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Cumulative quantum mechanics and the method of generalized mathematical transfer for describing the interaction of an electric field with matter in nanostructures of chemically doped diamond

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Abstract: We analytically (based on cumulative quantum mechanics (CQM) and the method of generalized mathematical transfer (MGMT)) study quantum nanometer cumulative-dissipative structures (CDS) and the forces arising in them, focusing nanostructures into regular, fractalized systems—cumulative dissipative standing hydrogen-like excitons (hydrogen-like atoms, molecules, lines, surfaces) and flickering crystals discovered by the author. The CQM turned out to be useful in describing “mysterious” CDS with sizes of 10^{-15} – 10^{26} m. In CDS, cumulation and dissipation of masses, energies, momenta and fields occur simultaneously (or with a certain delay) and in accordance with fairly general laws (the virial theorem works). Using the CQM and MGMT, in this paper we describe cumulative and dissipative phenomena in diamonds doped with boron (atoms from group III of the periodic table). Problems in crystals with chemical doping with foreign atoms (with their introduction into the crystal lattice) were solved. We: (1) discovered the Vysikaylo’s standing excitons formed on inhomogeneities of the permittivity $-\varepsilon(r)$ in diamond in the nanoregion of a foreign atom; (2) for the first time we solved the problem of measuring the $\varepsilon(r)$ profiles in inhomogeneous nanoscale structures using Raman spectra (RS) (with an accuracy of up to 99.9% of $\varepsilon(r)$ and a step of up to 0.3 nm depending on the distance from the impurity atom (boron)); (3) based on our theory of the Vysikaylo’s standing excitons, we prove the observation in the experiments described in the literature of degeneracy of the electron spectra in standing excitons with respect to the principal quantum number n and $n-1/2$. Comparing the theory and experimental observations of RS in diamonds doped with boron, we solve (formulated by us earlier) the problem between the de Broglie hypothesis and the classical new quantum mechanics of Dirac (which limits the ψ -functions-prohibits symmetric de Broglie half-waves in spherically and cylindrically symmetric quantum hollow resonators) in favor of the de Broglie hypothesis. Based on the works of Vanier and Mott, we have refined the definition of the permittivity of nanocrystals as a coefficient in electric potentials ($U(r) \rightarrow \varepsilon(r)U(r)$), rather than electric fields ($D(r) = \varepsilon(r)E(r)$). We have constructed the most complete theory of chemical doping of crystals (using the example of crystals of group IV elements with atoms of groups III and V from the periodic table. For the first time, the question of quantum cleaning of crystals or cumulation of dopant atoms to each other has been raised.

Keywords: physical and chemical doping of nanostructured materials; Vysikaylo’s cumulative-dissipative structures; cumulative quantum mechanics; cumulation of de Broglie waves; unlimited cumulation of de Broglie wave $\psi_{n-1/2}$ -functions; chemical doping; spherically and cylindrically symmetric quantum hollow resonators; exciton

1. Introduction

General knowledge obtained in solving some problems can and should be used in describing seemingly completely different phenomena described by partially

modified mathematical models. Much in common (analogous), discovered in some areas of science, can and should be discovered, studied and applied in other sciences. We generalize mathematical analogs for describing phenomena in femto-, nano-, meso- and macro-worlds and the concept of “dark spaces” and points of cumulation (L_1) of energy-mass-momentum flows between Coulomb attractors of any scale. In this paper, we reveal the essential role of these areas and cumulation points in the transfer of energy-mass-momentum flows in flickering crystals from nanometer attractors. Cumulation (self-focusing) of charged particles in inhomogeneous media with electric fields is a universal property of CDS. To describe such phenomena in crystals doped with foreign atoms, we modify the CDS model for hollow quantum polarizing resonators. It is based on CQM. Cumulation (self-focusing) of charged particles in inhomogeneous media with electric fields is a universal property of a number of CDS with typical dimensions from 10^{-15} to 10^{26} m [1–5]. The Vysikaylo’s CDS include neutral and charged structures such as neutrons, atomic nuclei, atoms, molecules, lightning, tornadoes, stars, galaxies, intergalactic lightning, states, ethnic groups, living organisms, etc. The analogy of processes in CDS allows us to apply the method of generalized mathematical transfer (MGMT) of the most complete mathematical models to describe similar phenomena from well-studied areas of science in less thoroughly studied areas of natural sciences. When transferring mathematical models in this way, we must take into account the specifics of the phenomena being described. The use of nanosystems in the transformation of atomic energy and electromagnetic radiation into other types of energy is very promising.

In this paper we use the method of general mathematical transfer (MGMT) to describe the phenomena of cumulation and dissipation of electromagnetic radiation in the region of a foreign atom chemically doping diamond crystals. This method consists of transferring mathematical models and their solutions from well-studied areas of natural sciences to less well-studied areas of natural sciences.

1.1. Definition of cumulation

The most intelligible definition of cumulation is given in the preface by Zeldovich [6]: “Cumulation is the concentration of force, energy or another physical quantity in a small volume.” We will rely on this definition in the future, implying that we are talking about the cumulation of a certain parameter (dynamic or static order). When using this definition, it should be understood that during the cumulation of one parameter, “dissipation” (scattering, decrease, etc.) of another parameter can be observed. Thus, when a shock wave expands into the atmosphere, its speed increases due to a decrease in the density of the atmosphere with height, while the energy density decreases. In this case, we can talk about the cumulation of speed in the wavefront, but there is no need to talk about the cumulation of energy. The main results of the study of the phenomenon of unlimited cumulation are presented in Zababakhin’s study [6] in the form of separate problems.

The difference between Vysikaylo’s CDS and dissipative structures (Kolmogorov-Turing-Prigogine and their followers—DS) is the consideration of convective processes of focusing (cumulation) of energy-mass-pulse flows (EMPF) to points, lines or surfaces of cumulation [1–5]. The difference between CDS discovered

by Vysikaylo [1] and the DS (in particular, from Busselator) is that in CDS there is a generation of new degrees of freedom (NDF: rotation, pulsations, violation of electroneutrality, generation of electric and magnetic fields, etc.) on account of a cumulation of convective flows. Vysikaylo's CDS have a number of common properties that differ from the properties of DS. We will focus on a number of common properties of positively charged (+) CDS (cathode spots, atomic nuclei, stars, galaxies, etc.).

1.2. Method of generalized mathematical transfer

The analogy of a number of processes in the CDS allows us to apply the method of generalized mathematical transfer (MGMT) of the most complete mathematical models and their solutions to describe similar phenomena from well-studied areas of science in less thoroughly studied areas of natural science. When transferring mathematical models in this way, we must take into account the specifics of the phenomena being described. MGMT has long been used and is still used to accelerate the development of philosophy and natural sciences in various areas of natural science. This method was used by:

- 1) Newton. He studied and generalized the description of gravitational forces on Earth and in space. On 29 March 1696, Isaac Newton was appointed Warden (and from 1699, Director) of the London Mint. In this position, he saved England from chaos due to the debasement of money by counterfeiters;
- 2) Louis de Broglie described quantum phenomena using his hypothesis. He put forward the hypothesis: "Particles behave like waves" and received the Nobel Prize for developing this idea. He introduced the concept of de Broglie wave for particles;
- 3) Einstein to describe the photoelectric effect. He applied the opposite idea: "Electromagnetic waves behave like particles" and received the Nobel Prize for explaining the photoelectric effect;
- 4) Niels Bohr. He successfully applied the planetary model of Copernicus and his followers to describe electric potentials and electron orbits to explain the spectra of the hydrogen atom. In 1922, Niels Bohr was awarded the Nobel Prize "for his services to the study of the structure of atoms and the radiation emitted by them";
- 5) Schrödinger with Dirac. They created a new quantum (wave) theory that takes into account the wave statistical properties of an electron in a hydrogen atom and its wave passage through two slits. As a result, in 1933, Schrödinger and Dirac received the Nobel Prize "for the discovery of new productive forms of atomic theory" (and thus for the transfer of Bohr's theory to the status of a pseudoscientific theory);
- 6) Vysikaylo to the discovery and description [1–5] of:
 - a) Shock waves of an electric field and plasma nozzles (analogs of Laval nozzles) in a plasma with current (1985–2024). For this, he constructed the most complete theory of perturbations in a plasma with current and thus obtained a modified Navier-Stokes equation, which made it possible to analytically and numerically describe all the phenomena observed (in the experiments organized by him) in an inhomogeneous plasma with current;

- b) 5 points of cumulation and libration for electrons in a system of two rotating positively charged Coulomb attractors (two atoms in a molecule). Here he took into account the generality of the laws of cumulation $\sim 1/r$ in gravitational and Coulomb potentials. He related the number of electron cumulation points in a molecule with the valence of atoms;
- c) Points, lines and surfaces (charged strata—running and standing shock waves of the electric field) of libration and cumulation for free electrons between positively charged structures of plasma with current. He proved that Coulomb (electric) potentials in 4D space-time function similarly to gravitational potentials;
- d) Endoelectrons in fullerenes and, accordingly, all spectra of negatively charged fullerenes. He proved (based on a number of experiments [2]) the formation of symmetric de Broglie waves of electrons in hollow fullerenes and the corresponding degeneration of spectra in hollow fullerenes with respect to the principal quantum number n and $n-1/2$. Such similar cos-waves are formed (when drops fall into cylindrical glasses with liquid, when studying sonoluminescence phenomena, etc.) in ordinary hydrodynamics in hollow spherically and cylindrically symmetric resonators.
- e) Standing Vysikaylo's excitons formed in the area of chemical doping of diamonds with boron atoms. Here he modified the Wannier-Mott model (with a uniform value of the permittivity $\varepsilon = \text{const}$ throughout the crystal) into a model that takes into account the polarization of crystals ($\varepsilon = \varepsilon(r)$) when a foreign atom is introduced into their crystal lattice. And thus he proposed a new method for determining the profile of the relative permittivity $\varepsilon(r)$ from the distance to the center of a standing exciton using Raman spectra [5];
- f) The mechanism of Coulomb fractalization of meteoroids and small asteroids by a plasma tail behind them. This mechanism was developed on the basis of a modification of the Vysikaylo's model of lightning impulse propagation observed in experiments by Shenland in 1934–1938. Shenland experimentally proved, using a Boyce chamber, that electrons, cumulating in lightning, run away from it forward (in the form of runaway electron jets) with an energy of up to 5 MeV, etc.

