

# Stresses occurring on a cylinder with annular B4C, Ti and AL (2024) materials

Hüseyin Fırat Kayıran

ARDSI Mersin Provincial Coordination Unit, Bahçelievler Neighborhood, Huseyin Okan Merzeci Boulevard, No: 507/A, 33140 Mersin, Turkey; huseyinfiratkayiran@gmail.com

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**ABSTRACT:** In this study, the calculated thermal stresses on a layered annular circular disk were analyzed analytically. At the end of the analysis, it was observed that the thermal tensile strength of the boron carburetor (B4C) with a high modulus of elasticity is higher than titanium, and titanium is more than aluminum. The results obtained are shared with the literature and presented with graphs.

**KEYWORDS:** mathematical formulation; annular disk; stress analysis

## 1. Introduction

It is very possible to come across studies related to discs in the literature. Recently, due to the increase in the use of machine parts in a very high-temperature environment, it has become important to study the thermo-mechanical behavior of different solids with temperature-dependent material properties. In general, when the studies conducted in this field are examined, the stresses occurring on the brake discs coated with  $\text{Cr}_2\text{O}_3$  using the plasma spray technique in one study and the thermal behavior of a disc modeled in the other study were examined<sup>[1,2]</sup>. In a different study, a thermomechanical analysis of the brake disc of a heavy vehicle with finite elements was performed. As a material choice, he used materials made of stainless steel, aluminum alloy, and cast iron. They have shared the obtained results with the literature<sup>[3]</sup>. In different studies, stresses occurring in different disc materials were shared with the literature in graphs<sup>[4-7]</sup>.

In a different study, heat-related changes occurring on a thick plate were investigated<sup>[8]</sup>. In a different study, again, the thermoelastic problem of a cylinder modeled as a multilayer cavity was solved. The results obtained are presented with graphs<sup>[9]</sup>. In another study, the effect of changing thermal expansion coefficients on thermal stress was studied. The obtained result has been shared with the literature<sup>[10]</sup>.

## 2. Material and method

In this study, a disk with AL (2024) material in the innermost part, titanium in the middle part, and boron carbide (B4C) material in the outermost part was modeled. Taking into account the equations given below with references to the equations, the stresses occurring on the cylinder have been determined by means of a computer program. The formulas derived for the multilayer annular circular disk are given below (**Figure 1**).

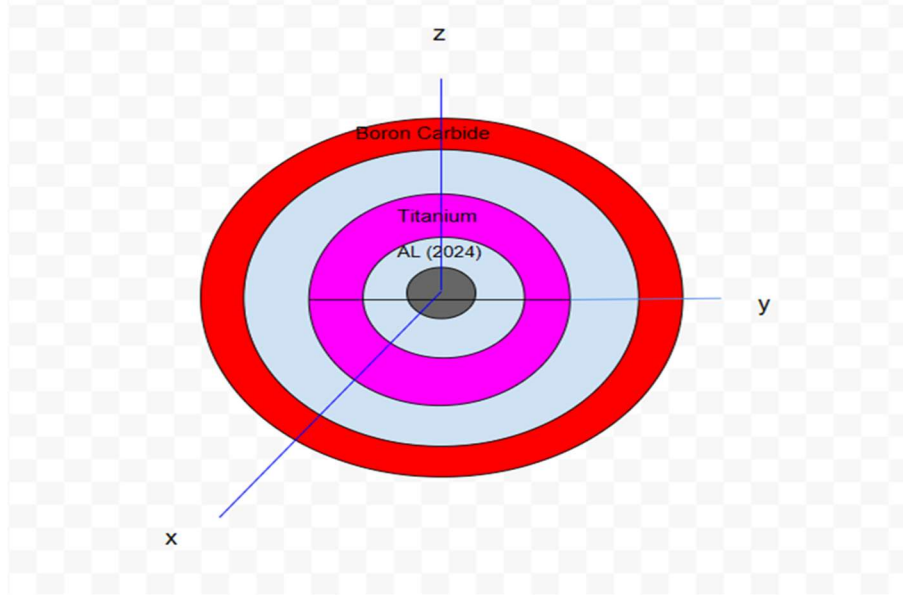


Figure 1. A ring-shaped disc.

The mechanical properties of the disc material are given in **Table 1** below.

Table 1. A composite disc with Kevlar (491PR-286)<sup>[11–13]</sup>.

	$E$ (GPa)	Thermal conductivity	Thermal diffusivity	$\alpha_r$ (1/°C)	$\nu_{er}$
<b>B4C (boron carbide)</b>	472	4.2	2.89	$9.4 \times 10^{-6}$	0.21
<b>Titanium</b>	120	11.4	2.53	$8.5 \times 10^{-6}$	0.36
<b>AL (2024)</b>	73.1	12.1	2.68	$2.32 \times 10^{-6}$	0.33

As shown in **Figure 1**, the inner layer is modeled as AL (2024), the middle layer is titanium, and the outer layer is boron carbide. The modulus of elasticity of the outer material is selected as high.

