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Original Research Article

Investigation of stresses in a Kevlar (491PR-286) material disc by the Chebyshev pseudospectral method

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ABSTRACT: In this study, a composite disc with Kevlar (491PR-286) material was modeled. Kevlar consists of very strong fibers of very light carbon origin. That is why they are used quite often in unmanned aerial vehicles and spacecraft. The disc has been subjected to thermal stress under a linearly increasing temperature distribution. The temperature limit conditions were applied as 25 °C, 50 °C, 75 °C, 100 °C, and 150 °C. The obtained findings were determined using a computer program, the psedudospectral Chebyshev method, and analytically in three different ways. The main difference between this study and other studies is that it investigates the thermal stresses occurring in circular discs using different methods. The results obtained are compared fairly among themselves and presented with graphs. It was determined that tangential stresses were higher than radial stresses at the studied temperature values. In the analytical study conducted, the radial stresses on the inner and outer surfaces of the disc were determined to be zero for the boundary conditions. Under the increasing temperature distribution from the inner surface to the outer surface, tangential stresses occurred as tensile stress on the inner part of the disc and compressive stress on the outer part. Under the decreasing temperature distribution from the inner surface to the outer surface, tangential stresses are pressed on the inner part of the disc, resulting in a tensile stress on the outer part. It is observed that with increasing temperature, there is an increase in radial and tangential stress values. At the end of the study, it was concluded that Kevlar (491PR-286) material discs can be used at high temperatures.

KEYWORDS: composite material; finite element; mathematical formulation; computer program; thermal stress; Kevlar

1. Introduction

This study is very important from the point of view of materials science. Discs are the reason for preference in the machine industry. It is of vital importance to know the reactions of different disc materials to different temperatures. When the studies conducted in this field are examined, the stresses of different disc materials have been investigated by different methods. However, in this study, a disc with Kevlar (491PR-286) material was modeled differently from other studies. When the literature review is done, rotating wipers are usually used on machine parts in the air industry. The tensile behavior of two different materials has been investigated. In addition, the studies conducted to understand this issue have been researched in detail. In this context, the effect of the Poisson ratio on functionally rated pressure vessels has been investigated in a study[1,2]. In another study, the stresses occurring in hollow cylinders with FGM material were investigated. The radial and tangential stresses obtained at the end of the study were shown separately[3]. In a different study, a study was conducted on finite element modeling of thermal stresses in aerospace structures made from polymer composite materials[4]. Stress analysis can be applied in every field. For example, in another study, stress analysis of a material modeled in a different field was performed[5]. With this study, the temperature increases linearly from the inner surface of the disc to the outer surface. The analytical solution, the ANSYS 2023 R1 program, which is a finite element method, and the pseudospectral Chebyshev method were used. Machine parts are used in different fields. For example, steel-curved shock absorbers were designed in a different study. This new damping system provides designers with an opportunity to control and improve the seismic performance of steel semi-rigid moment frames (SRF)[6,7]. In a different study, the fatigue behavior of non-patched aluminum-composite plates was investigated numerically. The results obtained are graphically shared[8]. In another study, the thermal stress distributions formed on a brake disc/pad were investigated. At the end of the study, it was determined that different friction force profiles depending on time caused tensile stresses on the friction surface of the brake disc. The stresses occurring on rotating discs were investigated, and the results were shared with the literature[9,10]. In a different study, thermal analysis was performed on different shock absorptions at 30° and 120° with different angles in December. It has been observed that higher frame strength and energy dissipation occur by increasing the damper angle from 30° to 60°[11]. Studies related to discs are found in the literature. There are various studies on rotating discs with isotropic and anisotropic material properties and studies conducted by applying a pressure effect to the inner and outer surfaces of the disc. There are two different innovations in this study. One of them is the usability of the kev disc at high temperatures, and the second innovation is the conclusion that the Chebyshev pseudospectral method can be preferred for stress analysis in discs. Temperature can be considered as one of the situations that significantly affects material behavior and where it is necessary for researchers to work continuously. In general, the tendencies of metallic materials towards temperature are different. In composite materials, this situation is completely different. A significant decrease or increase in temperature can lead to serious damage to machine parts. Since machine parts heat up during operation, stress analysis based on temperature is of great importance. Since there is a possibility of internal stress due to irregular cooling, sizing should be done taking this situation into account. In the literature research, no studies related to the investigation of stresses occurring in the kev material disc by different methods could be seen.

