





# Dynamic modeling of ball screw feed systems: A review and framework from physical mechanisms to a physics–data hybrid approach

Yongqiang Li<sup>1,2,\*</sup> , Saiful Bahri Bin Mohamed<sup>1</sup> , Mohd Shahir Bin Kasim<sup>1</sup> , Siti Nurul Akmal Binti Yusof<sup>1</sup> 

<sup>1</sup> Faculty of Innovative Design and Technology, Universiti Sultan Zainal Abidin, Kuala Nerus 21300, Malaysia

<sup>2</sup> School of Intelligent Equipment, Lanzhou University of Information Science and Technology, Lanzhou 730300, China

\* Corresponding author: Yongqiang Li, [liyongqiang0215@gmail.com](mailto:liyongqiang0215@gmail.com)

## CITATION

Li Y, Mohamed SBB, Kasim MSB, et al. Dynamic modeling of ball screw feed systems: A review and framework from physical mechanisms to a physics–data hybrid approach. *Sound & Vibration*. 2026; 60(3): 4089.  
<https://doi.org/10.59400/sv4089>

## ARTICLE INFO

Received: 26 February 2026

Revised: 29 March 2026

Accepted: 2 April 2026

Available online: 27 May 2026

## COPYRIGHT



Copyright © 2026 Author(s).  
*Sound & Vibration* is published by Academic Publishing Pte. Ltd. This work is licensed under the Creative Commons Attribution (CC BY) license. <https://creativecommons.org/licenses/by/4.0/>

**Abstract:** Ball screw feed systems demonstrate that pronounced nonlinearity, time variation, and multiphysics coupling arise under high speed, high acceleration, and long-duration operation, where neither purely physics-based nor purely data-driven models can simultaneously ensure interpretability, deployability, and cross-condition reliability. However, the significant challenges this presents show that a more integrated framework could address these limitations more effectively. Moreover, the findings from existing approaches indicate that a Coupled Mechanism–Model–Data (CMMD) framework could provide a systematic way to connect physical mechanisms with application-oriented modeling through a hierarchical and constrained architecture. In light of these key results, the evidence shows that following a unified logic of “mechanism coupling–model structure–data updating interface” appears to support more coherent analytical development. Research shows friction, stiffness, thermal deformation, and wear affect modal properties. Nevertheless, the important propagation effects indicate that formulating these as identifiable nonlinear and time-varying terms demonstrates that the framework retains critical physical interpretability. Furthermore, the evidence shows that hierarchical physics-based model families appear to differ significantly in representational capability, computational cost, and boundary sensitivity. Given that the findings demonstrate that conditions involving strong nonlinearity and parameter drift present particular analytical difficulty, LPV–NARX (Linear Parameter-Varying Nonlinear AutoRegressive model with eXogenous inputs), sparse identification, neural networks, and three hybrid paradigms—residual compensation, adaptive parameter updating, and physics-constrained learning—indicate that synthesis according to fusion location, updating objects, and failure boundaries could establish a constraint-driven hierarchical selection strategy. Data shows hybrid paradigms yield representative application scenarios.

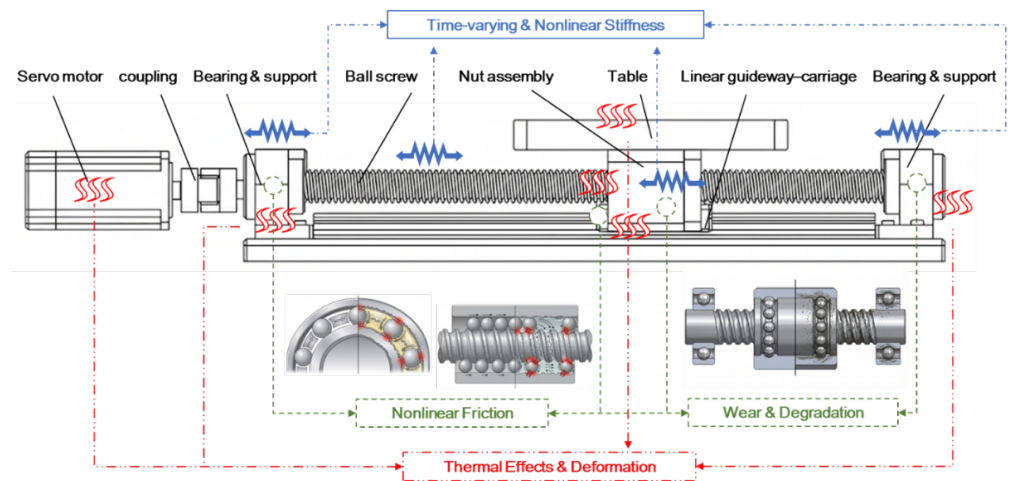
**Keywords:** ball screw feed system; nonlinear time-varying dynamics; multiphysics coupling; position-dependent stiffness; physics-informed hybrid modeling

## 1. Introduction

The ball screw feed system (BSFS) is the core actuator that enables high-precision linear motion in CNC machine tools. Its dynamic performance directly constrains the positioning and contour accuracy, response speed, and operational stability of the entire machine [1, 2]. Under high-speed, high-acceleration, and wide-frequency working conditions, the dynamic behavior shows nonlinear, time-varying, and multiphysics coupling characteristics. Therefore, traditional empirical tuning and single-model

approaches are difficult to balance interpretability, implementability, and reliability across different operating conditions [3–5]. Accordingly, establishing a dynamic modeling framework oriented toward complex conditions and long-term operation is an important foundation for control tuning, error compensation, structural design, and intelligent maintenance [5].

**Figure 1** illustrates BSFS and the coupling of its multi-source mechanisms at different time scales. Friction, position-dependent stiffness, thermal deformation, and degradation jointly shape the dynamic characteristics. They influence modal properties, vibration, and trajectory errors through parameter drift and boundary variation [6, 7]. Therefore, the main challenge of dynamic modeling is to transform these coupled mechanisms into model expressions that are embeddable, identifiable, and updatable, so that the requirements of real-time control and stability can be satisfied.

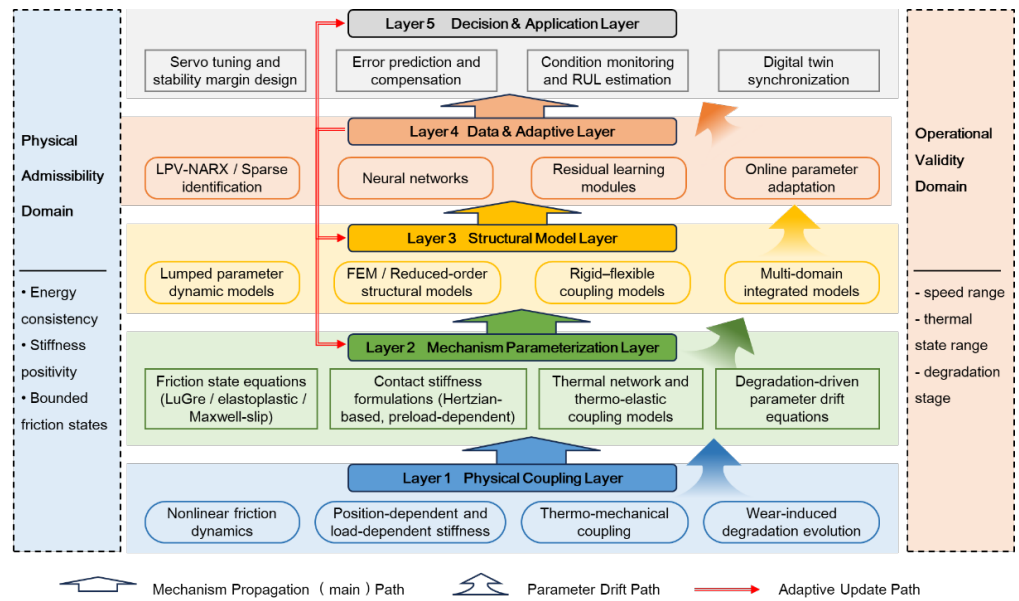


**Figure 1.** Structure of a BSFS and schematic illustration of multi-source physical mechanisms.

The evidence shows that existing studies have formed three key paths to address these challenges: physical modeling, data-driven modeling, and hybrid modeling. However, the significant findings indicate that physical models emphasize structural consistency and interpretable boundaries [3,4]. Furthermore, the results demonstrate that data-driven models focus on complex mappings and multi-source fusion [8]. In light of these findings, hybrid structures show that improved accuracy and cross-condition adaptability under stability constraints remain critical [9]. Studies show most reviews address modeling, control, vibration, or dynamic design of BSFS [1,2,10]. Moreover, significant evidence shows that studies integrating mechanism coupling, model structure, and data updating interface as a unified logical system appear limited. Additionally, the findings indicate that method comparison, application selection, and digital twin deployment remain critical elements that existing reviews have yet to establish within a coherent framework. Given that the results demonstrate this gap, the evidence shows that a more unified approach could provide important analytical support.

This paper shows that a Coupled Mechanism-Model-Data (CMMD) framework, as shown in **Figure 2**, could provide the systematic connection between physical mechanisms and application-oriented modeling. Moreover, the significant framework indicates that physical mechanisms, structural dynamic models, and data-driven

adaptive methods could demonstrate integration within a hierarchical and constrained architecture. However, the CMMD framework appears to establish that the mapping from mechanisms to parameters, the embedding relationship from parameters to structure, and the adaptive correction interface from structure to data explicitly define relationships that previous reviews could not support. In light of these key findings, the hierarchical structure shows that nonlinear friction, thermo-mechanical effects, stiffness evolution, and degradation processes might indicate systematic propagation across modeling layers. The framework shows mechanisms that affect prediction reliability and control robustness.



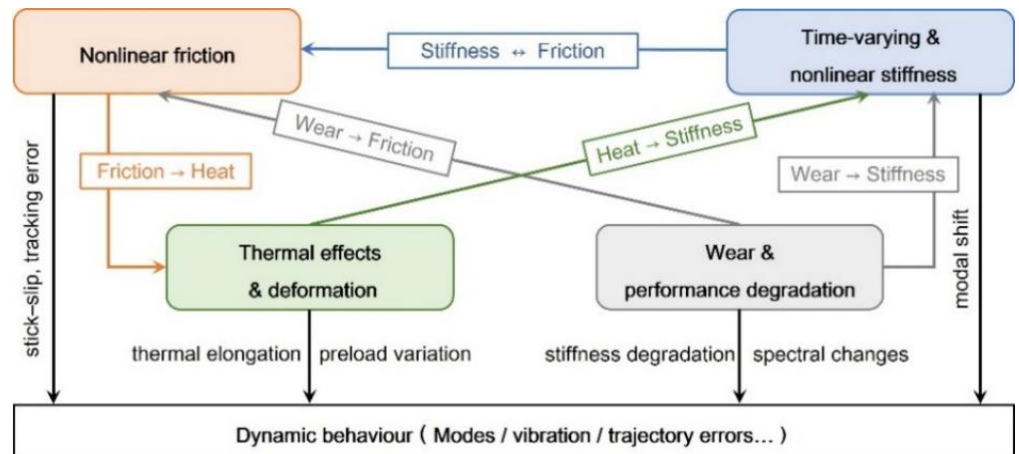
**Figure 2.** CMMD framework for the dynamics of BSFSs.

Based on this structure, this paper discusses key physical mechanisms (Section 2), physics-based modeling methods (Section 3), data-driven and hybrid modeling approaches (Section 4), constraint-driven hierarchical selection strategies and typical scenarios (Section 5), and finally provides conclusions and an outlook (Section 6).

