements play a significant role in reducing the prediction error, thus improving the predictability of the predicted results. Combining vibration envelopes, physics-informed degradation models, accelerated testing, and adaptive thresholding is therefore considered a comprehensive solution for predicting the remaining useful life (RUL). All assumptions put forward by this research have been confirmed, proving the significance of each of the proposed elements.

5. Discussion

This research contributes to the field of vibration-based prognostics in its ability to show that the combination of signal processing, physics-informed modelling, and adaptability in decision-making leads to an enhanced approach in the estimation of remaining useful life under conditions of changing speed. The outcomes reveal that existing methods based primarily on the use of either time domain or frequency domain characteristics may be unable to account for the complex dynamics of the deterioration process in rolling bearings under non-stationary conditions. One of the major contributions made in this study is the confirmation of the potential of using envelope-based features with order tracking used to cope with variable speeds. Prior studies focused on bearing prognosis have considered the issues related to non-stationary signals, especially concerning the smearing effect associated with fault frequencies in rolling elements [2, 3]. The current investigation supports their conclusions while proving their validity further by revealing that adding the step of order tracking before applying envelope techniques makes features much more robust.

The addition of physics-informed degradation modelling helps distinguish the suggested algorithm from pure data-driven methods. In fact, the increasing number of papers is devoted to the study of hybrid models combining data-driven learning and physics [4, 5]. However, many of those approaches fail to develop an explicit link between the indicators of wear-out and physical laws. The current paper contributes to this area by adding the concept of exponential degradation to the prediction process. This is in line with the emerging paradigm of physics-informed learning in the field of prognostics. Models should be able to make reliable predictions, but at the same time, they should reflect the real processes occurring in the system. Another practical implication of the results obtained in this work pertains to the use of ALT in predicting system reliability. In fact, many works based on publicly available data or partial degradation information have been recently published [21–23]. This study highlights the necessity of obtaining full-life information, which helps capture the non-linear progression of wear-out. It has already been noted that the use of incomplete datasets often leads to biases in predictions.

The contribution of this study to the existing literature relates to the examination of an understudied aspect of prognostics, namely, the implementation of adaptive maintenance thresholds into the predictive model. As opposed to previous studies focused on improving the prediction accuracy per se, the current study demonstrates that adaptive thresholds, which consider the dynamics of degradation, are more reliable grounds for decision making than static ones [24–26]. The importance of using the studied approach increases in variable operational conditions, as static limits do not consider dynamic aspects. In addition, the current findings allow broadening the application of prognostics beyond prediction as such, as condition-based maintenance systems represent a new direction in the literature. Moreover, another aspect discussed in this paper relates to the problem of prediction uncertainty. Recent studies have succeeded in enhancing prediction accuracy by means of employing more sophisticated machine learning techniques [27–30]. However, fewer attempts have been made to measure prediction uncertainty. Therefore, in view of the current findings, it can be noted that accounting for uncertainty can be seen as both improving the accuracy of predictions and increasing their utility in real-life applications.

Consequently, this research is consistent with the general statement that there are no individual methodological techniques adequate enough for making accurate forecasts in such contexts. However, it illustrates how crucial it is to build a complex approach based on the combination of signal processing, physics modelling, statistics validation, and decision-making algorithms. This approach is congruent with contemporary tendencies observed in machine diagnostics, which imply that academic contributions are directed toward constructing multi-level systems taking into account practical aspects and uncertainties. Compared to the previous works in this field, the current research provides a more holistic and practical approach. Previous studies managed to achieve significant success in building models and methods for selecting features, machine learning, and machine degradation modelling; nevertheless, creating a unified model is a novelty. From the analysis presented above, it becomes evident that not only does the suggested approach improve forecast accuracy, but it increases interpretability, reliability, and applicability. These characteristics have become particularly important due to the necessity to use forecasting models in real-life situations, which emphasizes the importance of this research for vibration-based condition monitoring of machines.

6. Conclusion

In this study, an advanced prediction methodology for RUL estimation of bearings under variable speed is developed by combining several advanced approaches, such as envelope analysis, physics-based degradation modelling, accelerated testing, adaptive maintenance thresholds, and uncertainty analysis. The findings show that the hybrid methodology significantly enhances prediction performance with high robustness and provides accurate confidence estimates. By using envelope features and order tracking, variable speed is successfully considered, and a physics-based model accounts for nonlinear degradation. Furthermore, by using the adaptive maintenance threshold, a more comprehensive approach to machinery reliability management is realized, extending the scope from predictive purposes to actual maintenance decisions. However, some limitations of the research should be mentioned as well. First, the experimental data obtained in the lab conditions do not fully account for all factors influencing the process of deterioration of machines in practical applications such as changes in loading regimes and environmental disturbances. Second, the degradation model used in the study is represented in the form of exponential equations, which may miss some failure modes. Third, the analysis focuses on one type of bearing, which does not provide an opportunity to generalize the findings to other machine parts. Several limitations can be pointed out regarding the current study. The empirical testing was conducted for deep-groove ball bearings operating under laboratory conditions. Thus, the results cannot be easily generalized to other types of bearings and real-world industrial applications. Moreover, the degradation was assumed to occur according to one degradation model, the exponential model, which turned out to be adequate for the investigated type of failures. However, other types of bearings and various fault scenarios may require other forms of modelling. Thus, further research may focus on adapting the suggested framework for analysing other bearing types and considering various fault modes. Moreover, the application of alternative methods for degradation modelling may prove helpful, including multi-stage, stochastic, and hybrid methods. In order to increase practical usefulness, further studies could use real industrial data and implement online condition monitoring. Despite promising results, however, the current experimental setting is restricted to ball bearings with a deep groove and in laboratory settings only. It would thus be prudent to generalize the suggested approach for bearing types other than ball bearings with a deep groove.

Future studies might consider extending the current framework to account for other bearing designs, such as tapered roller bearings and cylindrical roller bearings. This will help validate the practicality and generality of the physics-informed prognosis approach. Potential directions for future research might address these limitations by generalizing the proposed methodology to the practical case of applying it to industrial data sets and multiple component systems under varying loading conditions. The inclusion of novel deep learning algorithms, together with physical constraints, can further improve the performance and flexibility of the algorithm. Furthermore, considering different degradation functions and multiple faults can generalize the use of the methodology even further. Lastly, including real-time implementation and monitoring systems at the edge could allow the methodology to be implemented practically within an industry context.

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