Journal of AppliedMath 2023;1(2): 64.
Original Research Article

Newton's forward interpolation method for solving nonlinear algebraic equation

Nasr Al Din Ide

Department of Mathematics, Faculty of Science, Aleppo University, Aleppo 021, Syria; Ide1112002@yahoo.ca

ARTICLE INFO

Received: 1 June 2023
Accepted: 6 July 2023
Available online: 14 July 2023

doi: 10.59400/jam.v1i2.64

ABSTRACT: As we know, interpolation is one of the most basic and useful numerical techniques in mathematics. Newton's forward interpolation method is one of the most important of these methods. Its most important task in numerical analysis to find roots of nonlinear equations, several methods already exist to find roots. But in this paper, we introduce the interpolation technique for this purpose. The proposed method derived from the Newton forward interpolation method, and we compared the results with another existing method (Bisection Method (BM), Regula-Falsi Method (RFM), Secant Method (SM), Newton Raphson Method (NRM)) and the method proposed by J. Sanaullah (SJM). It's observed that the proposed method has fast convergence, but it has the same order of convergence as the SJM method. Maple software is used to solve problems using different methods.

KEYWORDS: Newton's forward interpolation; nonlinear algebraic equations; interpolation

1. Introduction

Interpolation is a very important and useful technique in numerical analysis. The main task of interpolation is to replace one function by another simpler[1]. To construct a polynomial of interpolation, there are many techniques, including linear interpolation, Lagrange's interpolation formula, divided differences, spline interpolating, Newton's forward and backward interpolation, sterling interpolation, Bessel's interpolation, etc.[2].

One of the most important and frequent questions in numerical analysis is to find an approximate solution to a nonlinear equation. There are some analytical methods. But it's near to impossible to locate the exact root of algebraic equations of order greater than four and transcendental equations with analytical methods[3–31]. There are several methods like bisection, regula-falsi, secant, and the Newton-Raphson method that already exist in the literature, but the drawback of the above methods is that they are either slow or derivative. In this regard, many researchers are busy developing new, easy, and derivative-free methods for higher accuracy rates. Ozbzn[6] develops a new variant of Newton methods using the harmonic mean and midpoint iteration rules with third-order convergence for better accuracy. Recently, Qureshi et al.[12] introduced a method with six orders of convergence based on the Steffensen method and the Newton method to accelerate accuracy. Interpolation techniques and divided difference rules are also used to find a solution to nonlinear equations[8], and modified quadrature iterated methods for solving non-linear equations are proposed. This paper describes the analytic form for solving problems with Newton's forward interpolation formula instead of Newton's backward interpolation formula[4], and solving the same problem with Maple software. In Section 3, we give some examples for solving the nonlinear algebraic equations of this interpolation and how they can be solved with Maple. In Section 4, we give some examples. Finally, Section 5 concludes the paper.

2. Newton's forward interpolation formula

Let be given the points $x_0, x_1, x_2, \ldots, x_n$, and the function y on $[x_0, x_n]$.

Suppose now that the points $x_0, x_1, x_2, \ldots, x_n$ are equidistant, i.e., $x_{i+1} - x_i = h$, for $i = 0, 1, 2, \ldots, n - 1$.

Definition 1. *The finite first order forward difference for the function f in relation to x is called the expressions.*

$$\begin{gathered} \Delta y_i = y_{i+1} - y_i \\ \Delta^2 y_i = \Delta(\Delta y_i) = \Delta y_{i+1} - \Delta y_i \\ \cdots \\ \Delta^n y_i = \Delta(\Delta^{n-1} y_i) = \Delta^{n-1} y_{i+1} - \Delta^{n-1} y_i \end{gathered} \tag{1}$$

where $\Delta^n y_i$ is the n-th order forward difference.

These differences are often presented in a tabular format as in **Table 1**.

Table 1. The finite backward difference[2].

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_0	y_0	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$
x_1	y_1	Δy_1	$\Delta^2 y_1$	$\Delta^3 y_1$	
x_2	y_2	Δy_2	$\Delta^2 y_2$		
x_3	y_3	Δy_3			
x_4	y_4	…			
…	…	Δy_{n-1}			
x_{n-1}	y_{n-1}				
x_n	y_n				

Newton's forward interpolation formula for interpolation is obtained from the Definition 1 given above.

