Journal of AppliedMath 2024, 2(4), 553.
https://doi.org/10.59400/jam.v2i4.553

Article

GeoGebra—A great platform for experiential learning, explorations and creativity in mathematics

Qamil Kllogjeri[1,*], Pellumb Kllogjeri[2]

[1] Global Savings Group, 2181927 Munich, Germany
[2] GeoGebra Institute of Albania, 3001 Elbasan, Albania
*** Corresponding author:** Qamil Kllogjeri, mili87k@yahoo.com

CITATION

Kllogjeri Q, Kllogjeri P. GeoGebra—A great platform for experiential learning, explorations and creativity in mathematics. Journal of AppliedMath. 2024; 2(4): 553. https://doi.org/10.59400/jam.v2i4.553

ARTICLE INFO

Received: 18 February 2024
Accepted: 26 June 2024
Available online: 27 August 2024

Abstract: In this paper we are presenting some examples of how GeoGebra is used in: a) explaining concepts of the first derivative, monotony, extremums; b) studying the properties of the function (strictly increasing/decreasing); c) demonstrating the mean value theorem. The results and the conclusions are based on the experiment carried out in the teaching and learning process in the chapter of derivatives in a third-year class of a secondary school in Albania. Also, there are some encouraging facts got by the use of GeoGebra: the double representation and the dynamic features of GeoGebra allow the students to quickly grasp the mathematical concepts and properties and be actively involved in further explorations. The use of GeoGebra tools is similar to the use of the tools of virtual games, and this is a great advantage to stimulate the students to learn mathematics and master their mathematical performance the same way they play games. On the other side, using GeoGebra, it is easier for the teachers to explain mathematical concepts, the properties of algebraic objects, to discuss about different situations and aspects of the subject under study and to methodically reason the results got. GeoGebra provides a very commodious environment for the students to effectively interoperate with each other. Our results showed that GeoGebra is effective in teaching and learning mathematics. GeoGebra software contributed in enhancing students' understanding of mathematical concepts and improved students' interest to learn mathematics. Also, we admit that not all the stages of implementing GeoGebra software in the classroom are flowing smoothly. Based on our experience and the other researchers, it is observed that the effectiveness is dependent on the way GeoGebra is integrated in the teaching and learning process, implying that the research must continue with the commitment of many researchers of mathematics, physics and other sciences.

Keywords: dynamic demonstration; visualization of the concept or property; false demonstration

1. Introduction

The double representation and the dynamic feature of GeoGebra are the best means that math teachers can now effectively use to teach mathematical concepts and properties [1,2]. Using GeoGebra it is possible to perform dynamic changes of the graphics or figures accompanied by the changes in their respective algebraic representations (equations), allowing this way the students to make observations and explorations. These are the main features of GeoGebra meeting the demands of many didactics and educators to provide as many representations forms as possible for the students [3]. Taking advantage of this double representation and dynamic feature of GeoGebra it is easier for the teachers to explain the mathematical concepts, the properties of algebraic objects and to methodically reason the result got; on the other hand, the students have the possibility to grasp faster and correctly a common model that is taught and to add more to their knowledge through their experience while they use GeoGebra. The purpose of this study is to enhance students' conceptual understanding of the notion of derivatives and its applications in studying the properties of the function by boosting students' visualization skills using GeoGebra software [4,5]

2. Methodology

In this section we present the effectiveness of GeoGebra software by allowing the students to quickly grasp the mathematical concepts and properties, by providing the tools and the conditions for research activity and by stimulating the students to actively be involved in further explorations.

2.1. Quick and correct grasping of the concept

Because of the double representation feature it is possible to perform dynamic calculus like functions in x, derivatives and integrals and draw conclusions about the properties of the algebraic objects within a short interval of time because there is a dependency between the algebraic object and its respective geometric object in the way that, a change done in the algebraic object is accompanied with the respective change in the geometric object. So, we can enter any function and show a visualization of generating the first derivative. Change $f(x)$ in the algebra window and have other functions. The fine thing is that the construction is so easy to do that it can be done together with the students. The double representation and dynamic feature allow the students to quickly grasp mathematical concepts. Here are several demonstrations with GeoGebra tools performed during the teaching process on the chapter of derivatives in the experiment carried out in a third year class of a secondary school in Albania.

Demonstration 1. *GeoGebra serves as a testing tool for many mathematical concepts or properties*.