2. Material and method

It has been taken as a reference that the temperature increases linearly from the inner surface to the outer surface of the composite disc. The temperature distribution varies in the radial direction. The composite disc is fixed, and its dimensions are determined as a = 50 mm and b = 100 mm (**Figure 1**). 25 °C, 50 °C, 75 °C, 100 °C, and 150 °C, different temperature values were taken as references.

While determining the mechanical properties of the composite disc, the areas of use of kevs were investigated first. Then, there have not been any studies in the literature related to the Kevlar (491PR-286) disc, which is a frequently preferred material today. Due to the fact that it is different from other studies, this material selection was considered appropriate. The material included in reference list No. 6 of the study and the mechanical properties that are indicated are modeled in this study. The discs are intertwined. This study was modeled as a fabrication. The inner radius of the disc is modeled as a, and the outer diameter as b.

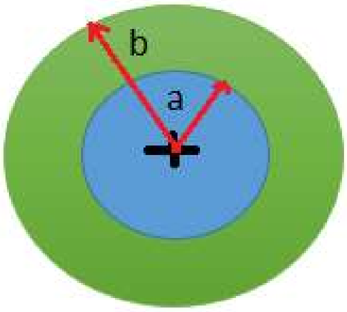

Figure 1. A composite disc modeled.

The mechanical properties of composite disc materials are given in **Table 1**.

Table 1. A composite disc with Kevlar (491PR-286)[2].

Disc	E_θ (GPa)	E_r (GPa)	*k*	α_r (1/°C)	α_θ (1/°C)	$\upsilon_{\theta r}$	a	b	t
	64.1	5.38	3.45	2.1×10^{-6}	63.6×10^{-6}	0.35	50 mm	100 mm	2 mm

The disc material is taken as a reference from the study mentioned above. The elasticity moduli, Poisson ratio, and thermal expansion coefficients of the disc material are given above.

2.1. Analytical solution

α_r, ve, and α_θ are the thermal expansion coefficients in the radial and tangential directions under conditions where the plane stress condition is valid for the composite thin disc. $a_{\theta\theta}$, a_{rr}, and $a_{r\theta}$ denote the constants of the elasticity matrix. The expressions of the elasticity constants in terms of engineering constants are as follows[2]. **Figure 2** shows the disc modeled under a linearly increasing temperature distribution from the inside to the outside of the disc. The coordinates of the disc are given.

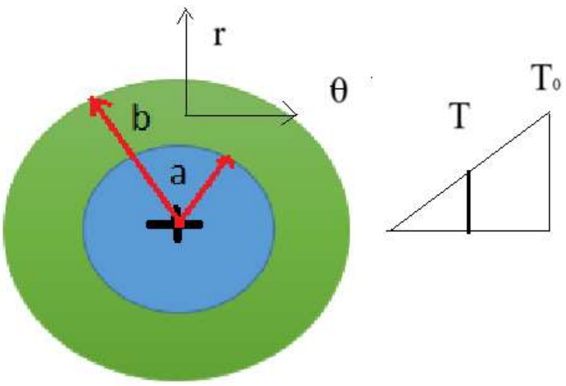

Figure 2. Temperature distribution occurring in the composite disc.

$\sigma_z = 0$ general equilibrium equation for a thin disc[12] is given in the form.

$$\frac{r(d\sigma_r)}{dr} + (\sigma_r) - (\sigma_\theta) = 0 \tag{1}$$

In Equation (1), r is the radius of the disc at any point, σ_r is the radial stress, and σ_θ is the tangential stress. Here, the disc material is taken as $i = 1$.

$$\varepsilon_r = \frac{du}{dr} \tag{2}$$

$$\varepsilon_\theta = \frac{u}{r} \tag{3}$$

here, u is the displacement in the radial direction. ε_r denotes radial deformation, and ε_θ denotes deformation in the tangential direction. Strain-stress relation.

$$\varepsilon_{ri} = \frac{1}{E}(\sigma_r - \upsilon\sigma_\theta) + \alpha_i T_r \tag{4}$$

$$\varepsilon_\theta = \frac{1}{E}(\sigma_\theta - \upsilon\sigma) + \alpha T_r \tag{5}$$

$$\sigma_r = \frac{F}{r} \tag{6}$$

$$\sigma_\theta = \frac{dF}{dr} \tag{7}$$

It shaped. Equations (6) and (7) are applied in Equations (4) and (5), respectively.

