Sound & Vibration 2025, 59(4), 3717.
https://doi.org/10.59400/sv3717

Article

Signal analysis of elastic waveguide-based techniques for monitoring bone fracture healing: application to structural state evolution in biological composites

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CITATION

Li W, Li X, Wu J, Liang X. Signal analysis of elastic waveguide-based techniques for monitoring bone fracture healing: application to structural state evolution in biological composites. Sound & Vibration. 2025; 59(4): 3717. https://doi.org/10.59400/sv3717

ARTICLE INFO

Received: 5 August 2025
Revised: 28 September 2025
Accepted: 17 October 2025
Available online: 21 October 2025

Abstract: This study explores the use of elastic waveguide propagation and signal analysis for monitoring structural evolution in heterogeneous media, with bone analogues employed as a case example. Synthetic models representing low, intermediate, and high stiffness states were examined using piezoelectric sensors to capture transmitted waveforms. Four parameters velocity, attenuation, dispersion index, and spectral entropy were extracted according to defined procedures. Results showed consistent trends: velocity increased, attenuation decreased, dispersion diminished, and entropy reduced as stiffness increased, confirming the sensitivity of wave-derived features to structural transitions. A Random Forest classifier was applied to these features, demonstrating highly accurate discrimination among the three states under controlled conditions. The integration of elastic wave descriptors with supervised learning highlights the potential of vibration-based diagnostics for tracking stiffness evolution in heterogeneous composites. While bone consolidation provides a compelling case study, the framework is generalisable to other composite systems, thereby reinforcing the contribution of elastic wave analysis to the broader field of sound and vibration.

Keywords: elastic waveguide; dispersion and attenuation; acoustic signal processing; vibration-based diagnostics; time–frequency analysis; machine learning classification

1. Introduction

Elastic wave propagation in complex and heterogeneous media remains a central theme in the fields of acoustics and vibration science. From the earliest developments in non-destructive testing (NDT) to the current generation of structural health monitoring (SHM) techniques, guided waves have been regarded as powerful carriers of information about the internal state of a material system [1,2]. Their sensitivity to density, stiffness, porosity, and microstructural irregularities makes them highly suitable for probing not only engineering composites but also natural materials that exhibit anisotropy and heterogeneity. Within the discipline of vibration and acoustics, extensive research has been devoted to understanding wave dispersion, attenuation, and mode conversion as they occur in layered structures, plates, shells, and porous media. These principles provide the theoretical basis for extending elastic wave methods to unconventional substrates, including biological tissues, where the same underlying physics governs signal propagation [2].

In vibration-based diagnostics, the measurement of wave velocity, attenuation, and dispersion provides a direct connection between dynamic wave behavior and the mechanical properties of the medium. Classical theories of wave propagation in rods and plates have been expanded to include guided wave modes in multilayered or porous structures, demonstrating that even subtle changes in stiffness or density can lead to measurable variations in signal features. Such insights have made guided wave analysis indispensable in aerospace composites, pipeline inspection, and civil infrastructure monitoring. Signal processing methods, including Fourier and wavelet-based approaches, have allowed increasingly precise characterization of dispersive waveforms, enabling researchers to separate overlapping modes and quantify frequency-dependent variations with high fidelity [2].

The application of these concepts to biological materials introduces both challenges and opportunities. Bone, in particular, represents a naturally occurring hierarchical composite with marked anisotropy, porosity, and evolving mechanical properties across its lifespan. Its cortical and trabecular regions differ in density and orientation, creating a medium where guided waves are strongly scattered and dispersed, which makes bone an intriguing, though complex, testbed for elastic wave analysis. Earlier investigations of ultrasound in bone primarily aimed at assessing density and diagnosing osteoporosis [3]. Broadband ultrasound attenuation (BUA) and speed of sound (SOS) were identified as sensitive to bone mineral content, leading to their adoption in clinical densitometry devices [4]. These approaches, however, relied on single parameters and were largely static in nature, offering limited insights into dynamic changes within the tissue.

Recent advances in time–frequency analysis have opened new avenues for interpreting elastic wave signals in heterogeneous media [2]. Unlike classical Fourier analysis, which assumes stationary signals, time–frequency methods such as short-time Fourier transform and wavelet transform allow localization of frequency content as it evolves over time, making them particularly valuable for dispersive waveforms. Wavelet analysis, in particular, has been widely adopted in vibration research for its multi-resolution properties, enabling simultaneous analysis of high-frequency transients and low-frequency trends. In the context of biological composites, these methods permit detailed characterization of how microstructural irregularities shape the frequency content of transmitted waves [2].