## 2. Key physical mechanisms and their effects on dynamic behavior

The dynamic characteristics of a BSFS, including modal properties, vibration, and trajectory errors, result from the coupling of multiple physical mechanisms under different time scales and operating conditions [11, 12]. As shown in **Figure 3**, friction, stiffness, thermal effects, and degradation interact with each other. Nonlinear friction directly affects low-speed stability and tracking error. At the same time, frictional heat changes the thermal field distribution [13–15]. Thermal deformation then causes preload drift and variation in contact state. As a result, the equivalent stiffness becomes time-varying and nonlinear, and modal migration may occur [16–18]. Long-term wear further alters friction levels and contact stiffness, which leads to slow drift of the dynamic characteristics [19, 20]. A three-layer transmission framework shows that cross-scale coupling among these mechanisms can be characterized quantitatively. Moreover, the parameter layer appears to establish that the friction force  $F_f$ , equivalent stiffness  $K_{eq}$ , thermal elongation  $\delta_{th}$ , and contact stiffness degradation  $\Delta K_{wear}$

serve as the primary quantifiable variables. Furthermore, the significant parameter variations at this layer indicate that modal frequency shifts  $\Delta f_n$ , damping ratio changes  $\Delta \xi$ , and mode shape distortions propagate through the structural layer. In light of these results, the error layer demonstrates that the cumulative contributions of all mechanisms to trajectory error  $E_{total}$  can be expressed as:  $E_{total} \approx E_{friction} + E_{stiffness} + E_{thermal} + E_{wear}$ , where each term represents the error contribution from the corresponding mechanism under specific operating conditions. Framework provides basis for cross-scale correlation analysis in the following subsections.



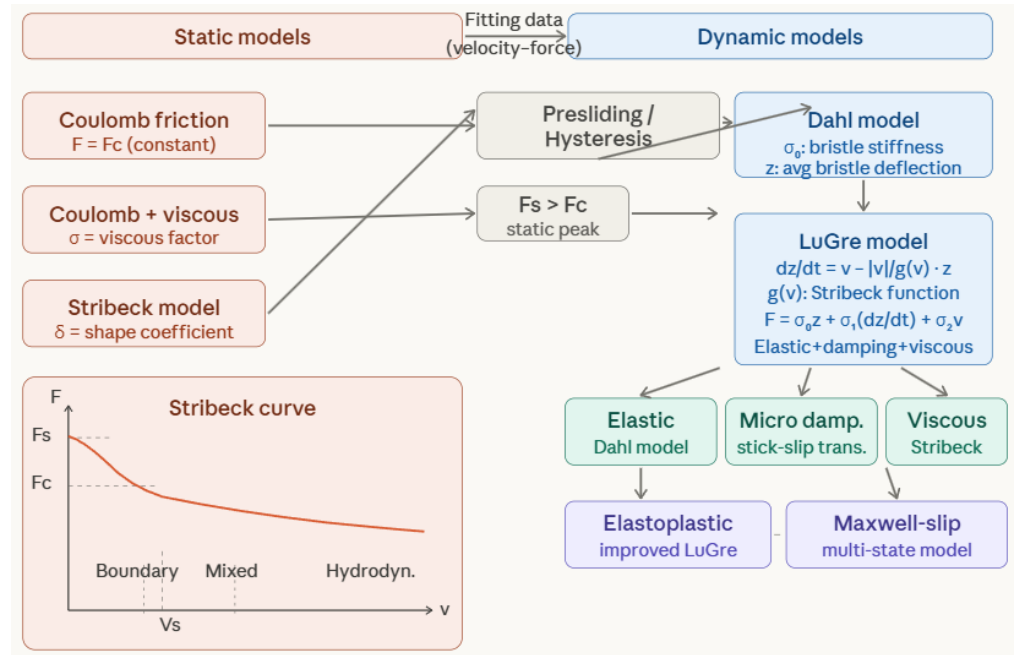
**Figure 3.** Coupled physical mechanisms in BSFS and their effects on dynamic behavior.

At the parameter level, these mechanisms appear as identifiable nonlinear and time-varying terms. At the structural level, they appear as the evolution of modal and damping properties. Therefore, they form the physical basis for subsequent modeling and updating strategy design [21–23].

### 2.1. Nonlinear friction characteristics

Friction in a BSFS mainly originates from the screw–nut pair, guideways, support bearings, and transmission accessories. These sources are superimposed, and the overall friction shows clear nonlinear behavior. In modeling and identification, the screw–nut pair and guideways are usually regarded as the dominant friction sources. The resistance from bearings and accessories is often incorporated into the total friction term or modeled as velocity-dependent damping. This treatment improves parameter identifiability and engineering applicability [24].

As shown in **Figure 4**, typical friction models and their dynamic characteristics can be described as follows. At the macroscopic level, friction behavior is commonly represented by Coulomb friction, viscous friction, and the Stribeck effect [12–14]. In the low-speed region, pre-sliding and stick-slip transitions are related to equivalent negative damping and structural mode coupling. These phenomena are key causes of low-speed vibration and crawling motion [25]. During long-stroke operation, friction also exhibits position dependence. Therefore, this feature should be included in parameter expressions [26].



**Figure 4.** Typical friction models and their dynamic characteristics in BSFS.

To overcome discontinuity near zero velocity and the limited transient prediction capability of static models, dynamic friction models such as the Dahl and LuGre models introduce internal state variables to describe the elastoplastic evolution at the contact interface [13, 14]. The LuGre model integrates the Stribeck effect and viscous terms within a unified framework [14, 15]. Therefore, it is more suitable for dynamic friction modeling in BSFSs. To address parameter drift and zero-speed drift, elastoplastic improved models and the generalized Maxwell-slip multi-state model further enhance low-speed stability and compensation accuracy [27, 28].

Temperature, load, and wear change lubrication conditions and contact characteristics. As a result, the Coulomb level, Stribeck characteristic velocity, and viscous coefficient evolve with time and operating conditions [28, 29]. Therefore, friction modeling usually needs to be coupled with structural flexibility and thermal effects [20]. In addition, online estimation or extended models are used to capture parameter time variation [21]. On this basis, combining mechanism-based models with data-driven representations can further improve prediction robustness [22].

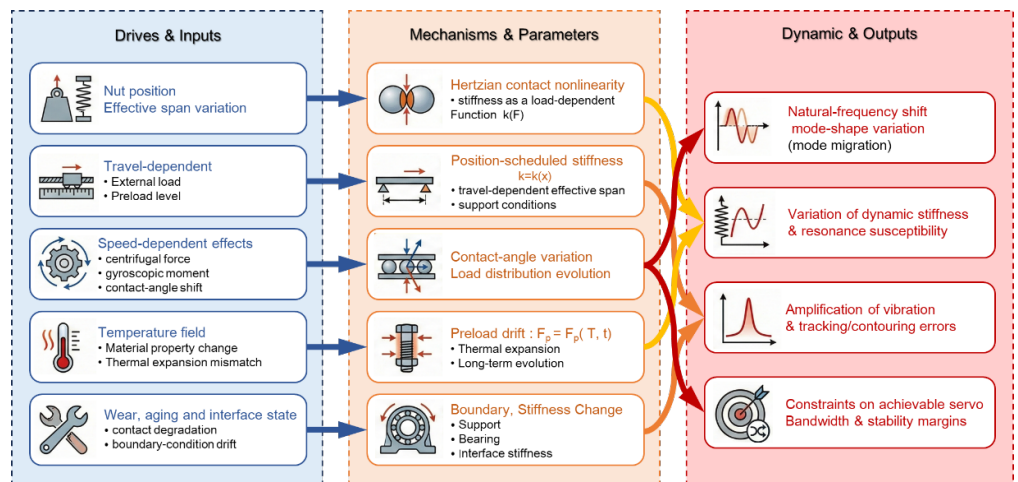
In summary, at the model level, friction mainly appears as nonlinear damping and time-varying parameter terms. It is the core source of low-speed dynamic error and transient response behavior.

## 2.2. Time-varying and nonlinear stiffness

The equivalent stiffness of BSFS is jointly determined by the screw shaft, rolling contact interfaces, support bearings, and interface connections. It shows load-dependent nonlinear characteristics, as well as position- and speed-dependent time-varying properties [3–5]. The ball-raceway contact stiffness can be described by Hertzian contact theory, which establishes a load-deformation relationship and represents a typical source of nonlinearity [4, 5]. Since the screw is a slender component, its effective span changes with nut position. Therefore, natural frequencies and mode

shapes show clear position dependence [3, 5]. Twin-screw structures and preload mechanisms further increase modal coupling and stiffness distribution complexity [5].

**Figure 5** shows that local stiffness variation propagates to global modal characteristics in ways that could demonstrate significant structural implications. Moreover, the findings appear to provide a structural basis for position-dependent dynamic modeling. Furthermore, the evidence indicates that increasing preload enhances axial stiffness and could suppress backlash across key operating conditions. Given that friction and heat generation also increase, the results show that a critical coupling trade-off exists between stiffness improvement and parameter drift [5]. Stiffness evolves with speed and load [23,24]. However, under high-speed operation, the significant centrifugal forces of rolling elements show that changes in contact angle could modify contact stiffness in important ways. Additionally, the evidence indicates that thermal deformation further alters equivalent stiffness and damping through material parameter variation and preload drift. In light of these key results, the findings demonstrate that stiffness time variation shows critical multiphysics coupling characteristics [7]. Stiffness shows multiphysics coupling. Nevertheless, in model representation, the evidence shows that low-order lumped parameter models could describe nonlinear and time-varying stiffness through load-dependent elastic elements and position-scheduled parameters [3,4]. Notwithstanding these results, high-fidelity models appear to explicitly introduce contact elements and boundary conditions that could represent preload and support stiffness in significant ways. Therefore, the findings indicate that medium- and high-frequency modes and coupling effects might be captured more accurately through these approaches [5, 6]. High-fidelity models show coupling effects captured accurately. In summary, the evidence shows that establishing identifiable expressions for rolling contact nonlinearity, position-dependent span variation, and high-speed coupling effects demonstrate that this foundation is critical for improving dynamic modeling accuracy and stability analysis.



**Figure 5.** Position-dependent stiffness and modal variation in BSFSs.

The stiffness variation shows that it directly governs the vibration behavior of the BSFS. Moreover, the shift in natural frequencies indicates that position-dependent resonance zones appear as the nut traverses the screw, within which excitation

from cutting forces or servo commands might trigger amplified vibration responses. Furthermore, the significant coupling between axial and torsional modes demonstrates that vibration amplitude intensifies under high-speed operation, leading to surface waviness and contour errors. In light of these findings, characterizing position-dependent stiffness indicates that it serves as a key prerequisite for vibration prediction and suppression in precision machining applications. Stiffness variation governs vibration, shapes resonance zones, and drives errors.

### **2.3. Thermal effects and thermal deformation**

Friction and transmission losses show that heat accumulates in the screw shaft and the supporting structures. However, the significant temperature rise and the thermal gradients indicate that axial elongation and structural deformation appear linked [7,25]. Moreover, the key findings demonstrate that noticeable positioning drift could emerge even before thermal steady state is reached [25,27].

The thermal field not only changes geometric dimensions, but also alters friction and stiffness characteristics through preload drift and changes in lubricant viscosity. Therefore, a thermo-mechanical-dynamic recoupling process is formed [7,26].

Common modeling approaches include empirical mapping models, lumped-parameter thermal network models, and thermo-mechanical coupled finite element models [25, 26, 28]. Empirical models are convenient for online implementation, but they are sensitive to changes in operating conditions. In contrast, thermal network models and thermo-mechanical finite element models can describe heat source distribution and thermal stress response in a unified way. They also provide reduced-order substructures for system-level modeling [7,25].

