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Article

Conformal theory of central surface density for galactic dark halos

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Abstract: Numerous dark matter studies of galactic halo gravitation depend on models with a core radius of r_0 and a central density of ρ_0. The central surface density product $\rho_0 r_0$ is found to be nearly a universal constant for a large range of galaxies. Standard variational field theory with Weyl conformal symmetry postulated for gravitation and the Higgs scalar field, without dark matter, implies nonclassical centripetal acceleration Δa, for $a = a_N + \Delta a$, where Newtonian acceleration a_N is due to observable baryonic matter. Neglecting a halo cutoff at a very large galactic radius, conformal Δa is constant over the entire halo, and $a = a_N + \Delta a$ is a universal function, consistent with a recent study of galaxies with independently measured mass, that constrains acceleration due to dark matter or to an alternative theory. An equivalent dark matter source would be a pure cusp distribution with a cutoff parameter determined by a halo boundary radius. This is shown to imply a universal central surface density for any dark matter core model.

Keywords: galactic dark halos; conformal theory; dark matter
PACS/MSC/JEL CLASSIFICATION: 04.20.Cv; 98.80.-k; 98.62.Gq

1. Introduction

Observed deviations from standard Newton/Einstein galactic gravitation have been modeled by distributed but unobserved dark matter (DM). Typical DM halo models imply centripetal radial acceleration $a = a_N + \Delta a$ as a function of radius in an assumed spherical galactic halo. DM Δa is added to the baryonic Newtonian a_N.

DM fits galactic rotation (orbital velocity vs. circular orbit radius) depending on the model parameters central density ρ_0 and core radius r_0 for a DM halo distribution. Observed data imply that the surface density product $\rho_0 r_0 \simeq 100 M_\odot pc^{-2}$ is nearly a universal constant for a large range of galaxies [1–3].

Assuming Weyl conformal symmetry for field action integrals gives an alternative explanation of observed Δa. Conformal gravity (CG) [4–8] and the conformal Higgs model (CHM) [9–13] introduce new gravitational terms in the field equations. The current updated conformal theory has recently been reviewed [14]. The nonclassical Δa of spherically averaged CG and the CHM replace the galactic radial acceleration attributed to dark matter.

A recent study of the rotational velocities of galaxies with independently measured galactic mass found total radial acceleration a to be a universal function of Newtonian acceleration a_N [15]. This constrains acceleration attributed to dark matter or alternative theory [16], requiring Δa to be a universal constant.

Conformal $v^2/c^2 = ra/c^2 = \beta/r + \frac{1}{2}\gamma r - \kappa r^2$ implies $\Delta a = \frac{1}{2}c^2\gamma - c^2\kappa r$, with constants defined by CG [5]. Values are fitted to the observed galactic rotation [4,14]. Neglecting halo cutoff $2\kappa r/\gamma$ for $r \ll r_H$ (halo radius), the CG acceleration constant γ predicts nonclassical acceleration Δa. $\gamma \simeq 6.35 \times 10^{-28}/m$ implies $\Delta a =$

$\frac{1}{2}\gamma c^2 \simeq 0.285 \times 10^{-10} m/s^2$ [12,13,16] for all CDM core models.

The uniform constant Δa puts a severe constraint on any DM model. The source density must be of the form ξ/r, a pure radial cusp [13,16], where constant $\xi = \Delta a/2\pi G = 0.06797 kg/m^2 = 32.535 M_\odot/pc^2$. CODATA Newton constant $G = 6.67384 \times 10^{-11} m^3 s^{-28} kg^{-1}$ [17]. The conflict between cusp and core DM models [18], may rule out DM for galactic rotation. Alternatives to CHM for Hubble expansion, such as introducing ad hoc curvature [4] or a cosmological constant, are not considered here.