$$\varepsilon_r = \frac{1}{E}(\frac{F}{r} - \upsilon\frac{dF}{dr}) + \alpha T_r \tag{8}$$

$$\varepsilon_\theta = \frac{u}{r}\varepsilon_{\theta i} = \frac{1}{E}\left(\frac{dF}{dr} - \upsilon\frac{F}{r}\right) + \alpha T_r \tag{9}$$

Eligibility equation for elongation is obtained,

$$r\frac{d\varepsilon}{dr} + \varepsilon_\theta - \varepsilon_r = 0 \tag{10}$$

the general Equation (11) is obtained by using the equilibrium Equations (1–7) in which the stress function can be defined as F.

$$r^2\frac{d^2F}{dr^2} + r\frac{dF}{dr} - F = -r^2\alpha_i E_i T_r' \tag{11}$$

From Equation (12),

$$r^2F'' + rF' - k^2F = \frac{(\alpha_r - \alpha_\theta)T}{a_{\theta\theta}}r - \frac{a_{\theta\theta}T'}{a_{\theta\theta}}r^2 \tag{12}$$

substituting T and T′ yields Equation (13).

$$r^2F'' + rF' - k^2F = \frac{(\alpha_r - \alpha_\theta)(r^2 - ar)}{a_{\theta\theta}(b-a)}T_O - \frac{a_{\theta\theta}T_O}{a_{\theta\theta}(b-a)}r^2 \tag{13}$$

In order to obtain Chebyshev points that have more mesh points at the boundary points, the Chebyshev differential matrix D can be calculated using Equation (13). In this way, high precision can be achieved: by multiplying the vector by a finite number of times in this method, the derivatives of the vector are obtained with high precision.

$$r_j = \cos\{j\boldsymbol{\pi}|N\},\ j = 0, 1, 2, 3, \ldots, N \tag{14}$$

$$F'(r_j) = (DF)_J \tag{15}$$

$$F''(r_j) = (D^2F)_J \tag{16}$$

$$F = [F_0 \ldots F_n]^T,\ r_j \tag{17}$$

r_j is numbered from right to left and can be defined in December [−1, 1]. The calculation of the derivative matrix can be done from the MATLAB m file[13]. When using first- and second-order derivatives, the Chebyshev differential matrix, in Equation (10).

$$\left[\frac{dF}{dr}(r_n)\right] \equiv D[F(r_n)] \tag{18}$$

$$\left[\frac{dF^2}{dr}(r_n)\right] \equiv D^2[F(r_n)] \tag{19}$$

If discretization is performed, such as Equations (11) the differential equation modeling the system is given below. Where L_1 is the linear coefficient, and RHSF is the right-hand side equation.

$$L_1 F = RHSF \tag{20}$$

When the boundary conditions $\sigma_r(a) = 0$ and $\sigma_r(b) = 0$ are applied in Equation (20), the non-obvious solution of RHSF is given below.

$$L_1 = r^2 D^2 + rD - k^2 r_j \tag{21}$$

$$RHSF = \frac{(\alpha_r - \alpha_\theta)(r^2 - ar)}{a_{\theta\theta}(b-a)} T_O - \frac{a_{\theta\theta} T_O}{a_{\theta\theta}(b-a)} r^2 \tag{22}$$

3. Conclusions and discussions

A computer program was used in this study. The thermal stresses occurring in a Kevlar (491PR-286) material disc were determined and calculated using different methods, and the resulting margin of error was examined.

As can be seen in **Table 2**, the stresses obtained on the inner surface of the disc are smaller than the stresses obtained on the outer surface, tangential stresses also increase as the temperature increases. The stresses obtained by three different methods are close to each other.

Table 2. Variation of radial and tangential stresses with temperature.

Temperature ΔT (°C)	Surface	Analytical solution		Finite element method		Pseudospectral Chebyshev method	
		σt (MPa)	σr (MPa)	σt (MPa)	σr (MPa)	σt (MPa)	σr (MPa)
25 °C	Inner	52.35	0	53.77	0	51.10	0
	Outer	–108.06	0	–109.83	0	–109.61	0
50 °C	Inner	69.81	0	68.19	0	70.08	0
	Outer	–144.09	0	–146.97	0	–142.89	0
75 °C	Inner	87.25	0	88.06	0	88.43	0
	Outer	–180.11	0	–180.25	0	–183.91	0
100 °C	Inner	104.71	0	102.87	0	101.59	0
	Outer	–216.12	0	–217.44	0	–215.63	0
150 °C	Inner	152.7	0	156.82	0	148.94	0
	Outer	–315.18	0	–313.08	0	–317.21	0

The solutions by finite element method (Ansys 2021-R1) are given in **Figure 2** and **Figure 3**.