In dynamic modeling, thermal effects mainly appear as slow time-varying parameters and structural boundary drift. Therefore, they are important factors for long-term accuracy retention and compensation strategy design.

Thermal deformation shows that slow drift in structural boundary conditions could indicate progressive shifts in natural frequencies and altered mode shapes during long-duration operation. However, the significant thermally induced dynamic variation appears to demonstrate time-varying vibration amplitudes and phase shifts in the servo response, making it difficult to maintain stable cutting conditions. Furthermore, the evidence indicates that identifying and compensating for thermally driven vibration drift could demonstrate direct engineering significance for maintaining dynamic accuracy. In light of these findings, the key results show that surface quality over extended machining cycles appears to require systematic attention to these thermally induced effects. Thermal drift affects boundary conditions, shifts frequencies, and degrades surface quality.

### **2.4. Wear and performance degradation**

Long-term cyclic loads and reciprocating motion show that wear on rolling contact surfaces and interface connections could indicate significant degradation over time. However, the findings demonstrate that increased friction, enlarged clearance, and reduced stiffness result from this accumulation. Moreover, the evidence indicates that

positioning accuracy and vibration characteristics appear to degrade gradually as wear progresses [29]. In light of these results, the early stage of wear indicates that changes in friction level and temperature rise are key indicators worth examining [25,26]. Wear shows coupled evolution characteristics [26,30]. Furthermore, the significant findings show that backlash expansion and modal frequency drift could appear in later stages of degradation [25,26]. Additionally, the evidence demonstrates that variations in temperature and load appear to accelerate lubricant degradation and surface damage. Thus, the results indicate that the degradation process demonstrates coupled evolution characteristics across multiple interacting mechanisms [26,30]. Wear links friction, preload, stiffness. Nevertheless, wear is usually represented by slow drift in friction parameters, preload, and contact stiffness at the modeling level [18,19]. Given that the evidence shows that trend models could demonstrate consistency with contact mechanics and wear laws [29]. Moreover, the significant findings indicate that these models appear to embed into dynamic models through online identification [21,30]. Models show life-cycle consistency [21,30]. In light of these results, the key evidence shows that vibration frequency shift, temperature rise, and error features may demonstrate utility in constructing state indicators [31,32]. Therefore, the findings indicate that remaining useful life prediction models appear to rely on these important features for health management purposes [31,32]. Thus, incorporating wear evolution into updatable parameter expressions demonstrates that long-term accuracy retention and reliable operation appear achievable.

### **3. Physical modeling methods**

#### **3.1. Lumped parameter models**

Lumped parameter models represent BSFS as a finite-degree-of-freedom network composed of masses, springs, and dampers. Moreover, concentrated parameters show that the inertia and flexibility of components such as the motor, coupling, screw shaft, nut seat, and worktable could demonstrate meaningful low-order behavior. Furthermore, this type of model indicates that low-order modes, axial vibration, and servo closed-loop dynamics appear accessible with significant efficiency. Given that evidence demonstrates broad applicability, the results show that system-level analysis and control design [33] could benefit from this approach. Models show torsional, axial, support, and guideway stiffness discretized as series or parallel elements. However, the findings show that boundary flexibility and transmission compliance could introduce important refinements to boundary conditions. In light of these results, prediction of resonance frequencies and transfer characteristics indicates that significant improvement appears achievable [34]. Additionally, rotational motion and linear motion demonstrate that a unified description with the servo system appears supported through lead conversion. Study shows the model expressed in state-space form supports identification, stability analysis, and controller tuning [35]. Nevertheless, the significant findings show that nonlinear effects such as friction, backlash, and preload indicate that nonlinear friction elements, clearance elements, or load-dependent elastic elements appear critical. Furthermore, these elements

demonstrate that low-speed crawling, reversal dead zones, and amplification of tracking errors [11, 14] could reflect key dynamic behavior. Given that evidence indicates position-dependent stiffness variation along the stroke, the results show that position scheduling or piecewise parameterization appears relevant. Evidence shows migration of low-order modes with position [16,23] captured. Therefore, the significant findings indicate that friction parameter drift and preload variation caused by temperature rise could demonstrate important thermal effects. Notwithstanding these results, the evidence shows that thermal deformation appears to influence dynamic response and error evolution [25,26].

### **3.2. Finite element modeling**

Finite element modeling shows that discretizing the screw shaft, nut seat, and support structures into beam elements or three-dimensional solid elements could indicate significant analytical value. Moreover, the findings appear to demonstrate that medium- and high-frequency modes and local flexibility can be described under complex boundary conditions. Thus, the evidence shows that this approach provides an important tool for analyzing structural vibration of BSFSs [34,35]. Given that the results indicate a clear procedural foundation, the typical procedure demonstrates that geometric modeling and mesh generation establish the necessary starting point.

FEM shows contact elements represent ball–raceway contact, preload, bearing support. Furthermore, the significant findings show that this structural model could support modal and dynamic response analysis [6]. In light of these results, the evidence appears to demonstrate that modal analysis, harmonic response analysis, or transient analysis can be carried out. Additionally, the key findings indicate that natural frequencies, mode shapes, and local responses can support structural optimization and dynamic performance evaluation [34].

Thermo-mechanical coupling shows a model that describes thermal stress and thermal deformation. However, the findings show that this model could also reflect their indirect influence on dynamic characteristics. Notwithstanding these results, the evidence appears to demonstrate that the model provides a significant basis for extracting flexibility distribution. Therefore, the key findings indicate that calibrating parameters in twin-screw or large-scale structures could demonstrate practical importance [6,7].

Finite element models show large degrees of freedom affect boundary condition sensitivity. Moreover, the significant findings show that substructuring and modal synthesis techniques could indicate an important reduction strategy in engineering practice. In light of the evidence, the results appear to demonstrate that the reduced model could support coupling with system-level models. Thus, the key findings show that both accuracy and computational efficiency can be balanced [36].

### **3.3. Rigid-flexible coupled multibody dynamics modeling**

Rigid-flexible coupled multibody dynamics models use multibody dynamics as the main framework. Components such as the worktable and nut seat are treated as rigid bodies, while slender components such as the screw shaft are represented

as flexible bodies. Through constraint relations and modal coordinates, rigid body motion and elastic deformation are coupled. In this way, system-level dynamics and structural flexibility can both be considered under controllable computational cost [34, 36]. Flexible bodies are usually obtained from finite element discretization. Component mode synthesis and modal truncation are then applied to extract dominant modes. These flexible bodies are assembled with rigid components such as the motor rotor and coupling to form the transmission chain model [37–39].

This type of model can explicitly describe screw bending-torsion coupling, critical speed, and the influence of position-dependent boundary and preload variation on dynamic response [6, 40]. Nonlinear friction and contact stiffness at rolling interfaces can also be superimposed to increase model realism under different operating conditions [37]. The main limitation is that parameter definition and constraint modeling are complex. When nonlinear contact is included, computational cost increases significantly [36]. Therefore, this model is more suitable as a high-fidelity submodel in offline virtual prototyping and digital twin applications, where it supports coordinated evaluation of structural and control parameters [26].

### 3.4. Multi-domain integrated modeling

Multi-domain integrated modeling couples servo motor electromagnetic behavior, power electronics, control loops, transmission chain dynamics, frictional heating, and thermal deformation within a unified framework [26]. In this way, energy and information flow can be tracked from power input to worktable motion output at the system level [29]. This approach is used for performance evaluation and cross-disciplinary optimization. Common implementations include object-oriented physical modeling based on Modelica, and co-simulation platforms that combine Matlab Simulink with AMESim or ADAMS. These approaches allow inverter dead zones, sampling delays, saturation, and other electrical non-idealities to be quantified together with structural vibration and thermal drift in one integrated model [26].

This type of model provides an extensible backbone for digital twins. It can also be combined with model order reduction and data-driven residual modules to reduce computational burden and calibration cost [38]. However, it contains a large number of parameters and strongly depends on experimental calibration. Therefore, hierarchical modeling and sensitivity analysis are needed to extract key parameters and improve maintainability [26].

### 3.5. Applications and comparison of physical modeling methods

As shown in **Table 1**, physical modeling methods form a hierarchical system from low-order to high-fidelity models and from single-domain to cross-domain representations. Lumped parameter models retain the dominant dynamic channels required for closed-loop control using a small number of identifiable parameters [33,35]. They are suitable for parameterizing key mechanisms such as friction and position-dependent stiffness, and they are convenient for online analysis and compensation [14, 15]. Finite element models explicitly represent boundary conditions, contact interfaces, and thermally induced deformation in a spatially distributed manner.

Therefore, they provide high-fidelity support for modal analysis and thermal drift evaluation during structural design [39]. Their reduced-order forms can further serve as flexible substructures embedded in system-level models [36].

**Table 1.** Applications and comparison of physical modeling methods for BSFSs.

Physical modeling method	Modeling level	Mechanism representation	Main frequency range & outputs	Interface for hybrid modeling	Typical applications
Lumped parameter model [35,39]	Component to subsystem: low dimension	<ul style="list-style-type: none"> <li>Parameterized friction</li> <li>Equivalent stiffness &amp; damping</li> <li>Time-varying terms [14,15]</li> </ul>	<ul style="list-style-type: none"> <li>Low-order modes &amp; resonance</li> <li>Reversal dynamics</li> <li>Closed-loop indicators</li> </ul>	<ul style="list-style-type: none"> <li>Online parameter update</li> <li>Residual compensation embedding</li> </ul>	<ul style="list-style-type: none"> <li>Controller tuning</li> <li>Vibration avoidance</li> <li>Feedforward &amp; error compensation</li> </ul>
Finite element model [39]	Structural level: distributed high dimension	<ul style="list-style-type: none"> <li>Spatially distributed boundaries</li> <li>Contact</li> <li>Heat sources &amp; thermal deformation</li> </ul>	<ul style="list-style-type: none"> <li>Mode shapes frequency response</li> <li>Thermal drift</li> <li>Deformation field</li> </ul>	<ul style="list-style-type: none"> <li>Reduced flexible substructure</li> <li>Physics-based prior constraints [36]</li> </ul>	<ul style="list-style-type: none"> <li>Modal evaluation</li> <li>Thermal drift analysis</li> <li>Structural design</li> </ul>
Rigid-flexible multibody model [6,36,40]	Assembly level: constraints + flexible modes	<ul style="list-style-type: none"> <li>Constraint coupling</li> <li>Assembly relations</li> <li>Nonlinear contact superposition</li> </ul>	<ul style="list-style-type: none"> <li>High-speed response</li> <li>Bending-torsion coupling</li> <li>Position dependence</li> </ul>	<ul style="list-style-type: none"> <li>Virtual prototype backbone</li> <li>Residual learning for contact friction</li> </ul>	<ul style="list-style-type: none"> <li>High-speed analysis</li> <li>Control strategy evaluation</li> <li>Virtual prototyping</li> </ul>
Multi-domain integrated model [26]	System level: cross-domain closed loop	<ul style="list-style-type: none"> <li>Energy &amp; information coupling</li> <li>Across electrical-mechanical-thermal domains</li> <li>Explicit non-idealities</li> </ul>	<ul style="list-style-type: none"> <li>Closed-loop error</li> <li>Vibration-thermal interaction</li> <li>System indicators</li> </ul>	<ul style="list-style-type: none"> <li>Digital twin backbone</li> <li>Physical backbone + residual correction interface</li> </ul>	<ul style="list-style-type: none"> <li>Coordinated optimization</li> <li>Digital twin deployment</li> <li>Drift evaluation</li> </ul>

Rigid-flexible coupled multibody models connect rigid-body motion and flexible-body dynamics through assembly constraints [36]. Under high-speed conditions, they allow unified analysis of bending-torsion coupling and contact nonlinearity. They also support control strategy evaluation based on virtual prototyping [6, 40]. Multi-domain integrated models show that electrical drive non-idealities, control loops, mechanical transmission, and thermal effects can demonstrate coherent closed-loop behavior within one unified framework [26]. However, the significant results appear to indicate that these models support system-level performance evaluation and coordinated optimization. Furthermore, the key evidence shows that such models could provide a standard interface for hybrid modeling, where a physical backbone is combined with data-driven residual correction. In light of these findings, the important structure indicates that key mechanism channels are retained while unmodeled dynamics or parameter drift are compensated by data modules [38]. Models show an integrated framework supports residual correction.