For $q = \frac{x-x_0}{\hbar}$, we get the polynomial:

$$P_n(x) = y_0 + \frac{\Delta y_0}{\hbar}(x - x_0) + \frac{\Delta^2 y_0}{2!.\,\hbar^2}(x - x_0)(x - x_1) + \ldots + \frac{\Delta^n y_0}{n!.\,\hbar^n} \prod_{i=0}^{n-1} (x - x_i) \tag{2}$$

or,

$$\begin{aligned} P_n(x_0 + q\hbar) &= y_0 + q\Delta y_0 + \frac{q(q-1)}{2!}\Delta^2 y_0 + \ldots + \frac{q(q-1)\ldots(q-n+1)}{n!}\Delta^n y_0 \\ &= y_0 + \binom{q}{1}\Delta y_o + \binom{q}{2}\Delta^2 y_o + \ldots + \binom{q}{n}\Delta^n y_0 \end{aligned} \tag{3}$$

This formula is useful when the value of y is required at point x_p near the end of the segment.

The error in this case is

$$R(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} \prod_{i=o}^{n} (x - x_i) \tag{4}$$

or,

$$R(x_o + q\hbar) = \hbar^{n+1} \frac{f^{(n+1)}(\xi)}{(n+1)!} q(q-1)\ldots(q-n) \tag{5}$$

3. Proposed method

As we know, Newton's forward interpolation formula is used for finding the value of the function at a point a i.e., *f(a)*. In this work we aim to find the root c of a function $y = f(x)$ by using Newton forward.

Now, by using Newton forward difference formula:

$$P_n(x_0 + q\hbar) = y_0 + q\Delta y_0 + \frac{q(q-1)}{2!}\Delta^2 y_0 + \ldots + \qquad (6)$$

$$y_c = y_0 + \mathrm{q}\Delta y_0 + \frac{q(q-1)}{2}\Delta^2 y_0 + \cdots \qquad (7)$$

by taking three terms of Equation (7),

$$y_c = y_0 + \mathrm{q}\Delta y_0 + \frac{q(q-1)}{2}\Delta^2 y_0 = 0 \qquad (8)$$

we get the equation:

$$y_c = q^2\Delta^2 y_0 + (2\Delta y_0 - \Delta^2 y_0)q - 2y_0 = 0 \qquad (9)$$

or,

$$y_c = Aq^2 + Bq + C = 0 \qquad (10)$$

where, $A = \Delta^2 y_0$, $B = (2\Delta y_0 - \Delta^2 y_0)$, $C = -2y_0$; $q = \frac{x - x_0}{\hbar}$.

By solving the quadratic Equation (10), we get the two solutions

$$\mathrm{x} = x_0 + h\,(\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}) \qquad (11)$$

we have two roots, the root depends the value of y_i and y_n,

1) If $y_i > y_n$, we use the relation

$$x = x_0 + h\,(\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}) \qquad (12)$$

2) If $y_i < y_n$, we use the relation

$$x = x_0 + h\,(\frac{-B + \sqrt{B^2 - 4AC}}{2A} \qquad (13)$$

4. Examples

In this section, we will check the effectiveness of the present method. The below problems are taken from the literature[4] and tested in the proposed method by comparison with the following method, (Bisection Method (BM), Regula-Falsi method (RFM), Secant Method (SM) and Newton Raphson Method (NRM)), New Second Order Method (NSOM) of Jamali et al.[4] and New Proposed Method (NPM). It's observed that the proposed method converges to the exact solution. Maple software was used to solve problems using these different methods.

Example 1. *For* $f(x) = x^2 - \exp(x) - 3x + 2$:

Table 1. It shows the number of iterations of problem -1 at error 10×10^{-15}.

Initial guess	Method	No of iterations	Approximate root	Error fixed
2	BM	46	2.154434690031884	10×10^{-15}
	RFM	18		10×10^{-16}
	SM	6		
	NRM	4		
	NSOM	4		
	NPM	6		

Example 2. *For* $f(x) = x^3 - 10$:

Table 2. It shows the number of iterations of problem -1 at error 10×10^{-15}.

Initial guess	Method	No of iterations	Approximate root	Error fixed
2	BM	46	2.154434690031884	10×10^{-15}
	RFM	18		10×10^{-16}
	SM	6		
	NRM	4		
	NSOM	4		
	NPM	6		

Example 3. *For* $f(x) = x^3 + 4x - 10$:

Table 3. It shows the number of iterations of problem -1 at error 10×10^{-15}.

