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Article

Analysis of an existing methodology for assessing vehicle-track interaction under reliability conditions

Shukhrat Djabbarov[*], **Bakhrom Abdullayev**, **Aziz Gayipov**, **Abdusaid Yuldashov**, **Nodir B. Adilov**, **Irina Soboleva**

Department of Wagons and Wagon Facilities, Tashkent State Transport University, Tashkent 100069, Uzbekistan

*** Corresponding author:** Shukhrat Djabbarov, shuhratassistent@gmail.com

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Abstract: This paper presents a focused analytical audit of two simplified vehicle-track interaction schemes reported in the State Standard for 1,520 mm Gauge Railways: (i) a wheel-on-rail stability scheme intended to characterize flange-climb resistance and (ii) a track lateral stability scheme intended to estimate sleeper-rail-grid shift under train loading. The aim is twofold: first, to identify the internal inconsistencies of these specific formulations; second, to derive general lessons for transparent and reproducible safety assessment in railway engineering. The audit is organized around three steps: reconstruction of the source free-body diagrams and symbol definitions, point-by-point consistency checks of the equilibrium relations, and formulation of corrected self-contained equations in a unified coordinate system. The revised presentation shows that the source schemes mix force and moment terms, use ambiguous coordinate conventions, and in several places produce self-referential or physically trivial coefficients. A compact worked example is provided to illustrate the numerical difference between a parameter-independent source-style ratio and the corrected parameter-dependent contact-equilibrium relation. The paper also clarifies the status of each figure as either adapted from the source schemes or reconstructed by the authors and condenses the normative background to the material strictly needed for interpretation. Overall, the revised workflow improves traceability, reproducibility, and engineering interpretability.

Keywords: analytical audit; wheel–rail interaction; flange climb; track lateral stability; corrected equilibrium equations; reproducibility

1. Introduction

Simplified hand-calculation schemes continue to be used in railway engineering because they provide rapid intuition about wheel-climb risk and track shift resistance. Their usefulness, however, depends entirely on transparent assumptions and internally consistent equilibrium statements. This paper focuses on two specific schemes reported in the State Standard for 1,520 mm Gauge Railways [1, 2]: a wheel-on-rail stability formulation and a sleeper–rail-grid lateral stability formulation. The purpose is not merely to criticize these sources, but to use them as concrete case studies from which general lessons can be derived for transparent and reproducible safety assessment.

The central contribution of the paper is therefore organized in three layers. First, the source schemes are restated in a neutral, audit-ready form. Second, the inconsistencies are identified point by point, including mixed frames of reference, ambiguous force definitions, force–moment non-equivalence, and self-referential expressions. Third, corrected formulations are given in a self-contained manner, with an illustrative worked example showing how the corrected result differs numerically from a source-style coefficient. The analytical background also draws on classical theoretical mechanics [3–5], general railway-track and assessment references [6–9], and standard railway vehicle dynamics texts [10–12].

Only the minimum normative context required for interpretation is retained. EN 14363 and UIC 518 are referenced because they define the acceptance quantities against which analytical outputs are usually interpreted, but the paper is not intended as a general review of standards. Likewise, the literature on ballast lateral resistance is used only to justify why the resistance term in track-shift checks cannot be treated as a universal constant [13,14].

Broader context on wheel/rail interaction and derailment criteria is drawn from Zeng and Wu, Marquis and Greif, Kuo and Lin [15–17], with contact-mechanics foundations from Kalker and Johnson [18, 19]. Track geotechnology and track-shift safety background are referenced from Selig and Waters and the U.S. Federal Railroad Administration [20–23], respectively. The recent review of the Prud'homme criterion [24], ballast lateral-resistance studies [25–28] and lateral-stability studies [29,30] are cited only where directly relevant.

The remainder of the paper is structured as follows. Section 2 defines the audit scope, notation, and reproducibility protocol. Section 3 analyzes the wheel-on-rail scheme, identifies the principal inconsistencies, and presents a corrected local equilibrium formulation together with a worked example. Section 4 applies the same logic to the track lateral stability scheme. Section 5 discusses the broader implications, and Section 6 summarizes the conclusions.

2. Research methodology

The audit methodology is designed to mirror the structure required for a reproducible critique: source-scheme description, inconsistency identification, corrected formulation, and illustrative comparison. Only the notation and background necessary to reproduce the revised equations are retained.

2.1. Audit scope and source schemes

The primary audit targets are the two calculation schemes reported in the State Standard for 1,520 mm Gauge Railways [1,2]: the wheel-on-rail stability scheme and the track lateral stability scheme. For each scheme, the original diagram, symbol set, and stated interpretation are reproduced as faithfully as possible before any criticism is introduced.

The audit distinguishes clearly between the physical quantity intended by the original authors and the quantity that can actually be justified from equilibrium. This distinction is important because some source expressions are only meaningful after additional assumptions are added, whereas those assumptions are not stated explicitly in the source text.

2.2. Notation harmonization and consistency checks

All symbols are rewritten in a unified notation, with explicit units, sign conventions, and reference directions. The reconstruction uses free-body diagrams and checks whether each unknown has a corresponding equilibrium relation and whether each term preserves dimensional consistency.

