Journal of AppliedMath 2024, 2(3), 532.
https://doi.org/10.59400/jam.v2i3.532

Article

Fundamental properties of the gyroscope oscillation

Ryspek Usubamatov

Kyrgyz State Technical University after I. Razzakov, Bishkek 720044, Kyrgyzstan; ryspek0701@yahoo.com

CITATION

Usubamatov R. Fundamental properties of the gyroscope oscillation. Journal of AppliedMath. 2024; 2(3): 532. https://doi.org/10.59400/jam.v2i3.532

ARTICLE INFO

Received: 1 February 2024
Accepted: 6 June 2024
Available online: 16 June 2024

COPYRIGHT

Copyright © 2024 author(s). *Journal of AppliedMath* is published by Academic Publishing Pte. Ltd. This work is licensed under the Creative Commons Attribution (CC BY) license. https://creativecommons.org/licenses/by/4.0/

Abstract: Despite partial solutions by famous scientists during the early Industrial Revolution, gyroscope problems remained unsolvable until the beginning of the twentieth century, when several fundamental physical laws were finally formulated to describe them. Today, the principles of classical mechanics enable the formulation and description of the physical processes involved in the rotation of any object. Gyroscopic devices are objects that rotate and exhibit oscillation, which has been a challenging problem in engineering mechanics. The oscillation of a gyroscope is caused by the interaction between external and inertial torques. This is different from other examples of oscillation, such as pendulums and springs, which have been well documented. The main difference in the physics of gyroscopic oscillation is that the spinning rotors of the gyroscopic devices are supported on one side, with their axes perpendicular to the axis of oscillation. The oscillation of gyroscopic devices is interrelated with the potential and kinetic energy of their components. However, the physics of the oscillation of such objects has not been fully described in publications until recently. The theory of gyroscopic effects for rotating objects has now been published and provides a solution to this problem. According to this theory, gyroscopic inertial torques represent the potential energy of the external torque and the kinetic energy of the spinning rotor. This paper demonstrates the distribution of inertial torques about the axes of Cartesian coordinates, which enables the computation of gyroscope motion and oscillation.

Keywords: oscillation; gyroscope; potential and kinetic energies; inertial torques

1. Introduction

Many helpful publications provide extensive information about mechanical gyroscopes, which are simple devices with a rotating disc that are used to maintain orientation based on the principles of the rotating disc's angular momentum. Gyroscope problems attracted scientists at the time of the Industrial Revolution [1,2]. The famous mathematician L. Euler (1707–1783) studied the properties of rotating objects and formulated equations of the rotational motion around a fixed pivot point. His theory of the motion of rigid bodies (1765) became a seminal contribution to the contemporary theory of gyroscopes, and his principle is based on the conservation of kinetic energy, which is still considered fundamental. The phenomenon of gyroscopic effects attracted the attention of many outstanding scientists, paving the way for the formulation of the known gyroscope theory.

The development of the theory of spinning bodies was based on the works of famous scientists and researchers such as I. Newton (1642–1726), J. L. R. D'Alembert (1717–1783), Lagrange (1736–1813), L. Poinsot (1777–1859), L. Foucault (1819–1868), S. D. Poisson (1781–1840), Lord Kelvin (1824–1907), and others. The French mathematician P. S. Laplace (1749–1827) was the first to propose a gyrocompass that is insensitive to a magnetic field. Other brilliant scientists investigated, developed, and added new interpretations of the gyroscopic effects.

The theories of gyroscopes were developed mainly in the 20th century, but the first attempts at practical applications were made much earlier. In the 19th century, L. Foucault tried to prove the Earth's rotation experimentally in 1852 using a gyroscope—a fast-spinning, axially symmetric body in a Cardan joint. However, the results were not always convincing due to technical and theoretical challenges that failed to describe gyroscopic effects. Scientific and engineering issues delayed practical applications of gyroscopes, which only emerged in the early 20th century.

In 1914, Greenhill presented a simplified treatment of gyroscope properties in his book, Gyroscope Theory [3]. Despite the advancements in modern gyroscope theories, it is still difficult to distinguish them from the theoretical principles formulated in the past centuries, and gyroscopic theories have failed to match practice [4–6]. The absence of a complete gyroscope theory for centuries is an unusual phenomenon in the science of classical mechanics, which is more accustomed to solving complex problems than defining mathematical models for torques acting on a simple rotating disc. Numerical modeling of the gyroscopic effects was developed by researchers such as Klein and Sommerfield in 1913, which was later used for engineering solutions and software to account for gyroscope motions and oscillations [7].