The MGMT goes back to the idea of Eratosthenes, which he applied (19 June 240 BC) when calculating the length of the Earth's meridian (geometric structures are similar, various theorems can be applied to them, etc.). The method, founded by Eratosthenes, has long been successfully used on Earth and helps in the discovery and quantitative description of "mysterious" phenomena, including CDS.

Here we used this general method (MGMT) to expand (transfer) the achievements of the Vysikaylo's cumulative quantum mechanics [2,5] (obtained by analytically describing the phenomena of cumulation of de Broglie waves of electrons in hollow quantum resonators, using the example of hollow fullerenes— $C_{60,70}$), to describe the Vysikaylo's standing hydrogen-like excitons discovered in diamonds (crystal groups 4 of the periodic table) during their chemical alloying with foreign atoms with a valence other than carbon (atoms from the 3rd and 5th groups of elements of the periodic table).

In physics, there are two ways of alloying materials: 1) physical alloying with atomic or molecular structures having a high affinity for free electrons [2]; 2) chemical alloying, in which a foreign atom is embedded (with the replacement of the support lattice atom) into the crystal lattice of the support crystal [5].

1.3. Classification of excitons

The concept of an exciton is close to the concept of a solitary wave, soliton or CDS. When electromagnetic energy (waves) accumulates (absorbs and localizes) in a crystal, electrons and holes (positive ions) are formed in space. At any distance, they experience Coulomb interaction, which is weakened by the polarization of the medium localized within the characteristic size of the exciton (the distance between the electron and the ion). The interaction leads to the fact that electrons and holes should be considered in 3D space as a bound electron-hole pair—an exciton. In this case, an exciton is a quasiparticle (a quantum pulsar in which an electron pulsates in the region of a positively charged hole in accordance with its total energy), arising during globally currentless excitations in semiconductors. Coulomb potentials act as lenses, focusing electrons that have received additional energy. By focusing electrons that have received energy from electromagnetic waves, these potentials thereby cumulate (localize) the energy of these electromagnetic waves in an exciton. The cumulation of electromagnetic waves by excitons leads to an increase in the total energy of an electron in an exciton, a decrease in its kinetic energy in the potential well, and a corresponding increase in the radius of its pulsation in the hole region. And, conversely, the release of electron energy in an exciton (in the form of electromagnetic waves) leads to a decrease in the effective radius of electron pulsation in an exciton and a corresponding increase in its kinetic energy. This is the general scheme of pulsation of a quantum Coulomb pulsar and the formation of electron levels of excitons [5]. This general scheme requires a certain amount of detail in the case of excitons in crystals doped with foreign atoms.

Depending on the nature of the bond, two types of excitons localized in potential wells are noted [7,8] (**Figure 1**). The first type is small-radius excitons (Frenkel's excitons) associated with a specific atom; their sizes do not exceed the interatomic distance in the crystal [7,8]. Frenkel's excitons can move along the supporting crystal (if they were formed on the basis of one of the atoms of the crystal lattice) or be localized if the energy spectra of the excited atoms do not coincide with the energy spectra of the surrounding atoms [5]. The second type is free hydrogen-like spherically symmetric excitons of large Wannier-Mott radius (WM excitons), the characteristic sizes of which reach 20 nm (tens and hundreds of interatomic distances). This type of exciton is formed from atoms or molecules of a homogeneous reference crystal (with constant values of relative permittivity $\epsilon(R) = \text{const}$) and therefore they can move freely along the crystal.

We will dwell on the discovery and description of the third new type of standing (localized) excitons of large size and the formation of large-radius molecules, lines, surfaces and flickering crystals (superlattices) from them. These Vysikaylo's standing excitons (hydrogen-like, molecular-like, linear-, surface- and crystal-like structures from excitons) are formed by chemical doping of crystal lattices with impurity atoms

with a valence different from the valence of the atoms of the reference crystal. We prove that chemical doping leads to local changes in the relative permittivity in crystals. Let us consider the scheme of chemical doping.

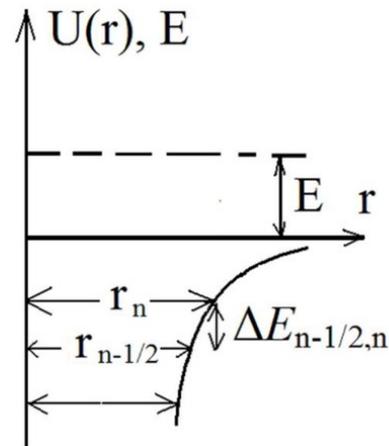


Figure 1. The de Broglie-Bohr-Schrodinger problem for hydrogen-like nanostructures.

The scheme of a stable (closed) quantum dot with an infinite potential Coulomb barrier surrounding it for particles with total energy less than zero is shown in **Figure 1**. To describe the energy spectrum of a quantum dot, a model of a hydrogen-like atom with an “orbit radius” is used r_n and $r_{n-1/2}$ are the characteristic radii of a quantum resonator with a Coulomb potential for quantum particles with a principal quantum number n and energies E_n (with asymmetric ψ_n) and $E_{n-1/2}$ (with symmetric $\psi_{n-1/2}$), respectively. $\Delta E_{n-1/2,n}$ is the Vysikaylo’s energy splitting of the higher levels with the main quantum number n [2].

1.4. Details schemes of Vysikaylo’s excitons and chemical doping of support crystals with acceptor impurities

Acceptor doping is the process of introducing an impurity into a material that locally lowers the Fermi level. This is achieved by doping the reference crystal with atoms of lower valence. Since the valence of the doping atom is lower, one of the valence electrons in the doped crystal is unpaired and, receiving additional energy, forms an exciton in the region of the doping atom. We will consider in detail the scheme of doping diamond (elements of group IV of the periodic table) with boron atoms (elements of group III).

Figure 2 shows the general scheme of chemical doping of diamond with an acceptor impurity—a boron atom. **Figure 2a** shows a flat 2D scheme of an unperturbed diamond crystal. In diamond, each carbon atom forms four paired strong sp^3 bonds. The distance between the centers of carbon atoms in diamond is 0.154 nm. In this case, the value of the relative permittivity of a homogeneous diamond is $\varepsilon = 5.7$.

When diamond is doped with a boron atom (its valence is 3), one electron of carbon (its valence is 4) in the region of the boron atom turns out to be unpaired and, receiving energy, can form a Vysikaylo’s standing exciton in this region [5]. **Figure**

2b shows a rough 2D diagram of diamond doping with a boron atom. As shown by numerical calculations performed using the DFT method (first-principles studies) [9], the distance between the boron and carbon atoms is significantly greater than the distance between the carbon atoms in pure diamond and reaches a value of -0.159 nm. This phenomenon was not taken into account in the classical works of the Collins group [10] and their followers, who were engaged in the alloying of diamonds. If this is so, then doping with a boron atom leads to a perturbation of the diamond crystal lattice in the region of the foreign atom (the boron atom pushes the carbon atoms apart) and thus leads to a significant local increase in the relative permittivity $-\epsilon$ and, consequently, leads to a change in the entire spectrum of the free WM exciton, making such an exciton a Vysikaylo's standing exciton [5]. A fast (relative to the velocity of electron movement in an exciton) movement of a hole (in the region of the doping atom) forms an exciton with a positive ion smeared in time and a hollow center (**Figure 2c**). Here we observe an analogy with the polarization capture of a free electron into a fullerene cavity [4,11]. Therefore (according to MGMT), we will transfer the technique of analytical calculations of the eigenenergy spectra in hollow spherically symmetric quantum resonators to the calculations of the energy spectra of Vysikaylo's standing excitons in boron-doped diamonds [5]. According to experimental and analytical studies of the resonant capture of electrons into hollow fullerene molecules, cos-wave spectra (with symmetric ψ -functions) appear in the eigenenergy spectrum of quantum resonators, i.e., spectra with half-integer principal quantum numbers $n-1/2$. This quantum degeneracy in the principal quantum number was discovered by the author on the basis of cumulative quantum mechanics and experimental data [2,5].