The feature of double representation and dynamic [1,2] one allows us to perform The First Derivative Test and the respective theorem. By using GeoGebra applet (see **Figure 1**) was so easily demonstrated that on the interval (x(D), x(E)), where the function is strictly decreasing, its first derivative represented by the ordinates of the points of the part of the graph of the first derivative of $f(x)$ corresponding to the mentioned interval, also the ordinates represented by the sign length of the segment A'H, is negative. The students could observe how the values of the first derivative change from positive to negative when point A is moving from the increasing interval to the decreasing one.

The demonstration was extended to the intervals (by moving the slider a) where the function is strictly increasing and accompanied with teacher–students discussion.

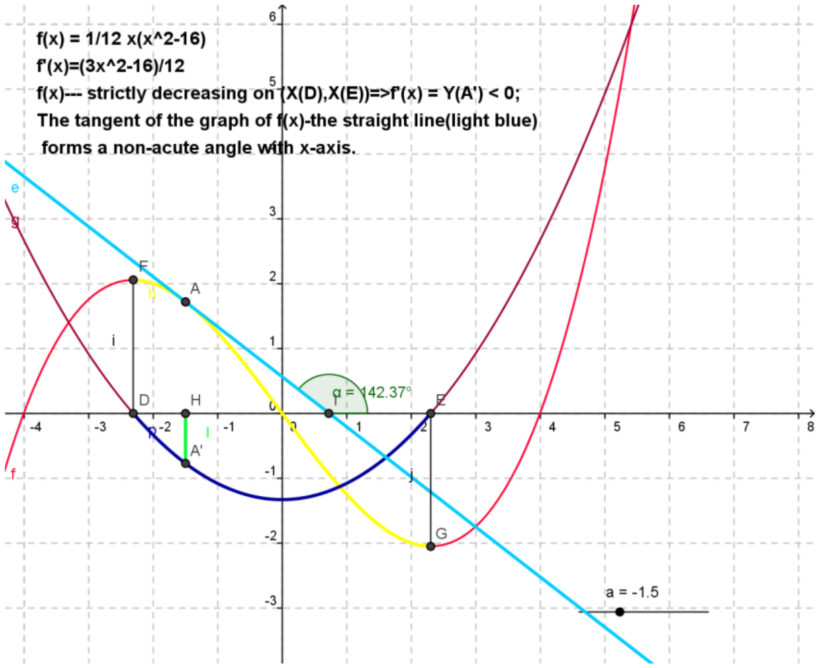

Figure 1. The relation between the first derivative of a function and its monotony.

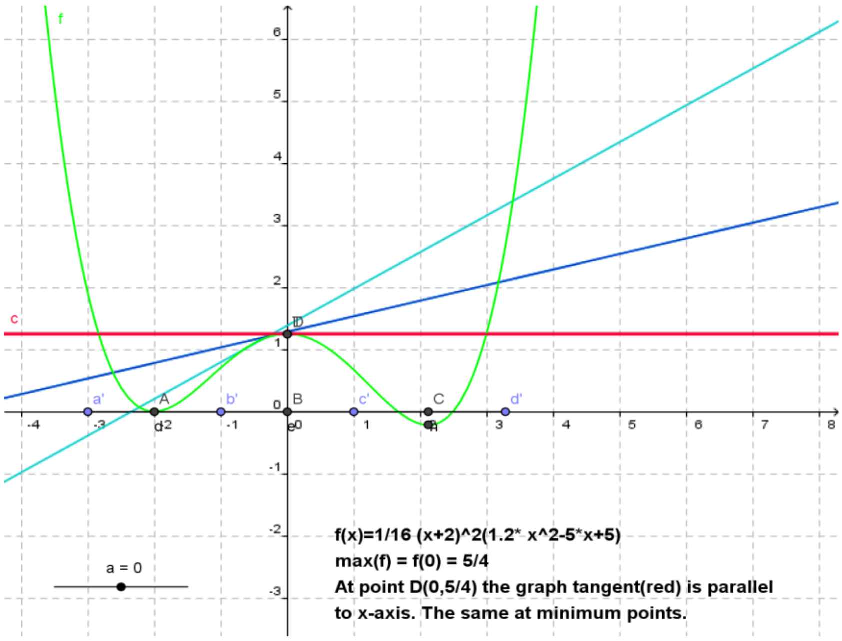

Figure 2. Illustration of the concept of local extremums.