### 3.6. Structural limitations of physics-based models

Physics-based models show that interpretability and causal consistency offer significant advantages, though the evidence indicates that parameter identifiability and sensitivity to boundary conditions constrain practical performance [33,35]. Moreover, lumped parameter models appear to demonstrate that structural simplification introduces important limitations in the results. Furthermore, the findings indicate that high-frequency modal interactions and contact nonlinearities could remain obscured, particularly under high-speed operating conditions [34, 39]. Given that simplified representations appear to show distributed flexibility, and complex modal coupling effects may not be captured, the evidence suggests that these models face key structural constraints. Models show fidelity limits under complexity. However, finite element models and rigid-flexible coupled models show that higher fidelity results are achievable, though the findings indicate that boundary conditions, preload assumptions, and contact stiffness estimation introduce significant sensitivity [36,39].

Thus, the evidence demonstrates that insufficient experimental calibration could produce important modeling uncertainty across the results. In light of these findings, the data show that nonlinear contact inclusion could substantially increase simulation cost, which appears to limit real-time implementation [36]. Additionally, significant evidence indicates that parameter drift caused by degradation might demonstrate treatment as a slowly varying disturbance rather than as structural evolution in the results. Drift simplification restricts long-term prediction reliability [25,29].

Therefore, although physics-based models remain essential for mechanism interpretation and structural analysis, their standalone use is limited by structural simplification, boundary sensitivity, and insufficient adaptability to parameter evolution.

#### 4. Data-driven and hybrid modeling methods

Compared with physics-based models that rely on mechanism assumptions and parameter calibration, data-driven identification can directly extract input-output mappings from operational data under strong nonlinearity and significant time-varying conditions. Therefore, it provides a useful supplement for rapid modeling and online updating of BSFSs [35,41]. However, engineering practice shows that a single data model often cannot satisfy interpretability, stability, and cross-condition reliability at the same time [38]. On the other hand, a single physics-based model is constrained by unmodeled dynamics, parameter drift, and boundary sensitivity [26], as well as by parameter drift during long-term operation [21,28]. Therefore, a more practical route is to retain key causal channels and implementable structures through physics-based models, and then introduce data modules for structural completion, residual absorption, or key parameter updating [8]. In this way, the model can maintain usable accuracy and stability under high-risk states such as low-speed pre-sliding, reversal transients, thermal drift, and degradation-stage transitions [21,28,30].

##### 4.1. LPV-NARX and sparse identification

The Linear Parameter-Varying Nonlinear AutoRegressive model with eXogenous inputs (LPV-NARX) method is based on structural preservation and parameter variation driven by scheduling variables. It aims to balance low-dimensional implementability and operating-condition adaptability over a wide working range [41, 42]. For BSFSs, scheduling variables such as nut position, velocity, load, and temperature can correspond to key mechanisms, including position-dependent stiffness migration, friction variation, contact state redistribution, and thermal drift gain [16,18,23,25,26]. In this way, the model can support gain scheduling and predictive control through interpretable parameters. It can also be tuned and analyzed together with closed-loop models under a unified state-space representation [35, 41, 42]. When parameters evolve continuously with operating conditions and overfitting must be suppressed, regularization and smoothing constraints can be introduced. This enables identifiable parameter-varying NARX representations. As a result, coverage capability is improved while structural compactness is maintained [42].

Sparse identification, including Sparse Identification of Nonlinear Dynamics

(SINDy) and related sparse regression methods, aims to select structural terms directly from data [43]. By constructing an over-complete candidate function library and imposing sparsity constraints, a small number of dominant terms can be selected from many candidates. This approach shows that compact dynamic equations yield clear mechanistic interpretation [43, 44]. Furthermore, the evidence indicates that position-dependent stiffness terms, velocity-dependent friction terms, and temperature-dependent damping terms might be explicitly included in the candidate library within BSFSs [16, 18, 23, 25]. Moreover, the identified model shows that nonlinear term completion and equation correction appear achievable through this framework. Given that the findings demonstrate that low-order implementable models could serve embedded control systems, the results indicate that digital twin cores benefit significantly from such formulations [45, 46]. Sparse identification shows screw drives link position-dependent dynamics to nonlinear friction. Additionally, the significant findings show that dictionary design and tailored identification strategies may support more interpretable low-order model expressions in representative studies [45].

It should be noted that both approaches have clear structural failure modes. LPV-NARX is highly sensitive to scheduling variable selection and data coverage. When operating dimensions expand, such as position-velocity-temperature-load coupling, parameterization complexity increases and extrapolation performance may degrade [41, 42]. Sparse identification is sensitive to excitation richness and noise level. Improper candidate libraries may introduce spurious structures or omit key nonlinear terms [46]. Therefore, these methods are more suitable for scenarios with continuously adjustable operating conditions, limited drift amplitude, and the need for control embedding [35, 41, 42]. They are not suitable as standalone solutions for full life-cycle prediction under strong domain shift or abrupt degradation transitions [30–32, 41–45].

## 4.2. Neural networks

Neural networks rely on high-capacity nonlinear mappings. They can learn complex input-output relationships from multi-source operational data. Therefore, they are advantageous in strong nonlinear modeling, coupled error prediction, and automatic feature extraction [47, 48]. In single-axis and multi-axis scenarios, command position, velocity, acceleration, and historical errors are commonly used as inputs. NARX-type networks or recurrent networks are adopted to describe the temporal evolution of errors. These models are then used for contour error prediction, feedforward correction, and trajectory optimization [9, 47, 48]. For state estimation and degradation recognition, Convolutional Neural Network (CNN) and Long Short-Term Memory (LSTM) combinations can utilize both frequency-domain correlations and temporal dependencies [49, 50]. Multi-sensor fusion frameworks have also been applied to degradation identification and classification of BSFS [50]. Domain adaptation and transfer learning show that domain-invariant features can improve cross-condition diagnostic capability when operating conditions change and data distribution shifts occur [51]. However, the key risk of neural networks appears to involve structural opacity, given that physical consistency cannot be automatically guaranteed [49]. Furthermore, significant evidence indicates that low-speed pre-sliding, reversal

transients, and thermal drift states could lie outside the training distribution [15, 25]. In light of these findings, extrapolation uncertainty demonstrates that risks in error compensation and state decision-making could be amplified in such cases [52]. Neural networks show structural opacity amplifies risk. Therefore, the important results show that practical implementation could require physical priors and credibility evaluation mechanisms [49, 52]. Moreover, one approach indicates that mechanism structures combined with neural networks demonstrate that residual terms, which are difficult to describe explicitly, are modeled by the network. While parameters appear constrained within feasible physical domains [49]. Additionally, the significant findings show that this approach may provide a more robust balance between accuracy and generalization [49]. Given that the evidence demonstrates that uncertainty quantification supports confidence estimation for outputs, another approach indicates that gated updating and risk control could be supported [52]. Results show uncertainty quantification aids risk control [52]. Notwithstanding these considerations, the key results show that neural networks could demonstrate greater suitability as compensation layers or estimators built upon a physical backbone [49, 52]. Thus, the important evidence appears to indicate that unconstrained global replacement models might represent a less suitable alternative [49, 52].

### 4.3. Hybrid modeling

A single physics-based model or a single data-driven model often cannot satisfy interpretability, real-time performance, and cross-condition accuracy at the same time [26, 38, 49, 52]. Therefore, hybrid modeling has become an important approach for high-fidelity modeling and online deployable updating [53–56]. As shown in **Figure 6**, three hybrid modeling paradigms are classified according to fusion location, updating object, and failure boundary. This classification provides reusable criteria for subsequent model selection.

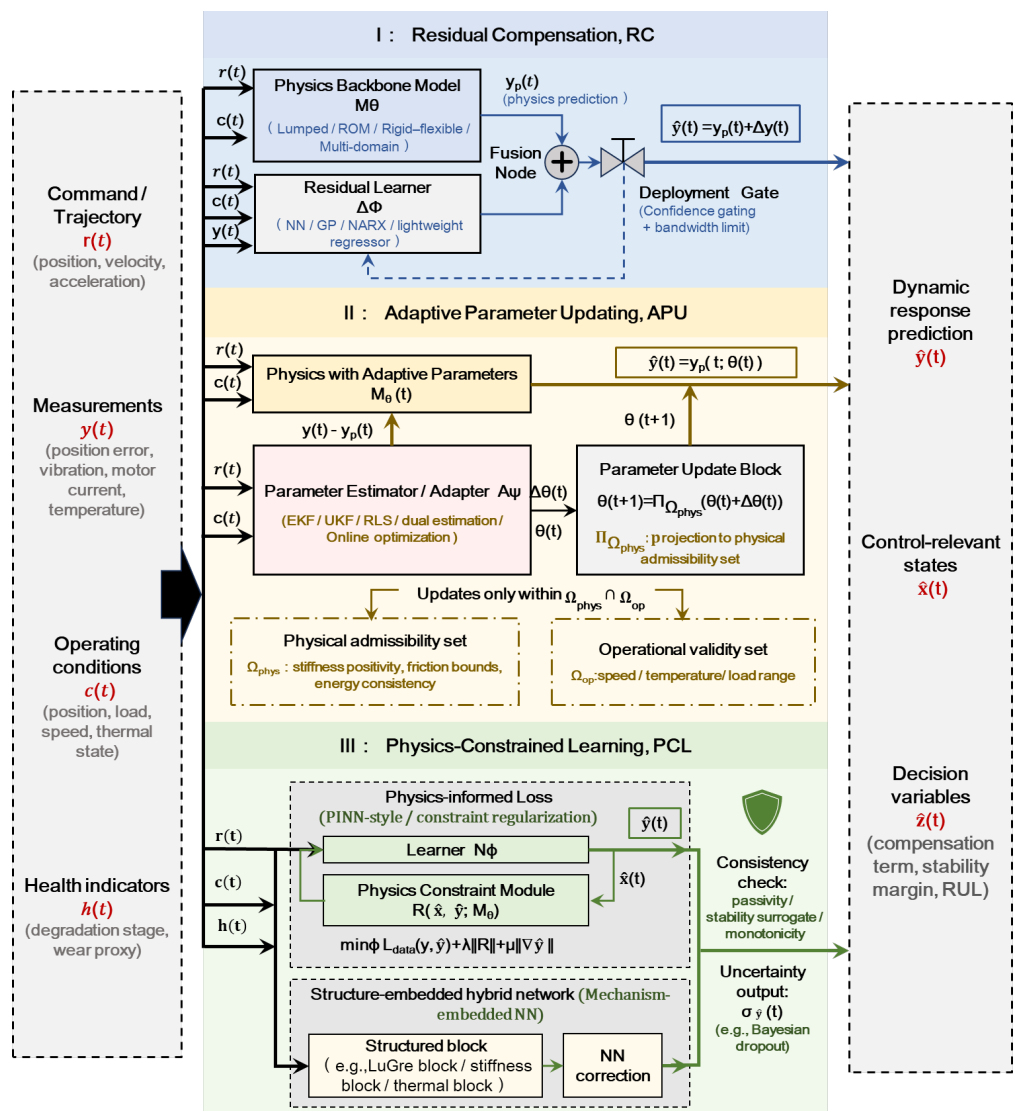
#### 4.3.1. Residual compensation

Residual compensation retains the physical foundation and introduces a data module at the output level to learn and correct prediction residuals [8, 35, 53–56]. This approach is fast to deploy. Accuracy improvement is direct, and the main physical structure is not altered. It is suitable for absorbing weak unmodeled nonlinearities, assembly variations, and environmental disturbances. It can also be applied in online compensation and feedforward correction scenarios. If update bandwidth and action channels are not constrained, the residual channel may amplify noise or disturb physical damping, which can lead to long-term drift. Therefore, confidence gating, update rate limitation, and stability envelope constraints are required [28, 52, 55].