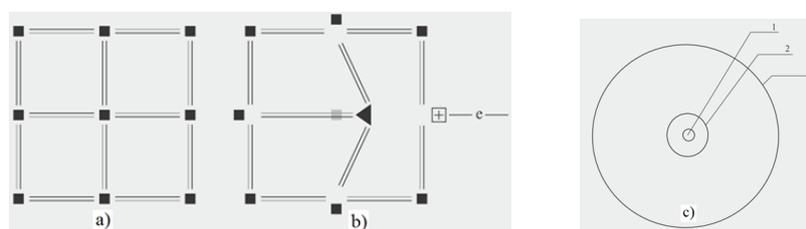


Figure 2. Geometric 2D scheme of formation of a quantum dot (socialization of an electron by the volume of a nanocrystal): **(a)** ideal crystal lattice of an element from group IV of the periodic table of D.I. Mendeleev; **(b)** geometric 2D scheme of modification of an ideal crystal lattice by introducing a boron atom into the crystal lattice (triangle). The impurity atom replaces the atom in the original (ideal) crystal lattice. e is the electron shared by the volume of the quantum dot. $+$ is a hole moving around the boron atom at a speed corresponding to the kinetic energy of the valence electrons of carbon atoms (the positive ion moves quickly). The gray square is the node of the ideal lattice to which the shared electron cumulates; **(c)** a spherically symmetric quantum dot in diamond with a boron atom replacing a carbon atom: 1—an internal cavity in the region of a node of an ideal crystal lattice, caused by pressing a boron atom to only three carbon atoms; 2—a sphere of motion of a positively charged hollow hole with a radius of $d \approx 1.59$ Å; 3—a boundary within which an electron is localized in a stable spherically symmetric quantum dot—a standing Vysikaylo's exciton (or color center) with a radius of a_n .

1.5. On the identification of the electronic spectra of Vysikaylo's excitons observed in experiments

The author suggests that a number of long-observed Raman spectra in boron-doped diamonds belong to the electron spectra of Vysikaylo's standing excitons [5]. However, for a number of reasons, these spectra have not yet been properly identified due to the lack of understanding of the basics of cumulative quantum mechanics (CQM) among experimenters [2]. When comparing with experimental observations of the energy spectra of Vysikaylo's standing excitons, we will mainly focus on the spectra established in the works of Collins and his co-authors. These spectra are recognized as reliable by most researchers of boron-doped diamonds. Collins in 1994 claimed that there are no adequate theoretical descriptions of any of the processes observed in the experiments: temperature dependences of scattering mechanisms, the contribution of the split-off valence band, and the population of excited states in the conductivity of doped diamond crystals [10]. In this case, for both polycrystalline and single-crystal homoepitaxial CVD diamond, measurements of electrical properties can be completely nullified due to the presence of a surface layer of non-diamond carbon. The task of describing all possible emission and resonant absorption spectra in doped diamonds is a rather complex and ambiguous task. We will apply MGMT to describe the electron spectra of Vysikaylo's hydrogen-like standing excitons [5]. This will prove that the description of Vysikaylo's standing hydrogen-like exciton atoms and molecules (using MGMT) can be carried out similarly to the description of the spectra of hydrogen atoms and molecules.

Before we proceed to the description of the electron spectra of Vysikaylo's standing hollow excitons (**Figure 2c**), we will briefly dwell on the achievements of cumulative quantum mechanics, based on the analytical [5] and experimental [11–13] study of the polarization resonance capture of free electrons into the internal cavity of various types of fullerenes ($C_{60,70}$). This will give us grounds to take into account a number of additional spectra involving the principal quantum numbers $n-1/2$. Thus, we will apply the basics of Vysikaylo's CQM in our study and interpretation of Raman spectra in diamonds doped with boron.

2. Achievements of the Vysikaylo's cumulative quantum mechanics

Along with the discovery of new materials, new technologies are being developed and attempts are being made to create mathematical models capable of describing phenomena in hollow quantum resonators—quantum dots, lines, and other cumulative-dissipative three-dimensional structures of nanometer dimensions. Such goals can be achieved by testing and modernizing the foundations of Dirac's new quantum wave mechanics (NQWM) and supplementing it with the foundations of Vysikaylo's cumulative quantum mechanics (CQM), which describes real cumulative and dissipative phenomena in nanometer structures within the framework of de Broglie's quantum mechanics [5]. This modification of the NQWM, confirmed by experiments in the nano-world in theoretical and experimental [2], studies of the polarization capture of electrons into the internal cavity of fullerenes, turns out to be very useful in describing the “mysterious” cumulative-dissipative phenomena in

structures with sizes from 10^{-15} to 10^{26} m [1–5]. The results of our study and the experiments available [2], can be formulated as follows:

1) It is necessary to limit everywhere only the probability (or the probability density $W(r) dr = \chi^2 |\psi|^2 dr$ taking into account the geometric coefficient $-\chi^2 = 2^k \pi^k r^k$, here $k = 1$ for cylindrical symmetry of a hollow resonator, $k = 2$ for spherical symmetry and $\chi^2 = 1$ for plane symmetry) of finding a particle in the volume of a resonator with different types of symmetry, and not the ψ -function, as Dirac [14] and his followers [15,16] do. They limit the ψ -function everywhere. We limit everywhere only the probability density of finding a particle— $W(r) = \chi^2 |\psi|^2$;

2) Modification of the Dirac's requirement and taking into account the geometric coefficient $-\chi$, normalizing the $\psi_{n-1/2}$ -functions ($\psi_{n-1/2} \sim \cos(kr)/r$ in spherical and $\psi_{n-1/2} \sim \cos(kr)/r^{1/2}$ cylindrical hollow resonators) transforms the forbidden eigen-energy spectra with energy $E_{n-1/2}$ with unlimited cumulation to the centers of hollow resonators (with $\psi_{n-1/2}$ -functions) into allowed ones. In this case, all spectra previously allowed only for resonators with planar symmetry [17] become allowed for spherically and cylindrically symmetric hollow quantum resonators, such as:

$$E_{n-1/2} = E_{ncos} = \pm (\hbar^2/2m)\pi^2(n-1/2)^{\pm 2}/(R + r_{ind})^2, n = 1, 2, 3, \dots \text{ и} \quad (1)$$

$$E_{n-1/2} \Leftrightarrow E_i; E_{n-1/2} \Leftrightarrow E_{i-1/2}$$

here “–” is for quantum wells, and “+” is for quantum resonators surrounded by potential barriers (polarization traps, etc.) [5], R is the characteristic size of the resonator, r_{ind} is the polarization length by which the resonator size increases, m is the classical electron mass;

3) The eigen-energy spectra (1) for hollow quantum resonators with spherical and cylindrical symmetries will be called the Vysikaylo's energy spectra, and the degeneracy of the electron spectra with respect to the principal quantum number n and $n-1/2$ will be called the Vysikaylo degeneracy. These spectra, obtained analytically in Vysikaylo's study [2], were in excellent agreement with the experimentally measured eigenenergy spectra of fullerenes C_{60} and C_{70} during the study of resonant captures of electrons by these molecules into their internal cavity [11–13]. Thus, negatively charged endo-ions $e_k@C_n$ with endo-electrons inside them are formed on the basis of hollow fullerenes – C_n . Here $k = 1, \dots, 6$ is the number of captured electrons, n is the number of carbon atoms in the fullerene [2];

4) The complete eigen energy spectra (taking into account the Vysikaylo's spectra) of hollow quantum resonators do not depend on the symmetry of the quantum resonator [2];

5) Our analysis of the experimentally obtained resonance spectra of electron capture (with resonance energy) by fullerenes [11–13] and comparison with our analytical calculations of these spectra within the framework of CQM [2] allows us to assert that the ψ -function of a particle appears only when boundary conditions are specified for the Schrödinger equation. This means that the particle always remains a particle with its mass, charge and other parameters, and the wave properties of the particle correspond to its statistical behavior in a quantum resonator and are determined by the boundary conditions, i.e. external factors.

Thus, in the case of experiments with “passing electrons through two slits” the interference pattern appears only as a statistical result. The representation of a free particle as a moving plane de Broglie wave is incorrect! The particle itself does not transform into a shell, a plane wave, etc. Statistically, the particle behaves like a wave in a certain resonator, and this behavior creates the effect of its transformation into a wave or shell. When the characteristic dimensions of quantum resonators change, the ψ -function of a quantum particle trapped in the resonator can be changed, for example, when an electron collapses into a proton in an atomic nucleus, etc. In this case, according to the virial theorem proved by Fock for quantum phenomena [18], half of the potential energy goes to increasing the internal energy of the proton turning into a neutron, and the other half goes to emitting a neutrino with an energy of about 0.85 MeV [5]. In this regard, the phenomena described within the framework of wave cumulative quantum mechanics have analogues in ordinary hydrodynamics [2,5].

In general, the cumulation and dissipation of energy, mass and momentum in CDS may not necessarily occur simultaneously. Thus, in an atom, the absorption of resonant electromagnetic energy and its subsequent re-emission usually occur with a certain time delay. However, the emission of an electromagnetic wave by an atom and the transition of an electron from an upper energy level to a lower energy level, according to the virial theorem, occur simultaneously [18]. Similarly, when an atom absorbs electromagnetic energy, the transition of an electron to a higher energy level also occurs simultaneously with the absorption of energy. Here, the processes of cumulation (focusing) and dissipation (increase in the characteristic dimensions of the structure) are co-organized. Here, the processes of cumulation (focusing) and dissipation (increase in the characteristic dimensions of the structure) are co-organized.

3. Basic provisions for the description of Vysikaylo’s standing excitons

In our papers we consistently solve the problems arising (as we believe, in describing the eigen-energy spectra of quantum nano-resonators of various natures) between the de Broglie hypothesis and the classical Dirac’s NQWM, which everywhere limits the values of ψ -functions. As we have proved (in our analysis of the available experimental and our theoretical studies [2]), it is a mistake to limit ψ -functions everywhere without taking into account the normalizing geometric coefficients χ^2 . In Vysikaylo’s study [2], it is shown how these problems and paradoxes (discrepancies between experimental observations and incomplete theories) caused by phenomena arising from the violation of electrical neutrality (in particular, polarization) in nanostructured composite materials are solved using CQM and taking into account the regularization of unbounded $\psi_{n-1/2}$ -functions by the normalizing geometric coefficient $-\chi^2$, which takes into account the symmetry of the quantum resonator [2]. The use of CQM made it possible to discover about 35 quantum-dimensional effects and describe a number of previously “mysterious” phenomena [2].