By applying the command of anti-derivative (which was to be learned in the next chapter) the students could observe that on the interval where the function (seen as derivative) was positive its anti-derivative was strictly increasing and so on. This observation done by using GeoGebra is very helpful in teaching and learning mathematics. It can be used before proving the above theorem: the students observe the relation under discussion for specific functions and later was jumped to generalization of this relation by proving the theorem for any function. We used it before proving the theorem and the result in regard with grasping the theorem and using it in applications was very successful. Using the example of the **Figure 1** was demonstrated the meaning of extremums at points F and G. Similarly, was demonstrated the meaning of the extremums at points A, C and D (**Figure 2**). This case corresponds to another function as you can see in the notes of **Figure 2**.

Demonstration 2. *The extreme value theorem says that there is a maximum (respectively minimum), and that there is at least one way of achieving that maximum (respectively minimum) within the interval [a’, d’]. The demonstration of the result of the important lemma that if x_E from (a, b) is abscissa of an extremum point E then $f'(i_E) = 0$, was performed by using the applet of **Figure 1**. The discussion was linked with the geometrical meaning of the first derivative. Using applet in **Figure 3** and leading the discussion with the students, the students were convinced that there is maximum at point M but the respective lemma is not satisfied (they could observe that the tangent at this point is undefined). The observation was accompanied by the discussion about the left and right limits of the derivatives. The left limit is 6, while the right one is −3.5 (both of them are very far from 0, so they are not equal). By moving the slider, they could see in algebra window that on the left side and close to M the values of the first derivative are positive and much greater than 0 and, on the right side and close to M the values of the first derivative are negative and much less than 0.*

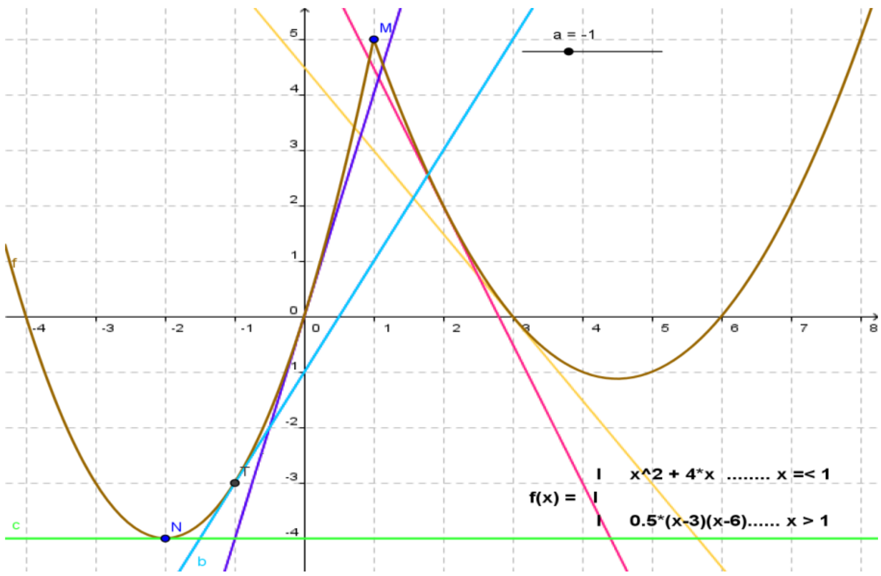

Figure 3. Tangent undefined at the local extremum(maximum).

At point N there is minimum (observation showed that tangent at N is parallel to *x*-axis). In the same time, for the students, was given the task of testing the sided limits at that point. Additional exercises were given to bring other examples of this type. After the proof of max-min theorem it is important that the teacher emphasize its practical use in many problems, that the second result, known as the extreme value theorem (or the max-min theorem), is linked with important application: “optimization problems”.

Demonstration 3. *In **Figure 4** is a GeoGebra Applet allowing the students to explore the conclusion of the MVT (mean value theorem) for derivatives. Our purpose is to demonstrate the employing of the common trick in proving this Calculus result which is achieved by taking the general function, and subtracting off the secant line by considering the assistant function: g(x) = f(x) − Secant line. Recall that MVT is proved* *by considering the distance function g(x) which is the magnitude of the vertical distance between a point (x, f(x)) on the graph of the function f and the corresponding point on the secant line through A and B, making g positive when the graph of f is above the secant, and negative otherwise.*