An inconsistency is recorded when the same quantity is counted simultaneously as a force and as a moment contribution, when components are mixed between coordinate systems without transformation, when support reactions are solved asymmetrically, or when a derived coefficient depends on itself or becomes trivial by construction.

Limiting-case checks are then used as a second diagnostic layer. Examples include zero friction, symmetric loading, and vanishing contact-angle limits for the wheel-on-rail problem, together with constant-resistance and displacement-dependent-resistance interpretations for the track lateral stability problem.

2.3. Minimal normative context and reproducibility

The corrected expressions are interpreted only against the acceptance quantities needed for engineering use: wheelset lateral force, vertical wheel load, guiding-force-related ratios, and track lateral resistance descriptors [13, 14, 24]. The standards are therefore used as an interpretive framework rather than as the main subject of the paper.

2.4. Corrected formulation strategy

For each audited scheme, the corrected formulation is written in a self-contained format: the coordinate system is stated explicitly, the force decomposition is shown, the equilibrium equations are grouped together, and the resulting assessment expression is derived from the same set of assumptions. This prevents the corrected model from becoming another partially implicit scheme.

2.5. Worked-example protocol

To demonstrate the engineering consequence of the corrections, a compact worked example is included for the wheel-on-rail scheme. The example is intentionally illustrative rather than certification-oriented: it uses representative values of contact angle, friction coefficient, and vertical load to show how a parameter-dependent equilibrium relation differs from a parameter-independent source-style ratio.

3. Wheel-on-rail scheme: Source description, inconsistencies, and corrected formulation

3.1. Source scheme and notation

The wheel-on-rail audit is intentionally focused on the source scheme reported in the standard [1] and the guideline [2]. **Figure 1** reproduces the essential logic of that scheme before any criticism is introduced.

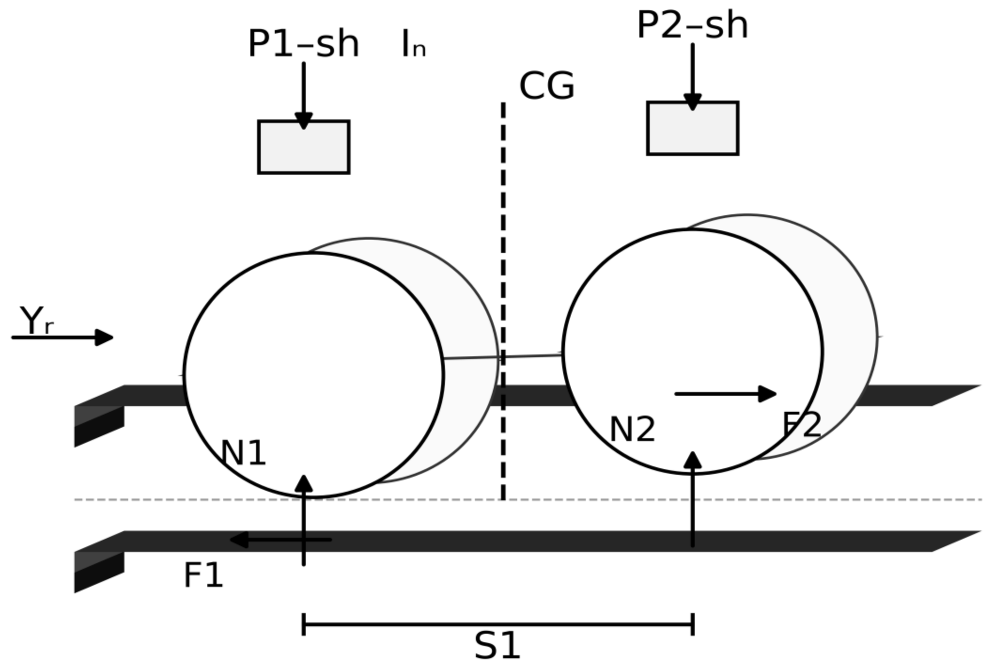

Figure 1. Source wheel-on-rail stability scheme reported in the State Standard for 1,520 mm Gauge Railways [1,2].
Note: Adapted from the original source scheme.

In **Figure 1**, the following notations are used [2]: P1-sh and P2-sh are the loads from the car body transmitted to the axle journals of the wheelset (kN); P1-r and P2-r are the wheel loads on the rails (kN). The purpose of **Table 1** is to restate these symbols in a unified notation before the inconsistency analysis begins.

The main symbols and units are summarized in **Table 1**.

Table 1. Main symbols used in the computational schemes.

Symbol	Description	Unit
P1-sh, P2-sh	Car-body load transmitted to wheelset axle journals	kN
P1-r, P2-r	Wheel load on rails	kN
Y_r	Lateral (frame) force	kN
N1, N2	Normal rail reactions	kN
F1, F2	Friction forces at the wheel-rail contact	kN
I_n	Inertial ("centrifugal") component (formal term)	kN
S1	Distance between rail axes (rail spacing)	m
S_sh	Distance between axle-journal midpoints	m
r_k	Wheel radius	m
r_sh	Axle-journal radius	m

3.2. Point-by-point inconsistencies in the source formulation

The scheme in **Figure 1** includes a bending moment applied at the axle journal as well as the forces P1-r and P2-r, which are treated as wheel loads on the rails and, simultaneously, as forces applied to the wheels.

