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Article

# Bifurcation analysis and multiobjective nonlinear model predictive control of forests global warming and carbon dioxide emission

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**Abstract:** Bifurcation analysis and multiobjective nonlinear model predictive control calculations are performed on problems involving forestation, human population growth, global warming and carbon dioxide emission. The bifurcation analysis confirms the existence of the oscillation causing Hopf bifurcations. An activation factor involving the tanh function is shown to eliminate the Hopf bifurcations. The multiobjective nonlinear model predictive control (MNLMPC) calculations were performed taking into account the inevitable human population growth and reduction in forest area to obtain control parameters that can be most beneficial. Bifurcation analysis was performed using the MATLAB software MATCONT while the multiobjective nonlinear model predictive control was performed by using the optimization language PYOMO.

**Keywords:** bifurcation; optimal control; multiobjective; global warming; carbon dioxide

## 1. Background

The negative effects of the increase in carbon dioxide in the atmosphere have led to global warming which in turn has caused extreme weather events like hurricanes and floods. Industrialization has created an increase in carbon dioxide [1,2]. global warming has also created health risks making human beings vulnerable to various fatal diseases [3–8]. In addition to industrialization, deforestation has been responsible for the carbon dioxide concentration increase in the atmosphere.

Fossil fuel burning has caused a tremendous increase in atmospheric carbon dioxide concentration [9]. The presence of forests has led to the sequestration of a considerable amount of carbon dioxide during photosynthesis. Hence the destruction of the forests because of human activities has also contributed significantly to the increase of carbon dioxide in the atmosphere [10–11]. Some strategies are being developed to reduce the amount of carbon dioxide in the atmosphere. However, it is important to develop strategies to minimize the amount of carbon dioxide being ejected into the atmosphere. Mathematical models have been developed to try and understand the dynamics of carbon dioxide in the atmosphere. The effect of excessive deforestation on the amount of carbon dioxide and the destabilizing effect of human activities on the atmosphere was computationally investigated [12–14].

Significant work involving mathematical models studying the dynamic relationship between forestation and atmospheric carbon dioxide was performed by Devi and Gupta [15,16]; Devi and Mishra [17]; Misra and Verma [18,19]; Misra et al. [20]; Shukla et al. [21,22]; Verma and Misra [23] and Panja [24]. The effects of deforestation on the increase of carbon dioxide and its effects on human life was investigated by several workers (Angelsen and Kaimowitz. [25], Pimm et al. [26], Angelsen et al. [27], Defries et al. [28], van der Werf et al. [29], Lonngren and Bai

[30], Ghommem et al. [31], Florides and Christodoulides [32], Newell and Marcus [33]. All this work involved bifurcation analysis and dynamic optimization. The bifurcation analysis revealed the existence of Hopf bifurcations that cause unwanted oscillations. Furthermore, all the dynamic optimization work involves single-objective optimal control.

The main objectives of this work are to conduct a rigorous bifurcation analysis and use an activation factor to eliminate the oscillation causing Hopf bifurcation points and perform rigorous multiobjective nonlinear model predictive control calculations on models involving forestation global warming, atmospheric carbon dioxide concentration increase taking into account the inevitable increase in human population and decrease in forest density.

## 2. Motivation

All previous work involving bifurcations of forest models demonstrate the existence of Hopf bifurcation points which cause inconvenient oscillations. No strategy has been so far provided to eliminate these Hopf bifurcations. The main motivation for performing the bifurcation analysis is to demonstrate the existence of Hopf bifurcations which cause oscillations and to use an activation factor to eliminate these oscillatory-causing bifurcations. All previous work optimization work regarding forests involved single objective optimal control calculations. No prior work has been done regarding multiobjective nonlinear model predictive control (MNLMPC) calculations. This motivates the performance of MNLMPC calculations of forests involving global warming carbon dioxide emission. This article is the first work where the oscillation-causing Hopf bifurcation points that occur in forest models have been eliminated with an activation factor and where multiobjective nonlinear model predictive control calculations were performed. Previous work [18–24] show the existence of Hopf bifurcations (but no strategy to eliminate these bifurcations) and perform single objective calculations.

## 3. Highlights of this work

The main highlights of this paper are:

1) To perform Rigorous bifurcation analysis and multiobjective nonlinear model predictive control calculations d on existing and modified models regarding forest density global warming and atmospheric carbon dioxide.
2) To use an activation factor to eliminate the undesirable oscillation causing Hopf bifurcations revealed by the bifurcation analysis.
3) To consider inevitable situations such as reduction in forest area and increase in the human growth population to obtain control values and minimize the damage.

This paper is organized as follows. The bifurcation analysis techniques and the multiobjective nonlinear model predictive control strategies are first discussed. This is followed by a description of the problems, results and discussion and conclusions.

## 4. Bifurcation analysis

The existence of multiple steady-states (caused by limit and branch point singularities) and oscillatory behavior caused by Hopf bifurcation points) in chemical processes has led to a lot of computational work to explain the causes of these nonlinear phenomena. n

MATCONT, (Dhooge and co-workers [34,35]) is a commonly used software to find limit points, branch points, and Hopf bifurcation points. Consider an ODE system

$$\dot{x} = f(x, \beta) \tag{1}$$

$x \in R^n$ The tangent plane at any point $x$ is$v_1, v_2, v_3, v_4, \dots, v_{n+1}$. Define matrix A given by

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} \cdots\cdots\cdots \frac{\partial f_1}{\partial x_n} & \frac{\partial f_1}{\partial \beta} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} \cdots\cdots\cdots \frac{\partial f_2}{\partial x_n} & \frac{\partial f_2}{\partial \beta} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \frac{\partial f_n}{\partial x_3} & \frac{\partial f_n}{\partial x_4} \cdots\cdots\cdots \frac{\partial f_n}{\partial x_n} & \frac{\partial f_n}{\partial \beta} \end{bmatrix} \tag{2}$$

With $\beta$ the bifurcation parameter. The matrix A can be written in a compact form as

$$A = [B| \quad \frac{\partial f}{\partial \beta}]. \tag{3}$$

The tangent surface must satisfy

$$Av = 0 \tag{4}$$

For both limit and branch points the matrix B must be singular. For a limit point (LP) the (*n*+1) th component of the tangent vecto $v_{n+1} = 0$ and for a branch point (BP) the matrix $\begin{bmatrix} A \\ V^T \end{bmatrix}$ must be singular., The function $\det\,(2f_x(x, \beta) \odot I_n)$ should be zero for a Hopf bifurcation point. $\odot$indicates the bialternate product while $I_n$ is the n-square identity matrix. A detailed derivation can be found in Kuznetsov [36,37] and Govaerts [38]. Sridhar [39] used Matcont to perform bifurcation analysis on chemical engineering problems.

Activation Factor:

The tanh activation factor is used in neural networks (Szandała, [40]; Kamalov et al. [41]; Dubey et al. [42]) and in optimal control problems to eliminate spikes in the optimal control profile (Sridhar [43–47]. Sridhar [48] showed that the presence of a singular points (limit and branch points) would enable the multiobjective optimal control problems to converge to the Utopia solution. Hopf bifurcation points cause oscillatory behavior. Oscillations resemble spikes. This work uses this tanh activation factor to eliminate the Hopf bifurcation points. The bifurcation parameter (control variable) $\xi$ is replaced by with $\frac{\xi \tanh(\xi)}{\varepsilon}$ where $\varepsilon$ is an arbitrary constant.

## 5. (Multiobjective Nonlinear Model predictive control) method

The multiobjective nonlinear model predictive control (MNLMPC) method was first proposed by Flores Tlacuahuaz et al. [49] and used by Sridhar [50]. This method is rigorous and it does not involve the use of weighting functions not does it impose additional parameters or additional constraints on the problem unlike the weighted function or the epsilon correction method (Miettinen; [51]. For a problem that is posed as

$$\begin{aligned} &min\, J\,(x,u) = (x_1, x_2 \ldots . x_k) \\ &subject\ to\ \ \frac{dx}{dt} = F(x,u); h(x,u) \leq 0; x^L \leq x \leq x^U; u^L \leq u \leq u^U \end{aligned} \tag{5}$$

The MNLMPC method first solves dynamic optimization problems independently minimizing/maximizing eac $x_i$ individually. The minimization/maximization of $x_i$ will lead to the values $x_i^*$ . Then the optimization problem that will be solved is

$$\begin{aligned} &min \sqrt{\ \{x_i - x_i^*\}^2} \\ &subject\ to\ \ \frac{dx}{dt} = F(x,u); h(x,u) \leq 0; x^L \leq x \leq x^U; u^L \leq u \leq u^U \end{aligned} \tag{6}$$

This will provide the control values for various times. The first obtained control value is implemented and the remaining discarded. This procedure is repeated until the implemented and the first obtained control value are the same.

The optimization package in Python, Pyomo (Hart et al. [52]) where the differential equations are automatically converted to a Nonlinear Program (NLP) using the orthogonal collocation method (Biegler, [53]) is commonly used for these calculations. The (5state of the art solvers like IPOPT (Wachter and Biegler, [54] and BARON (Tawaralmani and Sahinidis; [55]) are normally used in conjunction with PYOMO.

## 6. Description of problems

The problems investigated are 1) Misra and Verma [56] who developed d a mathematical model to study the dynamics of carbon dioxide gas in the atmosphere 2) Panja [57] who performed a modeling study about deforestation, carbon dioxide increase in the atmosphere and global warming 3) Verma and Verma [56] who studied the Effect of plantation of genetically modified trees on the control of atmospheric carbon dioxide 4) Misra and Lata [57] who developed a mathematical model to achieve sustainable forest management 5) A modified model of Misra and Lata [57] to include carbon dioxide sent into the atmosphere. A mathematical description of each of these problems will now be presented.

Problem 1:

The Variables are

- Xa(t) carbon dioxide concentration in the atmosphere
- Na(t) human population
- Fa(t) Forest biomass

- $\phi(t)$ deforestation rate-coefficient (bifurcation parameter and control variable)

The parameters are:

$\pi_1 = 0.01(\frac{ton}{ppm}); \pi = 0.01(\frac{person}{ton}); M = 2000(ton); u = 0.2(\frac{1}{month}); \theta = 0.00001(\frac{1}{ppm.month});$
$L = 1000(person); s = 0.01(\frac{1}{month}); \lambda_1 = 0.0001(\frac{1}{month.ton}); \alpha = 0.003(\frac{1}{month});$
$\lambda = 0.05(\frac{ppm}{person.month}); Q_0 = 1(\frac{1}{ppm.month})$

The equations are:

$$\begin{aligned} \frac{dX_a}{dt} &= Q_0 + \lambda N_a - \alpha X_a - \lambda_1 X_a F_a \\ \frac{dN_a}{dt} &= SN_a(1 - \frac{N_a}{L}) - \theta X_a N_a + \pi\phi N_a F_a \\ \frac{dF_a}{dt} &= uF_a(1 - \frac{F_a}{M}) - \phi N_a F_a + \pi_1 \lambda_1 X_a F_a \end{aligned} \tag{7}$$

Problem 2:

The second problem involves the same three equations as the first problem in addition to another fourth equation that considers the variation of the quantity of global warming at time, $G_a(t)$.

Hence the variables are:

- Xa(t) carbon dioxide concentration in the atmosphere
- Na(t) human population
- Fa(t) Forest biomass
- $\phi(t)$ deforestation rate-coefficient (bifurcation parameter and control variable)
- Ga(t) variation of the quantity of global warming at time.

The parameters are

$\pi_1 = 0.01(\frac{ton}{ppm}); \pi = 0.01(\frac{person}{ton}); M = 2000(ton); u = 0.2(\frac{1}{month}); \theta = 0.00001(\frac{1}{ppm.month});$
$L = 1000(person); s = 0.01(\frac{1}{month}); \lambda_1 = 0.0001(\frac{1}{month.ton}); \alpha = 0.003(\frac{1}{month});$
$\lambda = 0.05(\frac{ppm}{person.month}); Q_0 = 1(\frac{1}{ppm.month}), B = 0.6; \gamma = 0.2; \gamma_1 = 0.1; d = 0.1$

While the equations are:

$$\begin{aligned} \frac{dX_a}{dt} &= Q_0 + \lambda N_a - \alpha X_a - \lambda_1 X_a F_a \\ \frac{dN_a}{dt} &= SN_a(1 - \frac{N_a}{L}) - \theta X_a N_a + \pi\phi N_a F_a \\ \frac{dF_a}{dt} &= uF_a(1 - \frac{F_a}{M}) - \phi N_a F_a + \pi_1 \lambda_1 X_a F_a \\ \frac{dG_a}{dt} &= B + \gamma * X_a(t) + \gamma_1 * N_a(t) - d * G_a(t); \end{aligned} \tag{8}$$

Problem 3:

$$\begin{aligned}\frac{dC_a}{dt} &= Q + \lambda N_a - \alpha C_a - \lambda_1 + (\frac{\gamma_1 T_a}{k_1 + T_a})C_a F_a \\ \frac{dN_a}{dt} &= rN_a(1 - \frac{N_a}{k}) - \theta C_a N_a + \pi_1 \phi N_a F_a \\ \frac{dF_a}{dt} &= uF_a(1 - \frac{F_a}{M}) - \phi N_a F_a + \beta T_a F_a \\ \frac{dT_a}{dt} &= \nu(M - F_a) - \nu_0 T_a\end{aligned} \tag{9}$$

The variables are:

- Ca(t), atmospheric carbon dioxide concentration
- Na(t) human population
- Fa(t) forest area
- Ta(t) measure of plantation efforts

The parameters are:

$K_1 = 300(\text{million-dollars}); K = 11(\text{billion} - \text{persons}); M = 5900(\text{million} - \text{ha}); u = 0.005(\frac{1}{year});$
$\theta = 5.3765\, e - 08(\frac{1}{ppm(year)}); \lambda_1 = 7.5681\text{ e-07}(\frac{1}{\text{million-ha(year)}}); \alpha = 0.006(\frac{1}{year});$
$\beta = 3.e - 06(\frac{1}{\text{million-dollars(year)}}); \pi_1 = 0.001(\frac{\text{billion-persons}}{\text{million-ha}}); \lambda = 0.564(\frac{\text{ppm}}{\text{billion-person(year)}});$
$Q = 1.68(\frac{ppm}{year}); \gamma_1 = 5.e - 07(\frac{\text{million-ha}}{\text{year}}); \upsilon = 0.003(\frac{\text{million-dollars}}{\text{million-ha(year)}}); \upsilon_0 = 0.01(\frac{1}{year}); r = 0.026(\frac{1}{year})$
$\phi = 0.00042371(\frac{1}{\text{billion-person(year)}})$

Problem 4

The variables are:

- Ba is the density of forest resources
- Na is the density of the human population
- Pa is the demand of the population for forest-based product
- Ta is the measure of technological efforts to conserve forest
- Ea is the measure of efforts to reduce usage of forest resources

The equations are:

$$\begin{aligned}\frac{dB_a}{dt} &= SB(1 - \frac{B_a}{L}) - \alpha B_a N_a - \lambda_2 {B_a}^2 P_a + \phi_1 B_a T_a + \phi_2 {B_a}^2 T_a \\ \frac{dN_a}{dt} &= rN_a(1 - \frac{N_a}{K}) + \pi\alpha B_a N_a \\ \frac{dP_a}{dt} &= \lambda N_a - \lambda_0 P_a - \lambda_1 P_a E_a \\ \frac{dE_a}{dt} &= \psi P_a - \psi_0 E_a \\ \frac{dT_a}{dt} &= \phi(L - B_a) - \phi_0 T_a\end{aligned} \tag{10}$$

Parameters

$S = 0.6, \alpha = 0.008, L = 50, \lambda_2 = 0.0011, \phi_1 = 0.02, \phi_2 = 0.0006,$
$r = 0.1, K = 100, \pi = 0.01, \lambda = 0.1, \lambda_0 = 0.8, \lambda_1 = 0.1,$
$\psi = 0.2, \psi_0 = 0.1, \phi = 0.01, \phi_0 = 0.03$

Problem 5

The variables are

- Ba is the density of forest resources
- Na is the density of the human population
- Pa is the demand of the population for forest-based product
- Ta is the measure of technological efforts to conserve forest
- Xa amount of carbon dioxide emitted
- Ea is the measure of efforts to reduce usage of forest resources

The equations are:

$$\begin{aligned}
\frac{dB_a}{dt} &= SB(1-\frac{B_a}{L}) - \alpha B_a N_a - \lambda_2 {B_a}^2 P_a + \phi_1 B_a T_a + \phi_2 {B_a}^2 T_a \\
\frac{dN_a}{dt} &= rN_a(1-\frac{N_a}{K}) + \pi\alpha B_a N_a \\
\frac{dP_a}{dt} &= \lambda N_a - \lambda_0 P_a - \lambda_1 P_a E_a \\
\frac{dE_a}{dt} &= \psi P_a - \psi_0 E_a \\
\frac{dT_a}{dt} &= \phi(L - B_a) - \phi_0 T_a \\
\frac{dX_a}{dt} &= Q_0 + l_6 N_a - a_6 X_a - m_6 X_a B_a
\end{aligned} \tag{11}$$

Parameters

$S = 0.6, \alpha = 0.008, L = 50, \lambda_2 = 0.0011, \phi_1 = 0.02, \phi_2 = 0.0006,$
$r = 0.1, K = 100, \pi = 0.01, \lambda = 0.1, \lambda_0 = 0.8, \lambda_1 = 0.1,$
$\psi = 0.2, \psi_0 = 0.1, \phi = 0.01, \phi_0 = 0.03, l_6 = 0.05, a_6 = 0.003, m_6 = 0.0001, q_0 = 1$