The GeoGebra applet demonstrates the "lifting" of the graph of *f*(*x*) up or down or twisting it .... With this applet was demonstrated that by geometrical transformation of the graph of *f*(*x*) the students could be convinced that there is a point N on the graph of *f*(*x*) and between the points A and B, at which the tangent of the graph is parallel to the secant AB. The transformation was performed in such a way that the tangent of *g*(*x*) could touch the graph of *f*(*x*) where was done possible to plot the intersection point (N). It is clear that during the transformation the tangent remains always parallel to the respective secant. This is provided by one of the most important features of GeoGebra, by that of preserving the relative relationship between two objects where one is dependent on the other. Was needed the construction of a parallel to secant AB, a parallel passing through the point F on the graph of *f*(*x*) and close to point A. Then, the movement of the graph was done carefully until the secant CD rested on that parallel (look at **Figure 4**). It is important to add here that in all figures of this paper are shown freezing situations of the real dynamic situations demonstrated in applets by the use of virtual tools offered by GeoGebra software. How is defined the secant AB? Our purpose is not to prove MVT for a function defined in the interval (*a*, *b*) where the secant equation is the equation of a straight line passing through the points (*a*, *f*(*a*)) and (*b*, *f*(*b*)). We are focused only in the geometrical illustration of MVT, so the secant can be any straight line intersecting the curve of the given function in two points.

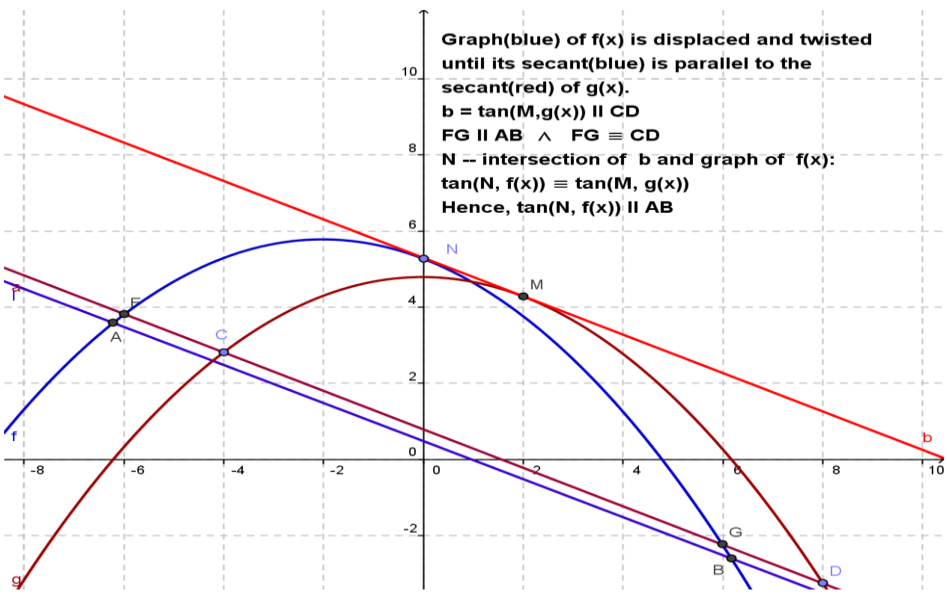

Figure 4. Picture of GeoGebra applet illustrating the transformation in MVT.

2.2. GeoGebra provides the tools and the conditions for research activity

GeoGebra is mainly used as a tool for teaching and researching. It is used as a checking tool to test and verify thinking, it is used as a demonstration tool to emphasize their impression. GeoGebra offers a very good place for practice and research work.

The teacher must draw the attention of the students to investigate different cases by using GeoGebra software if the tangent is defined at any point of the graph. During the teaching we brought examples when in different parts of the domain the functional dependence is different and after plotting the graph it looks quite clear that at the junction point there is tangent and so it is. Also, there are examples when in different parts of the domain the functional dependence is different and after plotting the graph it looks quite clear that at the junction point there is tangent (false demonstration) but, in reality, it is not so (look at **Figure 5**).