The machine dynamics and vibration analysis in existing publications do not adequately describe the oscillation of gyroscopes due to the complexity and specificity of the process [8,9].

Mathematical modes for gyroscope oscillation and vibration are simplified and give partial solutions [10–12], Most publications focus attention on gyroscope designs, technology, and practical applications [13–15]. The quality of the control of the vehicles in space and the corrections of the flight by the algorithmic improvements of the parameters are considered in publications [16–18]. There are many publications describing the improvement of gyroscopes, but direct answers to solving the main problems will be difficult to find. Nevertheless, the physical principles of classical mechanics enable solving all gyroscopic effects [19,20].

Recent studies show the oscillation of gyroscopes is achieved through the interaction of external and inertial torques, which differ from the examples of pendulum and spring oscillation. The primary difference in the physics of oscillation lies in the fact that gyroscopic devices have spinning rotors exhibiting kinetic energy. The potential energy of the gyroscopic device interacting with the kinetic energy of the spinning rotor during oscillation has been a complex problem for physicists for over a century. However, new mathematical models for the system of interrelated inertial torques of the spinning disc and principles of conservation of potential and kinetic energy have been developed to solve this problem. Publications related to the theory of gyroscopic effects for rotating objects contain fundamental principles that enable the description of the properties of gyroscopic oscillation [21].

In gyroscope theory, the equations describe the inertial torques and their effects on rotational motions but do not provide a complete understanding of the torques involved. In addition to the main inertial torques, two other torques arise when the gyroscope disc rotates around its diametrical line along two axes. The values of the torques are relatively small because the angular velocities around these axes are low. They are equal in magnitude but act in opposite directions around a single axis, which cancels out their effect on the overall rotation. These two additional torques represent a portion of the kinetic and potential energies that are important physical parameters in gyroscope theory.

The motion of a gyroscope around the axes of the Cartesian coordinates is influenced by both its potential and kinetic energies, as well as the action of the resistance and precession inertial torques. These inertial torques indirectly represent the kinetic energy of the spinning disc, the kinetic energy of the gyroscope's rotation around axes, and the potential energy of the external torque. It is important to note that the potential and kinetic energies of a gyroscope remain constant, which demonstrates the principle of mechanical energy conservation. The above characteristics of the gyroscope are explained by the new fundamental properties of its oscillation.

2. Methodology

The diagram in **Figure 1** is a presentation of the published research on gyroscopic effects [21] that clearly illustrates the torques acting on gyroscopic devices. The spinning disc's axle is horizontal, and an external torque *T, as* noted by the bold line, is applied to it. This generates a system of inertial, interrelated torques in the Cartesian coordinates, causing the gyroscope to move about its axes. However, the figure in previous publications does not display the vectors of the angular momentum H_x and H_y of the disc around axes *ox* and *oy*.

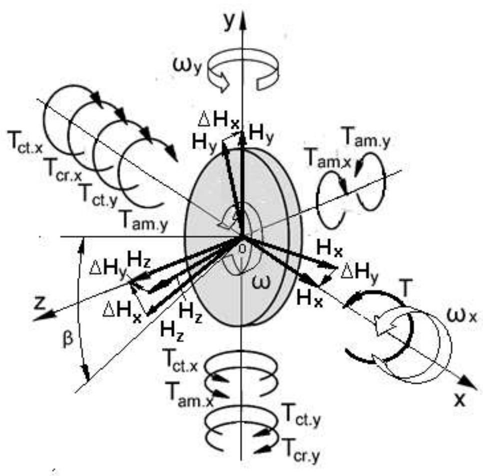

Figure 1. Torques act on the spinning disc and its motions around the axes of Cartesian coordinates.

For the readers, the actions of all inertial torques $T_{j.i}$ with indices presented by the thin lines and the spinning disc angular velocities denoted by ω_i and presented by the contour lines around axes *i* are explained in detail in **Figure 1**. The action of the load torque *T* and directions of angular velocities are represented in the counter-clockwise rotation around axes *ox, oy*, and *oz*. The external and inertial torques cause the spinning disc to rotate around these three axes. The rotations of the disc's mass around axes *ox, oy*, and *oz* are represented by the vectors of the angular momentums H_x, H_y, and H_z, respectively. The changes in the disc's angular momentums are represented by the vectors ΔH_x and ΔH_y around axes *ox* and *oy*, respectively.