## 7. Results

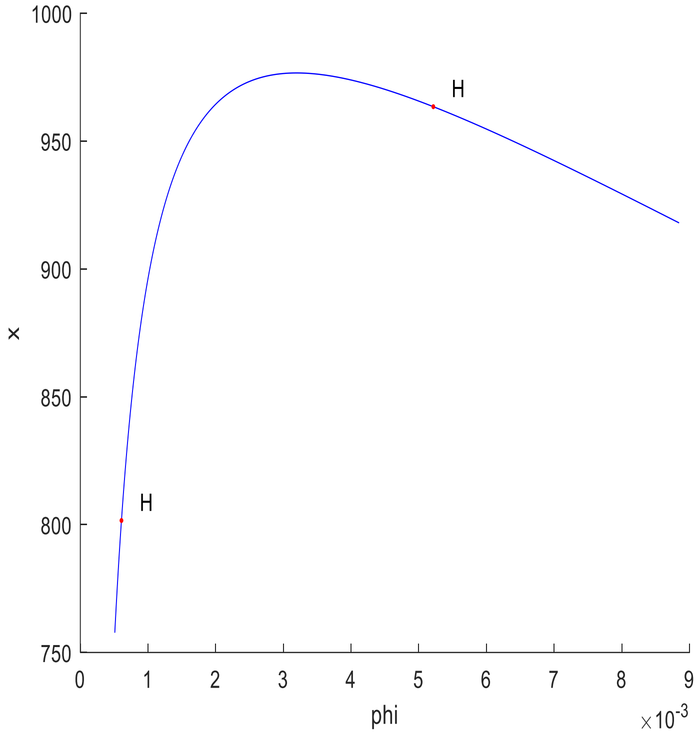
Problem 1

Bifurcation analysis

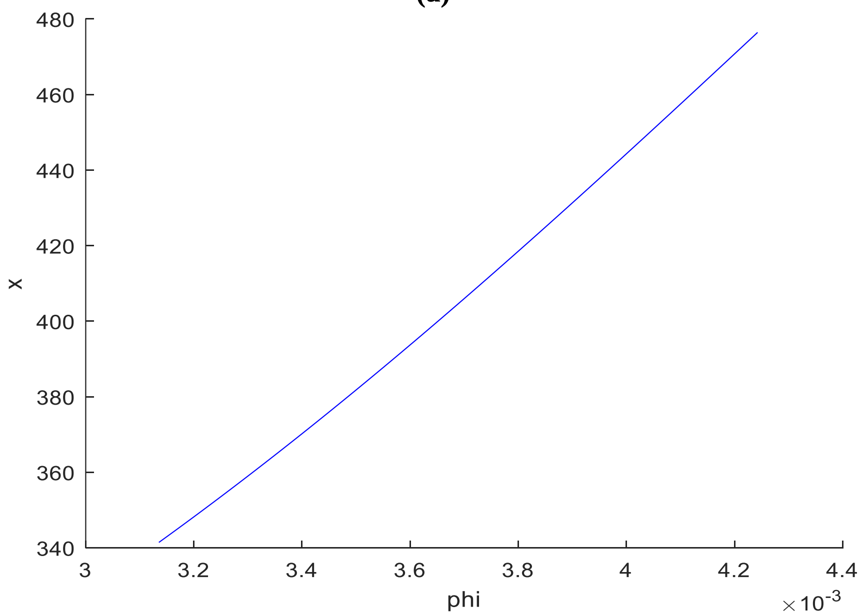
$\phi$ is the bifurcation parameter. Two Hopf bifurcations were found at values of $[X_a, N_a, F_a, \phi]$ of $x$ = (801.605410, 302.326612, 171.050672, 0.000608) and (963.420875, 38.523612, 0.372817, 0.005216) (**Figure 1a**) When an activation factor of $\frac{\tanh(\phi)}{0.09}$ was used the Hopf bifurcation disappears (**Figure 1b**).

MNLMPC

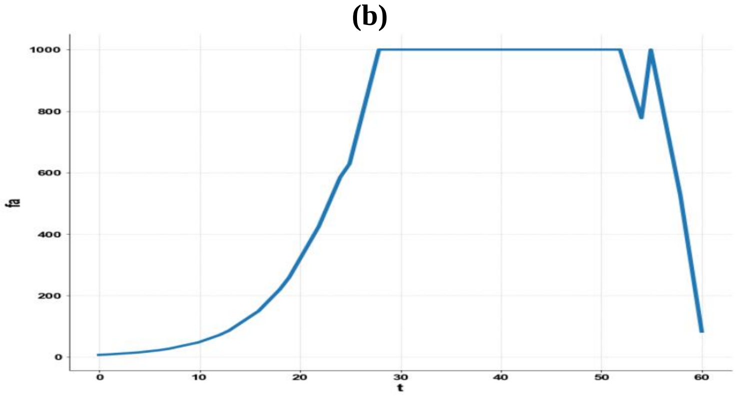
The individual single optimal control involves the minimization of $\Sigma X_a(t)$ and $\Sigma F_a(t)$which resulted in values of 18.765 and 0 while the maximization of $\Sigma N_a(t)$ that resulted in a value of 1880.65. The multiiobjective optimal control involved a minimization of $(\Sigma N_a(t) - 1880.65)^2 + (\Sigma F_a(t) - 0)^2$ , The MNLMPC control value obtained was 1.5933 × $10^{-8}$. The two-dimensional plots of the variables with time are shown in **Figure 1c–f** while the three-dimensional surfaces are shown in **Figure 1g–i**.


1000
950
900
850
800
750
x
H
H
0
1
2
3
4
5
6
7
8
9
phi
×10-3

**(a)**


480
460
440
420
400
380
360
340
x
3
3.2
3.4
3.6
3.8
4
4.2
4.4
phi
×10-3

**(b)**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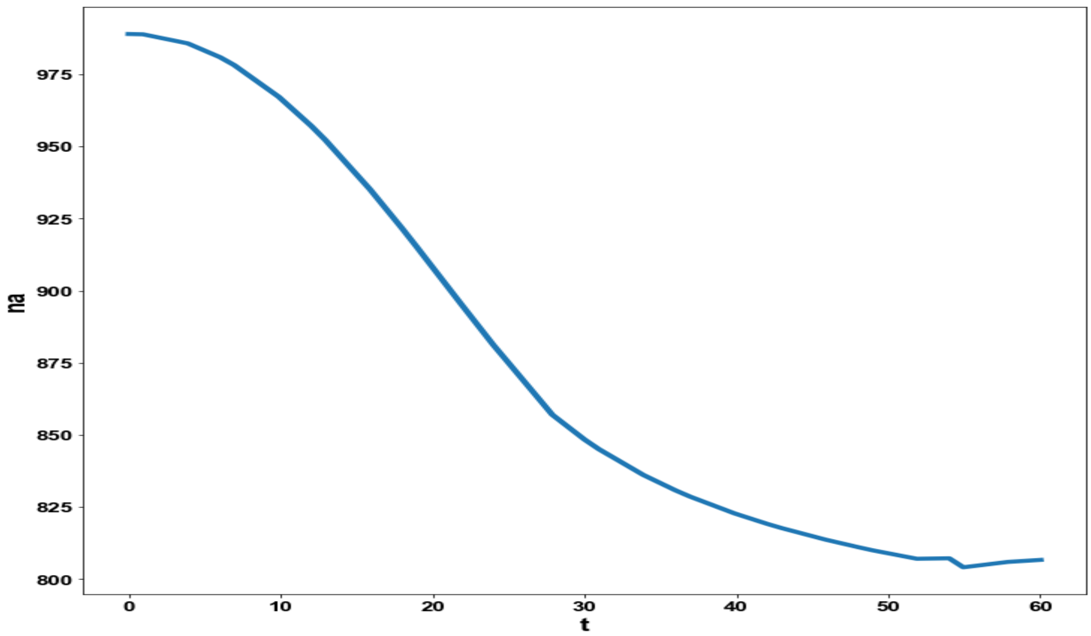
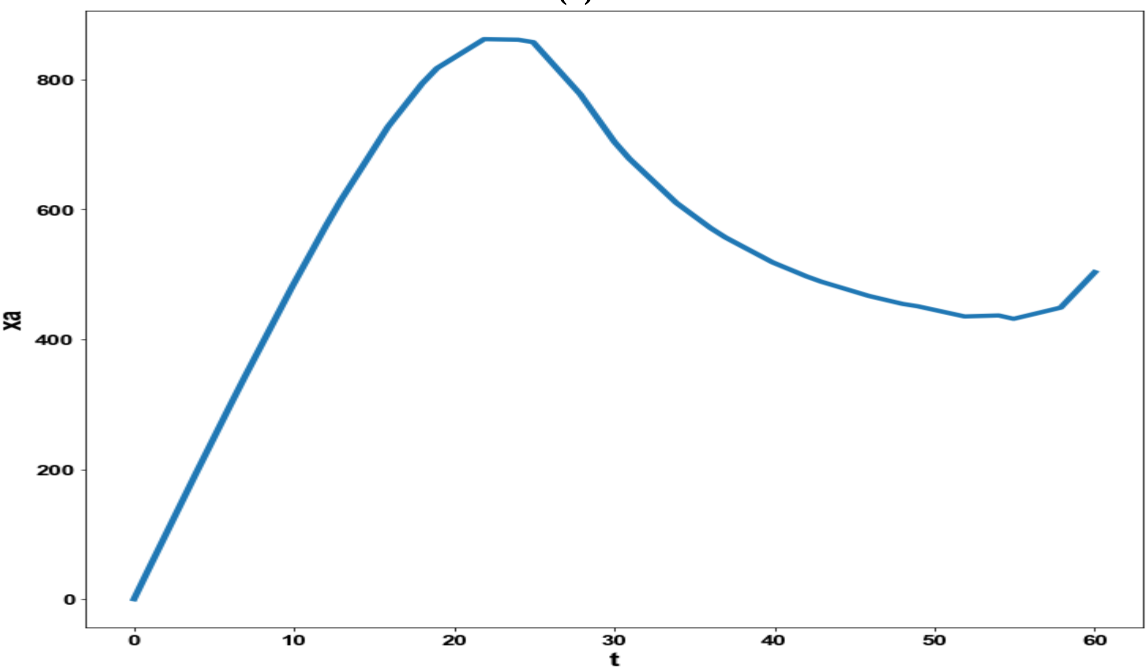
800
600
400
200
0
fa
0
10
20
30
40
50
60
t

**(c)**


na
975
950
925
900
875
850
825
800
0
10
20
30
40
50
60
t

**(d)**


xa
800
600
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0
0
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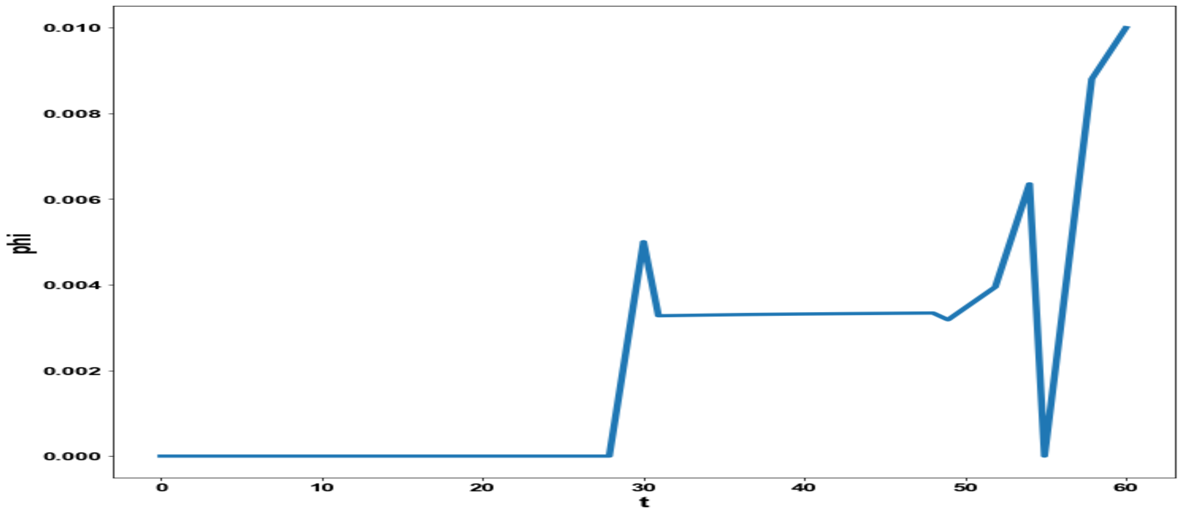
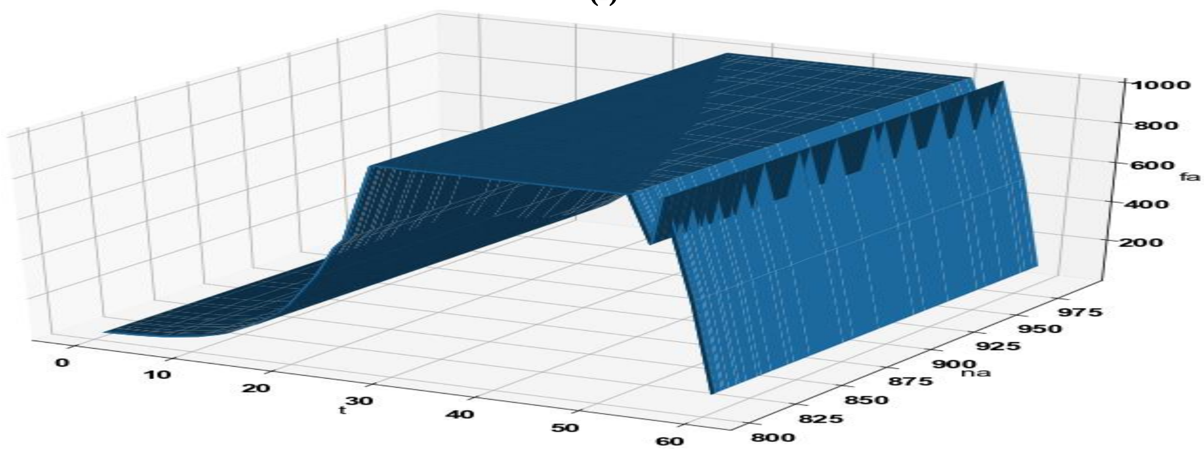
30
40
50
60
t

**(e)**


0.010
0.008
0.006
0.004
0.002
0.000
phi
0
10
20
30
40
50
60
t

**(f)**


1000
800
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200
fa
0
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40
50
60
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800
825
850
875
900
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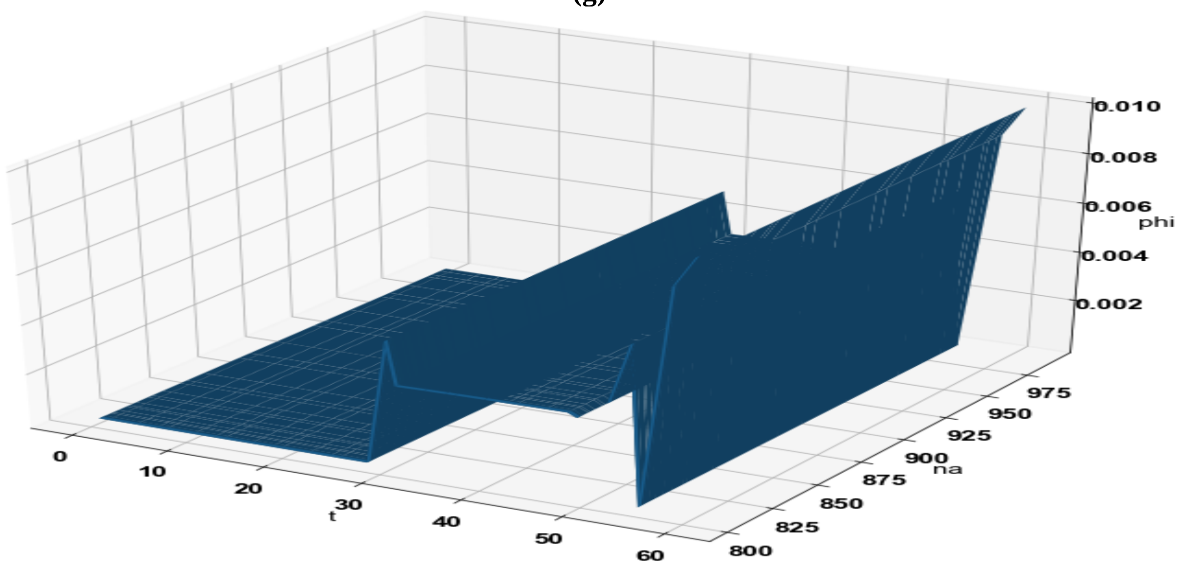
950
975
na