In **Figure 3**, the kev material disc is modeled with the ANSYS 2023-R1 program and divided into nodes. The inner and outer half diameters of the disc are sequentially. a = 50 mm, b = 100 mm. The thickness of the composite disc is determined as 2 mm. In **Figure 4**, the tangential determined on the disc obtained by ANSYS 2023 R1.

As can be seen in **Figure 4**, the tangential stresses obtained on the outer surface of the disc are greater than the tangential stresses obtained on the inner surface, tangential stresses increase in the parts shown in red. **Figure 5** shows the Radial stresses calculated by ANSYS 2023 R1.

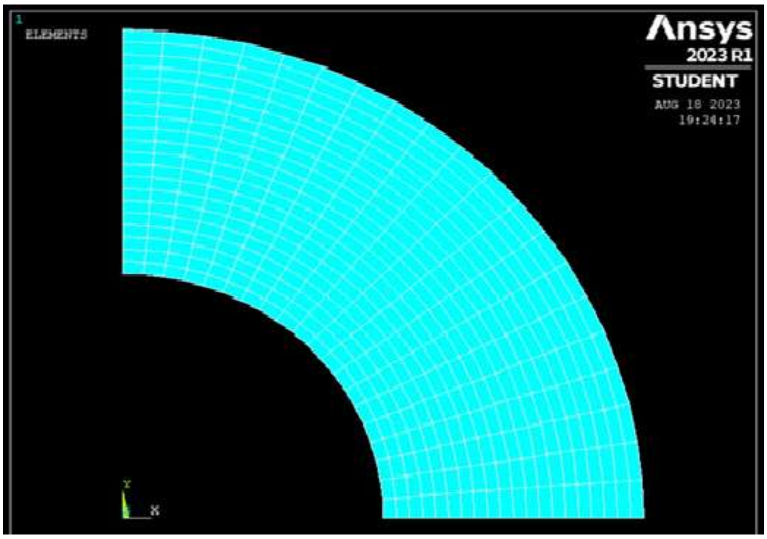

Figure 3. A composite disc modeled by the finite element method (ANSYS 2023 R1).

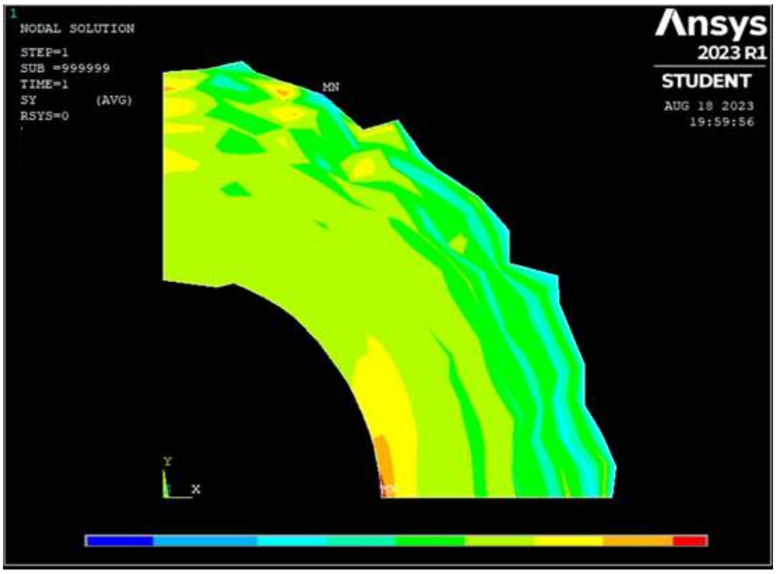

Figure 4. Distribution of tangential stresses by the ANSYS 2023 R1.