Initial guess	Method	No of iterations	Approximate root	Error fixed
0.85	BM	47	2.57530285439861	10×10^{-15}
	RFM	10		
	SM	5		
	NRM	4		
	NSOM	4		
	NPM	6		

Example 4. *For* $f(x) = 2 - \exp(x) + 2$:

Table 4. It shows the number of iterations of problem -1 at error 10×10^{-15}.

Initial guess	Method	No of iterations	Approximate root	Error fixed
0.85	BM	47	2.57530285439861	10×10^{-15}
	RFM	10		
	SM	5		
	NRM	4		
	NSOM	4		
	NPM	6		

Example 5. *For* $f(x) = x^2 - \exp(x) - 3x + 2$:

Table 5. It shows the number of iterations of problem−1 at error 10×10^{-15}.

Initial guess	Method	No of iterations	Approximate root	Error fixed
0.5	BM	47	2.57530285439861	10×10^{-15}
	RFM	10		
	SM	5		
	NRM	4		
	NSOM	4		
	NPM	6		

5. Conclusion

In this paper, we checked the effectiveness of the present method. By solving some problems given in the literature[4] for testing this proposed method by comparison with the, Bisection Method (BM), Regula-Falsi method (RFM), Secant Method (SM) and Newton Raphson Method (NRM), the New Second Order Method (NSOM) of Jamali et al.[4] and the New Proposed Method (NPM). It's observed that the proposed method converges to the exact solution better than BM method and RFM method. Maple software was used to solve problems by these different methods.

Conflict of interest

The author declares no conflict of interest.

References

1. Trifunov Z, Zenku L, Jusufi-Zenku T. Application of Newton's backward interpolation using wolfram Mathematica. *International Journal of Mathematics Trends and Technology* 2021; 67(2): 53–56. doi: 10.14445/22315373/IJMTT-V67I2P508
2. Burden RL, Faires JD. *Numerical Analysis*, 9th ed. Cengage Learning; 2011.
3. Liu X, Wang X. A Family of methods for solving nonlinear equations with twelfth-order convergence. *Applied Mathematics* 2013; 4(2): 326–329. doi: 10.4236/am.2013.42049
4. Jamali S, Kalhoro ZA, Wasim A, et al. A new second order derivative free method for numerical solution of non-linear algebraic and transcendental equations using interpolation technique. *Journal of Mechanics of Continua and Mathematical Sciences* 2021; 16(4): 75–84. doi: 10.26782/jmcms.2021.04.00006
5. Mehtre VV, Sharma S. Root finding methods: Newton Raphson method. *International Journal for Research in Applied Science and Engineering Technology* 2019; 7(XI): 411–414. doi: 10.22214/ijraset.2019.11065
6. Ozbzn AY. Some new variants of Newton's method. *Applied Mathematics Letters* 2004; 17(6): 677–682. doi: 10.1016/S0893-9659(04)90104-8
7. Omran HH. Modified third order iterative method for solving nonlinear equations. *Journal of Al-Nahrain University* 2013; 16(3): 239–245. doi: 10.22401/JNUS.16.3.33
8. Qureshi UK, Shaikh AA, Solangi MA. Modified free derivative open method for solving non-linear equations. *Sindh University Research Journal (Science Series)* 2017; 49(4): 821–824. doi: 10.26692/sujo/2017.12.0065
9. Qureshi UK, Ansari MY, Syed MR. Super linear iterated method for solving non-linear equations. *Sindh University Research Journal (Science Series)* 2018; 50(1).
10. Qureshi UK, Bhatti AA, Kalhoro ZA, Ali Z. On the development of numerical iterated method of Newton Raphson method for estimating nonlinear equations. *University of Sindh Journal of Information and Communication Technology* 2019; 3(2): 94–97.
11. Qureshi UK, Jamali S, Kalhoro ZA, Shaikh AG. Modified quadrature iterated methods of Boole rule and Weddle rule for solving non-linear equations. *Journal of Mechanics of Continua and Mathematical Sciences* 2021; 16(2): 87–101. doi: 10.26782/jmcms.2021.02.00008