Entropy-based descriptors have also gained traction as non-model parameters for quantifying signal complexity. Spectral entropy, derived from information theory, measures the degree of disorder in the frequency distribution of a waveform. In vibration science, entropy has been used to detect faults in rotating machinery, delamination in composites, and crack initiation in metallic structures. Applied to bone or similar porous media, entropy provides an indirect measure of microstructural irregularity: higher entropy implies a more disordered medium with irregular scattering, while lower entropy corresponds to greater uniformity and consolidation [5]. By combining traditional wave parameters such as velocity and attenuation with advanced descriptors like dispersion index and entropy, researchers can build a more holistic picture of how waves interact with complex media.

Parallel to advances in signal processing, machine learning has transformed the landscape of acoustics and vibration diagnostics. Whereas classical analysis often relied on thresholding or simple regression models, machine learning algorithms are capable of recognizing subtle, nonlinear relationships in multidimensional feature spaces. Random forests, support vector machines, and neural networks have all been successfully applied to tasks ranging from vibration-based fault detection in mechanical systems to acoustic classification in materials characterization [2]. These methods excel in integrating diverse features, mitigating noise sensitivity, and providing automated decision-making frameworks. Their adoption within the vibration community underscores a broader trend: the shift from manual interpretation of signals to intelligent systems that can autonomously infer material states from complex datasets.

The convergence of elastic wave theory, time–frequency analysis, and machine learning offers a powerful toolkit for studying wave behavior in heterogeneous composites. Bone, when viewed through the lens of acoustics, can be treated as a porous waveguide whose stiffness evolves during processes of consolidation. As the medium transitions from a low-stiffness disordered state to a more consolidated and mineralized structure, its elastic properties change in ways that directly influence wave propagation. The wave speed increases with stiffness, attenuation decreases as scattering diminishes, dispersion reduces as heterogeneity is resolved, and entropy falls as frequency content becomes more ordered [2]. These predictable relationships suggest that vibration analysis can serve as a sensitive proxy for monitoring stiffness evolution in such media.

Despite this theoretical potential, relatively few studies have examined stiffness evolution in biological composites from the perspective of acoustics and vibration. Most existing work remains situated within the biomedical domain, focusing on radiographic or conventional ultrasound imaging. While these approaches provide valuable anatomical detail, they do not exploit the full capacity of elastic wave analysis for dynamic, real-time monitoring of material state. Moreover, conventional imaging is often limited by cost, radiation exposure, or accessibility, particularly in settings where repeated monitoring is required. By reframing bone as a composite waveguide and applying acoustic signal analysis techniques, it becomes possible to extend vibration-based diagnostics into new domains, thereby linking established engineering methods with emerging applications in biological materials [2].

The novelty of the present work lies in integrating elastic wave propagation analysis with machine learning classification to track stiffness evolution in bone-mimicking media. Synthetic models representing different structural states were interrogated with high-frequency guided waves, and signal features were extracted using both time-domain and time–frequency methods. Parameters including velocity, attenuation, dispersion index, and spectral entropy were then combined within a supervised classification framework, demonstrating that acoustic features alone can reliably distinguish between stages of consolidation. Importantly, the emphasis here is not limited to clinical benefit but rather to advancing the understanding of how elastic waves behave in non-uniform, anisotropic composites and how their features can be harnessed for state classification [2].

By situating bone as an exemplar of heterogeneous materials and applying advanced vibration analysis techniques, this study contributes to the broader field of sound and vibration research. It illustrates that guided wave diagnostics, already well established in aerospace and civil engineering, can be extended to biological substrates without altering their underlying theoretical foundation. The implications reach beyond medicine, offering insights into how elastic wave features can inform non-destructive evaluation, intelligent sensing, and adaptive monitoring systems across a variety of composite structures. This dual relevance advancing acoustic theory while demonstrating an applied case in biomedical composites positions the work within the scope of vibration science while maintaining interdisciplinary value.

2. Literature review

Effective monitoring of structural state evolution in heterogeneous media has long been a challenge in both engineering and biomedical domains. Traditional imaging and diagnostic tools provide valuable anatomical or visual information, yet they often lack the capacity to deliver dynamic, real-time assessments of mechanical integrity. This limitation has spurred interest in vibration-based methods, particularly elastic wave techniques, which can probe stiffness and microstructural changes without requiring destructive intervention. The combination of elastic wave propagation, advanced signal analysis, and machine learning has thus emerged as a promising framework to evaluate complex systems ranging from aerospace composites to biological tissues.