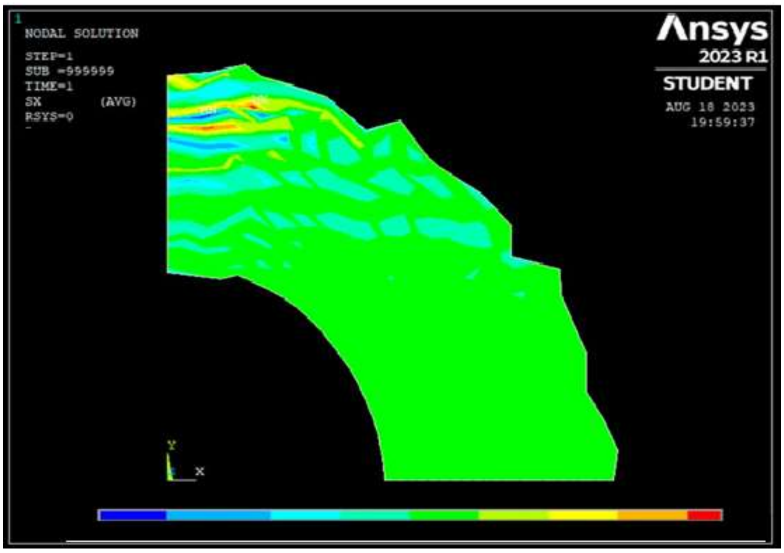

Figure 5. Distribution of radial stresses.

Figure 6 shows the radial stresses obtained as a result of different methods.

As seen in **Figure 6**, radial stresses increase from the inner to the middle part of the disc. Radial stresses are greater in the parts indicated by red.

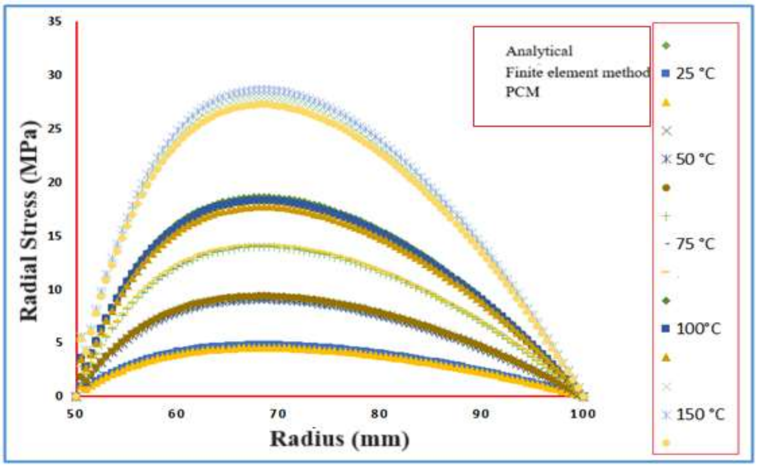

Figure 6. Showing the radial stresses occurring in the composite disc.

The radial stress on the disc was affected by increasing from the inner surface of the disc to the *r* = 70 mm region. From this region, the stress values towards the outer region of the disc decreased and took the value of zero in the outer region.

Figure 7 shows the tangential stresses.

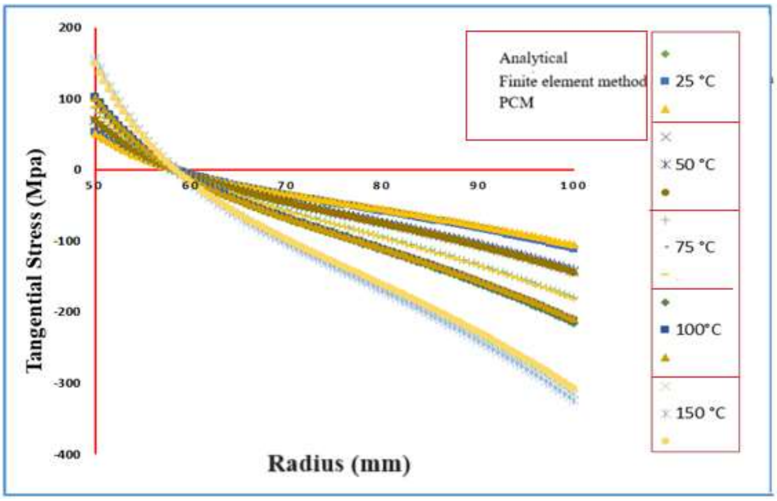

Figure 7. Showing the tangential stresses occurring in the composite disc.

As can be seen from **Figure 7**, the tangential stress components were determined as tensile stress on the innermost surface of the disc and compressive stress on the outermost surface. Tangential stresses were calculated as –108.06 MPa on the outer surface of the disc while they were 52.35 MPa on the inner surface of the disc at a temperature of 25 °C under linear increasing temperature. At a temperature of 150 °C, the tangential at the innermost part was calculated as 152.70 MPa, while at the outermost part it was –315.18 MPa.

