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Article

Relativistic light clocks: Arbitrary orientation in uniform motion and hyperbolic motion analysis

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Abstract: In this paper, we address the general case of a light clock in uniform translational motion parallel to itself and perpendicular to its uniform velocity *v*, as well as the case of the light clock in relativistic hyperbolic motion. Neither case has been previously addressed in the specialized literature, which typically restricts itself to canonical orientations where the light clock moves parallel to either the vertical or horizontal axis with uniform velocity, without acceleration. Therefore, it becomes interesting to study the more general case where the clock has an arbitrary orientation and/or is accelerated. Our paper is divided into two main sections. The first section deals with the light clock moving with constant velocity, oriented at an arbitrary angle with respect to the x-axis. We prove that the moving clock exhibits a standard time dilation, identical to that of a light clock moving in a canonical orientation. The second section deals with the light clock moving with constant acceleration, i.e., in hyperbolic motion. For the light clock in hyperbolic motion, we derive the period as measured from the perspective of an inertial frame and draw parallels with the case of uniform motion, outlining a term that is similar (but not identical) to the γ factor of uniform motion. We also point out that this factor depends not only on acceleration but also on the height of the light clock. This dependency on the dimension of the light clock distinguishes the accelerated case from the case of uniform motion. The first three sections deal with the theoretical aspects of light (optical) clocks, while the fourth section addresses the experimental implementations of optical clocks.

Keywords: light clock; relativistic motion; time dilation; hyperbolic motion; uniform velocity
PACS Classification: 03.30.+p

1. Introduction

A considerable amount of literature has been dedicated to the relativistic light clock in uniform translational motion parallel to itself and perpendicular to the direction of its uniform velocity *v* (see **Figure 1**) [1]. The existing literature extensively demonstrates that the canonical motion of the light clock results in standard time dilation when observed from an inertial frame with respect to which the clock is moving. Much less literature has been devoted to the case of the clock moving in the direction of its uniform velocity *v* (the “Langevin light clock”) [2], which shows the same amount of time dilation. No literature exists for the case of the light clock moving at an arbitrary angle with respect to its uniform velocity *v*. In this paper, we address this situation and demonstrate the important fact that time dilation is independent of the clock’s orientation.

For this purpose, we consider two inertial frames of reference: frame *F* co-moving with the clock and frame *F’*, with respect to which the clock is moving with velocity *v*. All calculations are performed from the perspective of frame *F’*. This outcome is expected since time dilation depends on speed, not on velocity. However, as we will see, the proof is by no means trivial since the transition times for light in each direction ("up" and "down") depend on the orientation of the light clock.

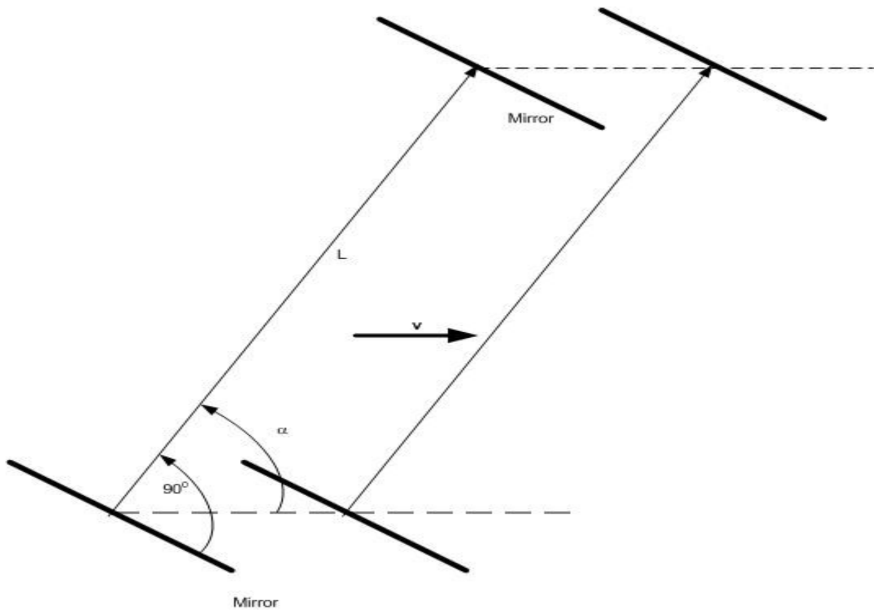

Figure 1. Light clock in translation motion.