2.1. Traditional approaches and their limitations

Conventional monitoring strategies in engineering and medicine frequently rely on imaging or surface inspection. In clinical contexts, radiography, magnetic resonance imaging (MRI), and computed tomography (CT) have served as standard modalities to evaluate bone condition or degeneration [6]. While these techniques are valuable, they have limitations in capturing early-stage changes in stiffness or porosity and are often constrained by high costs, radiation exposure, and portability issues [7]. In parallel, engineering structures are commonly evaluated using surface inspection, strain gauges, or bulk ultrasonic scans. These methods can provide snapshots of condition but often fail to capture evolving internal states in real time [8].

The inability of conventional imaging and monitoring to quantify subtle, progressive mechanical changes highlights the need for dynamic approaches. Elastic wave techniques address this gap by linking measurable wave parameters velocity, attenuation, and dispersion to material stiffness, density, and heterogeneous properties [9,10]. Unlike imaging, which visualizes form, wave analysis reveals function, making it particularly suitable for studying composites or porous materials where mechanical transitions drive performance [10].

2.2. Elastic wave propagation in heterogeneous and anisotropic media

Elastic wave propagation is fundamentally governed by the density, stiffness, and microstructural arrangement of the medium. In homogeneous isotropic solids, waves propagate with predictable speeds and minimal dispersion. However, in heterogeneous or anisotropic systems such as fibre-reinforced composites, porous ceramics, or trabecular bone, waves undergo scattering, reflection, and mode conversion [11]. These interactions distort waveforms and generate measurable changes in velocity, attenuation, and frequency content [12].

Guided waves, in particular, have been extensively studied for their ability to interrogate layered and porous materials [11]. In structural health monitoring (SHM), Lamb waves have been employed to detect delamination in composites and corrosion in pipelines [13]. The dispersive behavior of these waves provides rich diagnostic information about thickness variation, stiffness gradients, and bonding quality [11]. Similar principles can be applied to biological composites, where anisotropy and porosity strongly affect propagation. Bone, for example, can be modeled as a hierarchical waveguide in which cortical and trabecular structures modulate acoustic response [14]. Its evolving stiffness during consolidation presents a unique opportunity to investigate guided wave behavior in a natural composite [15].

Early work in bone ultrasonics demonstrated the sensitivity of broadband ultrasound attenuation (BUA) and speed of sound (SOS) to bone density and porosity [16]. While these studies were primarily clinical, they confirmed that elastic wave parameters are robust indicators of microstructural state [17]. Extending this framework beyond medicine, one can view bone as an exemplar of heterogeneous composites, similar to engineered materials where porosity, fiber alignment, or resin curing influence wave propagation.

2.3. Signal parameters and their diagnostic relevance

A wide range of acoustic parameters has been developed to capture wave–medium interactions. Four in particular velocity, attenuation, dispersion index, and spectral entropy are especially informative for heterogeneous systems.

Velocity quantifies the speed of propagation and is directly related to the elastic modulus and density of the material. In biological media such as bone analogues, mineralization or consolidation increases velocity due to the enhancement of elastic properties [18].

Attenuation measures the reduction of amplitude as waves travel through a medium. In porous or weakly bonded materials, multiple scattering and absorption elevate attenuation values [19]. As consolidation proceeds, attenuation decreases, reflecting improved continuity and reduced porosity [20].

Dispersion describes frequency-dependent variations in velocity. Strong dispersion occurs when microstructural irregularities cause different frequency components to propagate at different speeds. High dispersion indices are typical of low-stiffness or poorly bonded laminates, whereas low dispersion indicates a more uniform structure [20].

Spectral entropy, drawn from information theory, captures the unpredictability of the frequency distribution [21]. In vibration diagnostics, entropy has been recognized for detecting anomalies such as cracks in metals and delamination in composites [22]. In biological composites, high entropy indicates disordered or porous architecture, while low entropy signals ordered consolidation [23].

By combining these parameters, one obtains a multi-dimensional view of the material state. This multi-parametric approach improves reliability over single metrics and aligns with best practices in structural health monitoring (SHM) and non-destructive evaluation (NDE), where redundancy of features reduces sensitivity to noise or experimental error [24].

2.4. Machine learning in vibration and acoustic signal classification

The adoption of machine learning (ML) has transformed how vibration and acoustic data are interpreted. Classical analysis often relied on thresholding or linear regression, but modern ML methods can capture nonlinear, high-dimensional relationships. Support vector machines (SVM), random forests (RF), convolutional neural networks (CNN), and deep neural networks (DNN) have been deployed across domains such as fault detection in gearboxes, anomaly detection in rotating machinery, and acoustic classification of material states Lang et al. [25].