#### 4.3.2. Adaptive parameter updating

Adaptive parameter updating shows that using data to update key mechanism parameters in the physics-based model online [21, 28, 30, 55, 56] could demonstrate significant practical value. However, the findings indicate that parameter drift becomes explicit while interpretability is preserved [21, 28, 30]. Moreover, the evidence shows that this approach appears suitable for friction evolution, preload-related stiffness variation,

thermal drift gain change, and slow degradation processes [18, 21, 23, 25, 28–30]. In light of these results, the critical challenge indicates that identifiability and convergence remain important considerations. Updating shows that parameters must stay feasible. Furthermore, the results show that updating must remain consistent with observable quantities, and the evidence indicates that parameters might otherwise drift into nonphysical regions and accumulate errors [28,55]. Given that significant findings from studies on mechanism-data hybrid digital twin frameworks for BSFS demonstrate that constrained real-time adaptive parameter updating may improve simulation accuracy, the results appear to support important generalization gains. Nevertheless, the evidence shows that its upper performance bound appears still limited by calibration quality and observability [56]. Performance shows calibration limits remain.



**Figure 6.** Three hybrid modeling paradigms for BSFS: Residual compensation, adaptive parameter updating, and physics-constrained learning.

### 4.3.3. Physics-constrained learning

Physics-constrained learning shows that embedding physical consistency constraints into the model structure or loss function could demonstrate significant extrapolation capability. However, the findings appear to indicate that this approach

reduces distribution shift risk in relevant scenarios. Moreover, the evidence shows that it appears suitable for scenarios with high requirements on safety boundaries and extrapolation reliability [55]. Notwithstanding these results, the approach could demonstrate sensitivity to constraint specification, where improper constraints may introduce systematic bias. Constraints increase deployment cost. Therefore, the findings show that this approach appears more appropriate when high-value extrapolation or strong constraint requirements are present [55]. Given that broader reviews of hybrid modeling in manufacturing systems demonstrate that validation could indicate significant structural consistency considerations, the evidence shows that interpretability and uncertainty assessment appear critical. Thus, the results indicate that relying solely on fitting accuracy may demonstrate important limitations [57].

Prediction accuracy could be examined through three complementary indicators that may suggest distinct aspects of the significant performance envelope. The root mean square error  $RMSE = \sqrt{(1/N \cdot \sum(\hat{y}_i - y_i)^2)}$  might indicate the primary single-step prediction accuracy metric, where the key acceptable thresholds for BSFSs are  $RMSE \leq 2 \mu\text{m}$  for positioning error and  $RMSE \leq 5 \mu\text{m}$  for contour error under normal operating conditions. The maximum trajectory error  $E_{max} = \max|e(t)|$  shows that worst-case contour tracking performance appears critical, with engineering requirements typically specifying  $E_{max} \leq 10 \mu\text{m}$  for precision machining applications. Given that the long-term drift rate  $D_{drift} = \Delta E / \Delta t$  (unit:  $\mu\text{m/h}$ ) could demonstrate important prediction stability over the full life cycle, the evidence indicates that  $D_{drift} \leq 0.5 \mu\text{m/h}$  appears acceptable for continuous operation beyond 1,000 h. Findings show that three hybrid paradigms affect performance outcomes differently. Moreover, the residual compensation approach indicates that RMSE reductions of 20%–40% over the physics-only baseline may appear achievable under stable operating conditions. Furthermore, significant evidence shows that adaptive parameter updating demonstrates that  $D_{drift}$  falls within 0.2–0.8  $\mu\text{m/h}$  under slow degradation scenarios. Nevertheless, the findings indicate that physics-constrained learning could demonstrate the most stable  $E_{max}$  performance under domain shift, notwithstanding the results that typically show error growth limited to within 15% of the training-domain baseline. Results show physics-constrained learning limits error growth best.

Engineering practicability is evaluated through three indicators. The real-time computation load ratio  $R_{comp} = T_{compute} / T_{sample}$  shows that the model satisfies the servo control cycle requirement, where  $T_{sample}$  is typically 0.5–2 ms for CNC feed drives and  $R_{comp} \leq 0.3$  is recommended to leave sufficient margin for other control tasks. Furthermore, the significant findings indicate that residual compensation models typically achieve  $R_{comp} = 0.05$ –0.15, making them the most suitable for real-time implementation. However, the results demonstrate that adaptive parameter updating models require  $R_{comp} = 0.10$ –0.25 depending on the number of updated parameters, while physics-constrained learning models involve higher offline training cost but comparable online inference load. In light of these findings, the evidence shows that the parameter identifiability index  $I_{ident} = \text{rank}(\text{Fisher information matrix}) / n_{param}$  could demonstrate the effectiveness of parameter updating. Findings show  $I_{ident} \geq 0.8$  indicates well-conditioned identification, and  $I_{ident} < 0.5$  signals convergence failure.

Moreover, the key results indicate that the deployment complexity index  $C_{deploy}$  appears to function as a composite score incorporating parameter count, calibration experiment requirements, and online memory footprint. Additionally, significant evidence shows that lumped-parameter-based hybrid models could demonstrate lower deployment complexity than neural-network-based counterparts. Given that the findings demonstrate this pattern, the results indicate that  $C_{deploy}$  could appear more favorable for simpler model architectures. Therefore, the important evidence shows that these deployment considerations appear critical for practical implementation. Studies show that complexity links directly to model type.

#### 4.4. Method comparison

**Table 2** shows that a systematic comparison framework for different method families could demonstrate significant analytical value. Moreover, the evidence appears to indicate that integrating object-level, mechanism representation, primary outputs, physical model interfaces, and risk types into a unified benchmark might establish a key foundation for evaluation. However, the findings show that the selection problem appears transformed from a simple performance comparison into a critical trade-off. In light of these results, the data indicate that structural explicitness, updating depth, and risk tolerance represent important dimensions of that trade-off. Study shows framework links explicitness, depth, and risk tolerance.

**Table 2.** Applications and comparison of data-driven and hybrid modeling methods for BSFS.

Method family	Object & level	Mechanism representation & constraints	Main outputs	Interface with physical models	Typical applications	Risks
<b>LPV-NARX</b> (including scheduling NARX) [41–46]	Subsystem: low dimension	<ul style="list-style-type: none"> <li>Parameter variation driven by scheduling variables</li> <li>Regularization constraints</li> </ul>	<ul style="list-style-type: none"> <li>Fast prediction</li> <li>Equivalent parameters</li> </ul>	<ul style="list-style-type: none"> <li>Parallel channel</li> <li>Online parameter updating</li> </ul>	<ul style="list-style-type: none"> <li>Gain scheduling</li> <li>Predictive control</li> <li>Error prediction</li> </ul>	<ul style="list-style-type: none"> <li>Coverage sensitivity</li> <li>Degraded extrapolation</li> </ul>
<b>Sparse identification</b> (SINDy/sparse regression) [43–46]	Mechanism equation: low dimension	<ul style="list-style-type: none"> <li>Explicit physical terms in candidate library</li> <li>Selection</li> </ul>	<ul style="list-style-type: none"> <li>Structural equations</li> <li>Dominant terms</li> </ul>	<ul style="list-style-type: none"> <li>Equation completion</li> <li>Nonlinear term embedding</li> </ul>	<ul style="list-style-type: none"> <li>Identifiable low-order models</li> <li>Mechanism interpretation</li> </ul>	<ul style="list-style-type: none"> <li>Noise sensitivity</li> <li>Library dependence</li> </ul>
<b>Neural networks</b> (NARX-NN/LSTM/CNN, etc.) [49,51,52]	Data mapping: medium to high dimension	<ul style="list-style-type: none"> <li>End-to-end fitting</li> <li>Transfer learning &amp; uncertainty handling</li> </ul>	<ul style="list-style-type: none"> <li>State estimation</li> <li>Error prediction</li> </ul>	<ul style="list-style-type: none"> <li>Residual learning</li> <li>Observer embedding</li> </ul>	<ul style="list-style-type: none"> <li>Compensation</li> <li>Prediction</li> <li>Health evaluation</li> </ul>	<ul style="list-style-type: none"> <li>Limited interpretability</li> <li>Domain shift</li> <li>Credibility evaluation</li> </ul>
<b>Hybrid modeling</b> (Physics + Data) [53–56]	System level: scalable	<ul style="list-style-type: none"> <li>Physical backbone consistency</li> <li>Residual learning or adaptive parameter updating</li> </ul>	<ul style="list-style-type: none"> <li>Reliable prediction</li> <li>Closed-loop indicators</li> </ul>	<ul style="list-style-type: none"> <li>Backbone + residual</li> <li>Online calibration interface</li> </ul>	<ul style="list-style-type: none"> <li>Digital twin engine</li> <li>Cross-condition calibration</li> </ul>	<ul style="list-style-type: none"> <li>Upper bound limited by calibration</li> <li>Fusion stability constraints</li> </ul>

LPV–NARX and sparse identification show that control embedding and online scheduling benefit from explicit structures, interpretable parameters, and feasible stability analysis. However, the findings indicate that sensitivity to coverage and limited extrapolation capability remain significant constraints [41–46]. Furthermore, the evidence shows that neural networks could demonstrate effectiveness for complex nonlinear mapping and multi-source fusion [47–51]. In light of these results, the data indicate that reliable deployment under domain shift requires uncertainty quantification and transfer mechanisms [49–52]. Neural networks show data coverage limits deployment. Hybrid modeling could provide a more robust balance, given that the physical backbone shows that causal consistency and structural stability are key outcomes. Moreover, the evidence appears to demonstrate that the data module could absorb residual dynamics or update important parameters [53–56]. Therefore, the findings indicate that this structure provides a standardized interface for closed-loop

updating. Additionally, significant results show that hybrid structures combining physical models with data-driven discrepancy terms may demonstrate improved online optimization and compensation performance [57, 58]. Hybrid structures show performance gains. Nevertheless, the key evidence indicates that Gaussian process residuals appear to support integration within such hybrid frameworks. In light of the findings, the results show that machine tool dynamic accuracy studies demonstrate that this approach could provide relevant structural insights. However, the significant data appears to indicate that further validation may support broader deployment of these hybrid methods. Thus, the evidence demonstrates that combining physical and data-driven components remains a critical direction for robust modeling. Research shows hybrid models improve accuracy.