The discovery of the Vysikaylo’s degeneracy by the main quantum number n and $n-1/2$ in hollow quantum resonators significantly changes the parameters of quantum stars (white dwarfs, neutron stars and black holes) [1].

In this paper, we will consider similar problems in describing nanostructures that arise during chemical doping of crystals of group IV of the periodic table. We will do this using the example of studying nanostructures—Vysikaylo’s standing excitons: large hydrogen-like atoms, hydrogen-like molecules, lines, surfaces and flickering crystals of Vysikaylo’s standing excitons in inhomogeneous diamond doped with boron. For this purpose, we will modify the Wannier-Mott’s theory, constructed by them for homogeneous crystals, to describe standing excitons in inhomogeneous crystals [5].

3.1. Large running (free) Wannier-Mott’s excitons

Wannier-Mott excitons [7,8,19,20] exist in semiconductors due to the high permittivity of the latter. High permittivity $-\varepsilon > 5$ leads to weakening of electrostatic attraction between the electron and positively charged hole, which leads to a large radius of the exciton (a large hydrogen-like atom). The characteristic dimensions of Wannier-Mott’s excitons increase by ε times, and the energy spectra decrease by ε^2 times in relation to the parameters of the hydrogen atom:

$$E_n = -(m_r e^4 / (8\varepsilon_k \varepsilon_0 h)^2) / n^2 \quad (2)$$

where m_r is the reduced mass of the system (electron and hole) in a cubic crystal; ε_k is the constant permittivity of the crystal under consideration; n is the principal quantum number of the exciton. Here Wannier and Mott applied and detailed the method of Eratosthenes (MGMT) for predicting and describing large-radius excitons using a quantum model for the hydrogen atom.

The hydrogen-like electron spectrum of Wannier-Mott excitons was first observed in the absorption spectrum of Cu_2O in 1952 by Gross and Karyev and independently by Hayasi and Katsuki, but the excitonic interpretation was absent in the work of the Japanese authors. In all these studies, the permittivity was considered constant $\varepsilon = \text{const}$. Since such large-radius excitons are formed by the ionization of atoms or molecules of the supporting crystal, such excitons move freely along this crystal and are running (free) Wannier-Mott’s excitons.

3.2. Determination of relative permittivity for nanostructures

In 1937–1939, Wannier and Mott [19,20] proposed a model in which they predicted and described hydrogen-like structures of free excitons of large radius (up to 20 nm) in homogeneous crystals. For this, in the Bohr model of the hydrogen atom, they took into account the relative permittivity $(2) - \varepsilon$ (**Figure 3**). In the case of the Wannier-Mott (WM) model, the relative permittivity $\varepsilon(r) = \varepsilon_k = \text{const}$ for a pure crystal was considered constant throughout the crystal [7,8]. Under these assumptions, Wannier and Mott [7,8,19,20] were the first to show that the hydrogen atom model, taking into account the relative dielectric polarization, which weakens the Coulomb potential by a factor of $\varepsilon_k = \text{const} > 1$, can be successfully applied to describe free hydrogen-like excitons of large radius in homogeneous dielectric crystals [7,8].

Our modification of the Vanier-Mott’s model

A number of researchers mistakenly associate the definition of the relative permittivity ε_k with the wavelengths of electromagnetic radiation. They forget that

electromagnetic waves accumulate on electrons and thus increase the size (when they are absorbed by electrons) or decrease (when they are emitted by electrons) of the region of electron pulsation near the hole. (Within the framework of the old Bohr theory, the radius of the electron orbit in the exciton changes). The wavelength of electromagnetic radiation changes from zero to infinity. Therefore, already within the framework of the Wannier-Mott's model, the relative permittivity should be associated with the nanostructure corresponding to the characteristic orbit of the electron, and not with the wavelength of electromagnetic radiation. In the case of inhomogeneous crystals, such an understanding of the relative permittivity allows us to propose a method for determining the relative permittivity for nanosized structures [5].

In physics, there are two definitions of the relative permittivity ε : through the weakening coefficient of the electric field strength $-E$ ($D = \varepsilon E$), with all the assumptions [21], and through the weakening of the Coulomb potential $-\varphi(r) \rightarrow \varphi(r)/\varepsilon$ [21]. Such a definition for local potentials $\varphi(r)$ allows us to introduce a local concept for the relative permittivity in nanocrystals $\varepsilon(r)$ and $\varphi(r) \rightarrow \varphi(r)/\varepsilon(r)$ [5]. These two different definitions coincide only in the case of a constant value of $\varepsilon(r)$, i.e., in the case of a spatially homogeneous dielectric with $\varepsilon = \text{const}$ throughout the volume of the crystal. In the case of doping a crystal with a foreign atom, the crystal, as we have seen, becomes inhomogeneous in the doping region, and its relative permittivity can locally change significantly (**Figure 2b**). This can significantly change the resonance electron spectrum of Wannier-Mott's excitons in a crystal doped with a foreign atom [5]. Therefore, to solve problems in the nanoworld, where the Schrödinger equation includes the Coulomb potential profile, we will use the definition of relative permittivity as a coefficient weakening the Coulomb potential of a charge in a crystal at a distance r from this charge by $\varepsilon(r)$ times [5]. This definition allowed us to apply the Wannier-Mott's model modified by me to explain the Raman spectra obtained in a number of works [22–28], and to calculate the local values of $\varepsilon(r_n)$, where r_n is the radius of the Bohr orbit of the hydrogen-like standing Vysikaylo's exciton, with the principal quantum number n , and to obtain the profile of $\varepsilon(r_n)$ at $n = 1, 2, 3, \dots$ in the region of the doped atom [5]. In large-radius excitons, as well as in the hydrogen atom or the Frenkel's exciton, the processes of absorption of electromagnetic waves are possible—their cumulation into an atom or exciton, as well as the processes of their generation—the dissipation of excitation energy into the surrounding space. Therefore, such structures can be classified as CDS [1–5]. When studying the flows of dissipation from such spherically symmetric quantum structures, according to the principles of quantum (wave) mechanics, it is possible to construct an internal discrete quantum structure of the CDS and its modification, both during energy cumulation and during its dissipation (in particular, during the emission of electromagnetic waves) (**Figure 3**). In this case, as noted, we will take into account the weakening of the Coulomb potential in the dielectric using the spherically symmetric coefficient $\varepsilon(r)$. According to Landau and Lifshits [21] and the principle of superposition of Coulomb potentials, this coefficient is determined by internal charges and their location in a sphere of radius r , and not by parts of the crystal external to this sphere (**Figure 3**). Here we prove the absence of the manifestation of Mach's principle for the properties of standing Vysikaylo's excitons (on the influence of all atoms surrounding the exciton on its properties).

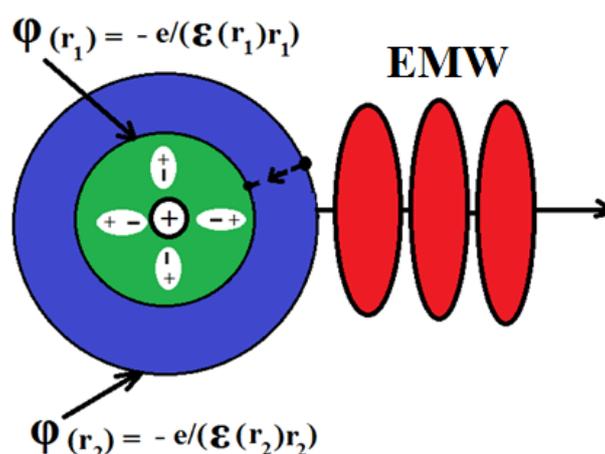


Figure 3. Scheme of the transition of an electron from an energy level with potential energy $\varphi(r_2)$ to a level with $\varphi(r_1)$ with the emission of an electromagnetic wave (EMW) carrying, according to the law of conservation of energy, information about the internal structure of the doped dielectric through $\varepsilon(r_1)$ and $\varepsilon(r_2)$.

Already in the Wannier-Mott model, the potential energy is weakened by the value of the relative permittivity $-\varepsilon_k$. This method requires a more precise definition of what the relative permittivity is for hydrogen-like nanometer structures and whether it is possible to take into account the inhomogeneity of doped crystals and thus modify the Wannier-Mott model by replacing $\varepsilon_k = \text{const}$ over the entire crystal with $\varepsilon(r)$, which depends only on the internal part of the crystal, located in the Vysikaylo's standing exciton. This is very important for studying the propagation and localization of large-radius excitons in inhomogeneous crystals.

So naturally for nanostructures—excitons, problems arose (similar to problems in atomic physics): 1) the cumulation of the energy of electromagnetic waves (the wavelengths of which can vary from thousands of nm to 100 nm) into the excitation energy—an exciton (the size of which can be less than 1 angstrom for Frenkel's excitons and about 20 nm for large Wannier-Mott's excitons), 2) localization and self-focusing of excitons and 3) the transformation of their energy, the transfer and accumulation of this energy in the battery, etc.

Within the framework of the new quantum theory, to describe hydrogen-like excitons of large radius, with a charge Z , the solution of the Schrödinger equation with a Coulomb potential weakened by ε_k times is used [7] (2). In Equation (2) in the Wannier-Mott's model, there is a discrepancy between the local value of the potential $U(r)$ and the excessive requirement of constancy of the relative permittivity $-\varepsilon(r) = \varepsilon_k = \text{const}$ throughout the crystal with any dimensions. This question was first posed in Vysikaylo's works [5].