**(g)**


0.010
0.008
0.006
0.004
0.002
phi
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20
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60
t
800
825
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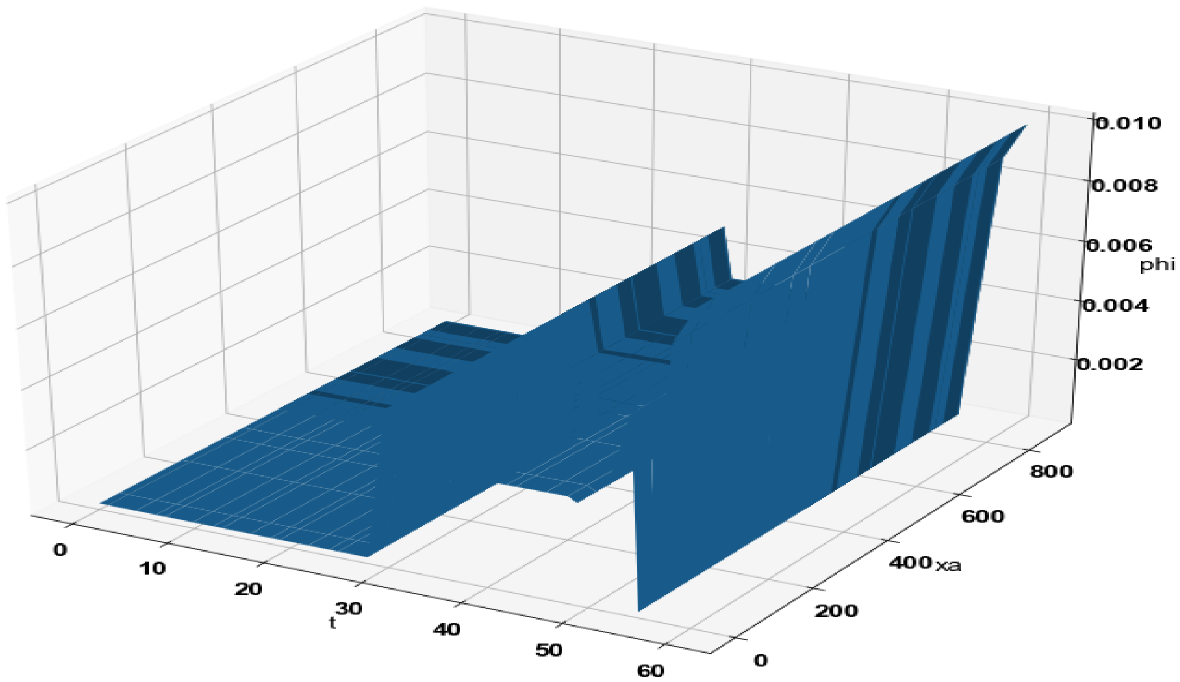
950
975
na

**(h)**



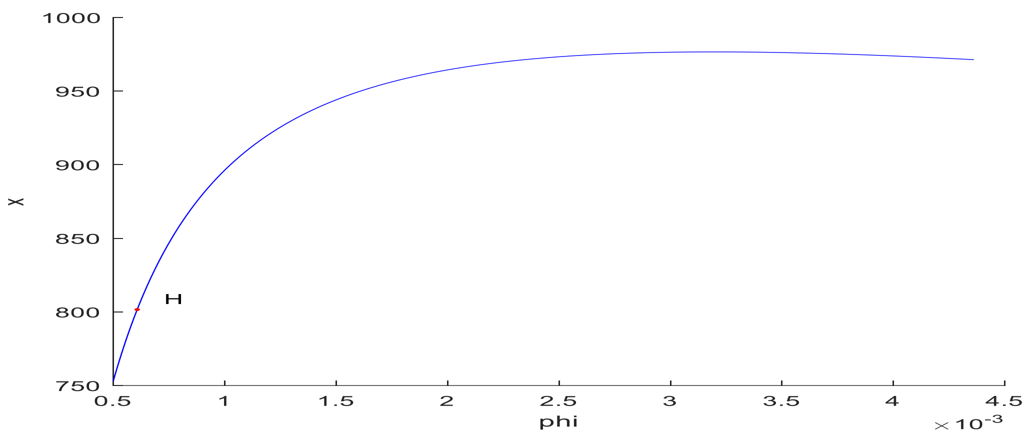

**(i)**

**Figure 1. (a)** Hopf bifurcation shown for problem 1; **(b)** hopf bifurcation eliminated for problem 1; **(c)** MNLMPC profile for problem 1(ba vs. t); **(d)** MNLMPC profile for problem (nz vs. t1; **(e)** MNLMPC profile for problem 1(xa vs. t; **(f)** MNLMPC profile for problem 1(phi vs. t) **(g)** MNLPMC surface for problem 1 (xa, na, t) **(h)** MNLPMC surface for problem 1(phi na, t); **(i)** MNLPMC surface for problem1 (phi, xa, t).

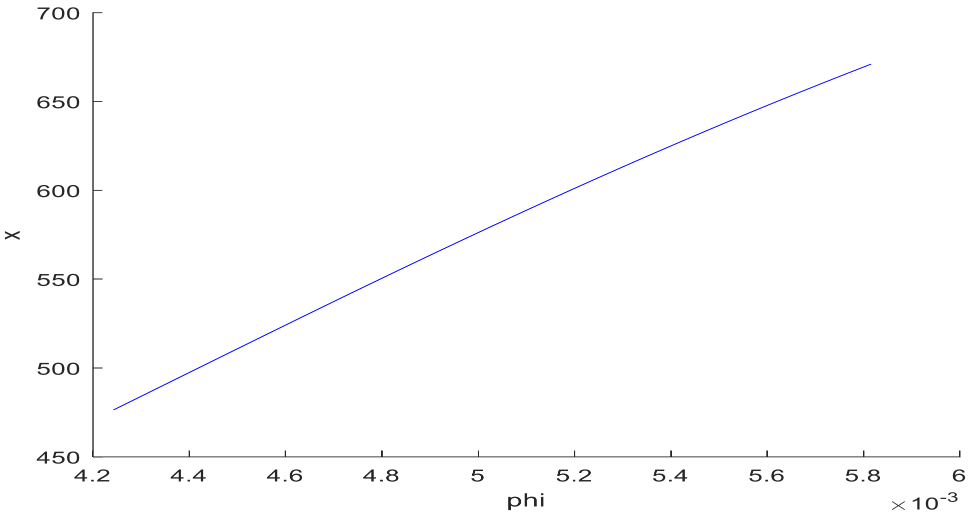
Problem 2

Bifurcation analysis

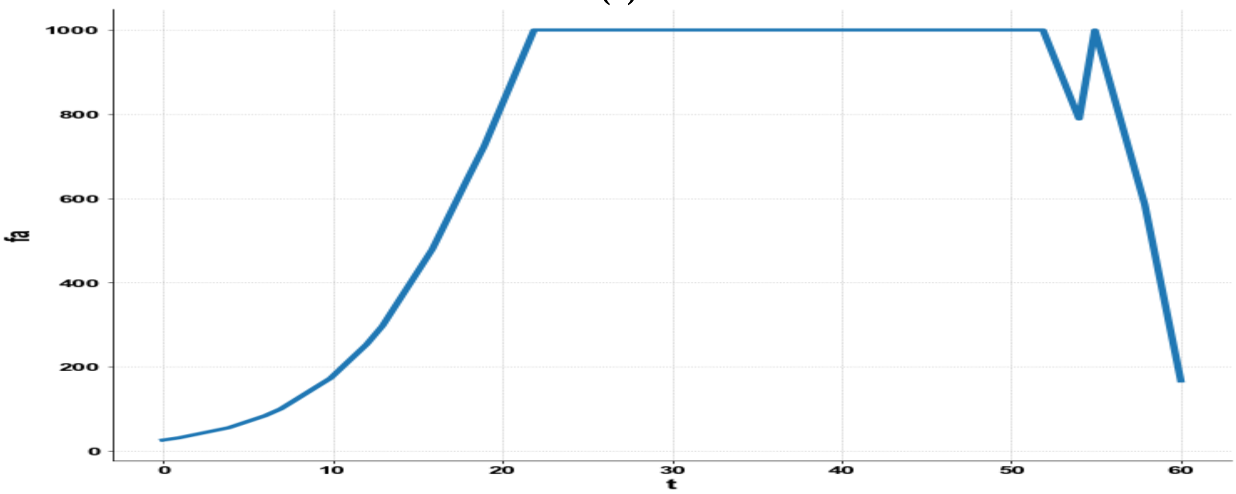
$\phi$ is the bifurcation parameter. One Hopf bifurcations were found at values of $[X_a, N_a, F_a, G_a, \phi]$ of $x$ = (801.605414, 302.326608, 171.050668, 1911.537436, 0.000608) and (**Figure 2a**) When an activation factor of $\frac{\tanh(\phi)}{0.09}$ was used the Hopf bifurcation disappears (**Figure 2b**).



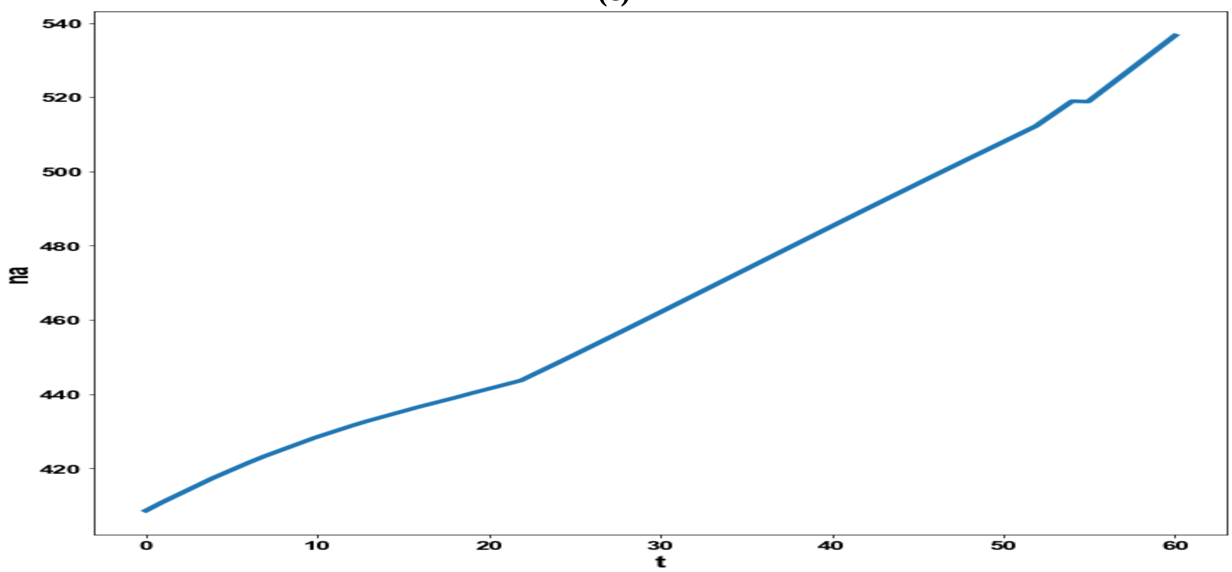

**(a)**



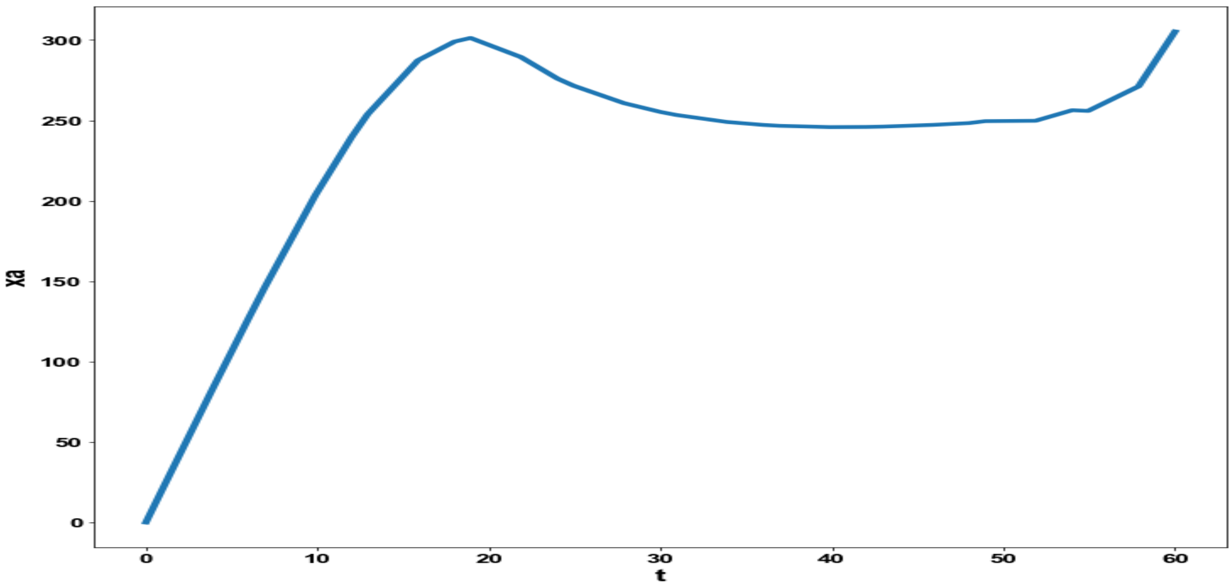

**(b)**




**(c)**




**(d)**


xa
t
0
50
100
150
200
250
300
0
10
20
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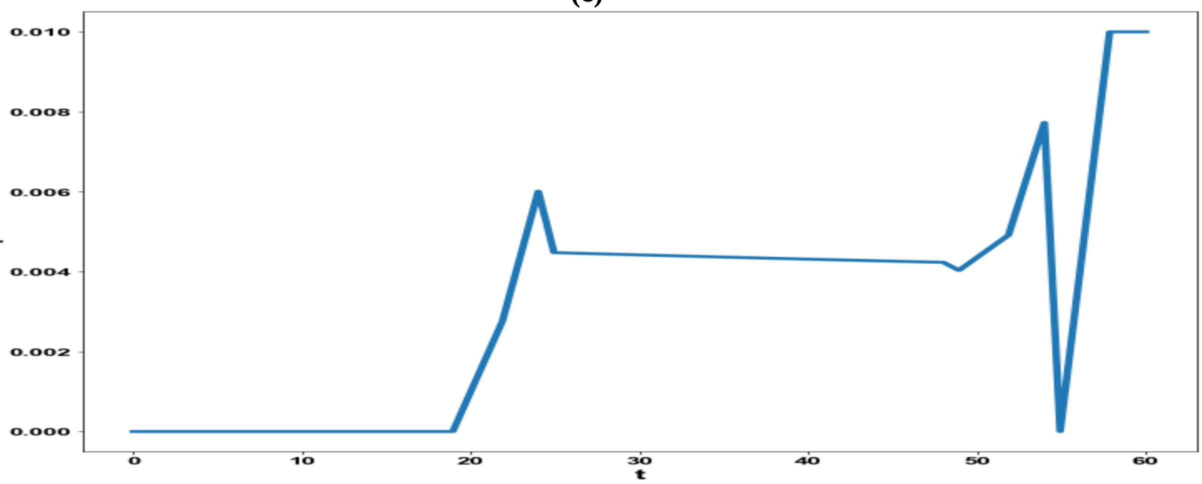
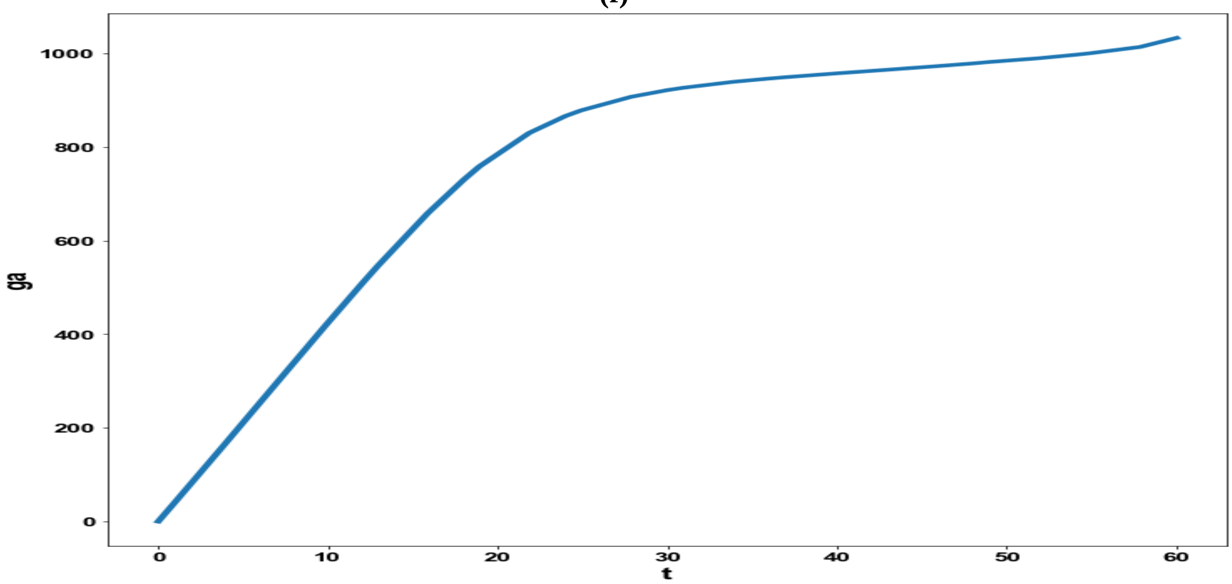
40
50
60