- The external torque, *T*, acts on the spinning disc around the axis *ox* and activates two inertial torques, $T_{ct.x}$, caused by centrifugal forces (index *ct*) and acting around axes *ox* and *oy*, respectively. The index "*x*" refers to the axis of origin, which is *ox*. The inertial torque, $T_{ct.x}$, acting around the axis *ox* is a resistance torque to the load torque, *T*, and around the axis *oy* is a precession torque. The moment of inertia of the disc around the axis *oz* is represented as *J*. Meanwhile, the angular velocities of the disc around axes *oz* and *ox* are denoted as ω and ω_x.

- The external torque, *T*, generates the initial resistance torques of the Coriolis forces, $T_{cr.x}$ (index *cr*), which act around the *ox* axis in the clockwise direction. Additionally, it causes a change in the angular momentum, $T_{am,x}$ (index *am*), which acts around the *oy* axis in the counter-clockwise direction.
- The load torques $T_{ct.x}$ and $T_{am.x}$ acting around axis *oy* generate resistance and precession torques of the centrifugal forces, which act around axes *oy* and *ox*, respectively. Additionally, they generate the resistance torque of Coriolis forces $T_{cr.y}$ that acts around axis *oy*, as well a precession torque of the change in angular momentum $T_{am.y}$ that acts around axis *ox*. The precession torques of the centrifugal forces $T_{ct.y}$ and the change in angular momentum $T_{am.y}$ are added to the initial two inertial torques $T_{ct.x}$ of the centrifugal forces and the torque of Coriolis forces $T_{cr.x}$ acting around axis *ox*.
- The resulting torque acting around the axis *ox* $T_x = T - T_{ct.x} - T_{cr.x} - T_{ct.y} - T_{am.y} = T - (4\pi^2/9)J\omega\omega_x - (8/9)J\omega\omega_x - (4\pi^2/9)J\omega\omega_y - J\omega\omega_y$ generates the corrected precession torques $T_{ct.x}$ and $T_{am.x}$ acting around the axis *oy* that changing the values of the resistance torques of the centrifugal $T_{ct.y}$, Coriolis $T_{cr.y}$ forces acting around axis *oy*. The resulting torque acting around the axis *oy* is $T_y = T_{ct.x} + T_{am.x} - T_{ct.y} - T_{cr.y} = (4\pi^2/9)J\omega\omega_x + J\omega\omega_x - (4\pi^2/9)J\omega\omega_y - (8/9)J\omega\omega_y$. Two resistance and two precession torques act upon the *ox* and *oy* axes.
- The disc's rotations around the *ox* and *oy* axes create precession torques $T_{am.x} = J_y\omega_y\omega_x$ and $T_{am.y} = J_x\omega_x\omega_y$. Both torques act around the *oz* axis, where $J_x = J_y$ refers to the moment of inertia of the disc around the *ox* and *oy* axes, respectively. The inertial torques $T_{am.y} = T_{am.x}$ in the counter-clockwise and clockwise direction around the *oz* axis, are mutually subtracted and do not affect other inertial torques.

Figure 1 illustrates how torques interact with feedback to produce the angular velocities of the disc around the Cartesian coordinate axes. This picture shows the basic principles of gyroscope theory, which are further explained in **Table 1** [11].

Table 1. Physical principles of gyroscope theory.

1) Inertial torques acting on the spinning disc			
Generated by	**Equation**	**Action and resulting in motions**	
		Percentage, %	**Type**
Centrifugal forces, (index *ct*)	$T_{cr.i} = (4\pi^2/9)J\omega\omega_i$	41,141	Resistance
		41,141	Precession
Coriolis forces, (index *cr*)	$T_{cr.i} = (8/9)J\omega\omega_i$	8337	Resistance
Change in angular momentum, (index *am*)	$T_{am.i} = J\omega\omega_i$	9372	Precession
	$T_{am.y} = J_x\omega_x\omega_y$	0	
	$T_{am.x} = J_y\omega_y\omega_x$	0	
2) Mechanical energy conservation law			
Dependency of angular velocities of the spinning disc about axes of rotation: $\omega_y = (8\pi^2 + 17)\omega_x)$			

Table 1 and **Figure 1** show the expressions for the inertial torques produced by the rotating mass of the spinning disc. These torques are responsible for generating centrifugal and Coriolis forces as well as changes in angular momentum. The inertial torques have an index *i* that corresponds to their axis of origin and action *ox*, *oy*, or *oz*. The expressions for the torques of the change in the angular momentum related to the axis *oz* and the angular velocities of the spinning disc clearly indicate this index. The motions of the spinning disc are interconnected through the dependency of the angular velocities about the axes of rotation *ox* and *oy*.