Such a rough accounting of polarization in crystals (accounting for the relative permittivity in the Wannier-Mott's model) led to a corresponding decrease in all proper energy spectra of the hydrogen atom E_n by ε_k^2 times [7] and a corresponding increase in the characteristic sizes of hydrogen-like structures $-r_n$ by ε_k times in relation to the sizes of the hydrogen atom. In this case, the sizes of excited excitons, as in the hydrogen atom, increase with the growth of the principal quantum number—proportionally to n^2 [8]:

$$r_n = \varepsilon_k n^2 a_0 \quad (3)$$

So, the question, “Is it possible to take into account the local heterogeneity of the relative permittivity or the weakening of the Coulomb potential by $\varepsilon(r)$ times, at least in spherically symmetric problems, and thus determine this coefficient in experiments at the nanometer level?” is posed in Vysikaylo’s study [5]. There, this problem is solved on the basis of the principle of superposition of electric fields, according to which the weakening of the Coulomb potential by $\varepsilon(r)$ times is caused not by the entire crystal but only by the polarization of charges inside a sphere of radius r (**Figure 3**).

In Vysikaylo’s study [5], according to Equations (2) and (3) and the law of conservation of energy, transitions between the intrinsic energy electron levels E_n and E_m , accompanied by electron spectra of the EMW – $h\nu_{n,m}$ for Vysikaylo’s standing hydrogen-like excitons of large radius are written in the following general form (**Figure 3**):

$$h\nu_{n,m} = E_{n,m} = E_n - E_m = U_n(r) - U_m(r) = -Ze^2/(\varepsilon(r_n)\varepsilon_0 r_n) + Ze^2/(\varepsilon(r_m)\varepsilon_0 r_m) \quad (4)$$

According to Equation (4), within the framework of the Bohr model of the hydrogen atom, for hydrogen-like excitons, it is possible, by their electron spectra, to determine the local heterogeneity of crystals in the region of spherically symmetric perturbations [5].

The coincidence of the results of the old Bohr and new quantum theories in determining the eigenenergies of a quantum system—the hydrogen atom—lies in Bohr’s successful choice of the condition for quantizing the angular momentum of an electron on an allowed circular orbit – $mv_n r_n = n\hbar$ or the product of the orbit length and the particle momentum – $2\pi m v_n r_n = \lambda_n p_n = nh$. Here $h = 2\pi\hbar$ is Planck’s constant. It should be noted that the first radius of the Bohr orbit in the semiclassical Bohr theory of the hydrogen atom coincides with the value of the radius $r_1 = \Lambda/\alpha$, at which the probability of finding an electron ($2^2\pi r^2 |\psi|^2$) reaches its maximum value. Here Λ is the Compton wavelength of the electron, α is the fine structure constant. A similar coincidence is observed for all subsequent radii r_n corresponding to the principal quantum number n greater than 1. Only this coincidence provides a certain basis for retaining the term Bohr radius in the new quantum theory. In this case, within the framework of the new quantum theory for hydrogen-like structures in any media, the Bohr radius of the orbit [5]:

$$r_n = a_0 \varepsilon(r) n^2 \quad (5)$$

Here a_0 is the first-level brow radius, $\varepsilon(r)$ is the *local* spherically symmetric attenuation coefficient of the Coulomb potential or the *local* spherically symmetric *relative permittivity* of the medium (a small part of the crystal) in which a hydrogen-like structure with characteristic dimensions determined by Equation (5) is formed. For vacuum $\varepsilon(r) = \text{const} = 1$, i.e., the limiting transition from excitons in inhomogeneous crystals to a hydrogen atom in vacuum within the framework of this model, which generalizes the description of hydrogen-like structures in inhomogeneous crystals, occurs without significant problems.

If the relative permittivity of the crystal (or the local attenuation coefficient of the Coulomb potential) – $\varepsilon(r)$ changes with the distance to the dopant atom introduced into

the crystal lattice, then the resonant exchange of electron energies cannot occur. This localizes the exciton generated or focused by the Coulomb potential (**Figures 2 and 3**) in the region of the dopant, where $\varepsilon(r)$ has different values than in places remote from the dopant atom. Thus, in Vysikaylo's study [5] the possibility of the appearance of standing Vysikaylo's excitons is substantiated.

Taking into account Vysikaylo's degeneracy of the principal quantum number in hollow quantum resonators determines the possibility of the appearance of symmetric de Broglie waves (cos-waves). Moreover, within the framework of Vysikaylo's cumulative quantum mechanics for hollow hydrogen-like structures in any media, the Bohr radius of the orbit for such waves [5]:

$$r_{n-1/2} = a_0\varepsilon(r)(n-1/2)^2 \quad (6)$$

4. Ponderomotive forces in inhomogeneous crystals

In inhomogeneous crystals (with inhomogeneous permittivities) in electric fields, ponderomotive electric forces arise— $qU(r)\nabla\varepsilon(r)$, acting on charged particles due to the gradient of the relative permittivity:

$$F = q\nabla(\varepsilon(r)U(r)) = q\varepsilon(r)\nabla U(r) + qU(r)\nabla\varepsilon(r)$$

A detailed analysis of these forces remains to be carried out by us in the future.

5. Electronic spectra of Vysikaylo's standing excitons of large radius

Vysikaylo's large-radius standing excitons arise in the region of introduction of a foreign atom (with a valence different from the valence of the atoms of the reference crystal) into the crystal lattice of the reference crystal. The electron and the hole are localized in the region of the foreign atom [5]. The introduction of a foreign atom generally leads to local quantum oscillations, including such a parameter as the permittivity $\varepsilon(r)$. This also leads to the energy localization of standing Vysikaylo's excitons in the region of the introduction of a foreign atom into the crystal lattice of the reference crystal. Such phenomena occur, for example, during chemical doping of diamond (or other crystals from group IV of the periodic table) with boron atoms (or other atoms from group III of elements) or nitrogen (or other atoms from group V of elements). In this case, as we have shown, the permittivity in boron-doped diamond changes locally by up to 7% [5].

5.1. Spectra of Wannier-Mott's excitons in diamond at $\varepsilon = \text{const}$

Under these ($\varepsilon = 5.7$) assumptions, according to the Wannier-Mott theory, when introducing a foreign atom into the diamond crystal lattice, then, taking into account the (cos-waves) of the Vysikaylo's spectra Equation (1), we have the following spectrum for free (running) excitons of large Wannier-Mott's radius with constant permittivity (WM spectrum):

$$E_{n-1/2} = -(1.674(1-1/2); 0.186(2-1/2); 0.067(3-1/2); 0.034(4-1/2); 0.021(5-1/2));$$

$$0.014(6-1/2); 0.01(7-1/2); \dots \text{ [eV]}$$

$$E_n = -(0.4185 (1); 0.104 (2); 0.0465 (3); 0.026 (4); 0.0167 (5); 0.0116 (6); \quad (7) \\ 0.00854 (7); 0.0065 (8); 0.0052 (9) \dots) \text{ [eV]} \quad)$$

In our calculations we used the following values for the hydrogen atom: the radius of the hydrogen atom is $a_0 = 0.529 \text{ \AA}$; the ionization potential of the hydrogen atom is $I = 13.598 \text{ eV}$. Possible transitions from (1) ($E_{n-1/2} \approx E_m; E_{n-1/2} \approx E_{m-1/2}$) are not taken into account here.

5.2. Spectra of Vysikaylo’s excitons in diamond at $\varepsilon = \varepsilon(r)$

The author believes that the complexity of interpreting the absorption and emission spectra of Vysikaylo’s large-radius standing excitons in experiments is determined to a greater extent by the profiles of the relative permittivity $-\varepsilon(r)$ of the reference crystal perturbed by the dopant atom (diamond in our case) than by the spectra of the dopant atom. For this reason, the most accurate values of the electron spectra of Vysikaylo’s large standing excitons can be obtained at cryogenic temperatures of crystals, when the temperature and concentration effects are small, and the probe radiation is strictly collimated, including in energy [5]. Such experiments were carried out, for example, in Denisov et al.’s research [27]; see **Table 1**. Unfortunately, Denisov et al. [27] present the electron spectrum of Vysikaylo’s excitons with energies in the range from 2 to 43 meV. The most informative for the next proof of the validity of Vysikaylo’s cumulative quantum mechanics are the electron spectra of Vysikaylo’s standing excitons in the energy range from 400 to 50 meV; see **Table 1**.

Table 1. The dependence of the relative permittivity ε on r $\varepsilon(r)$ and the characteristic radii r_n of a hydrogen-like hollow quantum dot in boron-doped diamond (electron spectra of Vysikaylo’s standing excitons) were calculated in accordance with Vysikaylo’s theory.

№	n	E , meV WM theory	E , meV experiment	ε , relat. units Vysikaylo’s theory	r_n , Angstrom Vysikaylo’s theory
1	1 – 1/2	–1674	?		0.8
2	1	–418.5	–369 [24,25] Lyman-Vysikaylo’s series	6.07	3.2
3	2–1/2	–186	–161 ÷ –165 [22,23,26] Vysikaylo’s series	6.14 ÷ 6.05	7.15
4			–140 ÷ –142 [26]		
5	2	104.6	~ –92.54?	6.06	12.82
6			–85 ÷ –89 [26]		
7	3 – 1/2	–67	~ –60.47?	6.0	19.84
8	3	–46.5	–42, 9 [27] Pashen-Vysikaylo’s series	5.93	28.25
9			–37.2 [27]		
10			–35.08 [27]		
11	4 – 1/2	–34.1	–30.99 [27] Vysikaylo’s series	5.98	38.77

Table 1. (Continued).