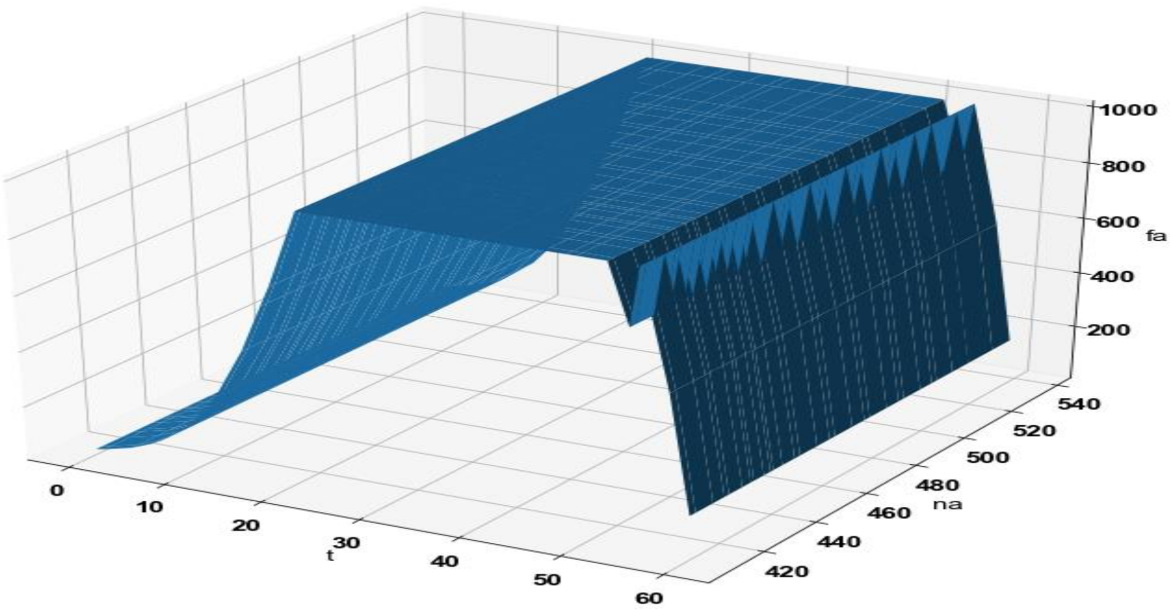
**(e)**


phi
t
0.000
0.002
0.004
0.006
0.008
0.010
0
10
20
30
40
50
60

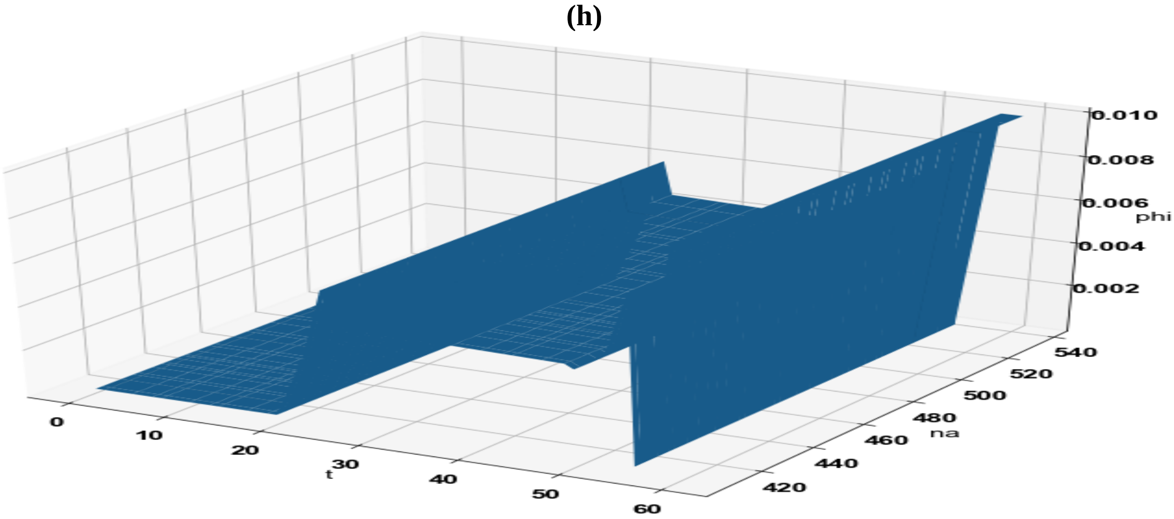
**(f)**


ga
t
0
200
400
600
800
1000
0
10
20
30
40
50
60

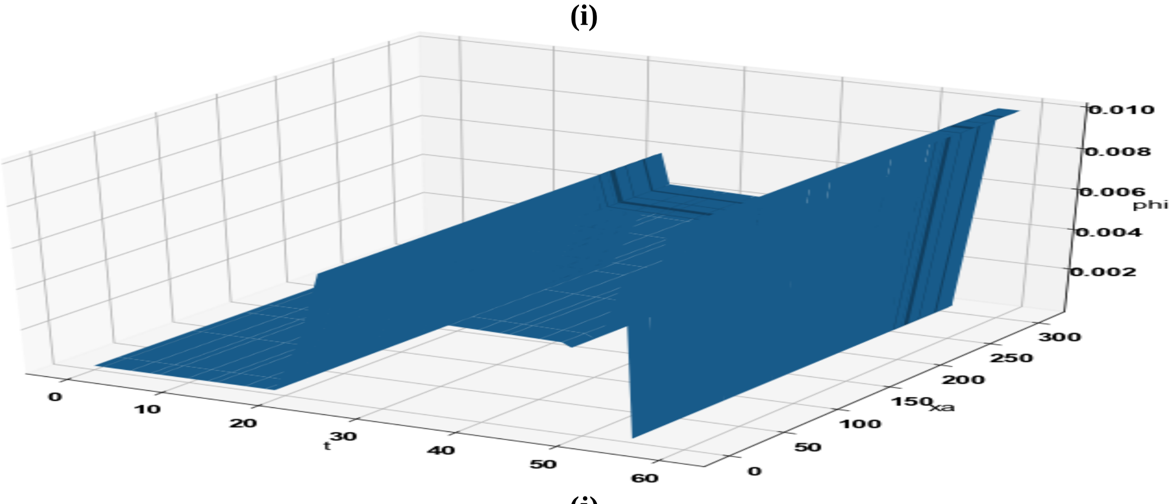
**(g)**


1000
800
600
fa
400
200
540
520
500
480
na
460
440
420
0
10
20
30
t
40
50
60

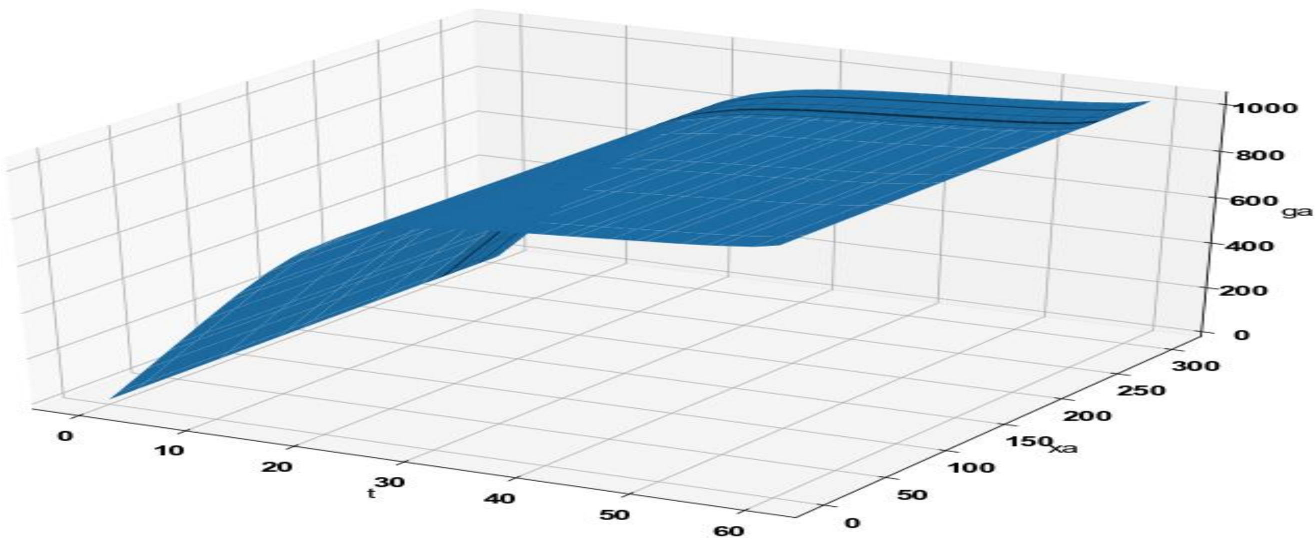
**(h)**


0.010
0.008
0.006
phi
0.004
0.002
540
520
500
480
na
460
440
420
0
10
20
30
t
40
50
60

**(i)**


0.010
0.008
0.006
phi
0.004
0.002
300
250
200
150
xa
100
50
0
0
10
20
30
t
40
50
60

**(j)**




**(k)**

**Figure 2. (a)** Hopf bifurcation in problem 2; **(b)** Hopf Bifurcation eliminated in Problem 2; **(c)** MNLMPC profile for problem 2(xa vs. t); **(d)** MNLPMC profile for problem 2(na vs. t); **(e)** (fa vs. t); **(f)** MNLPMC profile for problem 2(phi vs t); MNLPMC profile for problem 2; **(g)** MNLPMC profile for problem(ga vs. t); **(h)** MNLMPC surface for problem(xa, na, t); **(i)** MNLMPC surface for problem 2(phi, na, t); **(j)** MNLMPC surface for problem (phi, xa, t)2; **(k)** MNLMPC surface for problem 2(xa, ga, t).

MNLMPC

The individual single optimal control involves the minimization of $\Sigma X_a(t)$, $\Sigma F_a(t)$ and $\Sigma G_a(t)$ which resulted in values of 18.765, 0 and 78.606 while the maximization of $\Sigma N_a(t)$ that resulted in a value of 1880.65. The multiiobjective optimal control involved a minimization of $(\Sigma N_a(t) - 1880.65)^2 + (\Sigma X_a(t) - 18.765)^2 + (\Sigma F_a(t) - 0)^2 + (\Sigma G_a(t) - 78.606)^2$, The MNLMPC control value obtained was 1.5933e-08. The two-dimensional plots of the variables with time are shown in **Figure 2c–g** while the three-dimensional surfaces are shown in **Figure 2h–k**.

Problem 3

Bifurcation analysis

No bifurcation points were found.

MNLMPC

The individual single optimal control involves the minimization of $\Sigma C_a(t)$, $\Sigma F_a(t)$ which resulted in values of 82.3 and 0 while the maximization of $\Sigma N_a(t), \Sigma T_a(t)$ that resulted in a value of 750.6 and 8420.84. The multiiobjective optimal control involved a minimization of $(\Sigma C_a(t) - 82.3)^2 + (\Sigma N_a(t) - 750.6)^2 + (\Sigma F_a(t) - 0)^2 + (\Sigma T_a(t) - 8420.84)^2$, The MNLMPC control value ($\phi$) obtained was 0. The two-dimensional plots of the variables with time are shown in **Figure 3a–e** while the three-dimensional surfaces are shown in **Figure 3f–i**.

Problem 4

Bifurcation analysis

$\phi_2$ is the bifurcation parameter. One Hopf bifurcations were found at values of $[B_a, N_a, E_a, T_a, P_a, \phi_2]$ of (23.009573 101.840766 5.410829 10.821658 8.996809

0.002163) and (**Figure 4a**) When an activation factor of $\frac{\tanh(\phi)}{1.e+0.5}$ was used the Hopf bifurcation disappears (**Figure 4b**).

MNLMPC

The individual single optimal control involves the minimization of $\Sigma B_a(t), \Sigma E_a(t), \Sigma T_a(t)$, which resulted in values of 0,0,0. while the maximization of $\Sigma N_a(t), \Sigma P_a(t)$ that resulted in a value of 1165.02 and 209.187. The multiiobjective optimal control involved a minimization of $(\Sigma P_a(t) - 209.187)^2 + (\Sigma N_a(t) - 1165.02)^2 + (\Sigma B_a(t) - 0)^2 + (\Sigma T_a(t) - 0)^2 + (\Sigma E_a(t) - 0)^2$ , The MNLMPC control value ($\phi$) obtained was 0. The two-dimensional plots of the variables with time are shown in **Figure 4c–g** while the three-dimensional surfaces are shown in **Figure 4h–k**. The MNLPMC value obtained was 0.1.

Problem 5

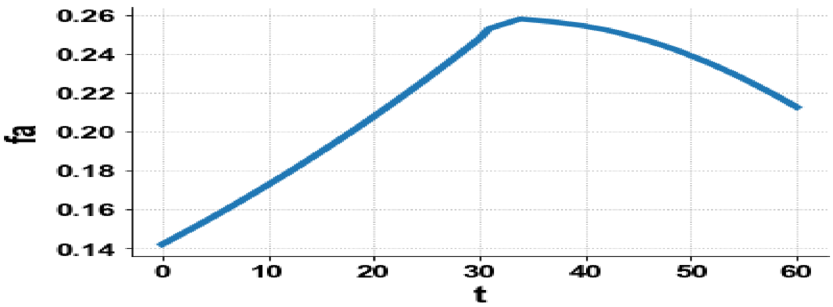
Bifurcation analysis

No bifurcation points were found.

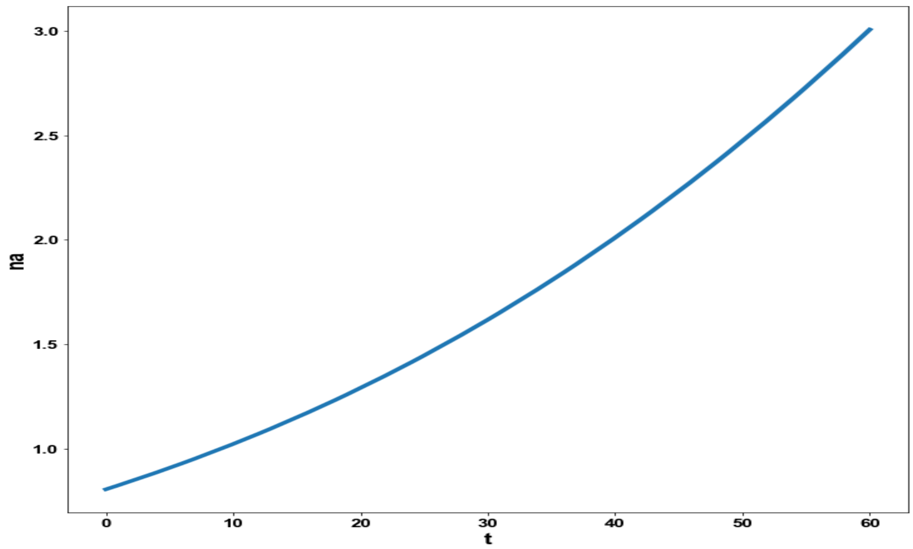
MNLMPC

The individual single optimal control involves the minimization of $\Sigma B_a(t), \Sigma E_a(t), \Sigma T_a(t), \Sigma X_a(t)$, which resulted in values of 0,0,0 and 54.9009. while the maximization of $\Sigma N_a(t), \Sigma P_a(t)$ that resulted in a value of 1165.02 and 209.187. The multiiobjective optimal control involved a minimization of $(\Sigma P_a(t) - 209.187)^2 + (\Sigma N_a(t) - 1165.02)^2 + (\Sigma B_a(t) - 0)^2 + (\Sigma T_a(t) - 0)^2 + (\Sigma E_a(t) - 0)^2 + (\Sigma X_a(t) - 54.9009)^2$,

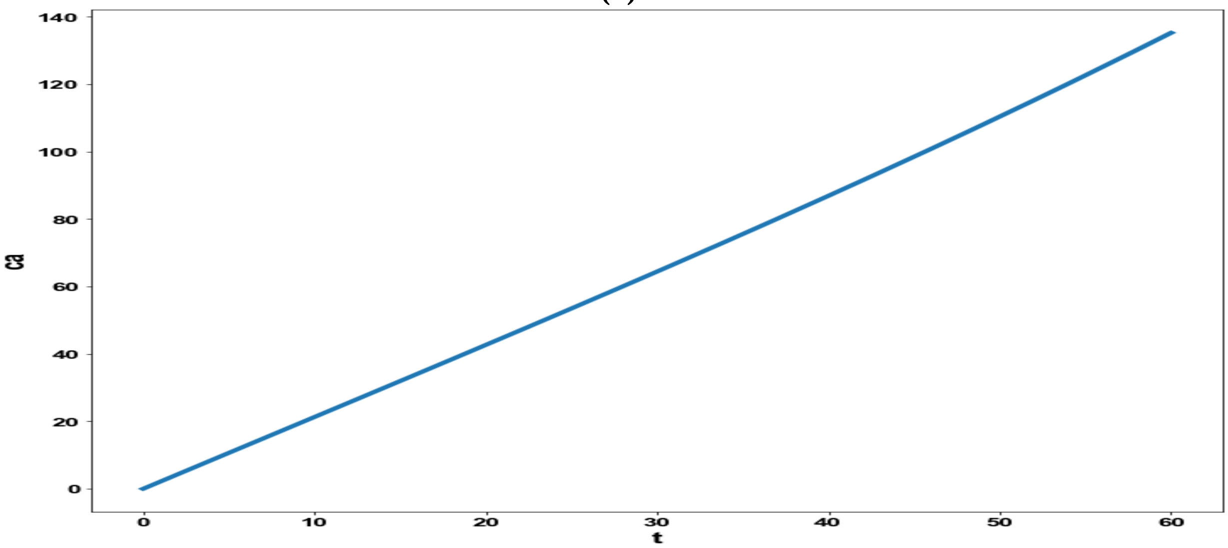
The MNLMPC control value ($\phi$) obtained was 0.05. The two-dimensional plots of the variables with time are shown in **Figure 5a–f** while the three-dimensional surfaces are shown in **Figure 5g–k**.