This formulation is inconsistent with classical theoretical mechanics because it violates the rules of force and moment reduction when replacing a load system with an equivalent one.

Therefore, the scheme in **Figure 1** cannot be regarded as a physically consistent model of a wheelset and must be reformulated with due consideration of force-system equivalence.

3.3. Corrected local equilibrium formulation

For a self-contained correction, the wheel–rail contact is represented in a single coordinate framework. Let L denote the lateral force in the track-fixed frame, V the vertical wheel load, N the normal contact reaction, T the tangential contact force, μ the friction coefficient, and δ the effective flange contact angle used in the adopted sign convention of **Figure 2**. At the limiting state, the tangential component is written as:

$$T = \mu N.$$

Resolving the contact force into the track-fixed frame gives the following self-contained equilibrium representation for the limiting contact state:

$$L = N \sin\delta - T \cos\delta,\ V = N \cos\delta + T \sin\delta,\ T = \mu N.$$

Substituting $T = \mu N$ yields the corrected parameter-dependent relation:

$$(L/V)\_crit = (\sin\delta - \mu \cos\delta)/(\cos\delta + \mu \sin\delta).$$

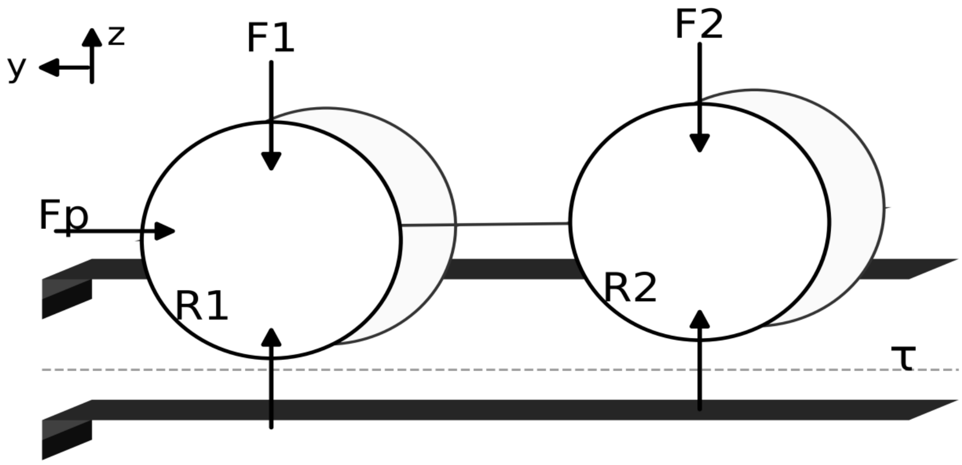

Figure 2. Reconstructed wheelset model used for the corrected discussion.
Note: Authors' reconstruction based on the State Standard for 1,520 mm Gauge Railways [1,2].

Unlike the source-style force ratio, this corrected relation depends explicitly on μ and δ and therefore cannot collapse to a universal constant. It should be read as an illustrative local equilibrium representation that makes its assumptions transparent rather than as a substitute for full approval testing.

The main value of the corrected formulation is methodological: every term now has a declared physical meaning, every unknown belongs to the same force system, and the parameter dependence can be reported explicitly in the manuscript or in subsequent calculations.

This self-contained representation also makes it possible to compare the corrected formulation numerically with the source-style coefficient without introducing hidden assumptions about missing lever arms or mixed coordinate definitions.

3.4. Worked example and parameter sensitivity

A compact worked example illustrates the numerical consequence of the correction. For $\mu = 0.30$, $\delta = 65°$, and $V = 100$ kN, the source-style force ratio remains $\eta\_F = 1.00$ by construction, whereas the corrected relation gives $(L/V)\_crit = 1.12$, and therefore, $L\_crit = 112.2$ kN.

The result is not presented as a universal threshold; its purpose is to show that the corrected formulation is parameter-dependent and numerically distinguishable from the source-style ratio even for one representative case. When μ and δ vary over realistic ranges, the predicted admissible lateral force changes accordingly, which is exactly why transparent parameter reporting is necessary.

Table 2 summarizes this illustrative comparison in the same notation used in the corrected formulation.

Table 2. Illustrative worked example comparing the source-style ratio and the corrected contact-equilibrium relation.

Case element	Symbol/Value	Source-style output	Corrected output
Friction coefficient	$\mu = 0.30$	not used explicitly	used explicitly
Effective contact angle	$\delta = 65°$	not used explicitly	used explicitly
Vertical wheel load	V = 100 kN	ratio unchanged	reference value
Illustrative result	—	$\eta\_F = 1.00$	$(L/V)\_crit = 1.12$; $L\_crit = 112.2$ kN

3.5. Relation to the source equation and equivalent-wheelset diagrams

The detailed reason why the source definition collapses to a trivial coefficient can be visualized directly. If the numerator is defined as the total force preventing lift and the denominator as the total force causing lift within the same static equilibrium statement, both quantities are equal at the limiting state and the ratio becomes unity.

This is why a force-ratio definition is not, by itself, a meaningful stability criterion in the present context.

$$\eta_F \geq \frac{F_{pr.}}{F_{ca.}}$$

Accordingly, the coefficient η_F remains 1.00 irrespective of the contact parameters and therefore cannot diagnose a change in physical safety margin.