N_0	n	E , meV WM theory	E , meV experiment	ϵ , relat. units Vysikaylo's theory	r_n , Angstrom Vysikaylo's theory
12			-28.02 [27]		
13	4	-26.15	-25.54 [27] Brackett-Vysikaylo's series	5.77	48.81
14			-22.56 [27]		
15	$5 - 1/2$	-20.67	-	5.92	63.42
16			-17.36 [27]		
17	5	-16.74	-15.13 [27]	6.0	79.29
18	$6 - 1/2$	-13.83	-13.51 [27]	5.77	92.3
19	6	-11.62	-11.41 [27]	5.75	109.57
20	$7 - 1/2$	-9.9	-9.3 [27]	5.88	131.48
21	7	-8.54	-7.93 [27]	5.92	153.3
22	$8 - 1/2$	-7.44	-		174.67
23	8	-6.54	-		195.0
24	$9 - 1/2$	-5.79	-5.58 [27]	5.8	222.0
25	10	-4.18	-3.97 [27]	5.85	309.6
26	11	-3.46			
27	12	-2.9			
28	13	-2.48			
29	14	-2.135			
30	$15 - 1/2$	-1.99	-1.98 [27]	5.71	635.
31	15	-1.86			
32	$5 - 1/2,$ $Z=2$	-41.3	-37.0 [28]	6.0	32.2
33	$5, Z=2$	-33.48	-28.02 [27]	6.2	41.2
34	$6 - 1/2,$ $Z=2$	-27.67		5.66	45.33
35	$6, Z=2$	-23.25	-22.56 [27]	5.78	55.1

Note: The dependence of the relative permittivity ϵ on r $\epsilon(r)$ was calculated according to Vysikaylo's theory (in a standing $\epsilon(r)$ wave). The own electron energy levels are $E_{n-1/2} = -13.56 ((n-1/2)\epsilon(r))^{-2}$ eV, $E_n = -13.56 (n\epsilon(r))^{-2}$ eV, and the characteristic radii $r_{n-1/2} = a_0\epsilon(r)(n-1/2)^2$ of a hydrogen-like hollow quantum dot in diamond doped with boron (electron spectra of Vysikaylo's standing excitons).

The spectra of Vysikaylo's standing excitons observed in experiments in diamonds doped with boron [10, 22–28], were identified by us as follows:

$$\begin{aligned}
 E_{n-1/2} = & ?(1-1/2); -(0,160 \div 0,165) (2-1/2); ?(3-1/2); 0,03099(4-1/2); 0,01351(6-1/2); \\
 & 0,0097(7-1/2); \dots \text{ [eV]}; \\
 E_n = & -(0,369(1); ?(2); 0,0429 (3); 0,02554 (4); 0,01513(5); 0,01139(6); \\
 & 0,00836(7); \dots \text{ [eV]}
 \end{aligned}
 \tag{8}$$

Previously, it was believed that an impurity atom perturbs the crystal at the level of inter-node dimensions (bound Frenkel's exciton). Now we see (**Figure 2, Table 1**) that there are standing (bound) excitons of large radius and they are caused by quantum (wave profiles $\epsilon(r)$) effects caused by the introduction of a foreign atom into the crystal

lattice of the doped crystal and this occurs in full accordance with the hypothesis of Louis de Broglie. Thus, in this work, based on the QCM, a completely new method for determining the relative permittivity profiles in nano-sized structures in doped crystals by Raman scattering is proposed for the first time (**Figure 2b**). According to the author's model Equations (1), (5), (6) and (8), based on the QCM, the step in determining the $\epsilon(r)$ profile is $\epsilon(r) - \Delta r \approx 0.529 \epsilon(r) (n-1/4)/Z$. Which is more than two times smaller than the step that follows from the classical Wannier-Mott model. The Z -charge is localized in the nano-structure; n is the principal quantum number of the standing exciton of a large radius formed on the $\epsilon(r)$ -structure.

5.3. Classification of electronic spectra of Vysikaylo's standing excitons of large radius

The electronic spectra of Vysikaylo's standing excitons can be similarly divided, like the spectra of the hydrogen atom, into the Vysikaylo-Lyman, Vysikaylo-Balmer, Vysikaylo-Paschen, Vysikaylo-Brackett lines, etc. (**Figure 4**). Unlike the Wannier-Mott's spectra, the following formulas should be used to calculate the Vysikaylo's spectra, taking into account the changes in the relative permittivity $\epsilon(r_i)$ of diamond in the regions of the doping atom [4]:

$$\nu = 1/\lambda = R(1/(\epsilon(r_i)i)^2 - 1/(\epsilon(r_m)m)^2) \tag{9}$$

where R is the Rydberg constant (it is equal to $109,737,31,568,539 \text{ cm}^{-1}$).

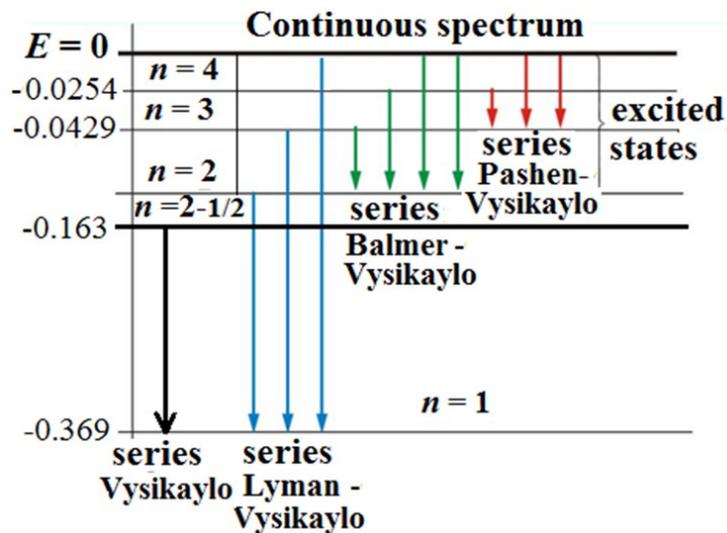


Figure 4. Electron levels during absorption and emission of electromagnetic waves by a Vysikaylo's standing exciton with principal quantum numbers n and $n-1/2$.

The author, on the basis of a number of experiments (in accordance with the works of Bohr, Schrödinger, Born, Wannier, Mott, Vysikaylo, and others), proves that in addition to electronic spectra with integer principal numbers Equation (5), in a number of experiments [22,23,26] a line is clearly observed, caused by an electronic level with the principal quantum number $i=2-1/2$ [5]. Therefore, to calculate all possible spectra of standing Vysikaylo excitons according to (5), it is necessary to take into account the Vysikaylo's degeneracy (in principal quantum numbers), both with integer

i and m , and with half-integer principal quantum numbers ($i = 1, 2^{-1/2}, 2, 3^{-1/2}, \dots; m = 1, 2^{-1/2}, 2, 3^{-1/2}, \dots$) and all possible spectra between transitions with integer and half-integer principal quantum numbers (**Figure 4**). Spectra with half-integer principal quantum numbers (cos-waves with symmetric ψ -functions) were first resolved for hollow quantum resonators with spherical and cylindrical symmetries within the framework of Vysikaylo's cumulative quantum mechanics [2], and this theory was confirmed by numerous experiments with resonant electron capture by fullerenes [11–13].

6. Oscillations of relative permittivity in Vysikaylo's excitons

From a comparison of the analytical WM spectra Equation (7) and the identified experimental ones Equation (8), it is possible to construct the oscillations of the permittivity $\varepsilon(r)$ depending on the distance $r_i = a_0\varepsilon(r_i)t^2$ to the center of the Vysikaylo's standing exciton (the dopant atom) (**Figure 2**). Here $i = 1/2, 1, 3/2, 2, 5/2, \dots$:

$$\varepsilon(r) = 6.07(1; 3.21\text{\AA}); 6.14 \div 6.05(2^{-1/2}; 7.15\text{\AA}); 5.93(3; 28.25\text{\AA}); 5.97(4^{-1/2}; 38.48\text{\AA}); 5.75(4; 48.86\text{\AA}); 6.0(5; 79\text{\AA}); 5.8(6^{-1/2}; 93\text{\AA}); 5.75(6; 109.5\text{\AA}); (10) \\ 5.787(7^{-1/2}; 129.4\text{\AA}); 5.768(7; 149.5\text{\AA})$$

Identification of the electron level spectra of standing (frozen into the crystal) hydrogen-like Vysikaylo's excitons of large radius is a complex arithmetic problem if we take into account all possible electron spectra analogous to the spectra of atomic hydrogen: Lyman, Balmer, Paschen and clearly observable Vysikaylo's spectra with principal quantum numbers $i = n^{-1/2}$ taking into account the oscillations of the relative permittivity $-\varepsilon_i(r_i)$. The fundamental role in substantiating the basics of the CQM for Vysikaylo's standing excitons is played by the long-observed experimental electron levels with principal quantum numbers $n = 1; 2^{-1/2}$. These spectra have been observed since 1968. Of interest are the spectra obtained by Denisov et al. [27] with low energy (significant excitation of standing excitons with large principal quantum numbers due to laser excitation). Of particular interest is the search for spectra theoretically predicted by us in the region of 0.2–0.06 eV ($i = 2, 3^{-1/2}$) and transitions between electron levels with $i = 1; 1.5; 2; 2.5; 3$.

7. Limits of applicability of the method for determining the electronic spectra of Vysikaylo's excitons

The radius of a quantum dot (QD), including a Vysikaylo's standing exciton $-a_n$ (or $a_{n-1/2}$) in a doped crystal depends on the profile of the relative permittivity of the crystal $-\varepsilon(r)$, which arises when an impurity is introduced into the crystal lattice and the value of the principal quantum number n (or $n^{-1/2}$) of the excited level of a hydrogen-like QD (the total energy of an electron in the QD).