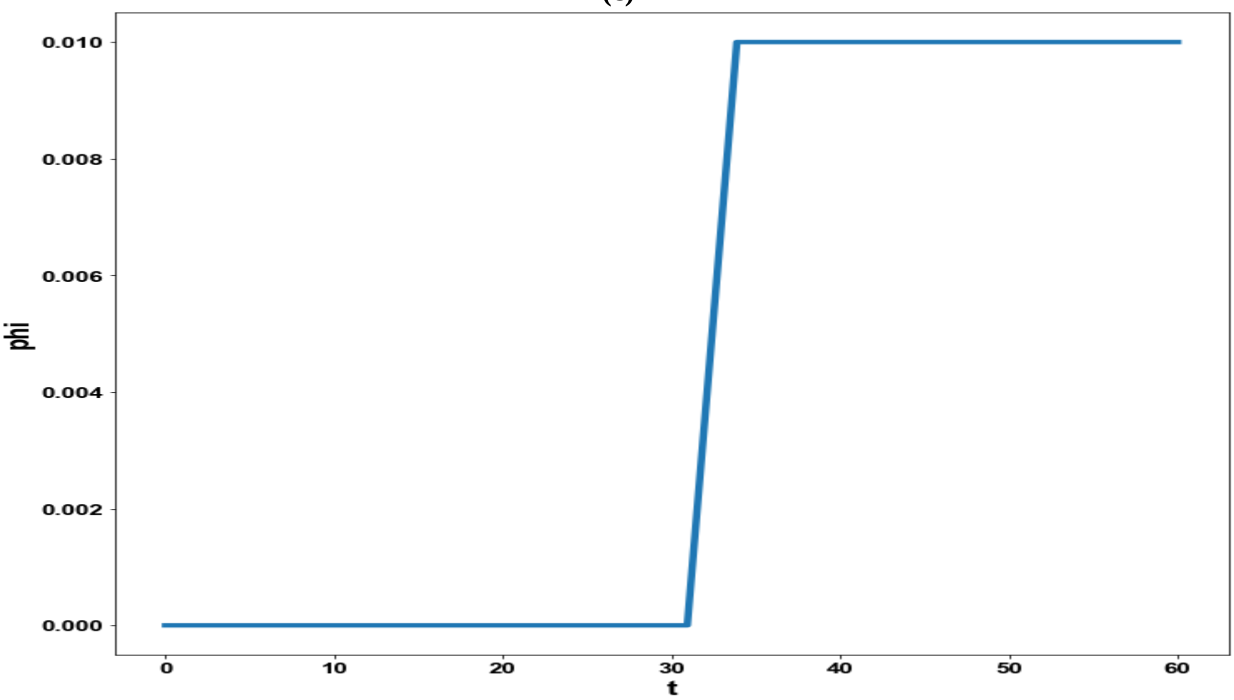

**(a)**


na
3.0
2.5
2.0
1.5
1.0
0
10
20
30
40
50
60
t

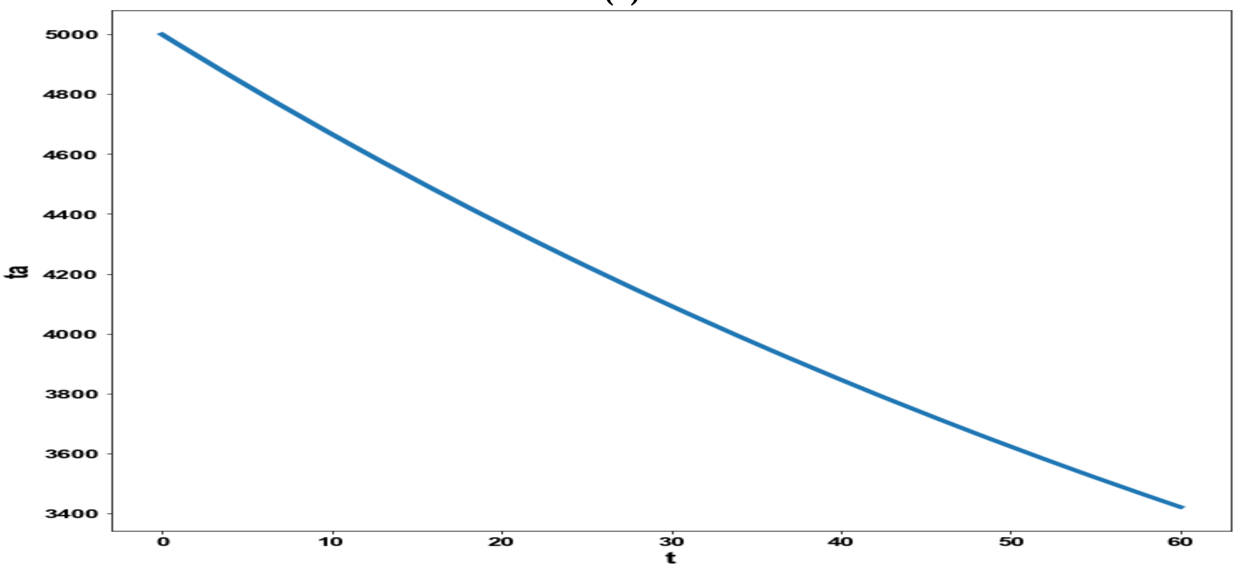
**(b)**


ca
140
120
100
80
60
40
20
0
0
10
20
30
40
50
60
t

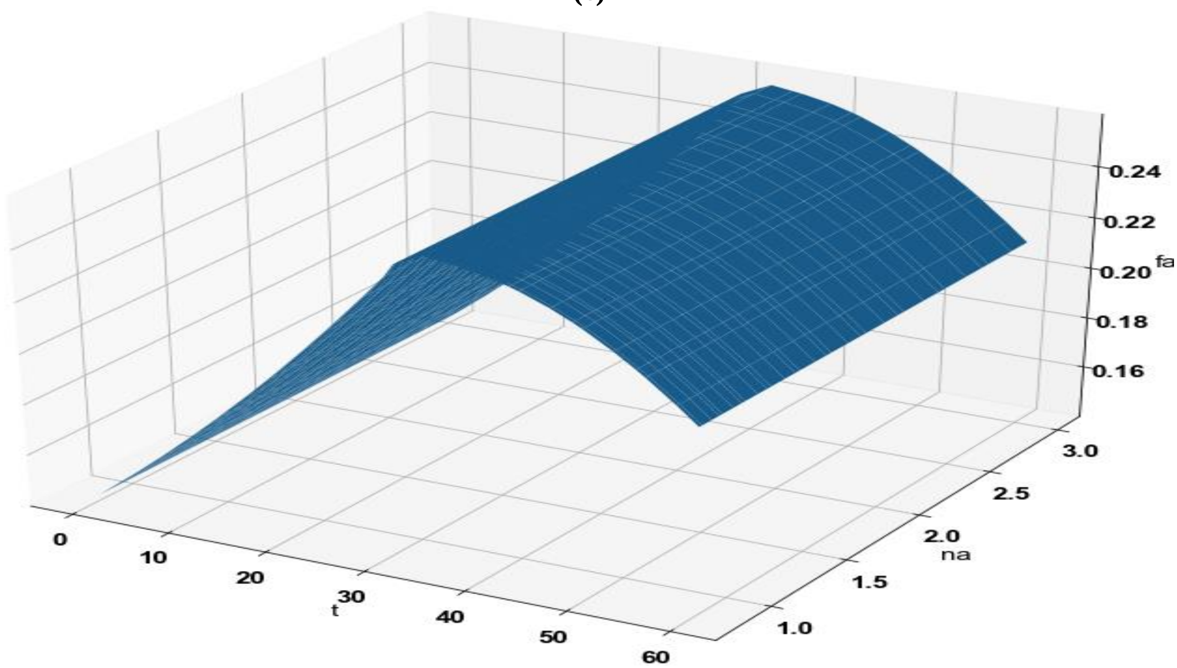
**(c)**


phi
0.010
0.008
0.006
0.004
0.002
0.000
0
10
20
30
40
50
60
t

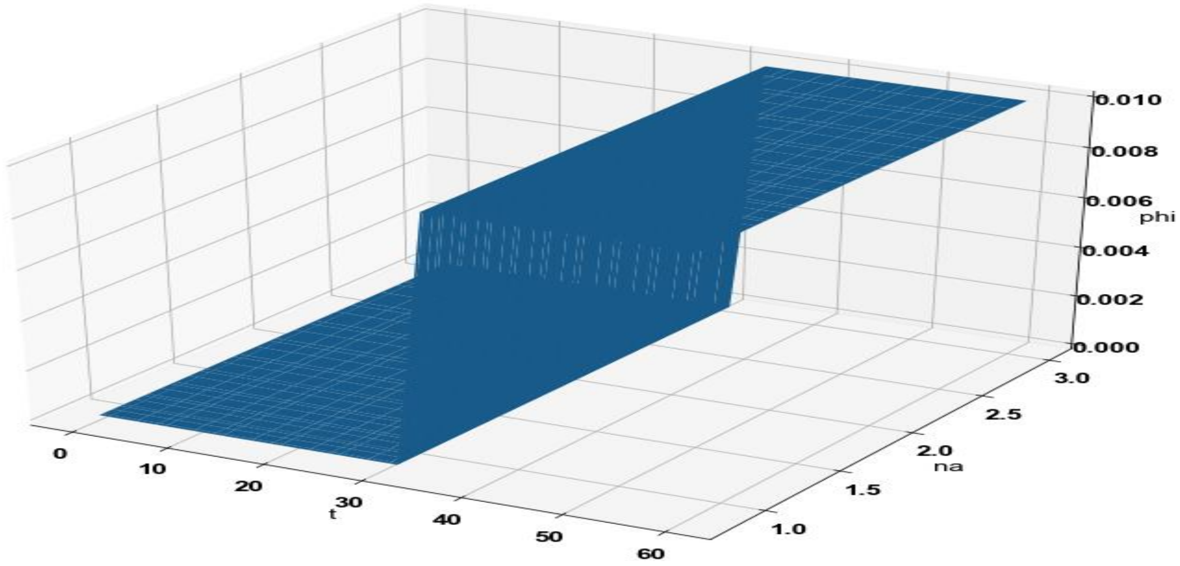
**(d)**


5000
4800
4600
4400
4200
4000
3800
3600
3400
fa
0
10
20
30
40
50
60
t

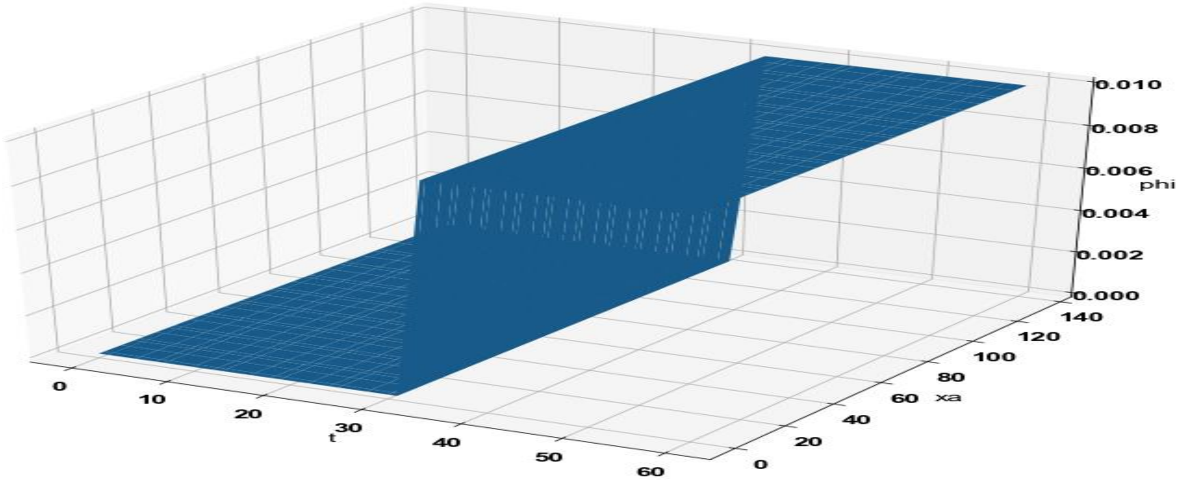
**(e)**


0.24
0.22
0.20
0.18
0.16
fa
3.0
2.5
2.0
1.5
1.0
na
0
10
20
30
40
50
60
t

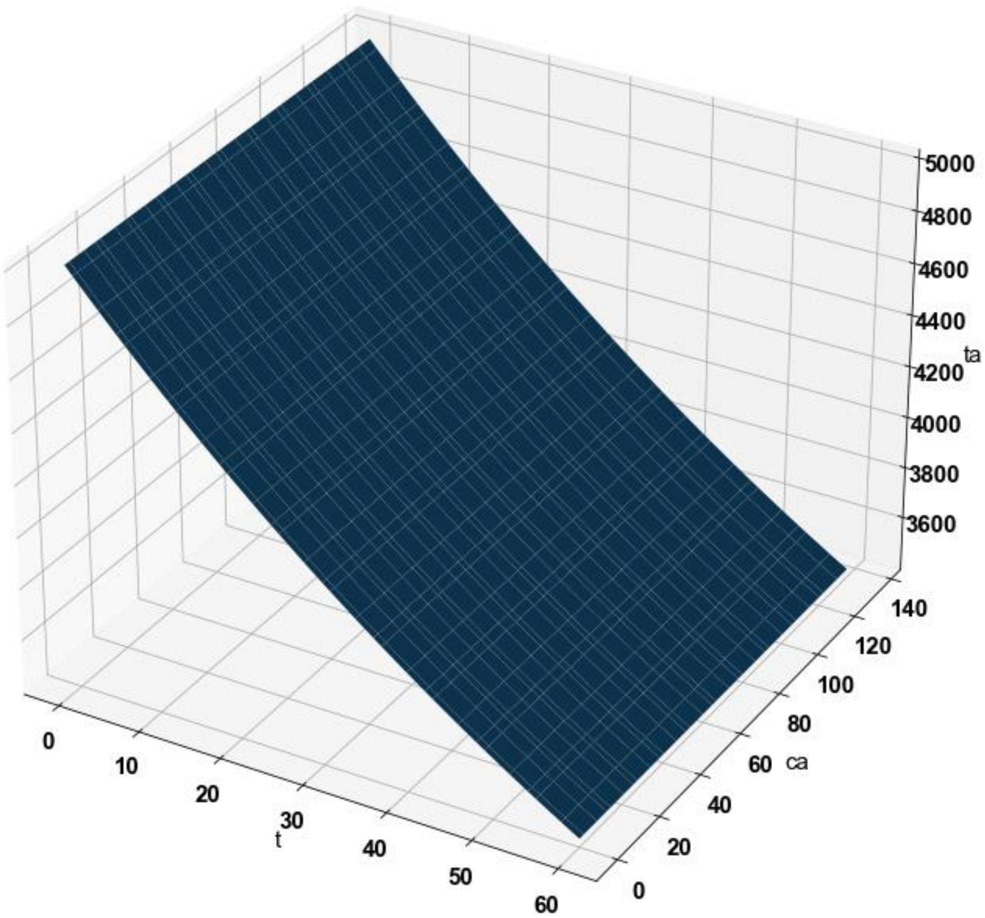
**(f)**


0.010
0.008
0.006
phi
0.004
0.002
0.000
3.0
2.5
2.0
na
1.5
1.0
0
10
20
30
t
40
50
60

**(g)**

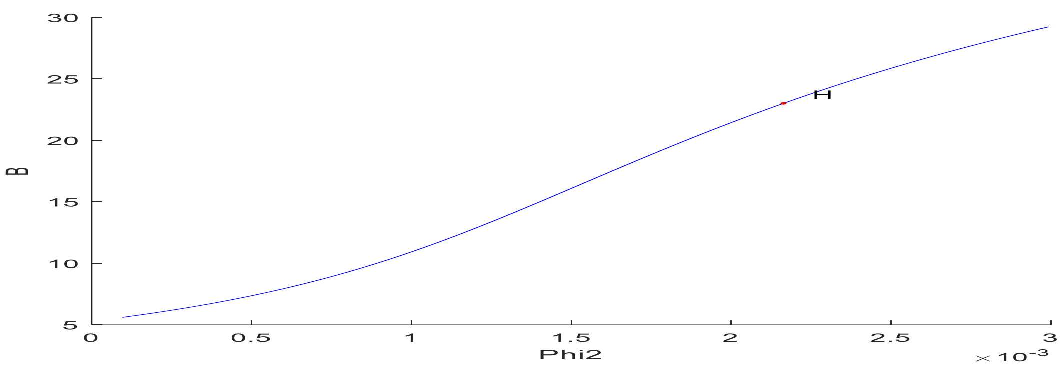

0.010
0.008
0.006
phi
0.004
0.002
0.000
140
120
100
80
60
xa
40
20
0
0
10
20
30
t
40
50
60