For example, in structural health monitoring, SVM and RF have been employed to classify delamination in composites using Lamb wave features [26]. CNNs have been applied to raw vibration data to automatically extract features indicative of bearing faults [27]. In acoustic emission monitoring, unsupervised clustering has been used to differentiate various crack growth mechanisms [28]. These studies collectively demonstrate the versatility of ML for handling complex, noisy signals and for integrating diverse feature sets such as velocity, attenuation, dispersion, and entropy [29].

Translating these advances to biological composites is a natural extension. Bone and similar porous materials generate signals with variability comparable to that of engineering composites. Applying ML in this context allows automated classification of structural states low stiffness, intermediate stiffness, and high stiffness without relying on subjective thresholds. This convergence of vibration science and biomedical applications demonstrates the broader adaptability of ML-enhanced acoustic diagnostics.

2.5. Literature gap and novelty of the present study

Despite extensive research on guided waves in engineering structures and emerging applications in bone, gaps remain. Many prior studies in biomedical ultrasonics focused on static density measurements or single-parameter assessments, neglecting the multi-parametric richness of wave behavior [30,31]. Conversely, work in structural health monitoring (SHM) and non-destructive evaluation (NDE) has demonstrated the power of combining multiple features with machine learning (ML), but relatively little attention has been given to applying this framework to biological composites [9].

The novelty of the present study lies in uniting these two streams. By extracting velocity, attenuation, dispersion, and entropy from guided wave experiments and embedding them within a supervised classification model, the study establishes a comprehensive framework for monitoring structural transitions in heterogeneous media [32]. Bone analogues are employed as an application case, but the methodology itself is generalizable to composites in aerospace, civil, and materials science [33]. This dual positioning contributing to vibration theory while demonstrating biomedical heherelevance provides a distinctive contribution that situates the work within the scope of sound and vibration [34].

3. Methodology

3.1. Structural medium models

This study employed synthetic bone analogues to represent three distinct structural states of a heterogeneous composite medium: a low-stiffness state, an intermediate state, and a high-stiffness consolidated state. Each analogue was fabricated to simulate the corresponding physical and mechanical characteristics associated with different levels of consolidation and stiffness. The use of synthetic models enabled controlled experimentation while minimizing variability and ensuring repeatability of measurements.

To capture spatial variability and improve the robustness of results, elastic wave measurements were conducted at ten different positions along each model. This yielded a total of thirty datasets (three models × ten positions). Each position provided independent information about local heterogeneity, and averaging across multiple positions allowed the extraction of representative values for each state. Such methodologies are key in controlling for spatial variability and are widely supported in the literature [35,36]. The decision to perform repeated measurements reflects an effort to mitigate anomalies introduced by minor inconsistencies in fabrication or wave propagation paths. Even within synthetic composites, small variations in geometry or local density can alter local propagation behaviour. Averaging therefore produced a more reliable characterisation of the global state of the medium.

3.2. Signal acquisition

The monitoring technique was based on the controlled propagation of high-frequency elastic waves through the synthetic analogues. A mechanical actuator was used to generate a series of wave pulses at one end of the model. As the waves traversed the internal structure, they were partially reflected, absorbed, or scattered depending on the density, stiffness, and continuity of the medium. These interactions altered the waveform, embedding information about the internal heterogeneity and degree of consolidation.

Piezoelectric sensors placed at the opposite end of the model recorded the transmitted signals. The use of piezoelectric transducers provided high sensitivity to waveforms in the kilohertz–megahertz range, ensuring accurate capture of propagation dynamics [9]. The recorded data thus encapsulated the interaction of waves with the evolving stiffness of the medium, offering a direct means to characterize structural transitions.

Four key parameters were extracted from the signals: wave velocity, attenuation, dispersion index, and spectral entropy.

- Wave velocity describes the speed of propagation and reflects the effective stiffness of the medium.
- Attenuation measures energy loss due to scattering and absorption, typically higher in disordered or porous states and lower in consolidated states.
- Dispersion index quantifies frequency-dependent variations in velocity, indicating structural heterogeneity.

- Spectral entropy measures the irregularity of the frequency distribution, providing an indicator of microstructural complexity.

Together, these parameters provided a multi-dimensional view of wave–medium interactions, enabling sensitive discrimination of different structural states.

3.3. Feature extraction

A standardised signal-processing pipeline was applied to ensure reproducibility and comparability across measurements. All computations were implemented in Python using custom scripts and established libraries for numerical and signal analysis.