The condition for the applicability of Equations (5) and (6), according to Landau et al. [7], consists of the requirement of "a sufficiently large value of the orbit radius of the Wannier-Mott's exciton" $-a_n \sim \hbar^2 \varepsilon n^2 / m e^2 \gg a_0$ (**Figure 2b**). This condition is obviously fulfilled for large n , but in crystals with large ε , it can also be fulfilled for $n \sim 1$ [7].

Similarly, we substantiate the condition of applicability Equations (1)–(10) for Vysikaylo’s standing excitons of large radius by the requirement: $a_{n-1/2} \gg a_0$ (or $a_n \gg a_0$) (**Figure 2b**) [5].

However, in the case of the formation of a standing exciton of large radius, this condition is modified to $a_{n-1/2} > d$ or $a_n > d$. Here d is the size between the nearest atoms in the crystal lattice of the supporting crystal (diamond, silicon, germanium, etc.). This condition is associated with the impossibility of forming lower energy electron levels in the structure of a standing exciton with profiled $\varepsilon(r)$ with characteristic sizes $a_{n-1/2} < d$ or $a_n < d$ (**Figure 2c**). Inside this region, there is no positive charge for the electron (**Figure 2b,c**), and therefore, a hydrogen-like exciton is not formed [5].

For diamond, $d \approx 1.54 \text{ \AA}$. Therefore, the lower energy state for a standing exciton of large radius with $\psi_{1-1/2}$, with energy $E_{1-1/2} \approx -1.476 \text{ eV}$ and with $a_{1-1/2} \approx 0.8 \text{ \AA} < d = 1.54 \text{ \AA}$, as a standing exciton of large radius, is not realized in diamond (**Table 1**). For a similar reason, the states of standing excitons of large radius with $Z = 2, 3$ with $a_{n-1/2} < 1.54 \text{ \AA}$ and, accordingly, with $a_n < 1.54 \text{ \AA}$ are not realized.

Standing excitons with $Z = 2, 3$ arise in the region of the dopant atom during the ionization of carbon atoms surrounding the boron atom. In this case, positively charged holes can intensively move around the boron atom along the nearest carbon atoms due to energy resonance.

8. Calculations of Vysikaylo’s exciton molecules and flickering crystals

By analogy with hydrogen molecules, flickering molecules and crystals from large-radius standing excitons can be formed by overlapping their excited electron shells. Using the MGMT, it is possible to calculate the concentration of the acceptor-dopant impurity during the formation of flickering molecules and crystals from Vysikaylo’s large-radius standing hydrogen-like excitons. The difference between standing exciton molecules and hydrogen molecules is as follows. Since the exciton atoms in exciton molecules are strictly fixed in crystals, they have no rotational degrees of freedom. Oscillations are possible due to the movement of a hole around the doping atom. In this case, the electron spectra of standing exciton molecules are significantly richer than those of hydrogen molecules. This is due to the electron spectra with half-integer principal quantum numbers (cos-waves with symmetric wave ψ -functions). Standing flickering molecules and crystals of standing excitons of large radius can be formed by overlapping excited electron shells of two or more nearby standing excitons and can be observed at concentrations of dopant (for example, boron):

$$N_{B,n} = 1/(2a_n)^3 = 10^{24}/(2 \cdot 0.529 \cdot \varepsilon(r) \cdot n^2)^3 \text{ cm}^{-3} \quad (11)$$

or it is for symmetric states of de Broglie waves of an electron at:

$$N_{B,n-1/2} = 1/(2a_{n-1/2})^3 = 10^{24}/(2 \cdot 0.529 \cdot \varepsilon_{n-1/2}(r) \cdot (n-1/2)^2)^3, \text{ cm}^{-3} \quad (12)$$

For diamond doped with boron, the formation of flickering crystals (superlattices), according to (11), is observed at $n = 5$ and concentration of boron

doped into the diamond crystal lattice, $N_{B,5} \approx 2.5 \times 10^{17} \text{ cm}^{-3}$, at $n = 4$, $N_{B,4} = 9 \times 10^{17} \text{ cm}^{-3}$, and at $n = 1$ $N_{B,1} = 4.7 \times 10^{21} \text{ cm}^{-3}$.

The formation of hydrogen-like molecules with overlapping orbits of excitons with different values of the principal numbers n and $m^{-1/2}$ is possible.

The main difference between hydrogen-like exciton molecules and hydrogen molecules is the absence of rotational and vibrational degrees of freedom in their spectrum (at least they are significantly stabilized in the crystal). This allows one to study quantum phenomena without taking into account rotational degrees of freedom and is an additional method for verifying the electronic spectra of ordinary hydrogen molecules and other hydrogen-like molecules.

On the basis of MGMT (the method of Wannier and Mott modified by us for the description of standing excitons of large radius) and the basis of Raman spectra, it is possible to calculate all the electronic spectra of standing flickering molecules with excited inside them flickering standing super-atoms (standing excitons of large radius) of two types: with symmetric and asymmetric wave ψ -functions. Such synergetic multiphase or multiphase solid-state systems (**Figure 5**), with splitting of levels with the principal quantum number n into two sublevels, with flickering (standing exciton) subsystems, may prove to be very promising in the utilization of thermal energy into electromagnetic energy and in other practical applications.

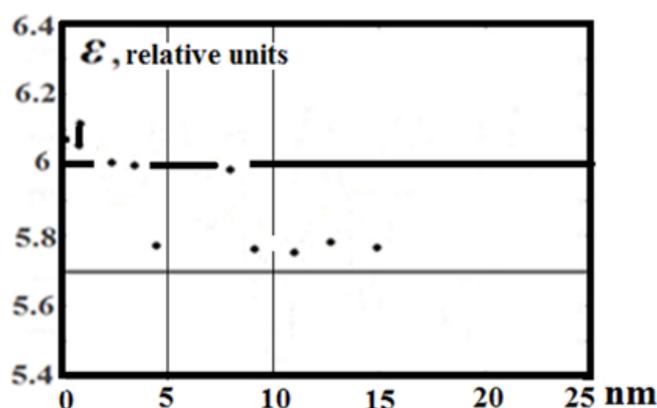


Figure 5. Vysikaylo's quantum oscillations $\varepsilon(r)$ in acceptor-doped diamond in the region of the introduction of a boron atom into the lattice [5].

MGMT, used by the author in this work, goes back to the works of Eratosthenes (this method was used by Vanier [19], Mott [20], Mandelstam and others). Mandelstam said in one of his lectures: "You all know such systems as a pendulum and an oscillatory circuit, and you know that from an oscillatory point of view, this is the same thing. Now all this is trivial, but the remarkable thing is that it is trivial." It turns out that a pendulum (in a cuckoo clock at a grandmother's village or a Foucault pendulum) and an oscillatory circuit in wireless telephony and combination scattering of light (oscillations in atoms) are the same thing (from a mathematical point of view).

The phenomenon of oscillation of the relative permittivity is associated with oscillations of the electron density in the region of the implantation of a foreign atom. In metals with defects, a similar phenomenon is called Friedel's oscillations. In this

case, the electrons screening the impurity charge form a halo around the defect center with alternating regions of condensation and rarefaction of their density.

In a gas discharge, the analog of this phenomenon is pulsating or standing striations in space, known to Faraday.

In the photographs, the running waves become standing (**Figure 6**).



Figure 6. Waves on water, according to de Broglie's hypothesis, are similar to $\varepsilon(r)$ waves in doped diamond in the region of the introduction of a boron atom into the lattice (see **Table 1** and **Figure 5**).

We similarly used the generalization of the solutions of the Poisson equation (first obtained by Euler in 1767, verified by Lagrange and Roche in celestial mechanics) to describe the interaction of electrons with positively charged Coulomb attractors—plasmoids or holes in crystals. In plasma, studies of such phenomena have already begun [5]. These Euler solutions, describing the role of potential long-range forces or their fields in the contraction of gravitating small particles to libration (cumulation) points in celestial mechanics, have not yet been used for a similar solution of a number of paradoxes in crystals with standing or running excitons.

The role of analogs in physics lies in prompting and verifying models and their solutions. Indeed, the mathematical generality of problems in any long-range potential fields ($\Delta\varphi = \rho$) gives grounds to use not only all discoveries in celestial mechanics to discover new 3D phenomena, which have not attracted the attention of researchers of inhomogeneous plasma (with violation of electroneutrality), but also in many other similar phenomena.

Molecular spectra of standing and running excitons, analogs of hydrogen molecules H_2 or positive ions H_2^+ have not yet been identified and their discovery requires experimental studies. However, even now it is possible to note the special role in the formation of the properties of flickering crystals from standing excitons of nanometer (large) radius of electron cumulation points $-L_1$ and the features of the electron flow through these points in flickering crystals. Similarly, on the basis of MGMT, one can expect hyper-conductivity effects of flickering systems upon excitation of standing excitons of large radius.

On the basis of MGMT, it is possible to predict and calculate the forces of attraction of dopant atoms to each other in the reference crystal during excitation of excitons in it, etc.

Vysikaylo's cumulative points L_1 are formed between any spatially separated attractors with long-range potentials. The presence of cumulation points in the plasma leads to the formation of a Faraday dark space between two positively charged Coulomb attractors [5].

The obtained profiles of relative permittivity pulsations (oscillations $\varepsilon(r)$) (Figure 5) allow us to calculate with good accuracy the concentrations of the dopant for the formation of large-radius exciton molecules, lines of standing excitons (Figure 7), and the parameters of flickering 2D surfaces and 3D crystals.