Figure 3 gives the authors' illustration of this triviality.

For traceability, the source equation reported in the State Standard for 1,520 mm Gauge Railways [1,2] is reproduced below in its original symbolic context.

$$k = [(P1 + P2 - N2) \sin\tau]/[F1 + (Y\_r - F2) \cos\tau]$$

Here, Y_r corresponds to F_r in **Figure 2**; P1 → F1, P2 → F2, F1 → Fτ1, F2 → Fτ2, and τ → α.

In equation above, the normal reaction N2 should be treated as a force causing wheel lift and therefore appear in the denominator rather than the numerator.

For comparison, **Figure 4** provides an illustrative example of the classical stability coefficient defined as the ratio of the stabilizing and overturning moments (for demonstration only).

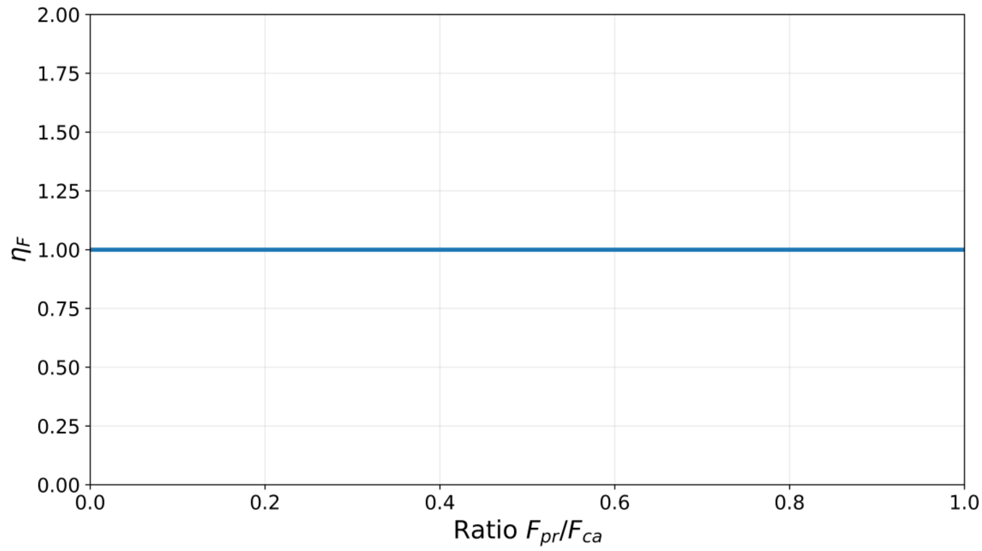

Figure 3. Illustration of the triviality of η_F when defined as F_prevent/F_cause.
Note: Authors' illustration.

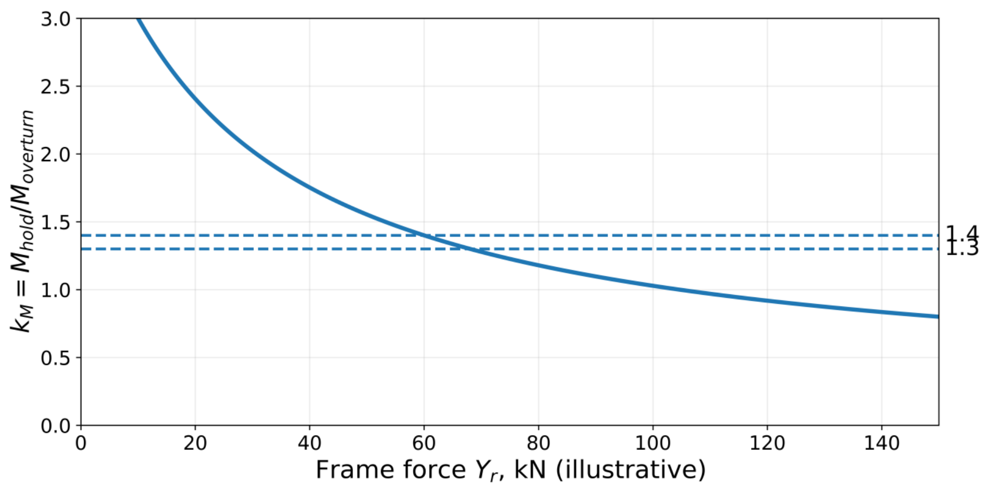

Figure 4. Illustrative dependence of the moment-based stability coefficient k_M = M_hold/M_overturn on the lateral force Y_r.
Note: Authors' illustrative plot.

A further methodological inconsistency should be noted: when determining the normal component N1 of the support reaction (outer rail thread A), the car-body loads acting on the axle journals are not accounted for symmetrically in the derivation of N1 and N2.

In **Figure 5**, h1 and hτ1 are the lever arms of forces N1 and Fτ1, respectively; rk is the wheel radius, equal to 0.475 m for a freight wagon.

Wheel-on-rail stability can be assessed by constructing a form of an equivalent wheelset model, using the corollary of the statics axiom: the system of forces may be shifted along their lines of action without changing the external effect, provided the equivalence conditions are preserved (**Figure 6**).