**(h)**

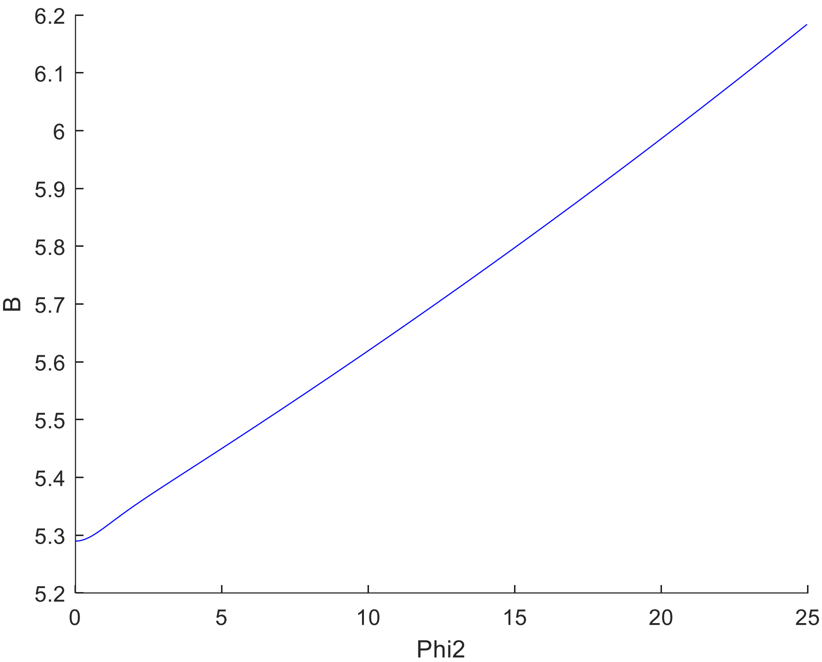



**(i)**

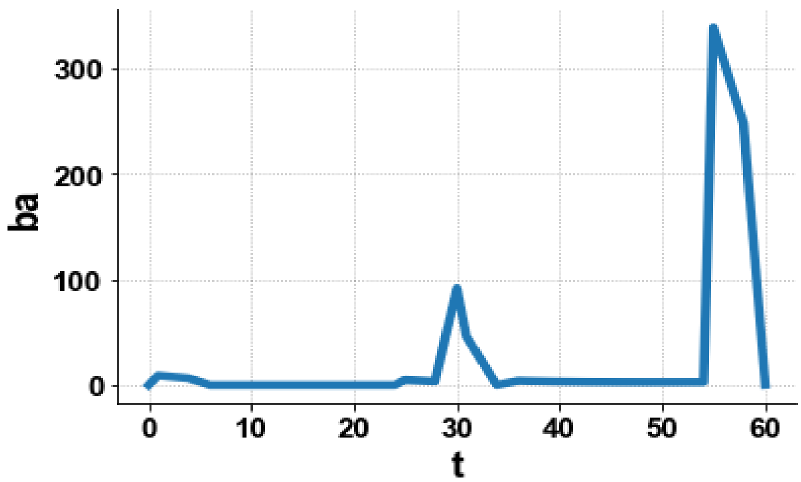
**Figure 3. (a)** MNLMPC profile for problem 3(fa vs. t); **(b)** MNLMPC profile for problem 3na vs. t); **(c)** MNLMPC profile for problem 3 (ca vs. t); **(d)** MNLMPC profile for problem 3 (phi vs. t); **(e)** MNLMPC profile for problem 3(Ta vs. t); **(f)** MNLMPC surface for problem 3 xa, na, t; **(g)** MNLMPC surface for problem 3 phi, na, t; **(h)** MNLMPC surface for problem 3 Phi, xa, t; **(i)** MNLMPC surface for problem 3(ta, ca, t).