### Analytical formulation

The heat conduction equations occurring in a multilayer circular disk depending on the temperature are derived below<sup>[14,15]</sup>:

$$\frac{1}{r} \frac{d}{dr} \left( r \lambda_1(T_1) \frac{d(T_1)}{dr} \right) + \frac{1}{r^2} \frac{d}{d\theta} \left( \lambda_1(T_1) \frac{d(T_1)}{d\theta} \right) + \frac{d}{dz} \left( \lambda_1(T_1) \frac{d(T_1)}{dz} \right) = \rho_i C_i T_i \frac{d(T_1)}{dT} \quad (1)$$

$T_i = 0, t = 0$ , Boundary conditions; inner surface of the first layer ( $i = 1$ ),

$$\lambda_1(T_1) \frac{d(dT_1)}{dr} + h_0 T_1 = 0 \quad (2)$$

outer surface of the  $k$ -th layer ( $i = k$ ), interface of the  $i$ -th layer ( $i = 2, 3, \dots, k$ )

The boundary conditions are determined by the basic equations in the cylindrical coordinate system. The equations are derived from each other.

$$\Lambda^2 \Lambda^2 \Lambda w^i = \frac{-1}{(1 - \nu_i) D_i} \Lambda^2 D M_T \quad (3)$$

$$D_i = \frac{E_i \Lambda h^3}{12(1 - \nu^2)} \quad (4)$$

Boundary conditions;  $r = r_0, r = r_k$ , moment and stresses are given below using Hooke's law.

$$\sigma_{rr} = \frac{1}{h} N_{rr+} + \frac{12z}{h^3} M_{rr} + \frac{1}{(1-\nu_i)} \left( \frac{1}{h} N_{T+} + \frac{12z}{h^3} M_T - \alpha_i(T_i) k^2 E_i T_i \right) \quad (5)$$

$$\sigma_{\theta\theta} = \frac{1}{h} N_{\theta\theta+} + \frac{12z}{h^3} M_{\theta\theta} + \frac{1}{(1-\nu_i)} \left( \frac{1}{h} N_{T+} + \frac{12z}{h^3} M_T - \alpha_i(T_i) k^2 E_i T_i \right) \quad (6)$$

$$M_T = E_i \int_0^h \alpha_i(T_i) T_i z dz; N_T = E_i \int_0^h \alpha_i(T_i) T_i z dz; ; \alpha_i(T_i) = \alpha_{i0} \exp(w_2 T_i) \quad (7)$$

### 3. Findings and discussion

At the end of this study, it is observed that the heat resistance of the material with a high modulus of elasticity depending on the tangential stress is excessive. The results obtained by numerical analysis of tangential stresses occurring in different directions are given below between **Figures 2–4** with graphs (**Figures 2–4**).

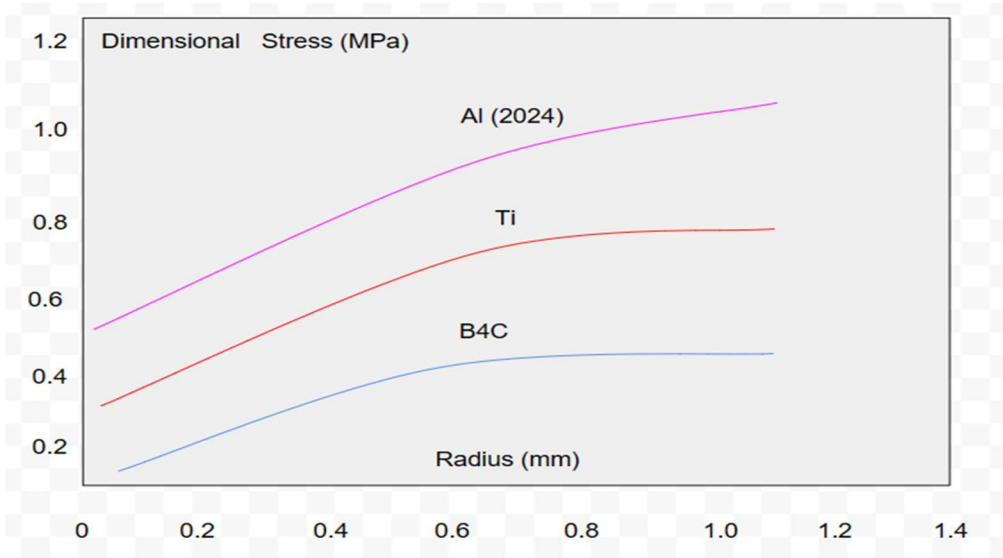


Figure 2. Plot of  $\sigma_{\theta\theta}$  along  $\theta$ .