2. Implied DM parameters

A DM galactic model equivalent to conformal theory would imply uniform DM radial acceleration $\Delta a = 2\pi G\xi$, attributed to radial DM density ξ/r for universal constant ξ, modified at large galactic radius by a halo cutoff function. Enclosed mass $M_r = 2\pi\xi r^2$ implies $r\Delta a/c^2 = GM_r/r$. DM models avoid a distribution cusp by assuming a finite central core density. A recent fit to the Milky Way rotation uses a DM core with a decreasing exponential cutoff [19]. For arbitrary r_0, asymptotic radial acceleration is unchanged if mass within r_0 is redistributed to uniform density $\rho(r)$ within a sphere of this radius. Conformal density ξ/r implies mass $M_0 = 2\pi\xi r_0^2$ in volume $V_0 = \frac{4\pi}{3} r_0^3$. For a DM spherical model core that replaces a central cusp density, conformal theory implies constant $\rho(r_0)r_0 = r_0 M_0/V_0 = 3\xi/2$. For assumed PI core DM density [1] $\rho(r) = \rho_0 r_0^2/(r^2 + r_0^2)$, central $\rho_0 = 2\rho(r_0)$. Hence, for a PI core, $\rho_0 r_0 = 3\xi = \frac{3\Delta a}{2\pi G} = 0.204 kg/m^2 = 97.6 M_\odot pc^{-2}$, independent of r_0. This value is proportional to $\rho_0/\rho(r_0)$ for other core models. DM studies indicate a mean value $141 M_\odot pc^{-2}$ [3]. MOND [20], assuming $a^2 \to a_N a_0$ as $a_N \to 0$, without dark matter, implies $\rho_0 r_0 \simeq 130 M_\odot pc^{-2}$ [21].

3. Conformal theory of Δa

For a central gravitational source with spherical symmetry, in the Schwarzschild metric, conformal gravity has an exact solution of radial Schwarzschild potential $B(r)$ [5–7]. Outside a source of finite radius [5],

$$B(r) = -2\beta/r + \alpha + \gamma r - \kappa r^2 \quad (1)$$

for constants related by $\alpha^2 = 1 - 6\beta\gamma$[7]. $B(r)$ determines circular geodesics with orbital velocity υ such that $\upsilon^2/c^2 = ra/c^2 = \frac{1}{2} rB'(r) = \beta/r + \frac{1}{2}\gamma r - \kappa r^2$. The Kepler formula is $ra_N/c^2 = \beta/r$, from a 2nd-order equation. The 4th order conformal equation introduces two additional constants of motion, radial acceleration γ and cutoff parameter κ. The parameter κ, unique to conformal theory, relates galactic baryonic mass to large radius r_H of a galactic halo, whose cosmic mass has been deleted by falling into the central galaxy [12]. Classical gravitation is retained at subgalactic distances by setting $\beta = GM/c^2$ for a spherical source of baryonic mass M [4]. For $r \ll r_H$, $\frac{\kappa r}{\gamma}$ can be neglected, so that $\Delta a = \frac{1}{2}\gamma c^2$.

The CHM [9,11,13,16] determines γ as a universal constant, independent of galactic mass. The Higgs scalar field acquires a gravitational term that implies a modified Friedmann equation for the cosmic scale factor $s(t)$ [9]. This implies dimensionless cosmic centrifugal acceleration $\Omega_q = \frac{s\ddot{s}}{\dot{s}^2}$. The Friedmann equation determines the observable radial acceleration parameter γ for massive objects within r_H [13]. Assuming an empty halo, due to the gravitational concentration of all mass inside the halo radius r_H to within galactic radius r_G, γ is determined by requiring continuous acceleration across r_H. Constant γ has a universal value throughout a depleted halo, proportional to uniform cosmic mass-energy density ρ_m [12]. Equating constants for the baryonic Tully-Fisher relationship and neglecting cutoff κ, constant $\Delta a = \frac{1}{2}\gamma c^2 = \frac{1}{4}a_0$, for MOND a_0 [12,13]. Determined by CG from observed galactic rotation [4,13], parameter $\gamma = 6.35 \times 10^{-28}/m$, using data for the Milky Way galaxy [22,23]. Hence, neglecting halo cutoff, $\Delta a = 0.285 \times 10^{-10} m/s^2$ and MOND $a_0 = 4\Delta a = 1.14 \times 10^{-10} m/s^2$.

4. Conclusions

Conformal theory, consistent with the finding [15] for galaxies of known mass that observed radial acceleration a is a universal function of baryonic a_N, explains the observed constancy of halo central surface density deduced from DM core models. Any DM core model can be considered an approximation to implied conformal Δa. Nonclassical Δa predicted by conformal theory would require a pure cusp mass-energy source plus halo cutoff, which may rule out an exact DM model. This requires reconsideration of the consensus LCDM paradigm.

Conflict of interest: The author declares no conflict of interest.

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