#### **4.5. Digital twin**

Digital twin applications show that models must maintain usable accuracy under real-time computational constraints over long-term operation. Moreover, the hybrid modeling approach indicates that it serves not only to improve fitting accuracy but also as the dynamic engine layer of the twin system [53–56]. In light of these results, multi-domain integrated models show that a traceable causal backbone remains important for the evidence [26]. Furthermore, residual compensation or online parameter updating demonstrates that synchronization between the virtual model and the physical entity appears necessary [26, 53–56]. Hybrid modeling shows twins need accuracy and update credibility. However, the physical backbone shows that structural stability and interpretability appear critical, while the data module indicates that it absorbs disturbances and drift in the findings. Given that the results demonstrate updating must remain constrained, the valid operating domain and confidence gating boundaries appear to support the significant evidence. Notwithstanding these findings, the updating mechanism itself indicates that instability could emerge if the key constraints are not maintained [28, 52, 55]. Therefore, the significant findings show that multi-source data fusion, traceable error modeling, and real-time adaptive control appear to be important directions for machining error management [59]. Additionally, hybrid modeling in digital twins demonstrates that the focus should extend not only to accuracy but also to update credibility and structural consistency in the results. Twin studies show hybrid models need credibility and consistency.

The study shows that two representative engineering cases could demonstrate the implementation details of virtual–physical synchronization in digital twin applications for BSFSs.

Case 1: Thermal error compensation digital twin for a high-speed machining center feed axis. However, the significant findings indicate that the implementation follows a five-step closed-loop workflow. Furthermore, the key evidence shows that Step 1 involves physics backbone construction, where a lumped-parameter thermal network model is established with the screw shaft, nut assembly, support bearings, and guideways as thermal nodes, yielding a system of thermal equilibrium equations that describe heat generation, conduction, and dissipation under varying spindle speeds and feed rates. In light of the significant results, the mechanical submodel demonstrates

that thermal elongation couples to axial positioning error through the relationship  $\delta_{th} = \alpha \cdot L \cdot \Delta T$ , where  $\alpha = 11.5 \times 10^{-6}/^{\circ}\text{C}$ ,  $L$  is the effective screw length, and  $\Delta T$  is the nodal temperature rise. Step 2 covers sensor deployment and data acquisition. Moreover, the evidence shows that four to six PT100 temperature sensors are mounted at key thermal nodes, including the fixed bearing seat, free bearing seat, nut housing, and mid-span screw surface, together with one linear encoder providing real-time positioning feedback at a sampling interval of 100 ms. Given that the significant data indicates that Step 3 addresses adaptive parameter updating, an online least-squares estimator with a forgetting factor  $\lambda = 0.98$  updates thermal resistance and thermal expansion parameters every 30–60 s, constrained within physically admissible bounds to prevent non-physical drift. Step 3 shows estimator updates parameters within bounds. Therefore, the significant findings show that confidence gating is applied such that parameter updates are accepted only when the residual between predicted and measured temperature falls below 0.5 °C. Notwithstanding the key evidence, the results indicate that Step 4 addresses compensation command generation. where the updated thermal model computes the predicted elongation  $\delta_{th}$  in real time and feeds the compensation offset to the CNC position loop as a feedforward correction. Additionally, the significant data demonstrate that Step 5 covers virtual–physical synchronization validation, where the digital twin state is compared against encoder measurements every 10 min, and synchronization is considered maintained when the prediction residual remains below 2  $\mu\text{m}$ . Thus, the important evidence shows that synchronization appears achievable under standard operating conditions. Findings show residual stays below 2  $\mu\text{m}$ . However, the results indicate that experimental evidence obtained on a representative machining center with a 1,200 mm feed stroke demonstrates that under continuous machining conditions of 4 h with a temperature rise of 15–25 °C, the uncompensated thermal positioning error of 35–50  $\mu\text{m}$  is reduced to 3–5  $\mu\text{m}$  after compensation, achieving an error suppression rate exceeding 90%. Given that the significant findings show that the root mean square synchronization error between the digital twin prediction and the physical encoder measurement is 1.6  $\mu\text{m}$  over the full test duration, the results demonstrate that the long-term stability of the virtual–physical synchronization mechanism appears well-established. Furthermore, the key data indicates that these findings could support the broader applicability of the proposed synchronization approach across similar high-speed machining contexts. Evidence confirms long-term stability of synchronization.

The implementation workflow shows that five distinct stages could demonstrate a coherent architecture for full life-cycle degradation monitoring. Moreover, the significant hybrid physics backbone indicates that a lumped-parameter dynamic model integrating friction evolution and contact stiffness degradation submodels appears to establish the digital twin foundation. Furthermore, the degradation submodel shows that contact stiffness follows  $K_{contact}(t) = K_0 \cdot (1 - \beta \cdot W(t))$ , where  $K_0$  is the initial contact stiffness,  $\beta$  is the wear sensitivity coefficient, and  $W(t)$  is the cumulative wear index estimated from operating history. In light of these significant findings, the multi-source state observation stage demonstrates that three sensing channels—motor current signals sampled at 10 kHz, triaxial accelerometers

mounted on the nut housing for vibration feature extraction in the 10–500 Hz band, and linear encoder signals—appear to provide the evidence needed for real-time friction, stiffness, and backlash estimation. Stage 3 shows rolling-window updates degradation parameters every 500 h. However, the rolling-window identification strategy shows that a constrained recursive least-squares algorithm could establish that  $K_{contact} \in [0.4K_0, K_0]$  and  $F_c \in [0.8F_{c0}, 2.5F_{c0}]$  appear to remain the physically feasible domain throughout the service life. Additionally, the significant remaining useful life prediction stage indicates that an exponential degradation model shows that the predicted RUL trajectory appears to yield uncertainty quantified through a 95% confidence interval derived from the parameter covariance matrix. Given that the key maintenance decision support results demonstrate that the predicted RUL threshold of typically 400–600 h triggers alerts transmitted to the shop-floor management system, the evidence indicates that proactive intervention becomes feasible. Notwithstanding this threshold condition, the important validation results show that the 5,000 h accelerated life test on a precision grinding feed axis appears to demonstrate that the positioning error prediction achieves an RMSE of 1.8  $\mu\text{m}$ . Validation shows RUL prediction error stays below 8% actual remaining life. Moreover, the significant maintenance decision lead time indicates that the results demonstrate that 200–400 h ahead of the actual failure threshold appears to enable proactive intervention without unnecessary downtime. Furthermore, the key virtual–physical synchronization residual shows that the evidence appears to indicate that values remain below 3  $\mu\text{m}$  throughout the full test duration. Thus, the findings demonstrate that the significant hybrid updating mechanism could establish that robustness appears to hold under long-term degradation conditions.

The two cases above share a unified synchronization architecture in which the core closed-loop process follows the sequence: sensor data acquisition  $\rightarrow$  state observation  $\rightarrow$  parameter updating  $\rightarrow$  error compensation  $\rightarrow$  synchronization validation. However, the confidence gate shows that each parameter update step satisfies both the physical feasibility constraint (parameters remain within pre-defined admissible bounds) and the statistical credibility constraint (update is accepted only when the innovation signal exceeds the noise floor but remains below the anomaly detection threshold). Furthermore, the significant update bandwidth indicates that imposing a maximum parameter change rate of 5% per update cycle appears to prevent instability induced by abrupt parameter jumps. In light of these findings, the unified synchronization framework demonstrates that the digital twin remains structurally consistent, physically interpretable, and auditable throughout the full operational life cycle of the BSFS. Framework supports integrated model–control–maintenance decision loops in intelligent manufacturing environments.

#### 4.6. Structural bottlenecks

The main limitation of existing studies shows that insufficient single-step prediction accuracy is not the key concern. However, the findings indicate that the lack of life-cycle-oriented updating mechanisms represents a more significant issue. Moreover, current models show that the coupled chain of

“friction-thermal-stiffness-degradation” cannot ensure updatable, constrained, and verifiable operation during long-term use [28, 31, 32]. In light of these results, residual compensation without bandwidth and channel constraints indicates that noise amplification could degrade stability. Models show parameter drift occurs without constraints. Furthermore, the evidence shows that parameter adaptation lacking identifiability support could allow parameters to drift into non-physical regions. Given that physics-constrained learning demonstrates sensitivity to constraint mis-specification, the results indicate that apparent generalization could appear at the cost of systematic bias [28, 55]. Evidence shows generalization masks bias. Therefore, the findings show that future development should focus on verifiable updating mechanisms as a critical priority. Additionally, significant results indicate that hierarchical updating architectures and bandwidth limitations represent important components of this approach. Notwithstanding these findings, the evidence shows that explicit definition of physically feasible domains and valid operating regions could demonstrate essential supporting roles [52, 55]. Thus, the key results indicate that uncertainty-based confidence gating strategies could provide important additional support. Research shows hybrid modeling shifts toward auditable deployment.

## 5. Selection strategy and typical scenarios

The previous mechanism analysis and method comparison show that dynamic modeling of BSFS could demonstrate the essential characteristics of a constrained hierarchical decision problem. Moreover, the findings appear to indicate that this process requires balancing structural explicitness, parameter updatability, and engineering complexity. Furthermore, the three hybrid modeling paradigms shown in **Figure 6** could reveal that the significant differences in fusion positions between data and physical models appear to influence the overall framework. In light of these results, **Table 2** shows that the evidence demonstrates important distinctions in object level, interface form, and risk characteristics. Model selection shows a hierarchical decision process applies. However, the significant findings indicate that this process should be coordinated under physical consistency, parameter identifiability, and stability constraints.

### 5.1. Selection strategy

Model selection depends on modeling objectives and time scale. However, the findings show that structurally explicit physical models or parameter-scheduled models could demonstrate clear advantages for controller tuning, low-order modal analysis, and real-time implementation [3–5,41]. Moreover, significant evidence indicates that these models directly reflect the influence of position-dependent stiffness and friction nonlinearity on stability. Furthermore, the results show that these models maintain consistency with control-oriented state-space representations.

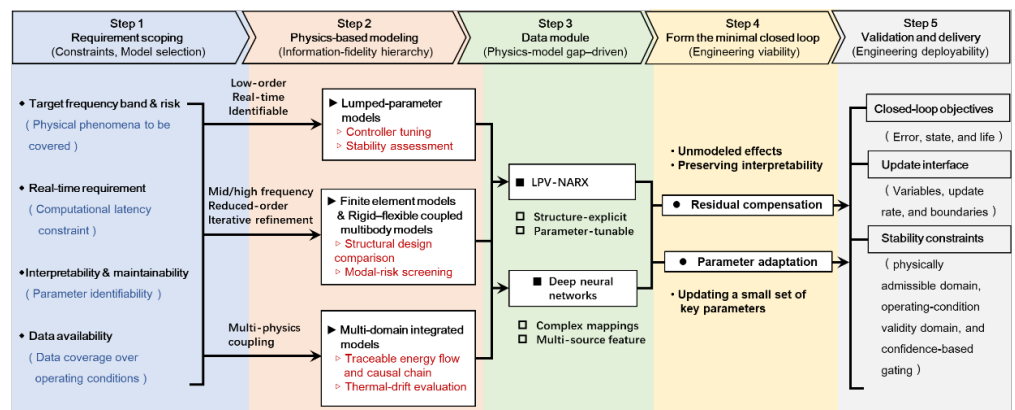
Fixed-parameter physical models show accuracy limits under time-varying conditions. When the system may suggest significant time-varying behavior or long-term parameter drift, the evidence indicates that fixed-parameter physical models alone appear insufficient to maintain accuracy [21, 28, 30]. Therefore, the

key findings show that online parameter updating or position scheduling mechanisms could demonstrate the capacity to capture the slow evolution of friction, stiffness, and thermal drift. Given that adaptive parameter structures could indicate better life-cycle consistency, the results show that identifiability and physically feasible domain constraints appear critical to ensure.

Data-driven models provide strong expressive capability for complex nonlinear prediction. However, the significant findings show that purely data-driven models could indicate unreliable extrapolation performance under strong domain shift or in low-speed sensitive regions [47–51]. Thus, the evidence indicates that these models could demonstrate greater reliability when combined with physical backbones through residual compensation or physics-constrained learning structures [49–52]. In light of these results, the data shows that this combination could indicate prevention of structural instability or parameter drift.