Solid-state flickering crystals (crystals of standing excitons), described by the Vysikaylo's model [5], contained inside the base crystal, can be very convenient and useful in practice and, especially, for scientific research, for example, general properties of metallic hydrogen-like crystals, study of the spectra of exciton molecules, etc. These molecules of Vysikaylo's standing excitons do not have rotational and vibrational degrees of freedom (they are suppressed by interatomic bonds in the crystal lattice of the supporting crystal). This property can be used to verify the electronic spectra of hydrogen molecules.

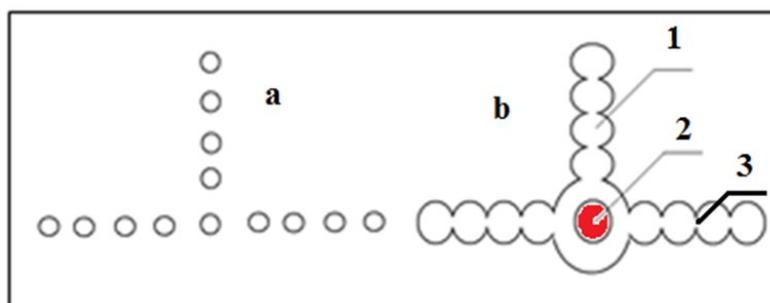


Figure 7. Formation scheme of a flickering crystal from standing excitons of large radius at different irradiation doses: *a*)—doped crystal at a weak energy source; *b*)—overlap of electron shells of standing excitons—quantum dots with variable size. (1) Channel for electrons (cumulation line) from quantum dots, (2) focusing energy into the center of its radiation; (3) formation of Vysikaylo's cumulation points – L_1 (for electron flows) between standing excitons of large radius [5].

9. Discussion of results

A number of researchers often believe that the value of the relative permittivity of a material is associated only with the wavelength of electromagnetic radiation. This is a mistake. According to our model of hydrogen-like standing excitons, the relative permittivity is a local main characteristic of a crystal ($\varepsilon(r)$ is a coefficient weakening the Coulomb potential of an exciton hole in a radially inhomogeneous crystal on a sphere of radius r in the doping region) and can change at the level of nanometer sizes, thereby showing how many times ($\varepsilon(r)$ times) the local Coulomb potential of an external charge located at a distance r from the positively charged center of an exciton changes. New nanostructured materials with nano-sized crystals with unique

properties are currently being developed. Of particular interest among them are materials formed from nano-crystallites with significantly different relative permittivities. Thus, to increase the efficiency of nanostructured luminophores, crystallites with a relative permittivity of $\epsilon \sim 1500$ are sintered with nano-graphite powder or fullerenes of nanometer dimensions. Models with a constant value of relative permittivity over the entire meso-structure are, in principle, inapplicable to describe excitons in such structures. We propose such a theory for nanostructures with large ϵ .

We have analyzed, on the basis of MGMT (based on the works of Vanier, Mott, de Broglie, Bohr, Schrödinger, Dirac, Vysikaylo, and others), many years of experimental observations of Raman spectra (laser radiation Raman scattering spectra) of diamonds doped with boron atoms, which began in the last century (before 1968) and continue to the present day [29,30]. Before the author's work, there was no worthy model of quantum transitions in such crystals [10]. This was due to the researchers' assumption about the constancy of the permittivity ϵ in the region of the introduction of a foreign atom into the crystal lattice and Dirac's erroneous prohibition of electron spectra with the principal quantum number $n^{-1/2}$ (cos-waves). As the author proves on the basis of theoretical foundations of quantum mechanics and numerous experimental observations, these provisions of Dirac [14] and his followers [15,16] should be substantially modified. In this paper, it is proved that the Dirac requirement modified by the author and the modified Wannier-Mott-Bohr model [5], taking into account oscillations (wave inhomogeneities, **Figure 5**) of the relative permittivity at the nanometer level Equation (9), allow for a satisfactory description of the Raman spectra Equation (8) obtained in experiments in the study of diamonds doped with boron (see the Table of experimental data in [22–28]).

Note that in this review we relied on experimental works performed at cryogenic temperatures and assumed spherical symmetry of Vysikaylo's standing excitons. Raman spectra of Vysikaylo's standing excitons can be modulated or accompanied by spectra of other structures similar, for example, to structures formed in doped hexaferrites with an average characteristic grain size of up to 6 μm and characteristic dimensions of the elementary lattice of up to $c = 2.3 \text{ nm}$ [31].

No less interesting experimental and theoretical works on nanoscale structures are presented [32–34].

10. Conclusions

The theoretical studies we have conducted have proven that:

- 1) We have identified and analytically described (based on Vysikaylo's cumulative quantum mechanics) practically all electron spectra (observed in experiments [22–28]) of Vysikaylo's standing excitons in diamond doped with boron atoms;
- 2) Our modified Wannier-Mott theory explains all electron spectra observed during irradiation of diamonds doped with boron atoms [5];
- 3) Dirac's requirement on the limitedness of the ψ -functions of de Broglie waves in excitons is excessive. It is sufficient to limit the probability of finding a particle in the volume of a standing Vysikaylo's exciton and take into account the normalization

geometric coefficient (taking into account the symmetry of the quantum resonator $-\chi^2 = 2^k \pi r^k$, $k = 2$ for spherical symmetry of the quantum resonator; $k = 1$ for cylindrical symmetry and $\chi^2 = 1$ for plane symmetry);

4) The Vysikaylo's degeneracy of the principal quantum number by n (asymmetric sin-waves) and $n-1/2$ (symmetric cos-waves) is observed in experiments not only during the capture of electrons into the internal cavity of fullerenes [2,5,11–13], but also during the formation of standing Vysikaylo's excitons [5,22–27];

5) The relative permittivity $\varepsilon(r)$ changes significantly (up to 7%) in the nano-region (up to 10 nm, **Figure 5**) of the doped atom embedded in the crystal lattice of the reference crystal (diamond, etc.);

6) In accordance with the change in the relative permittivity $\varepsilon(r)$ in the region of the dopant atom (at nanometer distances from it), a corresponding change in the Raman spectra (the spectra of combination radiation observed in the experiments [22–28]) occurs;

7) Using Raman spectra [22–27], it is possible to determine the nano-profiles of the relative permittivity $\varepsilon(r)$ of the reference crystal in the region of the introduction of a foreign atom into the crystal lattice (**Figure 5**) [5];

8) Experimental observations of Raman spectra [22–28] of Vysikaylo standing excitons prove the basic provisions of Vysikaylo's cumulative quantum mechanics [5];

9) The theory we have developed (the modified Wannier-Mott theory) of the formation of inhomogeneous permittivities in inhomogeneous crystals can be used for description at the level of nanostructures [5].

The knowledge we have obtained can be applied to the study of nanocomposite materials formed by chemical doping of dielectrics (diamond, silicon, germanium and other crystals from the IV group of elements) with foreign atoms from the III and V groups of elements of the periodic table. The unique technique we have developed, based on the Wannier-Mott's model modified by the author, allows, based on the experimentally established Raman spectra, to calculate the profiles of the relative permittivity at the nanometer level (from 0.3 to 20 nm) in chemically doped diamond. The results obtained indicate the erroneousness of the theoretical approach used [28], which does not take into account the wave (quantum) oscillations (oscillations) of the relative permittivity in the region of the doping center. The author believes that there is no point in advertising spin-orbital inventions [28]. It is better to modify the models of Wannier, Mott, Dirac, relying on the classical works of Euler, Frenkel, de Broglie, Bohr, Vysikaylo and others [2,5], verified by comparing experiments with full-fledged theories.

The obvious presence in experiments [11–13] of lines with the principal quantum number $n-1/2$ ($E_{n-1/2}$) in the resonance capture of free electrons by fullerenes and in the Raman spectra of boron-doped diamonds [22–27] once again proves the validity of the foundations of Vysikaylo's cumulative quantum mechanics [2,5] and thus proves that Dirac's requirement that ψ -functions are bounded everywhere is excessive and can be replaced by the requirement of the boundedness of the probability of finding a particle in a quantum resonator [5].

We have proved that quantum mechanics is a completely complete statistical theory capable of prediction if we do not attribute wave properties to particles.

Particles behave as waves in the statistical approach, but they remain particles if their behavior is considered as particles [2,5].

According to the Einstein-Podolsky-Rosen work and Gödel's incompleteness theorem, standard quantum mechanics is incomplete. This is true if the particle is assigned wave properties. However, if wave properties are assigned to the statistical behavior of the particle in the resonator, another point of view is formed, according to which quantum mechanics is a complete theory. It is substantiated on the basis of the law of conservation of energy and recognition of the probabilistic nature of the statistical behavior of the particle in the resonator.

In our works, we use classical parameters of electrons (mass and charge) and on this basis explain a number of previously "mysterious" statistical phenomena, including in experiments in chemically doped crystals. Studies of standing molecular excitons, flickering lines (nano-lightning), planes (nanostrata) and Vysikaylo's crystals (in chemically doped crystals) are of great scientific and practical interest for the further development of quantum mechanics. In such exciton molecules, lines, planes and crystals there are no external influences that lead to the excitation of vibrational and rotational degrees of freedom (they are significantly weakened in crystals by interatomic bonds).

The results obtained in this work prove the predictive power of Vysikaylo's cumulative quantum mechanics and the method of generalized mathematical transfer (MGMT) for studying and describing the structure of cumulative-dissipative systems at the nano-world level.

Conflict of interest: The author declares no conflict of interest.

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