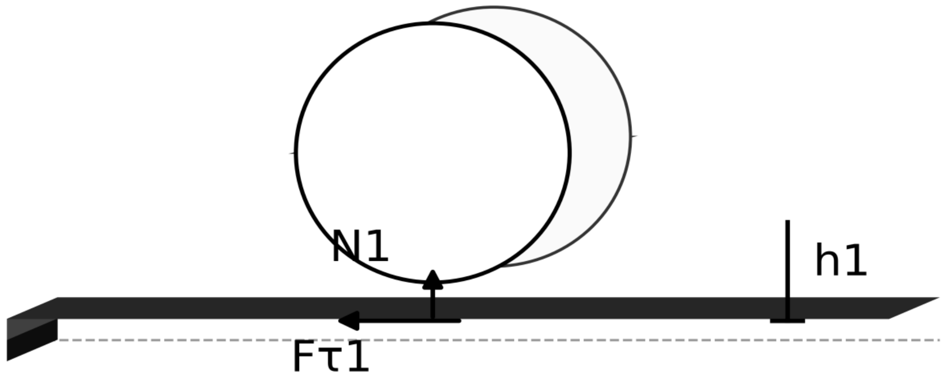

Figure 5. Equivalent wheelset model for a consistent reaction analysis.
Note: Authors' reconstruction based on the State Standard for 1,520 mm Gauge Railways [1,2].

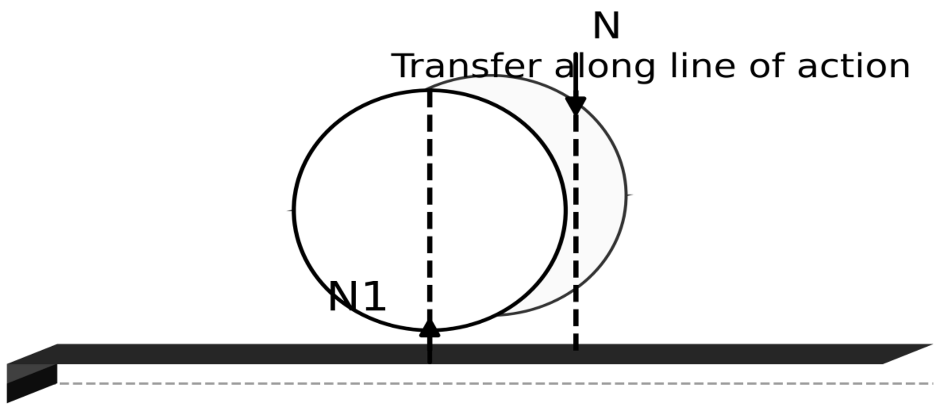

Figure 6. Alternative equivalent wheelset model preserving force-system equivalence.
Note: Authors' reconstruction based on the State Standard for 1,520 mm Gauge Railways [1,2].

Wheel-on-rail stability can also be assessed using another variant of the wheelset model based on reducing a system of forces to a given point and solving the equilibrium conditions consistently (**Figure 7**).

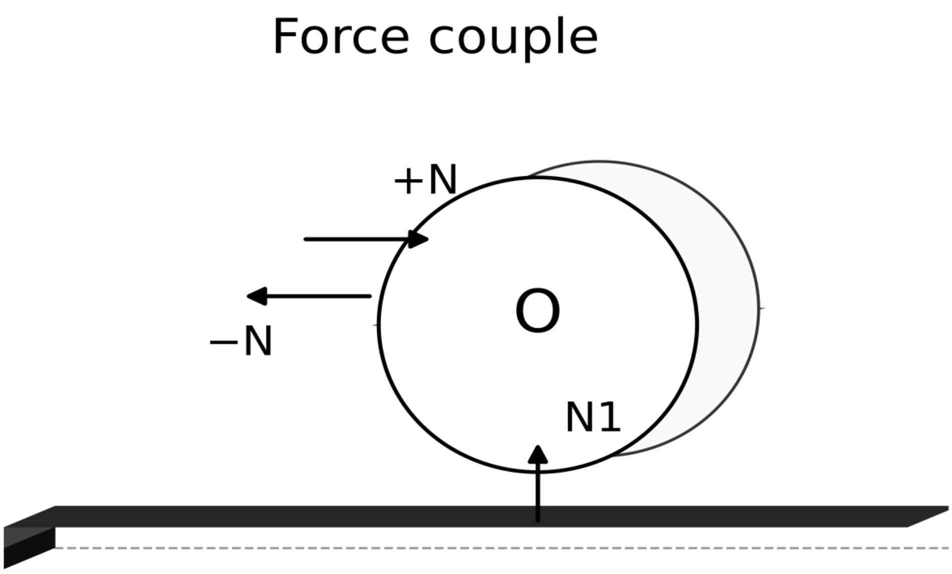

Figure 7. Point-reduction variant of the equivalent wheelset model.
Note: Authors' reconstruction based on the State Standard for 1,520 mm Gauge Railways [1,2].

A second error is the inconsistent choice of governing equations: the projection of forces onto the normal n-n is used to find N1, whereas N2 is obtained from a moment equation about a point, without a unified statement of the reaction problem.

Accordingly, the rail support reactions should be determined only by solving the equilibrium problem consistently, either using force projections onto selected axes or using moment equations about selected points (but not by mixing approaches without justification).

