Conformal theory of central surface density for galactic dark halos

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Abstract: Numerous dark matter studies of galactic halo gravitation depend on models with core radius $r_0$ and central density $\rho_0$. Central surface density product $\rho_0 r_0$ is found to be nearly a universal constant for a large range of galaxies. Standard variational field theory with Weyl conformal symmetry postulated for gravitation and the Higgs scalar field, without dark matter, implies nonclassical centripetal acceleration $\Delta a$, for $a = a_N + \Delta a$, where Newtonian acceleration $a_N$ is due to observable baryonic matter. Neglecting a halo cutoff at very large galactic radius, conformal $\Delta a$ is constant over the entire halo and $a = a_N + \Delta a$ is a universal function, consistent with a recent study of galaxies, with independently measured mass, that constrains acceleration due to dark matter or to alternative theory. An equivalent dark matter source would be a pure cusp distribution with cutoff parameter determined by a halo boundary radius. This is shown to imply universal central surface density for any dark matter core model.

Keywords: galactic dark halos; conformal theory; dark matter

1. Introduction

Observed deviations from standard Newton/Einstein galactic gravitation have been modeled by distributed but unobserved dark matter (DM). Typical DM halo models imply centripetal radial acceleration $a = a_N + \Delta a$, a function of radius in an assumed spherical galactic halo. DM $\Delta a$ is added to baryonic Newtonian $a_N$.

DM fits to galactic rotation (orbital velocity vs. circular orbit radius) depend on model parameters central density $\rho_0$ and core radius $r_0$ for a DM halo distribution. Observed data imply that surface density product $\rho_0 r_0 \simeq 100 M_\odot pc^{-2}$ is nearly a universal constant for a large range of galaxies [1–3].

Assuming Weyl conformal symmetry for field action integrals gives an alternative explanation of observed $\Delta a$. Conformal gravity(CG) [4–8] and the conformal Higgs model(CHM) [9–13] introduce new gravitational terms in the field equations. Current updated conformal theory has recently been reviewed [14]. Nonclassical $\Delta a$ of spherically averaged CG and the CHM replaces the galactic radial acceleration attributed to dark matter.

A recent study of rotational velocities of galaxies with independently measured galactic mass finds total radial acceleration $a$ to be a universal function of Newtonian acceleration $a_N$ [15]. This constrains acceleration attributed to dark matter or to alternative theory [16], requiring $\Delta a$ to be a universal constant.

Conformal $\nu^2/c^2 = \nu a/c^2 = \beta/r + \frac{1}{2} \gamma r - \kappa r^2$ implies $\Delta a = \frac{1}{2} c^2 \gamma - c^2 \kappa r$, with constants defined by CG [5]. Values are fitted to observed galactic rotation [4,14]. Neglecting halo cutoff $2 \kappa r/\gamma$ for $r \ll r_H$ (halo radius), CG acceleration constant $\gamma$ predicts nonclassical acceleration $\Delta a$. $\gamma \simeq 6.35 \times 10^{-28}/m$ implies $\Delta a = \frac{1}{2} c^2 \gamma^2 \simeq 0.285 \times 10^{-10} m/s^2$ [12,13,16] for all CDM
core models.

Uniform constant $\Delta a$ puts a severe constraint on any DM model. The source density must be of the form $\xi/r$, a pure radial cusp [13, 16], where constant $\xi = \Delta a/2 \pi G = 0.06797 \text{kg/m}^2 = 32.535 M_\odot / pc^2$. CODATA Newton constant $G = 6.67384 \times 10^{-11} \text{m}^3 \text{s}^{-2} \text{kg}^{-1}$ [17]. The conflict between cusp and core DM models [18], may rule out DM for galactic rotation. Alternatives to CHM for Hubble expansion, introducing ad hoc curvature [4] or a cosmological constant, are not considered here.