Later in the paper, we will also address the case of the light clock in relativistic hyperbolic motion. In this case, all calculations are performed from the perspective of the inertial frame *F'*, with respect to which the clock is moving with acceleration *g* oriented along the clock axis. We will derive a formula for time dilation that is similar, but not identical, to the time dilation for the case of uniform motion.

2. Inclined light clock in uniform motion

In this section, we analyze the period of a light clock in translation motion (parallel to itself) inclined at an angle α with respect to the x-axis. The motion is uniform with velocity v. The distance between the mirrors of the light clock is L. Let the frame commoving with the light clock be denoted as *F*, and the frame with respect to which the light clock is moving be denoted as *F'*. All the subsequent calculations are done from the perspective of the inertial frame *F'*.

The components of the "upward" moving light beam as measured in the "stationary" frame *F'* are (see **Figure 2**):

$$c'_x = c\frac{\cos\alpha + \beta}{1 + \beta\cos\alpha}$$
$$c'_y = c\frac{\sin\alpha}{1 + \beta\cos\alpha}\sqrt{1-\beta^2} \tag{1}$$
$$\beta = \frac{v}{c}$$

We can verify that the "upward" light speed as measured in frame *F'* is constant and frame invariant:

$$c' = \sqrt{c'^2_x + c'^2_y} = c \tag{2}$$

Armed with the above we can calculate the aberration angle of the "upward" beam of light:

$$\cos\theta' = \frac{c'_x}{c} = \frac{\cos\alpha + \beta}{1 + \beta\cos\alpha} \tag{3}$$

The “upward” light path length in frame *F’* is given by (see **Figure 2**):

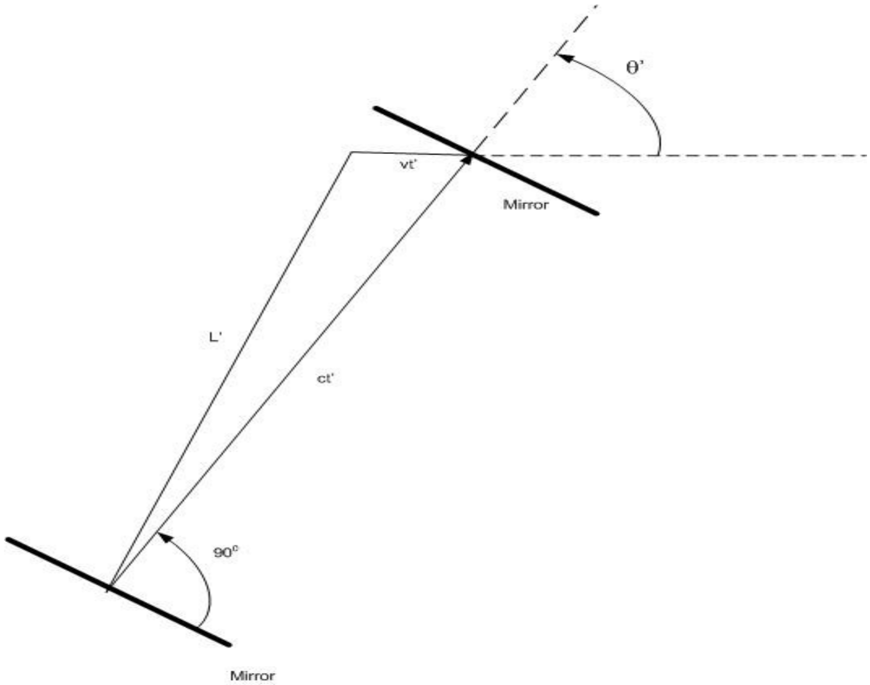

Figure 2. Upward light beam path.

$$L'^2 = (vt')^2 + (c't')^2 - 2c'vt'^2\cos\theta' \tag{4}$$

where [3]:

$$L'^2 = (\frac{L}{\gamma}\cos\alpha)^2 + (L\sin\alpha)^2 = L^2(1 - \beta^2\cos^2\alpha) \tag{5}$$