The previously mentioned inertial torques can be used to create mathematical models for various gyroscopic effects, including the oscillation and nutation of spinning objects. These are considered to be among the most complex problems in mechanical engineering. The oscillation processes of common objects such as a pendulum and spring are well-described in the sections on machine dynamics and vibration analysis of classical mechanics. Oscillations can be categorized into underdamped, damped, and critically damped types, each with distinct properties.

- An underdamped oscillation maintains a constant amplitude over time, while a damped oscillation gradually reduces the amplitude of the oscillations over time. A critically damped system returns to its original position without oscillation. These properties apply to the oscillation of a gyroscope, with its unique characteristics. They can be compared to the oscillation of an extensible spring with a hung weight. When a weight is hung from a spring, it stretches to a certain length and generates a force equal to the weight in magnitude. The amplitude of the oscillation of the spring with the weight is determined by the action of the weight, the inertial force generated by its fall, and the resistance force of the stretched spring.

When the gyroscope of one side support turns, it is brought down to a defined angle where the resistance torques generated by the rotating mass of the disc are equal to the components of the gyroscope weight. The amplitude of the gyroscope's oscillation is determined by the weight of the gyroscope, the inertial torque generated by its fall, and the change in the resistance inertial torques generated by the rotating mass of the disc due to the turn of the gyroscope. In both cases, the potential energies of the spring and gyroscope weight, as well as the spring stretch and the gyroscope resistance inertial torques, are converted into the kinetic energy of their oscillations, and vice versa.

- The damped oscillations of the spring and gyroscope exhibit the same level of specificity. In the case of a critically damped situation, the weight causes a minor stretch in the spring without any oscillation. When an external torque acts on one side of the gyroscope, it turns to an angle without returning to its initial position and oscillating. The potential energy of the weight of the spring is added to its stretch energy. The potential energy of the gyroscope weight decreases, and the kinetic energy of its resistance inertial torque decreases due to the turning of the gyroscope around its axis of rotation at a fixed angle.
- The differences in potential and kinetic energies of an extensible spring with a weight and a gyroscope on one side support, when subjected to short-term external torque, show that the gyroscope oscillation follows the same rules as those accepted in machine dynamics and vibration analysis of classical

mechanics. Mathematical models for the gyroscope's motion around axes should take into account the peculiarities of the changes in its potential and kinetic energies during oscillation.

The constant kinetic energy of a gyroscope's rotation around its axis depends on the spinning speed of the disc and its angular velocity of precession. This can be observed in the motions of a gyroscope that is suspended by a flexible cord on one side. If the spinning speed of the disc increases, the gyroscope's angular velocity of precession decreases around its axis. This is because the inertial torques generated by the spinning disc have a larger value, and so they resist the action of the external torque. However, the difference in the values of these torques is small. When the rotor's angular velocity is low, the gyroscope's motions around its axes are faster. This happens because the value of the inertial torques is less, and the action of the external torque produced by the gyroscope's weight is stronger. Although the mechanical energy transformation of gyroscopes is not the same as for physical objects like pendulums or springs, there are some similarities in the action of the resistance torques of the gyroscope and the spring and their oscillations.

- The gyroscope rotor's high angular velocity and the spring's stiffness create the critical damping property in a gyroscope supported on one side, preventing any oscillation process. This is not the only similarity between the two dynamic objects. When an external load of minor value is applied for a short duration, the gyroscope supported on one side shifts by an angle γ around the ox axis (as shown in **Figure 1**) due to the torque difference between the gyroscope weight and the resistance torque. In this condition, the gyroscope does not rotate around the oy axis and has zero angular acceleration εy because all resistance and inertial torques are deactivated. The ratio between the angular velocities of the gyroscope along two axes is not maintained due to the inertia of the gyroscopic mass. This is because the angular velocity of the gyroscope around axis oy is about a hundred times greater than around axis ox. When the external load ceases to act, a resistance torque is activated around the axis ox. However, the gyroscope remains shifted at an angle γ and begins to rotate around the axis oy. The external load briefly stretches the spring, but it quickly returns to its initial state.