2. Implied DM parameters

A DM galactic model equivalent to conformal theory would imply uniform DM radial acceleration $\Delta a = 2 \pi G \xi$, attributed to radial DM density $\xi/r$ for universal constant $\xi$, modified at large galactic radius by a halo cutoff function. Enclosed mass $M_r = 2 \pi \xi r^2$ implies $r \Delta a / c^2 = GM_r / r$. DM models avoid a distribution cusp by assuming finite central core density. A recent fit to Milky Way rotation uses a DM core with decreasing exponential cutoff $a$. For arbitrary $r_0$, asymptotic radial acceleration is unchanged if mass within $r_0$ is redistributed to uniform density $\rho(r)$ within a sphere of this radius. Conformal density $\xi/r$ implies mass $M_0 = 2 \pi \xi r_0^2$ in volume $V_0 = \frac{4}{3} \pi r_0^3$. For a DM spherical model core that replaces a central cusp density, conformal theory implies constant $\rho(r_0)r_0 = r_0 M_0 / V_0 = 3 \xi / 2$. For assumed Pl core DM density [1] $\rho(r) = \rho_0 \frac{r_0^2}{(r^2 + r_0^2)}$, central $\rho_0 = 2 \rho(r_0)$. Hence for a Pl core, $\rho_0 r_0 = 3 \xi / 2 G = 0.204 \text{kg/m}^2 = 97.6 M_\odot pc^{-2}$, independent of $r_0$. This value is proportional to $\rho_0 / \rho(r_0)$ for other core models. DM studies indicate mean value $\Delta 141 M_\odot pc^{-2}$ [3], MOND [20], assuming $a^2 \to a_N a_0$ as $a_N \to 0$, without dark matter, implies $\rho_0 r_0 \simeq 130 M_\odot pc^{-2}$ [21].

3. Conformal theory of $\Delta a$

For a central gravitational source with spherical symmetry, in the Schwarzschild metric, conformal gravity has an exact solution of radial Schwarzschild potential $B(r)$ [5–7]. Outside a source of finite radius [5],

$$B(r) = -2 \beta / r + \alpha + \gamma r - \kappa r^2,$$

for constants related by $\alpha^2 = 1 - 6 \beta \gamma$ [7]. $B(r)$ determines circular geodesics with orbital velocity $v$ such that $v^2 / c^2 = r \alpha / c^2 = \frac{1}{2} r B'(r) = \beta / r + \frac{1}{2} \gamma r - \kappa r^2$. The Kepler formula is $r \alpha N / c^2 = \beta / r$, from a 2nd order equation. The 4th order conformal equation introduces two additional constants of motion, radial acceleration $\gamma$ and cutoff parameter $\kappa$. Parameter $\kappa$, unique to conformal theory, relates galactic baryonic mass to large radius $r_H$ of a galactic halo, whose cosmic mass has been deleted by falling into the central galaxy [12]. Classical gravitation is retained at subgalactic distances by setting $\beta = GM / c^2$ for a spherical source of baryonic mass $M$ [4]. For $r \ll r_H$, $\frac{\kappa}{c^2}$ can be neglected, so that $\Delta a = \frac{1}{2} \gamma c^2$.

The CHM [9, 11, 13, 16] determines $\gamma$ as a universal constant, independent of galactic mass. The Higgs scalar field acquires a gravitational term that implies a modified Friedmann equation for cosmic scale factor $s(t)$ [9]. This implies dimensionless cosmic centrifugal acceleration $\Omega_q = \frac{a^2}{r^2}$. The Friedmann equation determines observable radial acceleration parameter $\gamma$ for massive objects within $r_H$ [13]. Assuming an empty halo, due to gravitational concentration of all mass inside halo radius $r_H$ to within galactic radius $r_G$, $\gamma$ is determined by requiring continuous acceleration across $r_H$. Constant $\gamma$ has a universal value throughout a depleted halo, proportional to uniform cosmic mass-energy density $\rho_m$ [12]. Equating constants for the baryonic Tully-Fisher relationship and neglecting cutoff $\kappa$, constant $\Delta a = \frac{\gamma}{2} c^2 = \frac{1}{2} a_0$,
for MOND $a_0$ [12,13]. Determined by CG from observed galactic rotation [4,13], parameter $\gamma = 6.35 \times 10^{-28}/m$, using data for the Milky Way galaxy [22,23]. Hence, neglecting halo cutoff, $\Delta a = 0.285 \times 10^{-10}m/s^2$ and MOND $a_0 = 4\Delta a = 1.14 \times 10^{-10}m/s^2$.

4. Conclusions

Conformal theory, consistent with the finding [15] for galaxies of known mass that observed radial acceleration $a$ is a universal function of baryonic $a_N$, explains the observed constancy of halo central surface density deduced from DM core models. Any DM core model can be considered an approximation to implied conformal $\Delta a$. Nonclassical $\Delta a$ predicted by conformal theory would require a pure cusp mass-energy source plus halo cutoff, which may rule out an exact DM model. This requires reconsideration of the consensus LCDM paradigm.

Conflict of interest: The author declares no conflict of interest.

References