From the above, we obtain the elapsed time necessary for the light front to move from the bottom mirror to the top mirror:

$$t' = \frac{L\gamma}{c}(1 + \beta\cos\alpha) \tag{6}$$

The light speed components of the “downward” moving light beam as measured in *F’* are:

$$\begin{aligned} c''_x &= c\frac{-\cos\alpha + \beta}{1 - \beta\cos\alpha} \\ c''_y &= -c\frac{\sin\alpha}{1 - \beta\cos\alpha}\sqrt{1 - \beta^2} \end{aligned} \tag{7}$$

Once again, we can verify that the “downward” light speed as measured in frame *F’* is constant and frame invariant:

$$c'' = \sqrt{c''^2_x + c''^2_y} = c \tag{8}$$

Armed with the above we can calculate the aberration angle of the “downward” beam of light:

$$\cos\theta'' = \frac{c''_x}{c} = \frac{-\cos\alpha + \beta}{1 - \beta\cos\alpha} \tag{9}$$

The “downwards” light path length in frame *F’* is given by (see **Figure 3**):

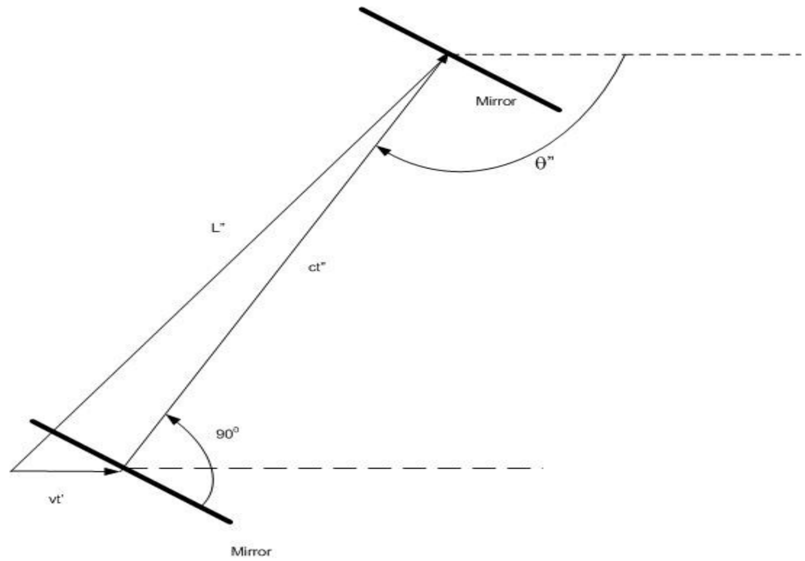

Figure 3. "Downward" light beam path.

$$L''^2 = L'^2 = (vt'')^2 + (c''t'')^2 - 2c''vt''^2 \cos\theta'' \tag{10}$$

From the above, we obtain the elapsed time necessary for the light front to move from the top mirror to the bottom mirror:

$$t'' = \frac{L\gamma}{c}(1 - \beta\cos\alpha) \tag{11}$$

The inclined light clock period as measured in frame *F'* is the sum of the transition times of the light beam from the bottom mirror to the top mirror and from the top mirror back to the bottom mirror:

$$t' + t'' = \frac{2L}{c}\gamma \tag{12}$$

On the other hand, the light clock period as measured in frame *F* is $\frac{2L}{c}$. Therefore, Equation (12) recovers the standard time dilation formula, that is, the dependence of the time dilation on the γ factor.

To recap, here are all the details of the transition times, the aberration angles, and the components of light velocity as viewed from the inertial frame *F'* in the "upward" and "downward" directions (**Table 1**), respectively:

Table 1. Details of clock transition times.