Physical-data integration shows advantages under strong multi-physical coupling. When multi-physical coupling appears strong and accuracy requirements may indicate elevated demand, the findings show that integrating physical models with data modules appears more advantageous [53–56]. Moreover, significant evidence shows that the physical backbone could demonstrate the capacity to ensure causal consistency and structural stability. Notwithstanding these results, the key data indicate that the data module could absorb unmodeled dynamics or update important parameters, suggesting that a balance between accuracy and interpretability appears achievable.

The model selection problem shows that a constrained hierarchical decision process could provide the key organizational framework. However, the significant findings indicate that the selection is divided into three levels, as shown in **Figure 7**. Moreover, the evidence demonstrates that the first level requires determination of the objective level and time scale. In light of these results, the second level indicates that physical consistency and parameter identifiability constraints appear critical to the evaluation. Research shows selection involves residual compensation, adaptive updating, or physics-constrained learning. Thus, the important data shows that this structure demonstrates that model selection is not a comparison of algorithms, but rather a coordination of structures under physical and stability constraints.



**Figure 7.** Constraint-driven hierarchical modeling selection strategy for BSFS.

## 5.2. Typical scenarios

The findings show that in low-speed and high-precision positioning scenarios, friction nonlinearity and pre-sliding behavior could indicate the main sources of error [15, 18]. Moreover, the model demonstrates that prioritizing accurate dynamic friction representation is critical to reliable performance. However, significant residual compensation or parameter updating indicates that suppressing zero-velocity drift may require careful integration. In light of these results, the evidence shows that strong low-speed stability and model continuity appear essential to such scenarios. Pure data-driven structures show physical constraints lacking. In high-speed and long-stroke machining scenarios, the significant findings show that position-dependent stiffness and thermal drift may indicate modal migration and dynamic variation [3, 7, 25]. Furthermore, the evidence demonstrates that position-scheduled stiffness representation and thermo-structural coupling modeling could appear particularly important in this case. Notwithstanding these results, the key data shows that parameter scheduling or online calibration appears necessary to maintain consistent dynamic response.

Vibration suppression shows that it represents a critical engineering objective in this context. Moreover, significant evidence indicates that position-dependent resonance migration requires adaptive notch filtering or gain scheduling strategies that track the evolving dynamic characteristics in real time. Furthermore, the findings indicate that thermal drift could demonstrate a destabilizing effect on the vibration response by shifting the effective stiffness boundaries. In light of these results, the modeling framework shows that it must not only predict steady-state dynamic behavior but also characterize transient vibration during acceleration, deceleration, and reversal phases. The framework addresses phases where excitation peaks and contour errors affect outcomes.

In long-term operation and life-cycle management scenarios, wear and preload variation show that slow parameter drift presents a significant challenge [29, 30]. Furthermore, the model could demonstrate updating capability to maintain cross-stage consistency across operational phases. Given that hybrid structures combine condition monitoring with parameter identification, the evidence indicates that prediction robustness appears enhanced while preserving physical interpretability [31, 32, 53–56]. Additionally, the significant findings show that coordinated structural design supports that these approaches yield more reliable outcomes. Models show drift needs updating.

In digital twin applications, the results show that real-time computation, virtual-physical synchronization, and long-term stability requirements appear simultaneously critical [53–56]. Moreover, the physical model demonstrates that the structural framework and energy transmission pathways provide that foundational basis for reliable simulation. In light of these findings, the data module indicates that environmental disturbances and structural deviations appear addressable through targeted compensation. Notwithstanding these considerations, the evidence shows that hierarchical updating mechanisms could maintain that consistency between simulation and physical entities. Synchronization links simulation to reality.

In summary, the significant findings show that different scenarios require different structural emphases across application domains. However, the key evidence indicates

that the common principle appears consistent: physical mechanisms demonstrate that they serve as that structural foundation across all cases. Therefore, the results show that data compensation or parameter updating mechanisms appear necessary according to time-varying intensity and application objectives. Given that stability and feasible domain constraints govern outcomes, the evidence indicates that the final structure might operate under that coordinated framework throughout deployment. Selection reflects hierarchy, not substitution.

## 6. Conclusion and Outlook

Dynamic modeling of BSFS shows that evolution from a single-mechanism description to a hierarchical organization problem defined by 'mechanism representation, structural embedding, constrained updating' could indicate significant structural complexity. Moreover, the key physical sources, including friction, position-dependent stiffness, thermal deformation, and degradation evolution, appear to demonstrate that the evidence supports a unified analytical logic. Furthermore, the complementary boundaries between physics-based modeling and data-driven approaches show that the significant findings clarify their hybrid integration paradigms. In light of these results, the modeling objective indicates that structural consistency, updating stability, and engineering maintainability across operating conditions and life-cycle stages demonstrate that single-step fitting accuracy is insufficient. Modeling shifts from fitting to structural consistency across life-cycle stages.

However, under multi-condition and long-term operation, the model structure shows that parameter observability and updating bandwidth appear to demonstrate critical requirements. Additionally, significant evidence indicates that slow time-varying effects should be compressed into low-dimensional update variables that might demonstrate projection into physically feasible domains and natural embedding into condition monitoring loops [18, 29–32]. Given that the findings demonstrate that online deployment environments impose dual requirements, the results show that computational efficiency and credibility appear to establish coordinated integration of model order reduction and uncertainty quantification as necessary. Notwithstanding these constraints, the evidence indicates that extrapolation risk might demonstrate quantitative control while preserving structural physical consistency [36, 39, 52–54]. Updating bandwidth links observability to deployment efficiency.

Thus, as physics-data hybrid structures show that the stability of the updating mechanism itself could indicate a core constraint for engineering implementation, the significant findings appear to demonstrate that embedded update channel selection becomes critical. Moreover, the evidence shows that frequency limitation and confidence gating mechanisms could demonstrate prevention of data-driven channels from disturbing damping characteristics and stability boundaries [28, 52, 55, 57]. Additionally, the results indicate that the key model structures appear to establish that these mechanisms support structural physical consistency. Given that the findings demonstrate that cross-machine and cross-condition operation is becoming common in intelligent manufacturing environments, the evidence shows that modeling frameworks could indicate the need for transferability and generalization capability. Frameworks

require transferability under structural consistency constraints.

Furthermore, the significant results show that domain adaptation and multi-source fusion mechanisms could indicate improved cross-domain robustness under structural consistency constraints [50,51,58]. However, the evidence appears to demonstrate that dynamic models in intelligent manufacturing systems indicate that prediction tasks represent only a limited role. Therefore, the key findings show that deep integration into controller tuning, error compensation, and health management decision-making processes could demonstrate a unified loop of model-control-maintenance. Notwithstanding current limitations, the significant evidence indicates that the updating mechanism appears to support that short-term accuracy optimization and long-term reliability management demonstrate complementary objectives [1,2,53–56]. Integration supports the model-control-maintenance loop reliability.

The CMMD framework shows that a structured pathway for vibration feature extraction, frequency-domain dynamic response prediction, and broadband vibration suppression strategy design could serve the core research interests of the target readership. Moreover, significant evidence indicates that embedding vibration-oriented observables—such as modal frequency trajectories, vibration amplitude envelopes, and spectral energy distributions—into the physics backbone as identifiable state indicators might demonstrate considerable analytical value. Furthermore, the findings show that this approach could enable the hybrid model to simultaneously serve control tuning, vibration monitoring, and health management functions within a unified and auditable framework. In light of these results, future work indicates that strengthening the engineering relevance of physics–data hybrid modeling could demonstrate measurable benefits for precision machine tool applications. Framework supports vibration suppression, monitoring, and health management via a unified physics–data hybrid approach.

**Author contributions:** Conceptualization, YL and SBBM; methodology, YL; formal analysis, YL; investigation, YL and MSBK; resources, SBBM and SNABY; data curation, YL; writing—original draft preparation, YL; writing—review and editing, YL, SBBM, MSBK, and SNABY; visualization, YL; supervision, SBBM; project administration, SBBM; funding acquisition, YL and SBBM. All authors have read and agreed to the published version of the manuscript.

**Funding:** This work received no external funding.

**Institutional review board statement:** Not applicable.

**Informed consent statement:** Not applicable.

**Data availability statement:** No new data were created or analyzed in this study; all referenced data are available in the cited published literature.

**Acknowledgment:** The authors would like to thank the Faculty of Innovative Design and Technology, Universiti Sultan Zainal Abidin, Malaysia, and the School of Intelligent Equipment, Lanzhou University of Information Science and Technology, China, for their institutional support during the preparation of this manuscript.

**Conflict of interest:** The authors declare no conflict of interest.

**AI use statement:** During the preparation of this work, the authors used ChatGPT (OpenAI) and Grammarly for grammar checking and language polishing of certain portions of the manuscript. The authors subsequently reviewed and edited the content as necessary and take full responsibility for the final content of the published article.

## References

1. Huang T, Kang Y, Du S, et al. A survey of modeling and control in ball screw feed-drive system. *The International Journal of Advanced Manufacturing Technology*. 2022; 121(5–6): 2923–2946. doi: 10.1007/s00170-022-09506-4
2. Yang H, Wang Z, Zhang T, et al. A review on vibration analysis and control of machine tool feed drive systems. *The International Journal of Advanced Manufacturing Technology*. 2020; 107(1–2): 503–525. doi: 10.1007/s00170-020-05041-2
3. Zhang H, Zhang J, Liu H, et al. Dynamic modeling and analysis of the high-speed ball screw feed system. *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture*. 2015; 229(5): 870–877. doi: 10.1177/0954405414534641
4. Xu M, Li C, Zhang H, et al. A comprehensive nonlinear dynamic model for ball screw feed system with rolling joint characteristics. *Nonlinear Dynamics*. 2021; 106(1): 169–210. doi: 10.1007/s11071-021-06815-3
5. Duan M, Lu H, Zhang X, et al. Dynamic Modeling and Experiment Research on Twin Ball Screw Feed System Considering the Joint Stiffness. *Symmetry*. 2018; 10(12): 686. doi: 10.3390/sym10120686
6. Duan M, Lu H, Zhang X, et al. Dynamic modeling and experimental research on position-dependent behavior of twin ball screw feed system. *The International Journal of Advanced Manufacturing Technology*. 2021; 117(11–12): 3693–3703. doi: 10.1007/s00170-021-07874-x
7. Yang J, Li C, Xu M, et al. Nonlinear dynamic characteristics of ball screw feed system under thermal deformation. *Nonlinear Dynamics*. 2022; 107(3): 1965–1987. doi: 10.1007/s11071-021-07072-0
8. Huang D, Duan J, Yang J, et al. A Fusion Modeling Method for Ball Screw Feed System of Machine Tool. In: *Mechanical Design and Simulation: Exploring Innovations for the Future, Lecture Notes in Mechanical Engineering*. Springer Nature; 2025. pp. 1097–1110. doi: 10.1007/978-981-97-7887-4\_96
9. Jiang Y, Chen J, Zhou H, et al. Nonlinear time-series modeling of feed drive system based on motion states classification. *Journal of Intelligent Manufacturing*. 2020; 31(8): 1935–1948. doi: 10.1007/s10845-020-01546-5
10. Wang X, Zhang D, Zhang Z. A review of dynamics design methods for high-speed and high-precision CNC machine tool feed systems. *arXiv preprint*. 2023. doi: 10.48550/ARXIV.2307.03440
11. Chen Y, Zhang S, Zhang Y, et al. Investigation on Sensorless Estimating Method and Characteristics of Friction for Ball Screw System. *Applied Sciences*. 2020; 10(9): 3122. doi: 10.3390/app10093122
12. Gagnon L, Morandini M, Ghiringhelli GL. A review of friction damping modeling and testing. *Archive of Applied Mechanics*. 2020; 90(1): 107–126. doi: 10.1007/s00419-019-01600-6
13. Canudas De Wit C, Olsson H, Astrom KJ, et al. A new model for control of systems with friction. *IEEE Transactions on Automatic Control*. 1995; 40(3): 419–425. doi: 10.1109/9.376053
14. Chen G, Li H. Dynamic Parameters Identification for the Feeding System of Commercial Numerical Control Machine. *Open Automation and Control Systems Journal*. 2013; 5: 161–166. Available online: <https://benthamopenarchives.com/contents/pdf/TOAUTO CJ/TOAUTO CJ-5-161.pdf>
15. Vahid-Araghi O, Golnaraghi F. Lead Screws. In: *Friction-Induced Vibration in Lead Screw Drives*. Springer; 2010. pp. 7–15. doi: 10.1007/978-1-4419-1752-2\_2
16. Maré JC. Friction modelling and simulation at system level: Considerations to load and temperature effects. *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*. 2015; 229(1): 27–48. doi: 10.1177/0959651814548440
17. Dupont P, Hayward V, Armstrong B, et al. Single state elastoplastic friction models. *IEEE Transactions on Automatic Control*. 2002; 47(5): 787–792. doi: 10.1109/TAC.2002.1000274
18. Al-Bender F, Lampaert V, Swevers J. The generalized Maxwell-slip model: A novel model for friction Simulation and compensation. *IEEE Transactions on Automatic Control*. 2005; 50(11): 1883–1887. doi: 10.1109/TAC.2005.858676