As can be seen from **Figure 8**, at the end of the analytical solution, the uniform stress on the innermost surface of the disc at a temperature of 25 °C is 52.35 MPa, and at the outermost part, it is 108.06 MPa. The results obtained by the Finite Element Method (ANSYS) are that the uniform stress on the innermost surface of the disc at a temperature of 25 °C is 53.77 MPa and –109.83 MPa at the outermost part. Materials that can be used manually with the pseudo spectral Chebyshev method have a uniform stress of 51.10 MPa on the innermost surface of the disc at a temperature of 25 °C, and –109.61 MPa on the outer part. Considering different temperatures, for example, 150 °C, at the end of the analytical solution, the uniform stress occurring on the innermost face of the disc is 152.70 MPa, and on the outermost part, –315.18 MPa. The results obtained by the Finite Element Method (ANSYS) are that the uniform stress on the innermost surface of the disc at a temperature of 150 °C is 156.82 MPa and –313.08 MPa at the outermost part. Materials that can be used manually with the pseudo-spectral Chebyshev method have a uniform stress of 148.94 MPa inside the disc at a temperature of 25 °C, and −317.21 MPa on the outside. In addition to all these and some other studies conducted[14–20], there are still serious questions in the literature about determining the appropriate choice of materials in this area, and more research is needed. Therefore, thermal elastic stress analysis was performed at a constant temperature in this study.

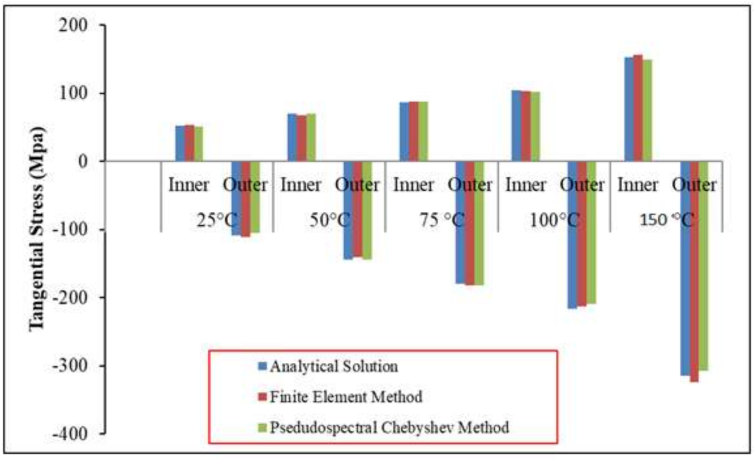

Figure 8. Stresses in the composite disc.

4. Conclusions

In this study, thermal stress analysis was performed by different methods under a linearly increasing temperature distribution for a Kev (491PR-286) material disc. The solution to the analytical method was made with the help of a mathematically developed computer program. The radial and tangential stress components occurring from the inner surface of the disc to the outer surface have been calculated for different temperature values. Assuming that the modulus of elasticity does not change with temperature, the solution was made, and the distributions of radial and tangential stresses occurring in the region from the inner surface of the disc to the outer surface were studied.

- Radial stresses are always zero on the outermost and innermost surfaces of the disc. Radial stresses take their highest value in the regions near the inner part of the disc.
- It has been observed that with increasing temperatures, there is also a significant increase in stress values.
- When the temperature increases linearly from the inner surface of the disc to the outer surface, radial stresses are formed in the form of pull stress. Tangential stresses, on the other hand, occurred in the form of tensile stress from the innermost part of the disc to the middle region, and from the middle part to the outer region in the form of compressive stresses.

- In this study, three different analysis methods were used. When these are compared among themselves, it is thought that the Chebsyhev method can give faster and more accurate results in December. The margin of error of the results obtained at the end of three different methods does not exceed 3%.
- The number of nodes obtained by the finite element method (ANSYS-2023) is higher than the number of nodes used by the Chebsyhev method.
- The tangential stress obtained by the computer program is 125.7 MPa at the innermost part of the disc and −315.18 MPa at the outermost part, while the change in stress obtained by the Finite Element Method (ANSYS) is about 2.69% and 0.66%, respectively, at the innermost and outermost parts. The percentage change in stress obtained by the Chebsyhev method is about 2.46% and 0.64%, respectively, at the innermost and outermost parts of the disc, for 150 degrees, the tangential stress obtained by the computer program is about 2.66% and 0.64%, respectively at the innermost and outermost part of the disc it has been reached.
- It is considered that the composite Kev (491PR-286) material is applicable.

Conflict of interest

The author declares no conflict of interest.

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