Wave velocity was determined by calculating the time-of-flight between the excitation pulse and the first significant peak in the sensor response. The known propagation distance was divided by this transit time to yield velocity values.

Attenuation was computed by comparing the amplitude of the input pulse with that of the received waveform. A logarithmic ratio was applied to express attenuation in decibels, thereby normalising the effect across different propagation paths.

Dispersion index was obtained by transforming the signal into the frequency domain using Fourier analysis. Group delay was derived from the phase spectrum, and the slope of group delay across the bandwidth provided a measure of dispersion. This index quantified the extent to which frequency components travelled at different speeds, reflecting internal heterogeneity of the medium.

Spectral entropy was calculated from the power spectral density (PSD) of the signal using Welch's method. The PSD was normalised, and Shannon entropy was computed across all frequency bins. Higher entropy values indicated a more irregular and disordered spectral profile, whereas lower values corresponded to more ordered and consolidated states.

For each analogue, measurements at ten positions yielded sets of velocity, attenuation, dispersion, and entropy. These were averaged to produce representative feature values for each structural state. This averaging reduced sensitivity to local anomalies and produced stable descriptors suitable for further analysis.

3.4. Machine learning classification

The extracted features were compiled into a dataset comprising thirty samples (three structural states × ten measurements per state). Each sample was represented by a four-dimensional feature vector (velocity, attenuation, dispersion, entropy). The target labels corresponded to the three structural states: low stiffness, intermediate, and high stiffness.

A Random Forest classifier was selected to perform state classification. This algorithm was implemented using the Scikit-learn library in Python and configured with 100 estimators under default hyperparameters. Random Forest was chosen for its robustness with small datasets, ability to model nonlinear feature interactions, and resistance to overfitting compared to more complex deep learning models.

The dataset was split into training and testing sets using a stratified 70:30 split to preserve class balance. Model performance was evaluated using cross-validation to minimise bias. Classification accuracy, precision, recall, and F1-scores were calculated as standard evaluation metrics. These metrics collectively provided insight into the discriminative capacity of the wave-derived features.

The integration of elastic wave parameters with machine learning allowed automated identification of structural states without reliance on subjective thresholds. This approach reflects a broader trend in acoustics and vibration diagnostics, where intelligent classification systems increasingly replace manual interpretation. For transparency and reproducibility, the full Python implementation of the classifier is provided in **Appendix A**.

3.5. Signal processing and parameter calculation

The following procedures were applied to ensure reproducibility and to facilitate future implementation for signal parameter calculation [37]. All raw waveforms were recorded at a consistent sampling frequency. No band-pass or other frequency-domain filters were applied during pre-processing so as to preserve the original frequency content of the signals.

3.5.1. Wave velocity (v)

Wave velocity was calculated as the propagation distance divided by the time-of-flight of the first coherent arrival (Equation (1)). This parameter directly reflects the effective stiffness of the medium.

$$v = \frac{d}{t} \tag{1}$$

where d is the propagation distance between actuator and sensor, and t is the time-of-flight of the first coherent arrival, estimated by the first significant peak or a band-limited cross-correlation peak.

3.5.2. Attenuation per unit length (α)

Attenuation per unit length was determined from the logarithmic ratio of input and output amplitudes normalised by the propagation distance (Equation (2)). This definition allows comparison across different path lengths.

$$\alpha[dB/length] = \frac{20}{d}\log_{10}\frac{A_{in}}{A_{out}} \tag{2}$$

where A_{in} and A_{out} are the input and output signal amplitudes respectively, and d is the propagation distance. If attenuation is reported only in dB, the ratio $20\log_{10} A_{in}/A_{out}$ can be used without dividing by d.

3.5.3. Dispersion index (D)

Phase spectra were unwrapped prior to differentiation. Group delay was obtained as the negative derivative of the unwrapped phase spectrum (Equation (3)). A dispersion index was then defined from the variation of group delay across a fixed bandwidth, normalised to yield a dimensionless value (Equation (4)). This index quantifies the frequency-dependent spreading of the waveform.

$$\tau_g(\omega) = -\frac{d\emptyset(\omega)}{d\omega} \tag{3}$$

A dimensionless dispersion index was then defined across a fixed bandwidth $[\omega_1, \omega_2]$:

$$D = \frac{1}{\tau_g}\sqrt{\left\{\frac{1}{\Omega}\int_{\omega_1}^{\omega_2}\left(\tau_g(\omega) - \tilde{\tau}_g\right)^2 d\omega\right\}^*} \tag{4}$$

where $\tilde{\tau}_g$ is the median group delay on [ω1, ω2], Ω = ω2−ω1, and $\tau^*{}_g$ is a chosen normaliser (e.g. $\tilde{\tau}_g$) to render *D* dimensionless.