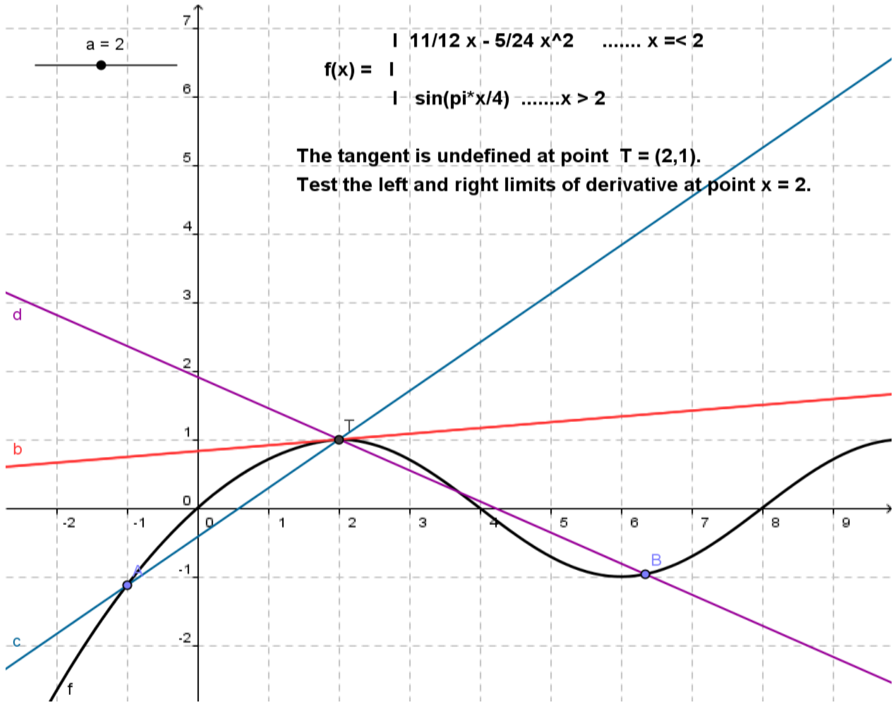

Figure 5. Picture of GeoGebra applet illustrating the non-existence of the tangent.

There is maximum at point T, moving the points A and B of the secants TA and TB towards the point T by the GeoGebra program is produced one single position at point T (the red line b), however the tangent doesn't exist at this point (the existence of the tangent is proved by testing the left and right-side limits of the derivative).

3. Conclusion

GeoGebra allows the teachers to easily and fast explain the mathematical concepts, the properties of algebraic objects and to methodically reason the results got. On the other hand, the students have the possibility to grasp faster and correctly a common model that is taught and to add more to their knowledge through their experience while they use GeoGebra; it helps them to do research work and explorations. GeoGebra provides independent learning, and it supports the present view related to education that "there is an acute issue to change the educational paradigm: leave traditional teaching, where the teacher is a person who provides ready-made facts, to one in which the student is an active recipient of knowledge"[6]. The students' views revealed that GeoGebra integrated with multi-teaching approaches increased their interest and motivation and boosted their imagination in mathematics learning [4].

GeoGebra provides good visualization to many specific examples of the theory to help students understand it. We could give example of how to use this software in the theory proof as well, but there is no room in the case of this paper. It is better and easier for the students to understand and operate with GeoGebra if we lay out the procedure or the steps (using bullet points in details) associated with due and clear explanations. This is a task to be considered in the future as a practical paper.

The implication of this study is that students are able to get resources in internet or learning material based on GeoGebra software. The students are motivated to explore new subjects and apply personalized learning. The result is: the students continually add more to their mathematical fund and they get a deeper understanding for the concepts and the methods of mathematics [7]. Our suggestion is to further coordinate and increase the cooperation among GeoGebra specialists and teachers by developing modules and videos about teaching and practicing with GeoGebra to help the new GeoGebra users in their study and research work. This is very important to attract more and more learners who are interested to learn from such available resources.

Author contributions: Conceptualization QK and PK, methodology QK; software, QK; validation, QK and PK; formal analysis, QK; investigation, QK; resources, QK; data curation, QK; writing—original draft preparation, QK; writing—review and editing, PK; visualization, QK; supervision, QK; project administration, QK. All authors have read and agreed to the published version of the manuscript.

Conflict of interest: The authors declare no conflict of interest.

References

1. Judith H, Markus H. Introduction to GeoGebra. Available online: www.geogebra.org (accessed on 11 February 2024).
2. Lavicza Z, Hohenwarter M, Jones K, et al. Establishing a Professional Development Network around Dynamic Mathematics Software in England. International Journal for Technology in Mathematics Education. 17(40): 178.
3. Josef B. Linking Geometry, Algebra and Calculus with GeoGebra. ACDCA, DUG and Technical University of Vienna; 2008.
4. Arcavi A. The role of visual representation in the learning of mathematics. Edu. Stud. Math. 2003; 52: 215-241.
5. Giaquinto M. Visual Thinking in Mathematics (An Epistemological Study). Oxford University, Press Inc.; 2007. pp. 163-185.
6. Competences for democratic culture. Living together as equals in culturally diverse democratic societies. Available online: https://rm.coe.int/16806ccc07 (accessed on 4 March 2024).
7. Grotzer TA. Expanding our vision for educational technology: Procedural, conceptual, and structural knowledge. Educational Technology. 2022; 42(2): 52-59.