Upward light beam	**Downward light beam**
$c'_x = c\frac{\cos\alpha + \beta}{1 + \beta\cos\alpha}$	$c''_x = c\frac{-\cos\alpha + \beta}{1 - \beta\cos\alpha}$
$c'_y = c\frac{\sin\alpha}{1 + \beta\cos\alpha}\sqrt{1 - \beta^2}$	$c''_y = -c\frac{\sin\alpha}{1 - \beta\cos\alpha}\sqrt{1 - \beta^2}$
$c' = c$	$c'' = c$
$\cos\theta' = \frac{c'_x}{c} = \frac{\cos\alpha + \beta}{1 + \beta\cos\alpha}$	$\cos\theta'' = \frac{c''_x}{c} = \frac{-\cos\alpha + \beta}{1 - \beta\cos\alpha}$
$L'^2 = L^2(1 - \beta^2\cos^2\alpha)$	$L''^2 = L^2(1 - \beta^2\cos^2\alpha)$
$t' = \frac{L\gamma}{c}(1 + \beta\cos\alpha)$	$t'' = \frac{L\gamma}{c}(1 - \beta\cos\alpha)$
$t' + t'' = \frac{2L}{c}\gamma$	

A quick sanity check shows that for $\alpha = \frac{\pi}{2}$ we recover the standard case of the light clock:

$$t' = t'' = \frac{L\gamma}{c} \tag{13}$$

and for $\alpha = 0$ we obtain Langevin's "horizontal light clock" [2]:

$$\begin{aligned} t' &= \frac{L\gamma}{c}(1+\beta) \\ t'' &= \frac{L\gamma}{c}(1-\beta) \end{aligned} \tag{14}$$

The above is the same as the equations derived for the "horizontal" light clock. Indeed, for the light beam "chasing" the mirror:

$$\begin{aligned} \frac{L}{\gamma} + vt' &= ct' \\ t' = \frac{L}{c\gamma}\frac{1}{1-\frac{v}{c}} &= \frac{L\gamma}{c}(1+\beta) \end{aligned} \tag{15}$$

For the light beam encountering the mirror, the equations are:

$$\begin{aligned} \frac{L}{\gamma} &= vt'' + ct'' \\ t'' = \frac{L}{c\gamma}\frac{1}{1+\frac{v}{c}} &= \frac{L\gamma}{c}(1-\beta) \end{aligned} \tag{16}$$

We can see that while both transition times t' and t'' depend on the angle of the light clock, its period does not. We have obtained a formal proof of time dilation isotropy.

3. The uniformly accelerated light clock (hyperbolic treatment)

When tidal effects (the variation of the acceleration g with the radial Schwarzschild coordinate) are negligible [4–38], then physics in the presence of a gravitational field is approximately equivalent to the physics in a uniformly accelerated coordinate system using the formalism of hyperbolic motion. At the time an electromagnetic pulse is emitted from the bottom of a uniformly accelerated light clock, the top of the clock is accelerated away from the direction of the light front with the uniform acceleration *g*. In this section, we denote the distance between the mirrors with *L* to maintain physical and mathematical coherence with the previous section. The light front encounters the top of the light clock after the time *t* given by the equation:

$$\begin{aligned} L + \frac{c^2}{g}\left(\sqrt{1+(\frac{gt_1'}{c})^2} - 1\right) &= ct_1' \\ t_1' &= \frac{L}{c}\frac{1-\frac{1}{2}\frac{gL}{c^2}}{1-\frac{gL}{c^2}} \end{aligned} \tag{17}$$

All variables in the above are measured from the perspective of the inertial system of coordinates with respect to which the light clock is being uniformly accelerated. In the downward motion, the equation changes to a more complicated form due to the fact that the "bottom" mirror had already been accelerating for a time t_1' before the light beam started its downward path:

$$L=\frac{c^2}{g}\left[\sqrt{1+(\frac{g\left(t_1^{''}+t_1^{'}\right)}{c})^2}-\sqrt{1+\frac{g^2t_1^{'2}}{c^2}}\right]+ct_1^{''} \tag{18}$$

$$t_1^{''}=f(g,L)$$

In Equation (18), the time has been explicated as a function of acceleration g and of the distance between the mirrors L.