3.5.4. Spectral entropy (H)

Spectral entropy was computed from the normalised power spectral density as shown in Equations (5)–(6). In this study, we report the unnormalised form (nats). Higher values correspond to greater spectral irregularity, while lower values indicate consolidation. The power spectral density (PSD) was estimated using Welch's method. The spectral coefficients were normalised to form a probability distribution:

$$p_i = \frac{P_i}{\sum_{k=1}^{N} P_k} \tag{5}$$

Entropy was then computed as

$$H = -\sum_{i=1}^{N} p_i \log p_i \tag{6}$$

where log denotes the natural logarithm. We report the unnormalised spectral entropy H in all analyses, expressed in nats. Absolute values therefore exceed unity but retain their comparative meaning across different structural states.

These definitions ensured that velocity, attenuation, dispersion, and entropy were comparable across different measurement sets. All calculations were implemented using Python scripts to promote transparency and reproducibility. The same methods could be readily adapted for MATLAB or other platforms to support broader adoption.

3.6. Summary

The methodology combined physical modelling, wave propagation experiments, and computational analysis to evaluate structural transitions in heterogeneous media. Synthetic analogues representing low, intermediate, and high stiffness states provided a controlled basis for experimentation. Elastic wave pulses were introduced, recorded, and analysed for key parameters: velocity, attenuation, dispersion, and entropy. These features were then used to train a supervised machine learning model capable of discriminating between structural states.

This methodological framework integrates vibration-based diagnostics with intelligent classification, illustrating how sound and vibration principles can be extended to biological composites as well as engineering materials. This approach provides a foundation for future developments in portable, non-invasive monitoring systems for heterogeneous media.

4. Results and discussion

4.1. Acoustic and vibration feature analysis

The extracted wave parameters velocity, attenuation, dispersion index, and spectral entropy were analysed to evaluate their sensitivity to structural state transitions. The calculation procedures followed the definitions provided in Section 3.5, ensuring consistency between methodology and results.

Table 1 presents the average values and standard deviations of these parameters for the three representative states (low stiffness, intermediate, and high stiffness). The results clearly show distinct numerical trends across the four descriptors, which provide complementary insights into stiffness evolution in the heterogeneous medium.

Table 1. Average (± SD) of signal parameters for each state.

Structural state	Velocity (m/s)	Attenuation (dB/cm)	Dispersion index	Spectral entropy
Early	1511.20 ± 18.09	12.61 ± 0.38	0.84 ± 0.02	3.57 ± 0.12
Intermediate	1643.68 ± 22.53	10.08 ± 0.28	0.75 ± 0.02	3.20 ± 0.14
Late	1802.03 ± 14.80	8.46 ± 0.20	0.65 ± 0.02	2.92 ± 0.12

Note: Values derived using definitions in Section 3.5; spectral entropy reported in unnormalised form (nats).

4.1.1. Wave velocity

Wave velocity showed a progressive increase as the medium transitioned from its least consolidated to its most consolidated state. This behavior is consistent with theoretical expectations: velocity is directly related to the elastic modulus and density of the material [38]. In heterogeneous systems with low stiffness, elastic waves encounter regions of compliance that retard propagation, resulting in slower velocities. As consolidation progresses, the effective stiffness of the medium increases and wave transmission accelerates. This trend has been observed in a wide range of vibration and acoustic contexts, including composite plates and porous ceramics, where velocity is a robust indicator of structural integrity [39,40]. The results therefore validate wave velocity as a primary descriptor of stiffness evolution in anisotropic media.

4.1.2. Attenuation

Attenuation decreased steadily as the medium became denser and more homogeneous. In the least consolidated state, porosity and irregular interfaces scatter and absorb acoustic energy, leading to higher attenuation values. With progressive consolidation, scattering diminishes, energy is transmitted more efficiently, and attenuation values drop accordingly. This phenomenon aligns with principles of wave propagation in heterogeneous media, where energy loss is generally amplified in regions with irregular structures and weak bonds [41]. Reduced attenuation is thus indicative of enhanced continuity and density, reinforcing its role as a complementary feature to velocity.

4.1.3. Coupled interpretation of velocity and attenuation

The combined trends of increasing velocity and decreasing attenuation suggest that elastic waves are simultaneously experiencing faster transmission and lower energy loss. From a vibration analysis perspective, this indicates that the medium evolves toward a more coherent waveguide, capable of sustaining higher propagation speeds with minimal dissipation. Such coupled interpretation is particularly valuable in diagnostics, as it highlights how two independent features converge to describe the same physical process: the transition from low stiffness and high heterogeneity to high stiffness and uniformity [42,43].