Thus, the threshold values of coefficient k reported in the State Standard for 1,520 mm Gauge Railways [1, 2] for locomotives, freight and passenger cars do not follow from a correct problem statement and should be revisited.

4. Track lateral stability scheme: Source description, inconsistencies, and corrected interpretation

4.1. Source scheme and physical meaning

The second audit target is the methodology used to calculate the transverse shift of the sleeper–rail grid under train loading. This phenomenon is safety-relevant because excessive shift either produces an immediately hazardous condition or accelerates the development of one.

Importantly, the set of lateral forces includes forces arising when rolling stock moves over track irregularity waves (i.e., the so-called lateral inertial forces associated with the transport motion of masses).

In an inertial reference frame, there is no separate external "centrifugal" force acting on the vehicle; the term appears when equations of motion are written in a non-inertial frame and should be interpreted accordingly.

Therefore, the normal inertial term in the physical and mathematical model of a loaded wagon (including the wheelset, **Figure 8**) should be introduced only as a formal component required by the chosen reference frame and kinematic description, rather than as a physically applied external load.

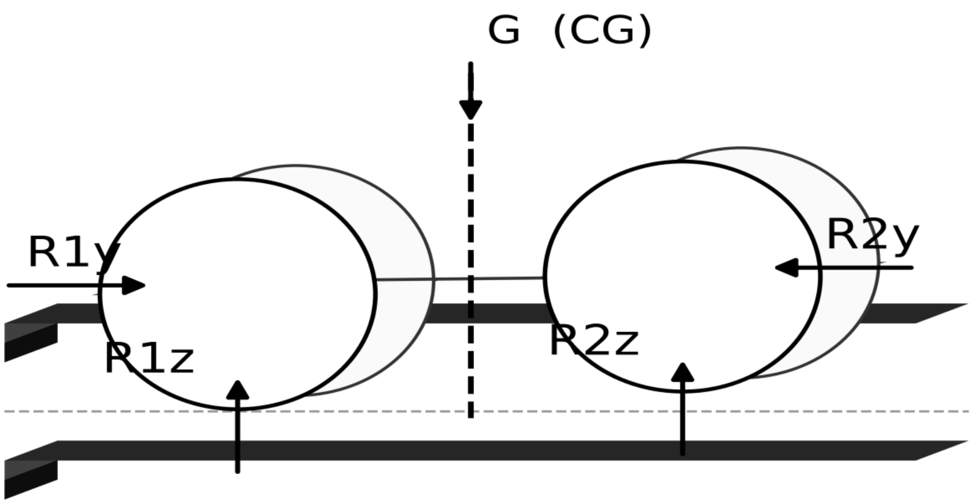

Figure 8. Physical model used to discuss axle loading and inertial terms in the track-shift scheme.

Note: Authors' reconstruction based on the State Standard for 1,520 mm Gauge Railways [1,2].

4.2. Why resistance cannot be treated as a universal constant

Transverse shift of the sleeper–rail grid is a structural response of the track system. While wheel–rail safety criteria focus on local contact, track shift is governed by the global lateral resistance of sleepers embedded in ballast, the restraint of fastenings, and the load distribution along the track panel. In ballasted track, lateral resistance is provided by a combination of friction at the sleeper bottom, passive resistance of the ballast shoulder against the sleeper ends, and interlock and confinement effects within the ballast bed. This resistance is strongly nonlinear with displacement: an initial stiffness region is followed by a peak and then a residual plateau once ballast is mobilized and begins to rearrange [25–28]. This nonlinear behavior is also supported by numerical and DEM studies [29,30].

Track lateral stability is also time-dependent. Repeated loading can progressively reduce confinement (ballast degradation, fouling, and loss of shoulder), and maintenance actions such as tamping can temporarily reduce resistance by loosening the ballast. Therefore, a meaningful track shift assessment must specify whether it is intended to prevent immediate unsafe shift under a single extreme load case or to limit cumulative degradation over many load cycles. This distinction is a core theme in high-speed track shift studies and state-of-the-art reviews [22,23].

4.3. Lateral resistance characterization: Experiments and models

Several experimental methods are used to quantify lateral resistance. The single-sleeper push (or pullout) test isolates a single sleeper and measures the force–displacement curve required to move it laterally; track-panel pullout tests assess a group of sleepers and capture interaction effects (often described as a "group pile" or interference effect) [26,27]. Laboratory-scale and field studies consistently show that sleeper shape, spacing, ballast shoulder geometry, and vertical preload influence the peak resistance and the initial stiffness.

Koike et al. proposed a numerical method calibrated against 1/5-scale model tests that explicitly accounts for sleeper shape and spacing and yields a resistance prediction with quantified uncertainty [26]. Koyama et al. extended this work to investigate ballast resistance under extended sleeper spacing and demonstrated that assuming constant lateral resistance per sleeper can be inaccurate because both rail weight distribution and sleeper interference change with spacing [27]. These results are particularly relevant when assessing modernized lines where sleeper type and spacing may differ from legacy design assumptions.

At the track scale, three-dimensional numerical simulations have been used to study ballast deformation and to perform parametric evaluations of shoulder width/height, ballast depth, and inter-particle friction. Kabo conducted 3D elastoplastic finite-element simulations to quantify how lateral resistance depends on ballast geometry and frictional properties [28]. Discrete element modelling (DEM) has also been used to reproduce single-tie push tests and to evaluate sensitivity to ballast porosity, particle friction, and shoulder geometry [30]. These studies support a key implication for analytical safety checks: the resistance term cannot be treated as a single universal constant; instead it should be scenario-specific and linked to measurable track condition descriptors.