When the rotor's angular velocity decreases, the gyroscope with one side support starts oscillating. The weight of the gyroscope creates a torque that shifts it at an angle β around the axis *ox* (**Figure 1**). This movement is defined by the resistance inertial torque. The spring's minor stiffness allows it to stretch under the weight of the gyroscope, enabling oscillation. If an external load of minor value is applied for a short duration, the gyroscope shifts at an angle γ_1 around the axis *ox*. This action increases the value of its resistance and inertial torques. Please refer to **Figure 1** for further clarification. When the external load stops moving, a resistance torque is activated around the axis *ox*, causing the gyroscope to oscillate back around the same axis. The gyroscope had initially shifted to angle β. Even though the external load stretches the spring, it eventually returns to its original state, allowing the oscillation process to continue.

The functions of a spring or flexible component in the analysis of mechanical oscillation are carried out by the inertial torques generated by the centrifugal and Coriolis forces and the torque of the change in the angular momentum forces of the rotating masses of the gyroscope. The gyroscope oscillates with specific properties that are represented by the different characteristics of the inertial torques and the spring in mechanical oscillation.

The gyroscope oscillates and produces vibration through the constant action of torques. This phenomenon can be described analytically using the principles of mechanical energy conservation and the work of torques. However, for the sake of simplicity in analyzing the gyroscope oscillation, the effect of frictional torques at the spinning disc supports and the one-sided support of the gyroscope is not taken into consideration.

3. Results and discussion

The gyroscope oscillation is governed by classical mechanics. Recent publications have presented the inertial torques that act on the gyroscope and mathematical models that validate its motion through practical tests. The oscillation of the gyroscope is determined by the interrelated inertial torques that act on it. The rotating mass of the spinning object generates an inertial torque that expresses its kinetic energy and is involved in the oscillations. Machine dynamics and vibratory analysis describe oscillations of objects mainly in terms of underdamped, damped, and critically damped types, which are characterized by specific properties of the pendulum and spring.

The following are the key properties that define the fundamental principles of oscillations for a gyroscope with one-side support:

When the angular velocity of the rotor of the gyroscope is high, expressing its stiffness, it is considered critically damped. This is similar to a spring of high stiffness which is also critically damped.

When the gyroscope experiences an external torque of short-time action, it shifts to a defined angle. As a result, the gyroscope's potential energy and precession velocity decrease without oscillation. However, the critically damped spring does not shift under the action of an external load, as it has one end fixed.

The oscillation properties of a gyroscope are more complex compared to those of a spring with weight. Gyroscope oscillation involves the kinetic energy of the spinning rotor and the weight of the gyroscope, which determine its stiffness. In contrast, the oscillation of a spring with weight is determined by the spring's stiffness. When the spring oscillates, its kinetic energy is converted into potential energy due to spring extension. Similarly, in a gyroscope, its kinetic energy is converted into potential energy of the resistance inertial torques. A comparative analysis of their properties shows similarities in their oscillations. The key properties defining the fundamental principles of oscillations for a gyroscope with one-side support are as follows: When the angular velocity of the gyroscope's rotor is high, indicating its stiffness, it is considered critically damped, similar to a spring of high stiffness, which is also critically damped.

When an external torque is applied to the gyroscope for a short time, it moves to a specific angle. This causes a decrease in the potential energy and precession velocity of the gyroscope without any oscillation. On the other hand, a critically damped spring does not move when an external load is applied, as one end of the spring is fixed. The potential energy from the external torque is converted into an increased resistance to the inertial torques generated by the spinning disc of the gyroscope. The external load compresses the spring and increases its resistance force.

The increased resistance inertial torques play a role in the potential energy of the gyroscope, which is converted into kinetic energy of its motions. This property is also possessed by the spring. When the gyroscope is in a horizontal position and has low angular velocity, the spinning disc is shifted to an angle where its resistance torque equals the torque produced by the component of its weight. At the same time, a highly flexible spring is stretched under the influence of an external load. The kinetic energy of the resistance inertial torques depends on the angular velocities of the spinning rotor and the precession velocity around the axis of rotation, which are determined by its stiffness formulated by the principle of mechanical energy conservation. This property is also demonstrated in the spring.