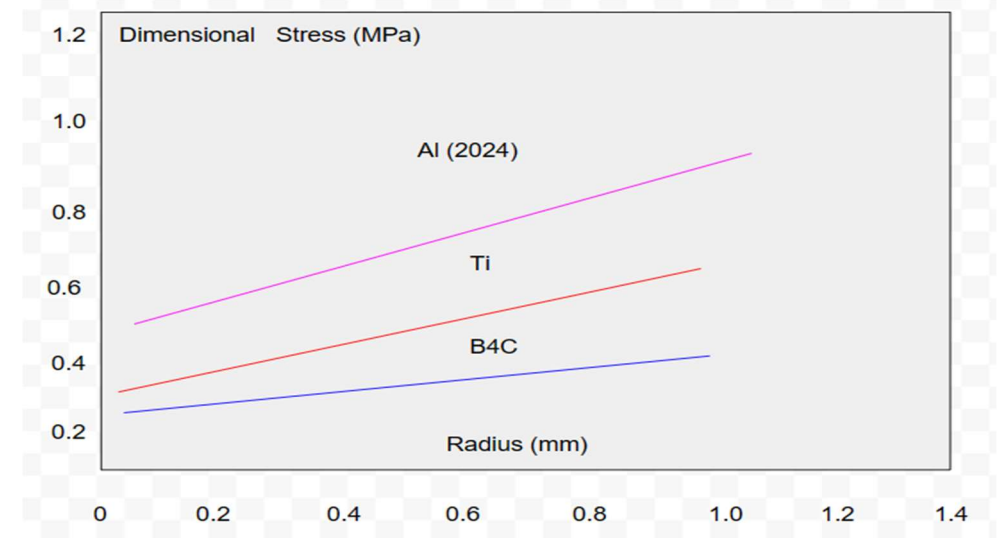


Figure 3. Plot of  $\sigma_{\theta\theta}$  along r.

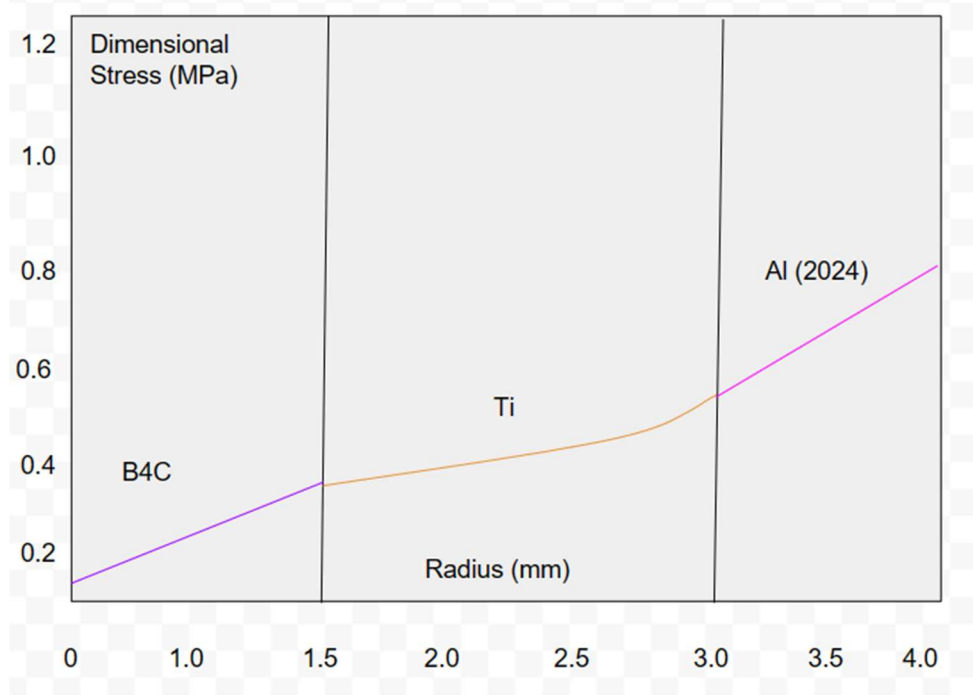


Figure 4. Plot of  $\sigma_{\theta\theta}$  along  $z$ .

It is thought that the results obtained at the end of this study may be compatible with other studies in the literature<sup>[16,17]</sup>.

## 4. Conclusion

In this study, the thermal behavior of materials with different mechanical properties depending on the temperature in a thin annular cylinder (disk) was investigated. It has been observed that the thermal tensile strength of boron carburetor (B4C) with a high modulus of elasticity is more than titanium, and titanium is more than AL (2024).

In this study, the temperature profile of an inhomogeneous multilayer annular disk with convective heating on it was investigated. Materials with different properties were selected in the study.

## Conflict of interest

The author declares no conflict of interest.

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