In addition, ballast degradation can substantially reduce lateral stability. Ngamkhanong et al. quantified the reduction of track lateral stability due to ballast degradation and fouling and highlighted that visually undetectable conditions (e.g., subsurface fouling) can undermine resistance [29]. This reinforces the need to treat resistance parameters as uncertain and to incorporate inspection/maintenance information into safety evaluation.

4.4. Acceptance context and corrected parameterization

In vehicle approval practice, excessive lateral wheelset forces are limited not only to prevent wheel–rail derailment mechanisms but also to reduce track damage and track shift. The so-called Prud'homme criterion is widely used in Europe as a limit on the sum of wheelset guiding forces, motivated by the desire to prevent progressive track panel shift under repeated axle passes. Recent literature reviews discuss how the criterion is applied in modern approval workflows and how its interpretation depends on speed, axle load, and the considered operating scenario [24].

High-speed track shift programs in the United States similarly distinguish between vehicle acceptance criteria and track shift criteria tailored to track segments with known parameters. State-of-the-art reviews and analysis reports emphasize that the track shift criterion aims to predict lateral residual deformation and degradation accumulation and thus requires explicit track parameterization rather than a one-size-fits-all threshold [22,23].

Within the audited scheme, the main methodological risk is mixing external lateral forces with inertial terms that arise only from a chosen non-inertial frame. To maintain physical meaning, the evaluation should either be formulated in an inertial frame (with only real external forces) or, if a moving frame is used, the inertial terms must be introduced consistently and never conflated with elastic resistance terms. This distinction is crucial when the scheme is used for safety justification because an apparent "increase" in lateral load may simply be an artefact of modelling choices rather than a measurable external force.

4.5. Corrected evaluation protocol

Based on the above evidence, a pragmatic protocol for analytical track shift checks is proposed. (1) Specify the track configuration (sleeper type, spacing, ballast shoulder, fastening system) and select or calibrate a lateral resistance curve C(y) from tests or validated models (single-sleeper and panel tests where possible) [26–30]. (2) Compute the distribution of lateral wheel loads along the track panel for the considered scenario (curving steady-state or transient due to irregularities), and separate real external loads from inertial terms if a moving frame is used (**Figure 8**). (3) Evaluate the peak displacement and compare both peak resistance and residual deformation criteria, distinguishing immediate safety from long-term degradation risk [22–24]. (4) Perform a sensitivity check over plausible ballast condition states to quantify the robustness of conclusions.

4.6. Specific inconsistencies in the source track-shift formulation

The vertical force set also includes forces arising when rolling stock moves over track irregularity waves (i.e., the so-called vertical inertial forces associated with the transport motion).

In calculations of track lateral stability, the holding force T_hold is commonly represented as the sum of the initial resistance to sleeper displacement in the absence of vertical load and the friction force.

However, this decomposition may introduce systematic error because C0 and the friction term depend on the combined action of lateral and vertical forces applied to the sleeper/ballast system.

In addition, in the State Standard for 1,520 mm Gauge Railways [1,2] the shifting force (T_shift) is evaluated using the friction force of the inner wheel on the inner rail (F_fr) computed from the average vertical load (P_avg = Fz_avg), which may be insufficient for dynamic load cases.

In the State Standard for 1,520 mm Gauge Railways [1, 2], a relation between the lateral (side) force Yb (equivalent to R1y) acting on the outer rail and the average vertical load P_avg (equivalent to R1_avg) is proposed in terms of an empirical dependence.

The inconsistency of the obtained relation is evident even formally because a function is defined through itself, i.e., P_avg = f(P_avg).

In reality, the initial resistance to sleeper displacement C0 is determined, among other factors, by the ratio of the horizontal and vertical components of the outer-wheel reaction (R1y and R1z), as well as ballast and fastening conditions.

Moreover, Vinogradov et al. [2] confuse the notions of “inertial forces” and “elastic forces”. For example, the maximum inertial force due to car-body oscillations on the suspension is estimated through the magnitude of an elastic force, which is an incorrect interpretation.

Similar inaccuracies regarding inertial forces are also noted in Glushko et al. [8].

The key identified inconsistencies and proposed clarifications are summarized in **Table 3**.

Table 3. Summary of identified inconsistencies and recommended clarifications.

Method element	Issue	Impact	Recommended clarification
Stability coefficient defined as a force ratio	Defining stability via F_prevent/F_cause is not valid for a force-equilibrium formulation.	Produces a trivial result η_F = 1 and does not represent a stability margin.	Use a moment-based criterion: ratio of stabilizing and overturning moments (kM = M_hold/M_overturn).
Accounting for car-body loads	Loads P1-sh and P2-sh are accounted for asymmetrically when deriving reactions N1 and N2.	Distorts rail reaction estimates and the assessment of wheel lift risk.	Account for P1-sh and P2-sh consistently in equilibrium and moment equations.
Method for determining reactions N1 and N2	Mixing different equations (force projections for N1 and moments for N2) without a unified formulation.	May yield mutually inconsistent results for the same input data.	Solve the reaction problem consistently: either by force projections or by moment equations about chosen points.
Interpretation of centrifugal force	A normal inertial term is introduced and interpreted as a “real” applied force.	Leads to an incorrect physical interpretation of forces in curves.	Treat it as a formal term associated with a non-inertial frame, without assigning it direct physical action in an inertial frame.
Track transverse (lateral) stability	The resisting and shifting forces are defined such that, for fixed input data, they become equal.	The criterion loses diagnostic value.	Formulate the criterion via the combined action of R_y and R_z and ballast/track-condition parameters.