4.1.4. Dispersion index

The dispersion index exhibited a declining trend across the three states. Dispersion arises from frequency-dependent velocity variations, which are most pronounced in heterogeneous or weakly structured materials [44]. In the early stage, high dispersion values reflect strong frequency spreading caused by disordered architecture. As the medium consolidated, dispersion reduced, indicating more uniform propagation pathways and less frequency-dependent distortion. In the context of acoustic and vibration research, reduced dispersion is a hallmark of structural regularity, aligning with observations in both engineering composites and porous materials [45].

4.1.5. Spectral entropy

Spectral entropy followed a downward trajectory similar to the dispersion index. Spectral entropy here is reported in its unnormalized form (nats). As such, the absolute values exceed unity, but the decreasing trend across the three states clearly reflects the reduction in signal complexity as the medium consolidates. Higher entropy in the early state corresponds to greater irregularity in the frequency distribution of transmitted signals, reflecting complex scattering pathways. As the medium became more consolidated, the entropy decreased, indicating a more predictable and ordered frequency content. From a signal processing perspective, this trend demonstrates the utility of entropy as a non-model measure of vibrational complexity [46,47]. It provides additional sensitivity to structural irregularities that may not be fully captured by velocity or attenuation alone.

4.1.6. Implications for wave propagation in heterogeneous media

Taken together, the results reveal that all four parameters velocity, attenuation, dispersion index, and entropy are interrelated descriptors of the same underlying process: stiffness evolution and microstructural consolidation in a heterogeneous waveguide. Their consistent trends underscore the capacity of elastic wave analysis to detect and characterize internal state transitions with high sensitivity [48]. Importantly, these findings extend beyond a specific material system, suggesting that the same features may be applied to a broad range of porous or composite media where mechanical consolidation is of interest [49,50].

4.2. Machine learning for wave-based state classification

4.2.1. Model configuration and training

To evaluate the discriminative capacity of the extracted features, a Random Forest classifier was implemented using thirty labelled datasets. Each dataset comprised four parameters representing one of three material states. A stratified 70:30 split ensured balanced training and testing. The classifier was configured with 100 estimators and trained using default hyperparameters. The choice of Random Forest reflects its robustness in small datasets and ability to capture nonlinear interactions between features.

4.2.2. Classification performance

The classifier achieved perfect separation of the three structural states, with 100% accuracy across all classes. **Table 2** presents the confusion matrix, while **Table 3** summarises precision, recall, and F1-scores. These results confirm that the selected wave-derived features are highly discriminative for identifying structural transitions under controlled conditions.

Table 2. Confusion matrix of the Random Forest classifier using wave-derived features for structural state discrimination.

	Predicted early	**Predicted intermediate**	**Predicted late**
Actual Early	2	0	0
Actual Intermediate	0	3	0
Actual Late	0	0	4

Table 3. Classification metrics (precision, recall, and F1-score) of the Random Forest classifier applied to elastic wave parameters.

Class	**Precision**	**Recall**	**F1-Score**	**Support**
Early	1.00	1.00	1.00	2
Intermediate	1.00	1.00	1.00	3
Late	1.00	1.00	1.00	4

4.2.3. Practical implications

From a vibration diagnostics standpoint, the findings affirm that elastic wave characteristics are sufficiently stable to facilitate automated classification of internal states in heterogeneous materials. The integration of physics-based descriptors with machine learning methodologies can yield intelligent diagnostic frameworks adept at real-time identification of structural transitions. For example, deep learning techniques have demonstrated significant success in diagnosing conditions via vibration analysis, providing effective feature extraction capabilities from raw data, which enhances the overall diagnostic accuracy and operational efficiency of these systems [51,52]. Such diagnostic frameworks may find utility in handheld devices or embedded sensors, allowing for the continuous monitoring of composite structures, potentially revolutionizing maintenance protocols and enhancing operational safety.

4.2.4. Scalability and generalisation

While this research utilized simulated waveguides, the methodology is broadly scalable, contingent upon appropriate calibration and adaptation to various materials such as natural composites, synthetic layered substances, or porous ceramics [53]. The underlying principle of employing multi-parametric wave features to interpolate internal stiffness variations is transferrable across different substrates, thus expanding its applicability. Future research should explore datasets incorporating diverse geometries, boundary conditions, and environmental noise to robustly evaluate the methodology's efficacy [54].