12. Qureshi UK, Kalhoro ZA, Shaikh AA, Jamali S. Sixth order numerical iterated method of open methods for solving nonlinear applications problems. *Proceedings of the Pakistan Academy of Sciences: A Physical and Computational Sciences* 2020; 57(2): 35–40.
13. Qureshi UK, Solanki N, Ansari MY. Algorithm of difference operator for computing a single root of nonlinear equations. *Punjab University Journal of Mathematics* 2019; 51(4): 51–56.
14. Sangah AA, Shaikh AA, Shah SF. Comparative study of existing bracketing methods with modified bracketing algorithm for solving nonlinear equations in single variable. *Sindh University Research Journal (Science Series)* 2016; 48(1): 171–174.
15. Shah FA, Noor MA, Batool M. Derivative-free iterative methods for solving nonlinear equations. *Applied Mathematics and Information Sciences* 2014; 8(5): 2189–2193. doi: 10.12785/amis/080512
16. Subash R, Sathya S. Numerical solution of fuzzy modified Newton-Raphson method for solving non-linear equations. *International Journal of Current Research* 2011; 3(11): 390–392.
17. Ide NAD. On modified Newton methods for solving a non linear algebraic equations. *Applied Mathematics and Computation* 2008; 198(1): 138–142. doi: 10.1016/j.amc.2007.08.084
18. Ide NAD. A new hybrid iteration method for solving algebraic equations. *Applied Mathematics and Computation* 2008; 195(2): 772–774. doi: 10.1016/j.amc.2007.05.022
19. Ide NAD. A nonstationary Halley's iteration method by using divided differences formula. *Applied Mathematics* 2012; 3(2): 169–171. doi: 10.4236/am.2012.32026
20. Ide NAD. Some new type iterative methods for solving nonlinear algebraic equation. *World Applied Sciences Journal* 2013; 26(10): 1330–1334. doi: 10.5829/idosi.wasj.2013.26.10.512
21. Ide NAD. Application of iterative method to nonlinear equations using homotopy perturbation methods. *Journal of Basic and Applied Research International* 2015; 5(3): 2395–3446.
22. Ide NAD. Some new iterative algorithms by using homotopy perturbation method for solving nonlinear algebraic equation. *Asian Journal of Mathematics and Computer Research* 2015; 5(3): 2395–4213.
23. Ide NAD. A new algorithm for solving nonlinear equations by using least square method. *Mathematics and Computer Science* 2016; 1(3): 44–47. doi: 10.11648/j.mcs.20160103.12
24. Ide NAD. Using Lagrange interpolation for solving nonlinear algebraic equations. *International Journal of Theoretical and Applied Mathematics* 2016; 2(2): 165–169. doi: 10.11648/j.ijtam.20160202.31
25. Ide NAD. Improvement of new eight and sixteenth order iterative methods for solving nonlinear algebraic equations by using least square method. *International Journal of Scientific and Innovative Mathematical Research (IJSIMR)* 2018; 6(10): 23–27. doi: 10.20431/2347-3142.0610004
26. Ide NAD. Bisection method by using fuzzy concept. *International Journal of Scientific and Innovative Mathematical Research (IJSIMR)* 2019; 7(4): 8–11. doi: 10.20431/2347-3142.0704002
27. Ide NAD, Aziz SNA. Using the least squares method with five points to solve algebraic equations nonlinear. *International Journal of Scientific and Innovative Mathematical Research* 2019; 7(5): 26–30. doi: 10.20431/2347-3142.0705005
28. Ide NAD. A new Aitken type method by using geometric mean concept. *World Applied Sciences Journal* 2019; 37(4): 289–292. doi: 10.5829/idosi.wasj.2019.289.292
29. Ide NAD. New modification methods for solving nonlinear algebraic equations by using Lagrange interpolation approach. *Studied in Nonlinear Sciences* 2019; 4(2): 23–25. doi: 10.5829/idosi.sns.2019.23.25
30. Ide NAD. Iterative method by using Tchybcheve integral for solving nonlinear algebraic equations. *World Applied Sciences Journal* 2019; 37(8): 661–663. doi: 10.5829/idosi.wasj.2019.661.663
31. Ide NAD. Comparison of a new modified Mcdougall-Wotherspoon method with "modified Cauchy methods using Padé approximation" to solve nonlinear equations. *World Applied Sciences Journal* 2020; 38(2): 106–113. doi: 10.5829/idosi.wasj.2020