5. Discussion

The audit reveals a recurring pattern: simplified schemes often attempt to compress a multi-degree-of-freedom vehicle–track interaction problem into a single "stability coefficient", but the compression is valid only under a carefully specified set of assumptions. When those assumptions are left implicit, later users may unknowingly change the physical meaning of the symbols (for example, by treating a normal contact force as a vertical load, or by interpreting an inertial term as an externally applied lateral force). The consequence is not merely a theoretical inconsistency: the resulting numerical thresholds can shift enough to affect engineering decisions about allowable speed, curving performance, or maintenance priorities.

From a methodological viewpoint, dimensional analysis and limiting-case checks are the most effective "first-line" tests for detecting such issues. If a derived ratio is intended to be dimensionless, every intermediate expression should preserve dimensions explicitly; if a term is a moment, its lever arm must be documented. Similarly, limiting cases provide intuition: for $\mu \to 0$ a flange-climb limit should not predict a finite admissible lateral force; for a vanishing contact angle, the lateral component should diminish. These checks are simple to implement and can be included in teaching materials and internal engineering notes without requiring complex simulations.

The second key implication is the role of uncertainty. Both wheel–rail contact and ballast resistance are variable in practice, and the variability is large enough that deterministic "single numbers" are rarely defensible. The reviewed literature on derailment criteria demonstrates sensitivity to friction, geometry, and operating conditions [16, 17]. The same conclusion is supported by experimental and numerical studies of track lateral resistance [25–28] and lateral stability [29, 30]. Therefore, a credible analytical assessment should report parameter ranges and provide sensitivity results, even if the final decision must be expressed as a pass/fail statement.

A practical checklist for authors and practitioners using simplified schemes is as follows. (1) Provide a figure of the coordinate system and explicitly state whether forces are expressed in track coordinates or in contact coordinates. (2) List all symbols with units and clarify whether a quantity is per wheel, per axle, or per wheelset. (3) Identify which equations are solved (force balance, moment balance) and which assumptions close the system (symmetry, neglected components, linearization). (4) Provide a reproducibility table with the chosen μ, δ (or profile family), and track resistance parameters. (5) Compare the derived assessment parameter to the quantities used in standard practice and discuss how the analytical check complements, rather than replaces, standardized test and simulation evidence [13, 14, 24].

Finally, limitations should be acknowledged. The present work is an analytical audit of specific published schemes rather than a full-scale validation study. It does not replace vehicle certification testing, nor does it attempt to recommend a new universal threshold for derailment or track shift. Instead, it provides corrected relations and a transparent workflow that can be used as a preliminary check, as an educational tool, or as a quality-control step when preparing manuscripts for peer-reviewed publication.

An additional benefit of aligning simplified checks with standards is improved interpretability of results. When analytical outputs are expressed in terms of the same assessment parameters that appear in EN 14363 or UIC 518 (e.g., wheelset guiding force, vertical wheel load, filtered peak values), it becomes easier to reconcile discrepancies between hand calculations and instrumented test runs. Conversely, if an alternative coefficient is introduced, it should be accompanied by an explicit conversion to the standard parameters so that reviewers and practitioners can assess whether the proposed threshold is compatible with accepted evaluation rules.

Future work should focus on two directions. First, the corrected relations should be embedded into an openly documented calculation template (spreadsheet or script) that enforces unit checks and produces sensitivity envelopes by default. Second, case studies using measured wheelset forces and independently measured track resistance curves would allow quantifying how often simplified analytical limits are conservative or non-conservative for real vehicles and track states. Such case studies would also support the development of "condition-aware" limits where track shift risk is evaluated using track inspection data (ballast condition, shoulder geometry, recent tamping) in addition to vehicle force measurements.

6. Conclusion

The revised paper has shown, in a deliberately structured manner, that the audited schemes from the standard [1] and the guideline [2] contain several reproducibility-relevant inconsistencies. For the wheel-on-rail problem, the main issues are the force-ratio definition that becomes trivial at the limiting state, the ambiguous separation of force and moment terms, mixed coordinate interpretations, and an incomplete or asymmetrically solved reaction problem. The corrected local equilibrium formulation presented here makes the dependence on μ and δ explicit and, through the worked example, shows that the resulting threshold is parameter-dependent rather than universal.

For the track lateral stability problem, the main issues are the treatment of resistance as a single constant, the self-referential dependence of average vertical load, and the conflation of inertial and externally applied forces. A reproducibility-oriented evaluation protocol is therefore recommended in which the resistance term is documented as a track-condition-dependent function and the mapping between model forces and acceptance quantities is stated explicitly. These revisions strengthen the manuscript's contribution to transparent and reproducible safety assessment in railway engineering.

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