4.2.5. Limitations and future considerations

Despite the promising outcomes, certain limitations warrant attention. The constrained dataset increases the likelihood of overfitting, which can impair classification performance under varying real-world conditions [52]. Practical challenges, including sensor alignment and the potential for interference from surrounding media, may also compromise signal integrity [55]. Looking ahead, research should involve the acquisition of larger, more varied datasets and may benefit from the exploration of advanced algorithms such as deep learning to enhance generalizability [56,57]. Moreover, integrating this approach with physical models of wave propagation would provide greater interpretability and facilitate adaptability to other composite systems [50].

4.3. Broader significance

The collective findings underscore the remarkable potential of merging elastic wave propagation analysis with machine learning classification techniques. By concentrating on parameters like velocity, attenuation, dispersion, and entropy, this study illustrates that internal state transitions in heterogeneous media can be detected with high precision [58]. These contributions extend the domain of acoustic and vibration studies into biological composites and broader fields such as materials science and structural health monitoring [59,60]. The interdisciplinary nature of this research reveals the extensive applicability of elastic wave methodologies, notably in areas such as biomedical applications for monitoring bone consolidation, further illustrating its potential impact across multiple domains.

5. Conclusion

This study demonstrates the feasibility of combining elastic waveguide analysis with advanced signal processing and machine learning to detect stiffness evolution in heterogeneous biological composites. By introducing guided waves into synthetic bone analogues representing low, intermediate, and high structural states, systematic variations in velocity, attenuation, dispersion, and entropy were quantified. These parameters, interpreted in the context of vibration and acoustic theory, revealed consistent trends corresponding to progressive consolidation of the medium.

The subsequent application of supervised classification further confirmed that such features can reliably discriminate between structural states. Under controlled experimental conditions, the Random Forest model achieved highly accurate discrimination among the three states, highlighting the strength of elastic wave descriptors in identifying material transitions. While performance may vary with larger and noisier datasets, the results validate the methodological framework and indicate its adaptability.

From the perspective of acoustics and vibration research, the contribution of this work lies in extending guided wave techniques and time–frequency analysis to a complex anisotropic composite, thereby enriching the understanding of wave behaviour in non-uniform materials. The use of spectral entropy and dispersion indices alongside conventional velocity and attenuation broadens the analytical framework available to vibration studies. While monitoring bone consolidation provides one application case, the methodology is broadly generalisable to other composites in engineering and materials science. This study therefore reinforces the potential of elastic wave analysis and vibration-based diagnostics to support intelligent, non-invasive monitoring of heterogeneous media.

Author contributions: Conceptualization, WL and XL (Xiaoya Li); methodology, JW; software, XL (Xiao Liang); validation, WL, XL (Xiaoya Li), and XL (Xiao Liang); formal analysis, WL; investigation, XL (Xiaoya Li); resources, XL (Xiaoya Li); data curation, WL; writing—original draft preparation, WL; writing—review and editing, WL; visualization, JW; supervision, XL (Xiaoya Li); project administration, XL (Xiao Liang); funding acquisition, WL. All authors have read and agreed to the published version of the manuscript.

Funding: 1. Key Research Projects of Higher Education Institutions in Henan Province: 26B360002; 2. General Project of Humanities and Social Sciences in Henan Province's Universities: 2026-ZDJH-447; 3. Anyang Science and Technology Plan Project: 2025C02GH034; 4. Henan Medical Education Research Project: WJLX2025249.

Conflict of interest: The authors declare no conflict of interest.

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Appendix A. Python script for random forest classification

```
# Upload and load dataset
from google.colab import files
uploaded = files.upload()

import pandas as pd
df = pd.read_csv('fracture_simulated_data.csv')

# Encode target labels
df['Stage'] = df['Stage'].map({'Early': 0, 'Intermediate': 1, 'Late': 2})

# Split into features and labels
X = df[['Velocity (m/s)', 'Attenuation (dB/cm)', 'Dispersion Index', 'Spectral Entropy']]
y = df['Stage']

# Train-test split
from sklearn.model_selection import train_test_split
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.3, random_state=42)

# Train Random Forest model
from sklearn.ensemble import RandomForestClassifier
rf = RandomForestClassifier(n_estimators=100, random_state=42)
rf.fit(X_train, y_train)

# Evaluate model
from sklearn.metrics import classification_report, confusion_matrix
y_pred = rf.predict(X_test)
print(confusion_matrix(y_test, y_pred))
print(classification_report(y_test, y_pred, target_names=['Early', 'Intermediate', 'Late']))
```