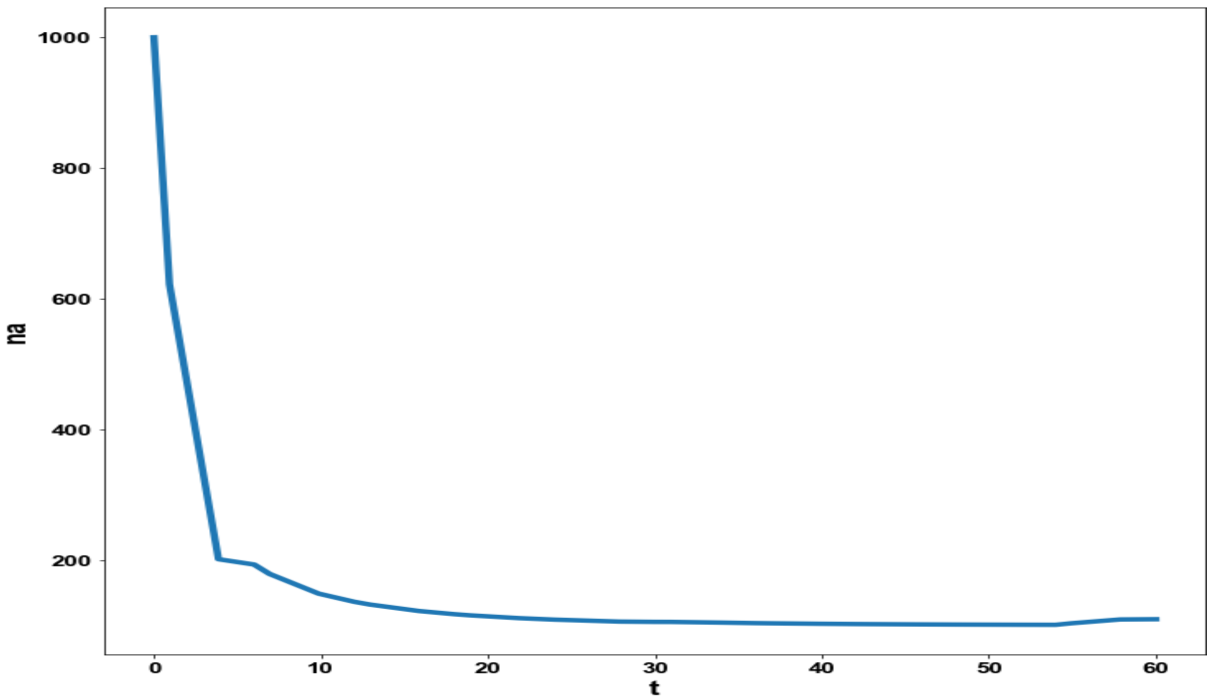

**(a)**


6.2
6.1
6
5.9
5.8
5.7
5.6
5.5
5.4
5.3
5.2
B
0
5
10
15
20
25
Phi2

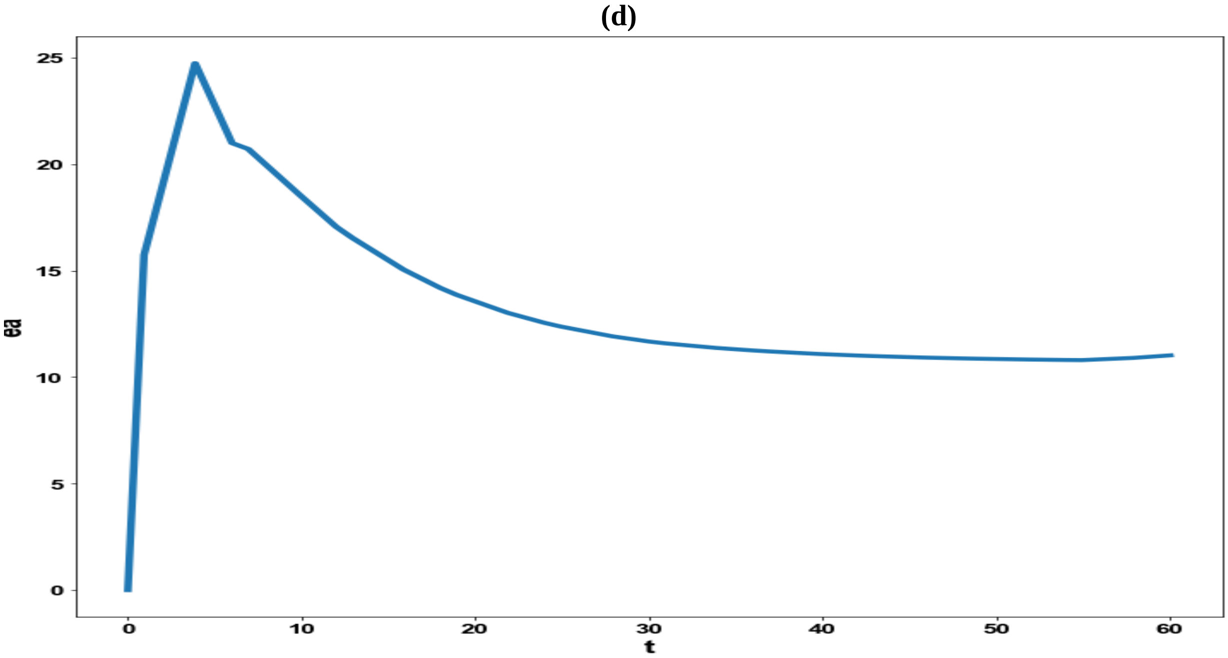
**(b)**


300
200
100
0
ba
0
10
20
30
40
50
60
t

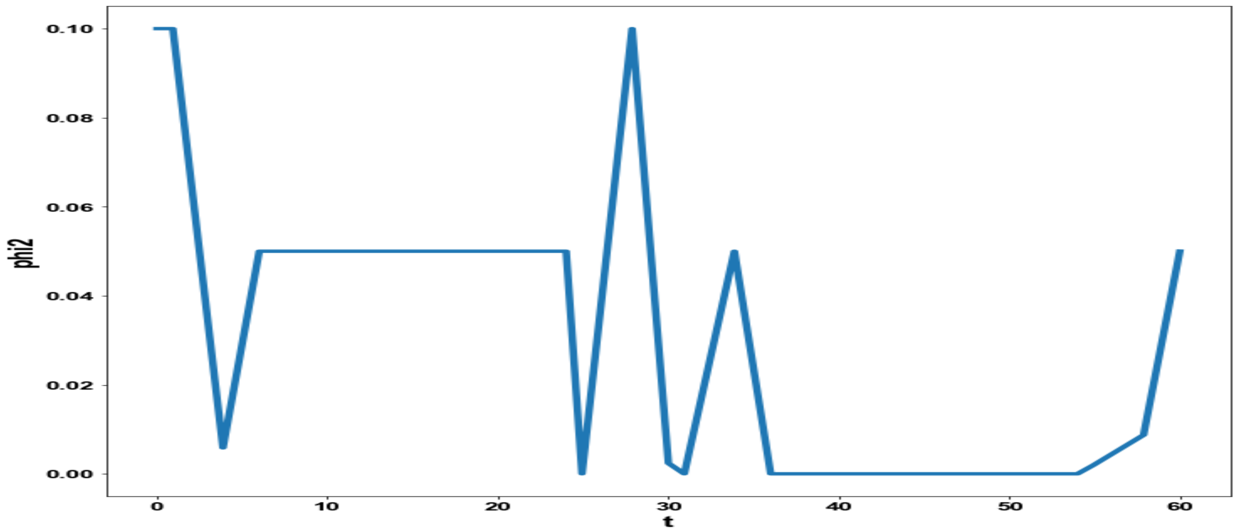
**(c)**


1000
800
600
400
200
na
0
10
20
30
40
50
60
t

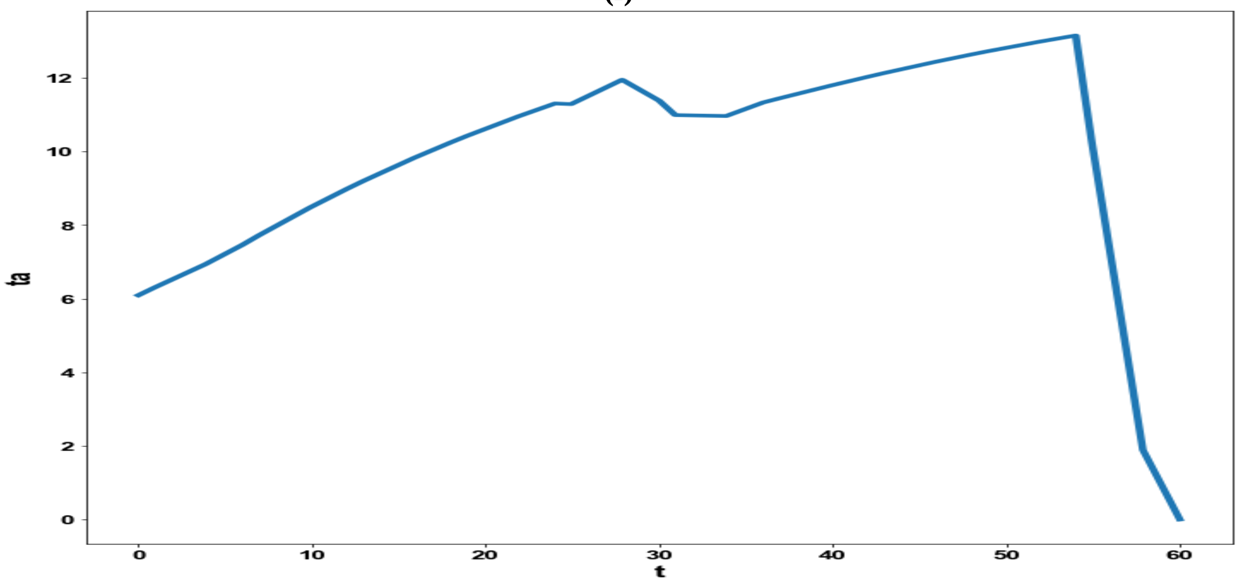
**(d)**


25
20
15
10
5
0
ea
0
10
20
30
40
50
60
t

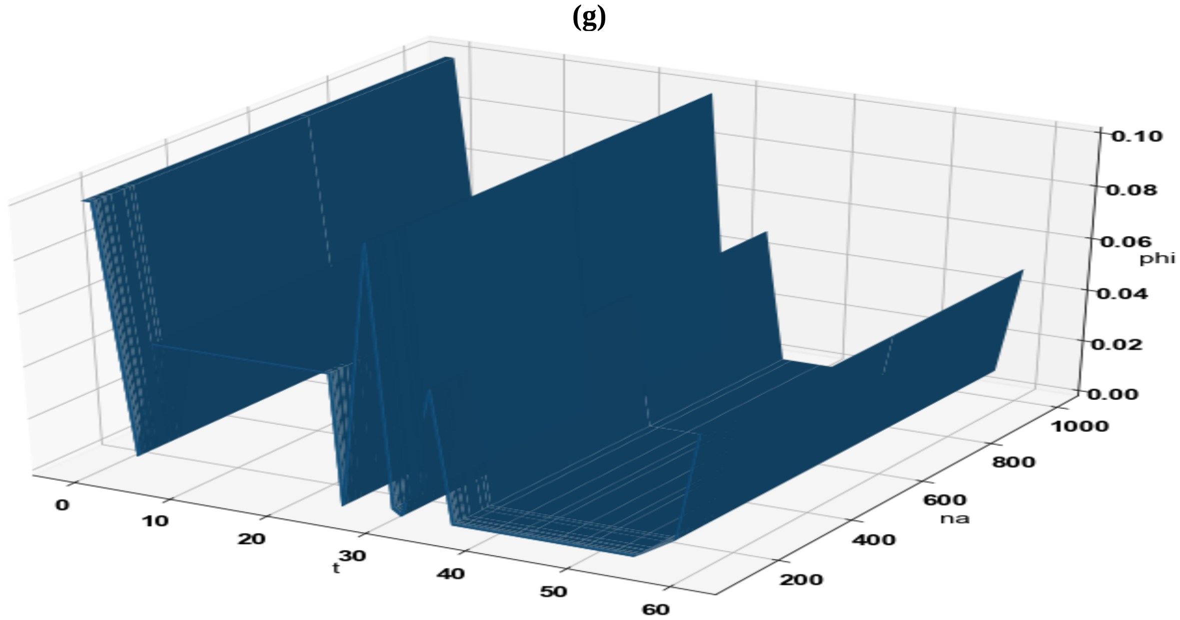
**(e)**


phi2
0.10
0.08
0.06
0.04
0.02
0.00
0
10
20
30
40
50
60
t

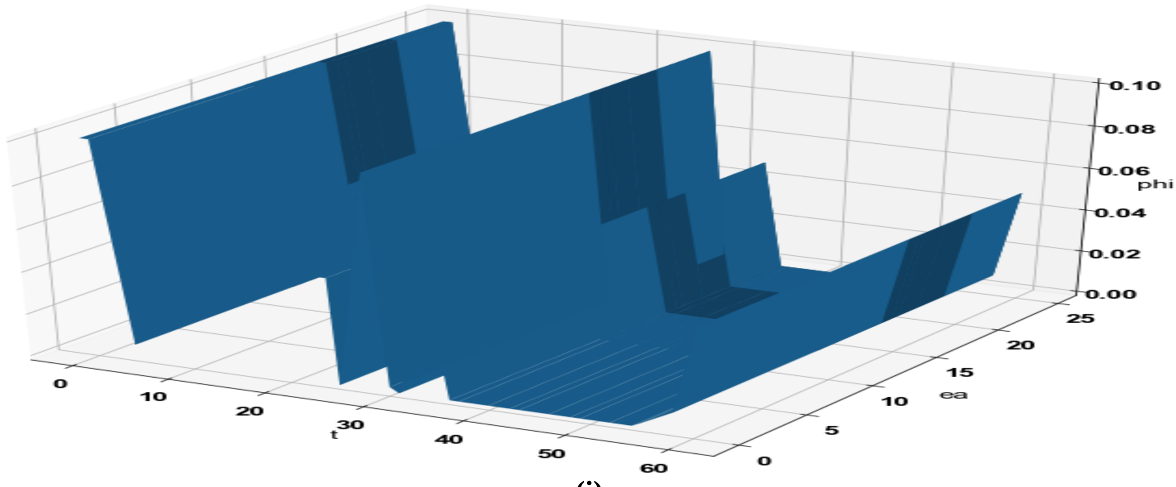
**(f)**


ta
12
10
8
6
4
2
0
0
10
20
30
40
50
60
t

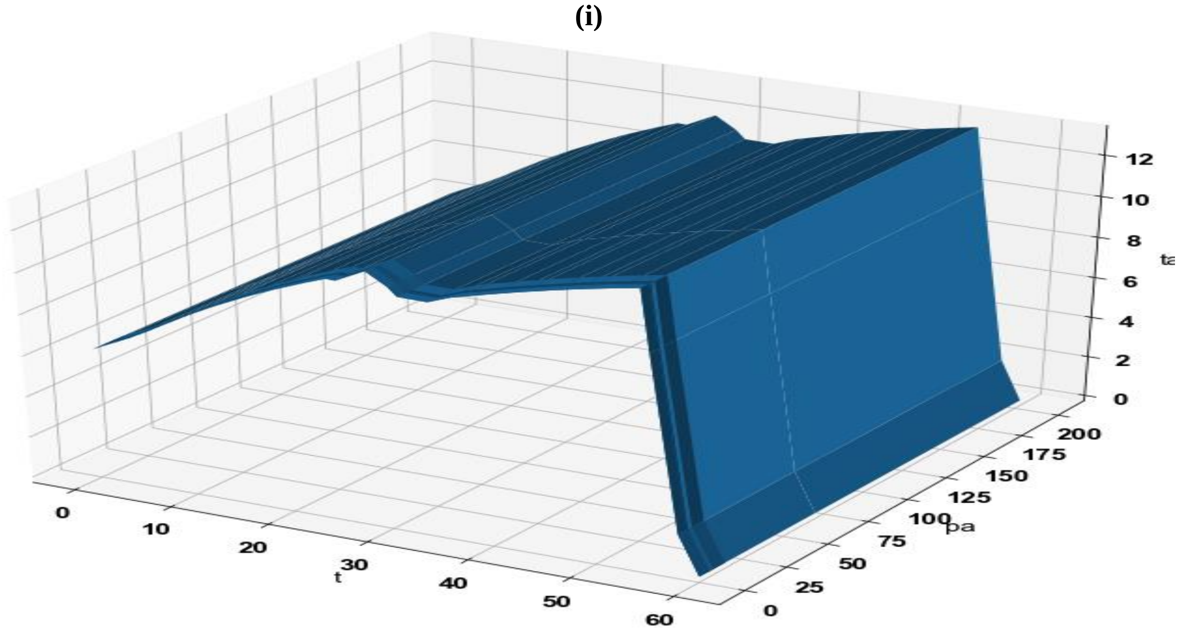
**(g)**


0.10
0.08
0.06
phi
0.04
0.02
0.00
1000
800
600
na
400
200
0
10
20
30
t
40
50
60

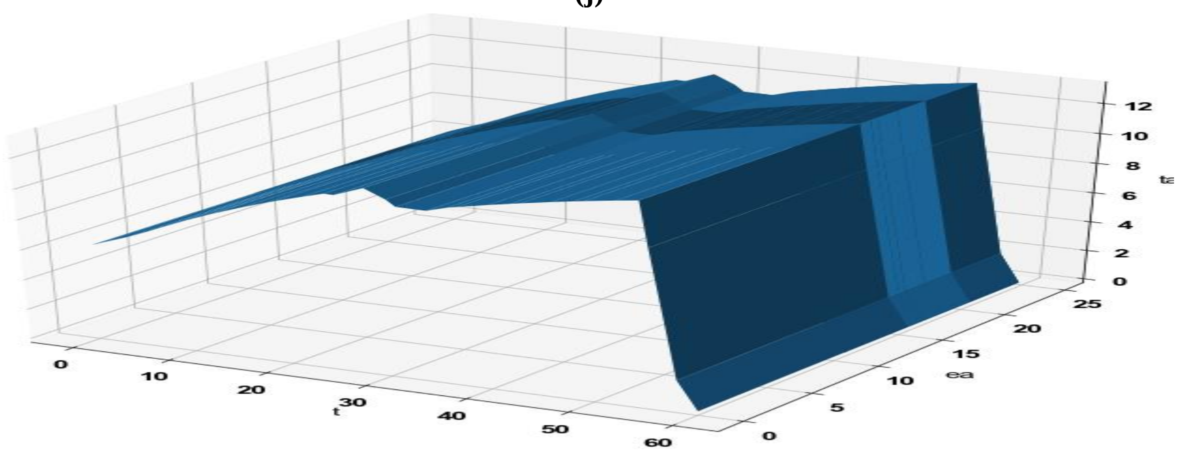
**(h)**




**(i)**

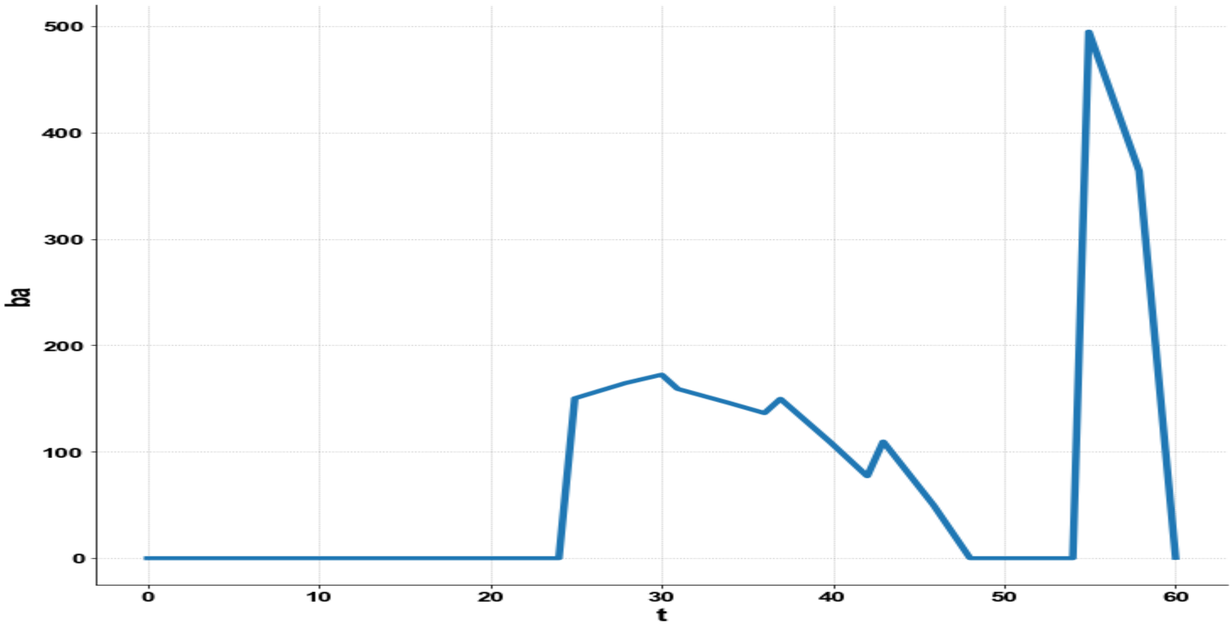



**(j)**



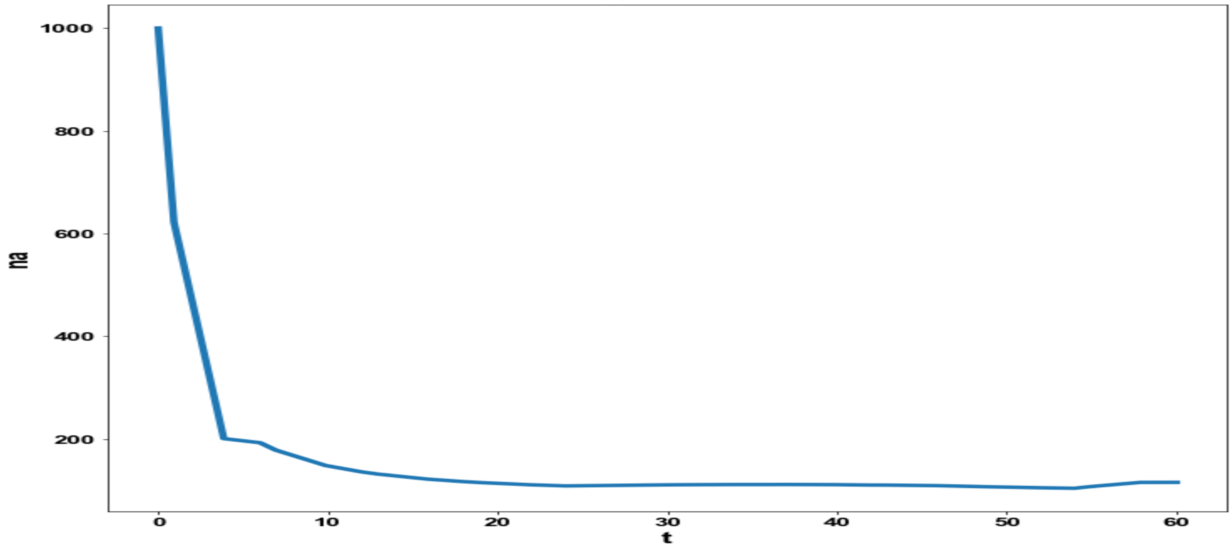

**(k)**

**Figure 4. (a)** Hopf bifurcation in Problem 4; **(b)** Hopf bifurcation eliminated in in Problem 4; **(c)** MNLMPC profile in problem 4 ba vs. t;**(d)** MNLMPC profile in problem 4 na vs. t; **(e)** MNLMPC profile in problem 4 Pa vs. t; **(f)** MNLMPC profile in problem 4 Ea vs. t ; **(g)** MNLMPC profile in problem 4; **(h)** MNLMPC surface in problem 4 (phi, na,t); **(i)** MNLMPC surface in problem 4(phi, ea, t); **(j)** MNLMPC surface in problem 4 phi, ba, t; **(k)** MNLMPC surface in problem 4 phi, ta, t.



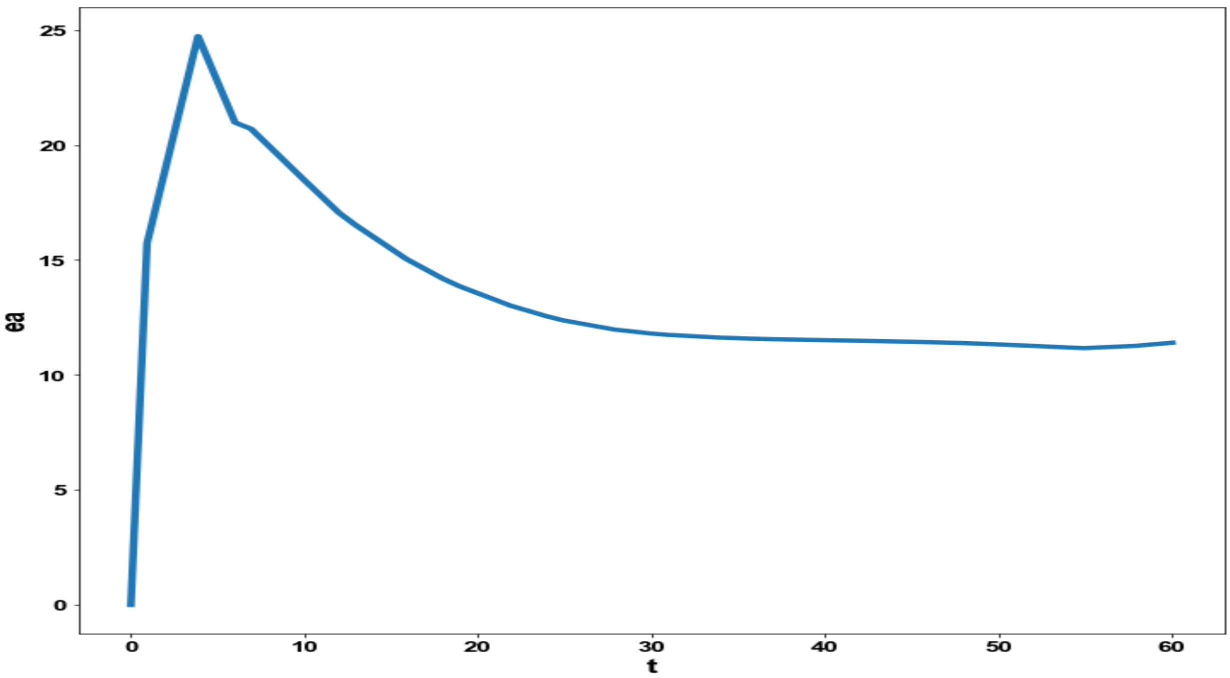
ba
500
400
300
200
100
0
0
10
20
30
40
50
60
t


**(a)**



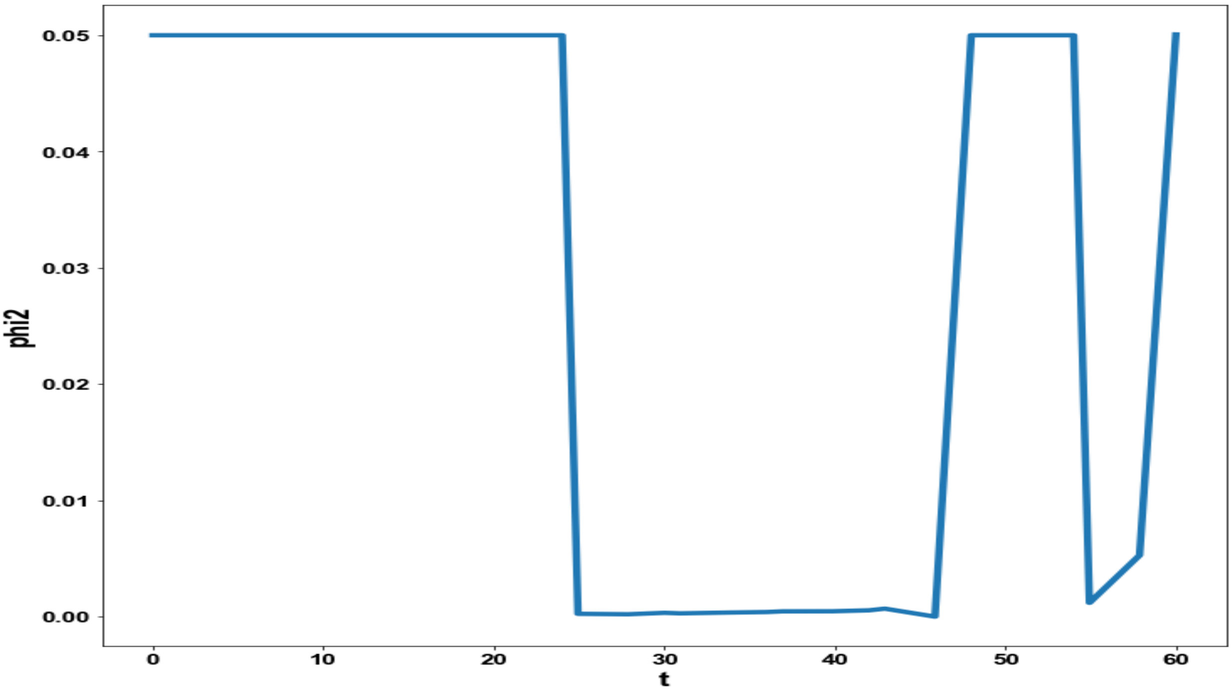
na
1000
800
600
400
200
0
10
20
30
40
50
60
t


**(b)**



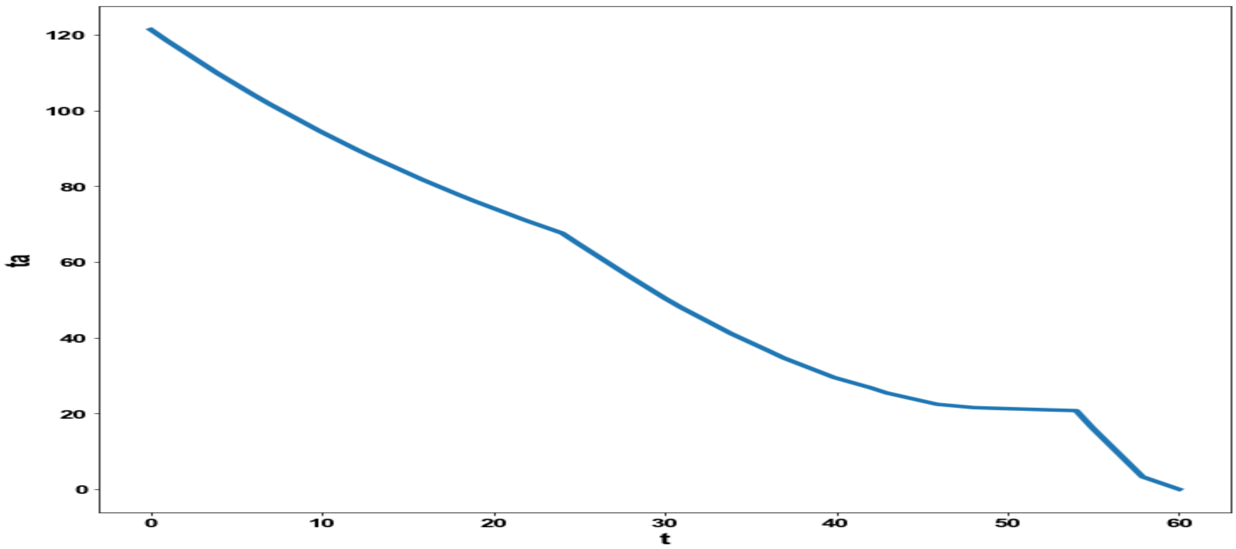
ea
25
20
15
10
5
0
0
10
20
30
40
50
60
t