The analysis of gyroscope oscillations enables their description through the principles of machine dynamics and vibration analysis of classical mechanics. Understanding engineering allows for solvable problems related to the oscillation processes of gyroscopic devices.

4. Conclusion

The textbooks on engineering mechanics provide a detailed analysis of the dynamic machine and vibratory properties of a pendulum and spring oscillations. Although these properties cannot be directly applied to gyroscope oscillation due to different physical principles, there are some similarities. For a long time, gyroscope oscillation lacked mathematical models due to this reason. Classical mechanics establishes the fundamental principle of object oscillation based on the conservation of mechanical energy. The conversion of potential energy to kinetic energy and vice versa can be applied to gyroscope oscillation. The main challenge lies in defining the forms of energy conversion and their properties. A physical analysis of the nature of the inertial torques generated by the rotating mass of spinning objects enables the definition of the fundamental principles of gyroscope oscillations and their properties. A new theory for gyroscopic effects can solve this problem and derive mathematical models for gyroscope oscillations.

Acknowledgments: The Kyrgyz State Technical University after I. Razzakov supported the research work for a publication that was performed as part of the employment and without financial support. The author did not use copyediting or translation services for the preparation of the manuscript.

Data availability: The author declares that the data supporting the findings of this study are available within the article.

Conflict of interest: The author declares no conflict of interest.

References

1. Bennett GG. An historical review of the development of the gyroscope. Australian Surveyor. 1970; 23(4): 244-252. doi: 10.1080/00050326.1970.10440242
2. Iurato G. On the Historical Evolution of Gyroscopic Instrumentation: A Very Brief Account. HAL Open Science; 2015.

3. Greenhill G. Report on Gyroscopic Theory. Palala Press; 2015.
4. Deimel RF. Mechanics of the Gyroscope. Dover Publications Inc; 2003.
5. Awrejcewicz J, Koruba Z. Theory of Gyroscopes. In: Classical Mechanics. Springer New York; 2012. doi: 10.1007/978-1-4614-3978-3
6. Turner G. Gyroscopes, 1st ed. Brightfusion Ltd.; 2007.
7. Klein F, Sommerfeld A. The Theory of the Top (Volume IV). Springer New York; 2014. doi: 10.1007/978-0-8176-4829-9
8. Hibbeler RC. Engineering Mechanics—Statics & Dynamics, 12th ed. Pearson Prentice Hall; 2010.
9. Germain P, Piau M, Caillerie D. Theoretical and Applied Mechanics. Elsevier; 2012.
10. Gregory DR. Classical Mechanics. Cambridge University Press; 2006.
11. Taylor JR. Classical Mechanics. In: University Science Books. University Science Books; 2005.
12. Aardema MD. Analytical Dynamics. In: Theory and Application. Academic/Plenum Publishers; 2005.
13. Passaro VMN, Cuccovillo A, Vaiani L, et al. Gyroscope Technology and Applications: A Review in the Industrial Perspective. Sensors. 2017; 17(10): 2284. doi: 10.3390/s17102284
14. Armenise MN, Ciminelli C, Dell'Olio F, et al. Advances in Gyroscope Technologies. Springer Berlin Heidelberg; 2011. doi: 10.1007/978-3-642-15494-2
15. Scarborough JB. The Gyroscope Theory and Applications. Nabu Press; 2011.
16. Wie B. Space Vehicle Dynamics and Control. American Institute of Aeronautics and Astronautics; 2005.
17. Sands T. Nonlinear-Adaptive Mathematical System Identification Algorithms. Computation. 2017; 5(4): 47. doi: 10.3390/computation5040047
18. Damrongsak B, Kraft M, Rajgopal S, et al. Design and fabrication of a micromachined electrostatically suspended gyroscope. Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science. 2008; 222(1): 53-63. doi: 10.1243/09544062jmes665
19. Jewett J, Serway RA. Physics for Scientists and Engineers, 10th ed. Cengage Learning; 2018.
20. Knight RD. Physics for Scientists and Engineers: A Strategic Approach with Modern Physics, 4th ed. Pearson; 2016.
21. Usubamatov R. Theory of Gyroscopic Effects for Rotating Objects. Springer International Publishing; 2022. doi: 10.1007/978-3-030-99213-2