19. Qiu YL, Zhou CG, Feng FT, et al. The influence of temperature rise on the friction torque of ball screw during low speed operation. *Results in Physics*. 2023; 51: 106703. doi: 10.1016/j.rinp.2023.106703
20. Jian ZW, Liu CC, Tang PY, et al. Investigation of interaction between friction and elastic deformation by using a novel feed drive model. *Mechatronics*. 2023; 95: 103052. doi: 10.1016/j.mechatronics.2023.103052
21. Xi T, Fujita T, Kehne S, et al. An extended LuGre model for estimating nonlinear frictions in feed drive systems of machine tools. *Procedia CIRP*. 2022; 107: 452–457. doi: 10.1016/j.procir.2022.05.007
22. Bui BD, Uchiyama N, Sano S. Nonlinear Friction Modeling and Compensation for Precision Control of a Mechanical Feed-Drive System. *Sensors and Materials*. 2015; 1. doi: 10.18494/SAM.2015.1169
23. Chen Y, Tang W. Dynamic contact stiffness analysis of a double-nut ball screw based on a quasi-static method. *Mechanism and Machine Theory*. 2014; 73: 76–90. doi: 10.1016/j.mechmachtheory.2013.10.008
24. Liu J, Ou Y. Dynamic axial contact stiffness analysis of position preloaded ball screw mechanism. *Advances in Mechanical Engineering*. 2019; 11(1): 1687814018819289. doi: 10.1177/1687814018819289
25. Min X, Jiang S. A thermal model of a ball screw feed drive system for a machine tool. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*. 2011; 225(1): 186–193. doi: 10.1177/09544062JMES2148
26. Mei Z, Chen L, Ding J. Multi-domain integrated modeling and verification for the ball screw feed system in machine tools. *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture*. 2020; 234(14): 1707–1719. doi: 10.1177/0954405420933704
27. Cao L, Park CH, Chung SC. Real-time thermal error prediction and compensation of ball screw feed systems via model order reduction and hybrid boundary condition update. *Precision Engineering*. 2022; 77: 227–240. doi: 10.1016/j.precisioneng.2022.05.017
28. Li Y, Yu M, Bai Y, et al. A Review of Thermal Error Modeling Methods for Machine Tools. *Applied Sciences*. 2021; 11(11): 5216. doi: 10.3390/app11115216
29. Altintas Y, Verl A, Brecher C, et al. Machine tool feed drives. *CIRP Annals*. 2011; 60(2): 779–796. doi: 10.1016/j.cirp.2011.05.010
30. Xi T, Kehne S, Fujita T, et al. Condition Monitoring of Ball-Screw Drives Based on Frequency Shift. *IEEE/ASME Transactions on Mechatronics*. 2020; 25(3): 1211–1219. doi: 10.1109/TMECH.2020.2969846
31. Zhang Li, Gao H, Wen J, et al. A deep learning-based recognition method for degradation monitoring of ball screw with multi-sensor data fusion. *Microelectronics Reliability*. 2017; 75: 215–222. doi: 10.1016/j.microrel.2017.03.038
32. Wen J, Gao H. Remaining useful life prediction of the ball screw system based on weighted Mahalanobis distance and an exponential model. *Journal of Vibroengineering*. 2018; 20(4): 1691–1707. doi: 10.21595/jve.2018.19099
33. Erkorkmaz K, Altintas Y. High speed CNC system design. Part II: Modeling and identification of feed drives. *International Journal of Machine Tools and Manufacture*. 2001; 41(10): 1487–1509. doi: 10.1016/S0890-6955(01)00003-7
34. Okwudire C. Finite Element Modeling of Ballscrew Feed Drive Systems for Control Purposes [PhD Thesis]. University of British Columbia; 2005.
35. Vicente DA, Hecker RL, Villegas FJ, et al. Modeling and vibration mode analysis of a ball screw drive. *The International Journal of Advanced Manufacturing Technology*. 2012; 58(1–4): 257–265. doi: 10.1007/s00170-011-3375-6
36. De Klerk D, Rixen DJ, Voormeeren SN. General Framework for Dynamic Substructuring: History, Review and Classification of Techniques. *AIAA Journal*. 2008; 46(5): 1169–1181. doi: 10.2514/1.33274
37. Zhang Y, Pu W. A tribo-dynamics model of ball screws based on flexible body element. *Mechanical Systems and Signal Processing*. 2025; 224: 111988. doi: 10.1016/j.ymssp.2024.111988
38. Fujita T, Xi T, Ikeda R, et al. Identification of a Practical Digital Twin for Simulation of Machine Tools. *International Journal of Automation Technology*. 2022; 16(3): 261–268. doi: 10.20965/ijat.2022.p0261
39. Zaeh MF, Oertli Th, Milberg J. Finite Element Modelling of Ball Screw Feed Drive Systems. *CIRP Annals*. 2004; 53(1): 289–292. doi: 10.1016/S0007-8506(07)60700-8
40. Li L, Zhang Q, Zhang T, et al. Vibration suppression of ball-screw drive system based on flexible dynamics model. *Engineering Applications of Artificial Intelligence*. 2023; 117: 105506. doi: 10.1016/j.engappai.2022.105506
41. Tóth R. Modeling and Identification of Linear Parameter-Varying Systems, *Lecture Notes in Control and Information Sciences*. Springer; 2010. doi: 10.1007/978-3-642-13812-6
42. Singh R, Sznaier M, Ljung L. An Atomic Norm Minimization Framework for Identification of Parameter Varying Nonlinear ARX Models. *IFAC-PapersOnLine*. 2019; 52(28): 1–6. doi: 10.1016/j.ifacol.2019.12.342

43. Brunton SL, Proctor JL, Kutz JN. Discovering governing equations from data by sparse identification of nonlinear dynamical systems. *Proceedings of the National Academy of Sciences*. 2016; 113(15): 3932–3937. doi: 10.1073/pnas.1517384113
44. Brunton SL, Proctor JL, Kutz JN. Sparse Identification of Nonlinear Dynamics with Control. *IFAC-PapersOnLine*. 2016; 49(18): 710–715. doi: 10.1016/j.ifacol.2016.10.249
45. Liu X, Li Y, Cheng Y, et al. Sparse identification for ball-screw drives considering position-dependent dynamics and nonlinear friction. *Robotics and Computer-Integrated Manufacturing*. 2023; 81: 102486. doi: 10.1016/j.rcim.2022.102486
46. Jacobs WR, Baldacchino T, Dodd T, et al. Sparse Bayesian Nonlinear System Identification Using Variational Inference. *IEEE Transactions on Automatic Control*. 2018; 63(12): 4172–4187. doi: 10.1109/TAC.2018.2813004
47. Huo F, Poo AN. Nonlinear autoregressive network with exogenous inputs based contour error reduction in CNC machines. *International Journal of Machine Tools and Manufacture*. 2013; 67: 45–52. doi: 10.1016/j.ijmachtools.2012.12.007
48. Zhang Y, Liang G, Cao C, et al. Short communication: Part contour error prediction based on LSTM neural network. *Mechanical Sciences*. 2023; 14(1): 15–18. doi: 10.5194/ms-14-15-2023
49. Huang D, Yang J, Xu G, et al. LuGre-Net: A hybrid neural network for friction modeling of feed systems in machine tools. *Nonlinear Dynamics*. 2024; 112(14): 11927–11943. doi: 10.1007/s11071-024-09674-w
50. Lei J, Jiang Z, Gao Z, et al. A state estimation method based on CNN-LSTM for ball screw. *Measurement and Control*. 2024; 57(10): 1417–1434. doi: 10.1177/00202940241241924
51. Azamfar M, Li X, Lee J. Intelligent ball screw fault diagnosis using a deep domain adaptation methodology. *Mechanism and Machine Theory*. 2020; 151: 103932. doi: 10.1016/j.mechmachtheory.2020.103932
52. Gal Y, Ghahramani, Z. Dropout as a Bayesian approximation: Representing model uncertainty in deep learning. In: *Proceedings of the 33rd International Conference on Machine Learning*; 19–24 June 2016; New York, NY, USA. pp. 1050–1059. Available online: <https://scispace.com/pdf/dropout-as-a-bayesian-approximation-representing-model-94gr4ig3gi.pdf>
53. Hänel A, Seidel A, Frieß U, et al. Digital Twins for High-Tech Machining Applications—A Model-Based Analytics-Ready Approach. *Journal of Manufacturing and Materials Processing*. 2021; 5(3): 80. doi: 10.3390/jmmp5030080
54. Zhang L, Liu J, Zhuang C. Digital Twin Modeling Enabled Machine Tool Intelligence: A Review. *Chinese Journal of Mechanical Engineering*. 2024; 37(1): 47. doi: 10.1186/s10033-024-01036-2
55. Huang Z, Fey M, Liu C, et al. Hybrid learning-based digital twin for manufacturing process: Modeling framework and implementation. *Robotics and Computer-Integrated Manufacturing*. 2023; 82: 102545. doi: 10.1016/j.rcim.2023.102545
56. Zhao R, Huang H, Mei L. A method for constructing digital twins of CNC machine tools feed systems based on hybrid mechanism-data. *Scientific Reports*. 2025; 15(1): 32186. doi: 10.1038/s41598-025-17587-w
57. Kasilingam S, Yang R, Singh SK, et al. Physics-based and data-driven hybrid modeling in manufacturing: A review. *Production & Manufacturing Research*. 2024; 12(1): 2305358. doi: 10.1080/21693277.2024.2305358
58. Xu H, Hinze C, Iannelli A, et al. Increasing dynamic accuracy of machine tools using predictive feedforward optimization with hybrid modeling. *Robotics and Computer-Integrated Manufacturing*. 2026; 98: 103137. doi: 10.1016/j.rcim.2025.103137
59. Fu X, Song H, Li S, et al. Digital twin technology in modern machining: A comprehensive review of research on machining errors. *Journal of Manufacturing Systems*. 2025; 79: 134–161. doi: 10.1016/j.jmsy.2025.01.005