**(c)**

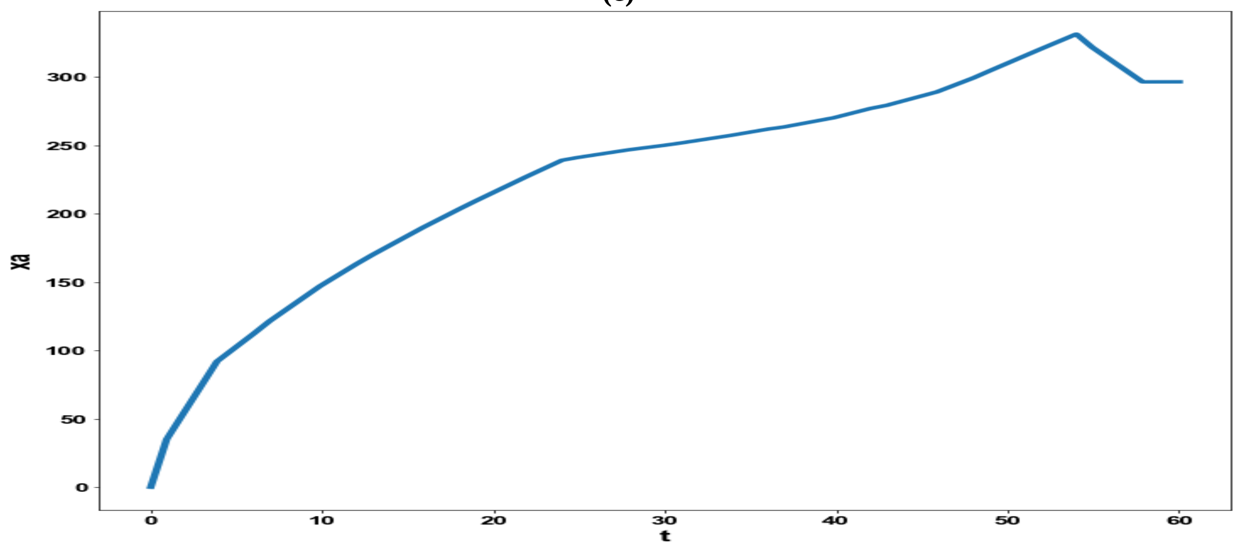


phi2
0.05
0.04
0.03
0.02
0.01
0.00
0
10
20
30
40
50
60
t

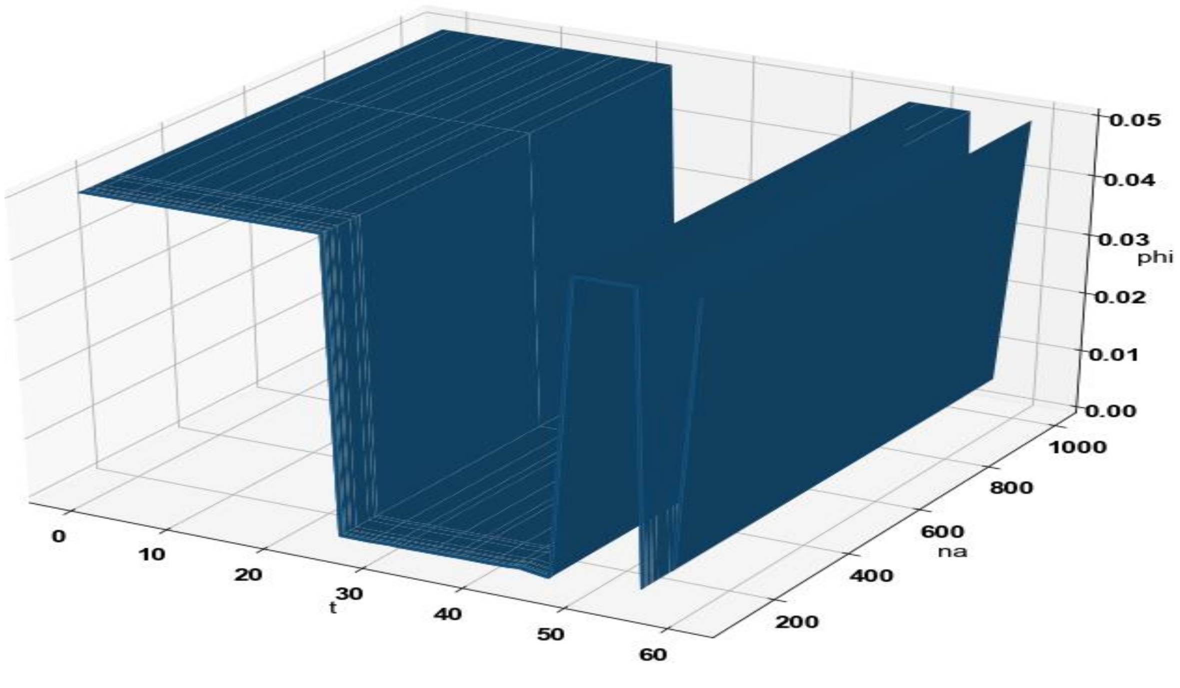

**(d)**


120
100
80
60
40
20
0
fa
0
10
20
30
40
50
60
t

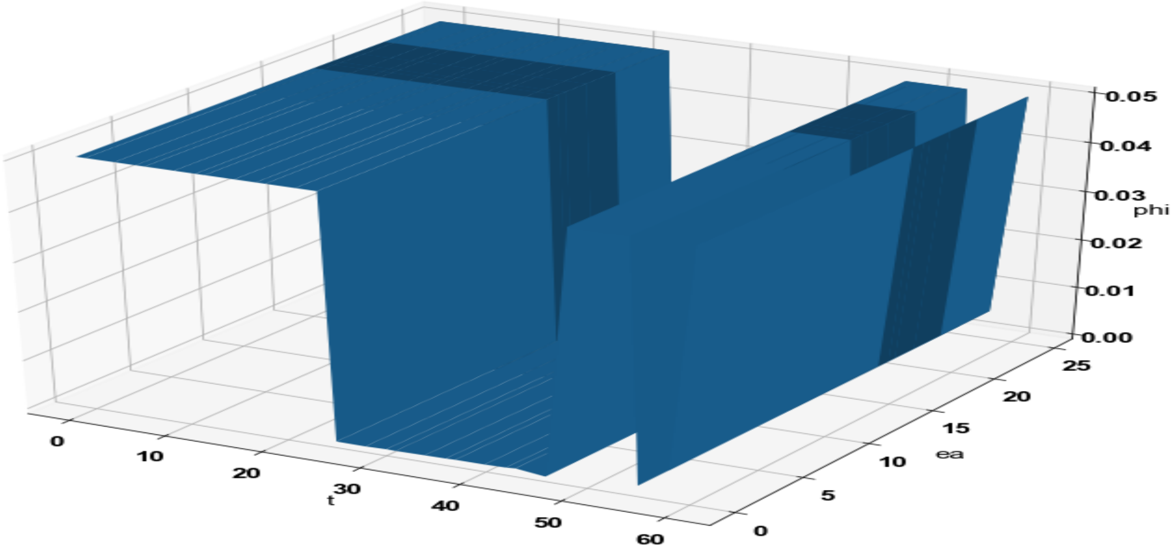
**(e)**


300
250
200
150
100
50
0
xa
0
10
20
30
40
50
60
t

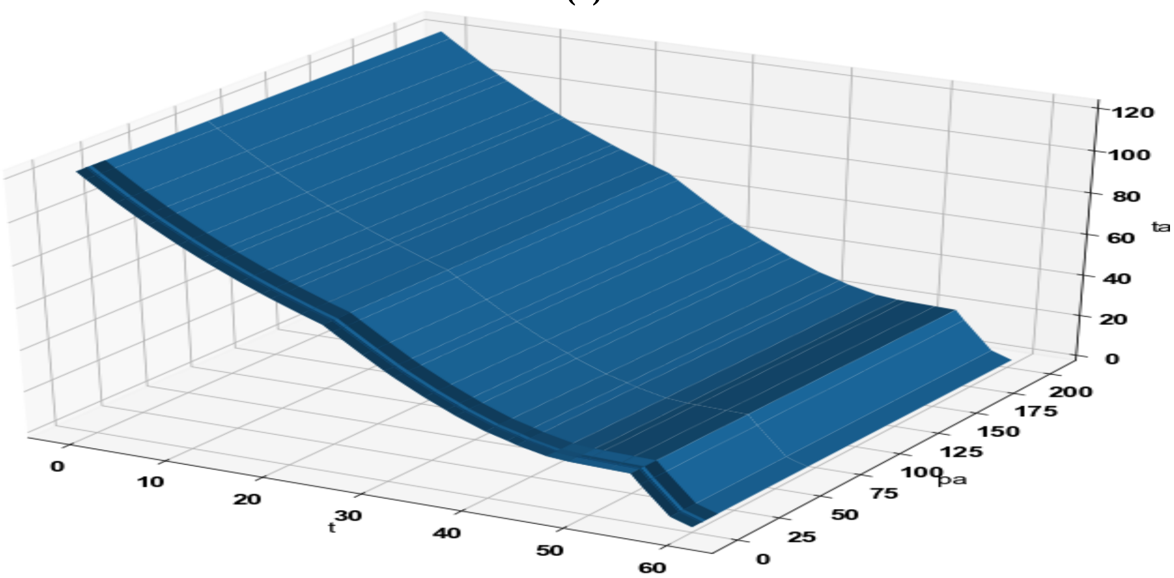
**(f)**


0.05
0.04
0.03
phi
0.02
0.01
0.00
1000
800
600
na
400
200
0
10
20
30
t
40
50
60

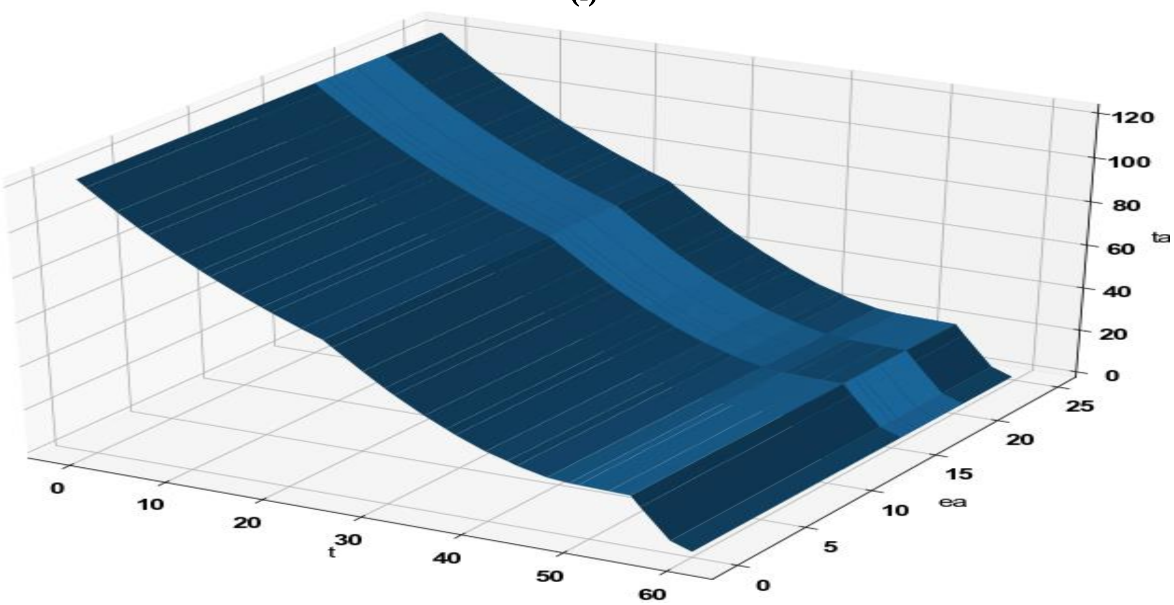
**(g)**


0.05
0.04
0.03
phi
0.02
0.01
0.00
0
10
20
30
t
40
50
60
0
5
10
15
ea
20
25

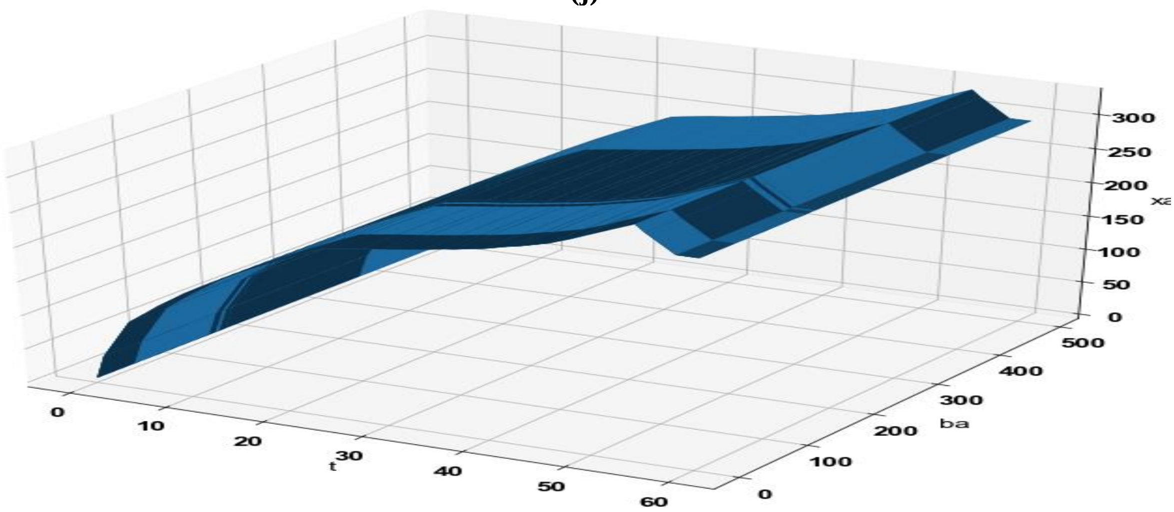
**(h)**


120
100
80
ta
60
40
20
0
0
10
20
30
t
40
50
60
0
25
50
75
100
pa
125
150
175
200

**(i)**


120
100
80
ta
60
40
20
0
0
10
20
30
t
40
50
60
0
5
10
15
ea
20
25

(j)




(k)

**Figure 5. (a)** MNLMPC profile in problem ba vs. t; **(b)** MNLMPC profile in problem 5 na vs. t; **(c)** MNLMPC profile in problem 5 ea vs. t;**(d)** MNLMPC profile in problem 5 phi vs. t; **(e)** MNLMPC profile in problem 5 ta vs. t; **(f)** MNLMPC profile in problem 5 xa vs. t; **(g)** MNLMPC surface in problem 5 )phi ea, t); **(h)** MNLMPC surface in problem 5(phi na t); **(i)** MNLMPC surface in problem 5(na,ba,t); **(j)** MNLMPC surface in problem 5 (.(xa ea t); **(k)** MNLMPC surface in problem 5(.(xa ba t).

## 8. Discussion of results

The results indicate that existing models involving forests and carbon dioxide emission can be modified to include other issues like global warming and carbon dioxide release. While adding new equations to a model can affect the presence of Hopf bifurcations, using an activation factor involving the tanh function eliminates the unwanted oscillation causing Hopf bifurcations. The modifications that involve the addition of new equations do not seem to affect the single objective optimal control calculations. The MNLMPC calculations provide strategies to minimize the increase of carbon dioxide in the atmosphere despite the inevitable rise in the human population. Minimizing the forest area and maximizing the human population, which are sometimes unavoidable, while performing the multiobjective calculations provides control values that can restrict the damage.

## 9. Conclusions

Rigorous bifurcation analysis and multiobjective nonlinear model predictive control calculations were performed on existing and modified models pertaining to forest density global warming and atmospheric carbon dioxide. An activation factor was used to eliminate the undesirable oscillation causing Hopf bifurcations that was revealed by the bifurcation analysis. Minimizing the forest area and maximizing the human growth population is sometimes inevitable, but these factors are taken into consideration to obtain control values that can still minimize the damage.

Previous work merely shows the existence of Hopf bifurcations and perform single objective calculations. This article is the first work where the Hopf bifurcation points that occur in forest models have been eliminated with an activation factor and where multiobjective nonlinear model predictive control calculations were performed.

**Conflict of interest:** The author declares no conflict of interest.

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