The light clock period is the sum of the "up" and "down" transition times:

$$T_1=t_1'+t_1'' \tag{19}$$

It is interesting to note that the period is dependent on both the acceleration of the light clock g and the distance between mirrors, L. The dependency on the dimension of the light clock sets the accelerated case apart from the case of uniform motion. We need to remember that the above calculation is done from the perspective of an inertial frame with respect to which the light clock moves at constant acceleration g. We remember that in another paper [9], we have shown that from the perspective of the instantaneously commoving frame, the light clock proper period is $\tau=2L/c$, independent of acceleration. In calculating the next period of the light clock, we need to take into account that the "top" mirror had been accelerating for a time T_1 before the light beam made its second upward trip:

$$L+\frac{c^2}{g}(\sqrt{1+(\frac{g(t_2'+T_1)}{c})^2}-\sqrt{1+\frac{gT_1^2}{c^2}})=ct_2' \tag{20}$$

As we can see, the light clock period keeps changing due to the fact that the "upwards" moving light beam time travel increases while the "downwards" moving light beam time travel keeps decreasing. The increase and the decrease are highly non-linear, leading to an ever more complicated formula for the period. A very good illustration of the time dilation in the light clock due to motion (either uniform or accelerated) is the so-called twin "paradox" [39–58].

4. Experiment vs. theory of optical clocks

The optical clock described in the previous sections is clearly an idealized description, totally impractical in terms of accelerating at the relativistic speeds where the relativistic time dilation can be observed. The mirror arrangement is way too fragile for practical experimentation. Nevertheless, on a practical level, the idea of using optical clocks dates back to the 1960s, when the idea of trapping atoms in an optical lattice using lasers was proposed by Russian physicist Vladilen Letokhov. The development of the first optical clock was started at NIST in 2000 and finished in 2006. These experimental setups had to take into consideration the effect of light and atom interaction and spin-orbit coupling [59–61]. In 2013, optical lattice clocks (OLCs) were shown to be as good as or better than caesium fountain clocks [62]. Two optical lattice clocks containing about 10,000 atoms of strontium-87 were synchronized with each other with a precision of at least 1.5×10^{-16}. There are two reasons for the possibly better precision: firstly, the frequency is measured using light, which has a much higher frequency than microwaves, and secondly, by using many atoms, any errors are averaged [63]. In 2018, JILA reported a 3D quantum gas clock that reached a residual frequency precision of 2.5×10^{-19} over 6 h. Recently, it has been proved that the quantum entanglement can help to further enhance the clock stability [64]. In 2020, optical clocks were researched for space applications like future generations of global navigation satellite systems (GNSSs) as replacements for microwave-based clocks. In fact, the theory of clocks in motion described in the previous sections finds direct application in the implementation of global positioning systems like GPS and GNSS in terms of frequency compensation at the launch of the onboard satellite clocks [26]. Currently, optical clocks are still primarily research projects, less mature than rubidium and cesium microwave standards. As the optical experimental clocks surpass the precision and stability of their microwave counterparts, this puts them in a position to replace the current standard for the time, the cesium fountain clock.

5. Conclusions and future work

We have derived the periods for a light clock in translation motion with an arbitrary angle and for a clock being uniformly accelerated along its axis. In both cases, the calculations are done from the perspective of an inertial frame F'.

In the current paper, we treated the general case of a light clock as seen in uniform translation motion parallel to itself perpendicular to the direction of its uniform velocity v as well as the case of the light clock in relativistic hyperbolic motion. Neither case has been treated previously in the specialty literature. The existent literature restricts itself to canonical orientation. The light clock moves parallel with either the vertical or the horizontal axis with uniform velocity; no acceleration is present. Therefore, it became interesting to study the more general case, whereby the clock has an arbitrary orientation and/or is accelerated. We proved that the moving clock exhibits the standard time dilation, identical to the light clock moving in canonical orientation. In the case of the light clock undergoing uniform acceleration. That is, for a light clock in hyperbolic motion, we have derived the period as measured from the perspective of the inertial frame, and we have drawn parallels with the case of uniform motion, outlining a term that is similar (but not identical) to the γ of uniform motion. We have also pointed out that the factor is not only dependent on acceleration but also on the height of the light clock. The dependency on the dimension of the light clock sets the accelerated case apart from the case of uniform motion.

We concluded by outlining the direction of our future research, that is, the case of a light clock having the beam oriented at an arbitrary angle with respect to the direction of its acceleration and a light clock positioned on a platform rotating at constant angular velocity. We will dedicate a future paper to the even more general case of a light clock having the light beam oriented at an arbitrary angle with respect to its direction of acceleration. We will also dedicate a separate section for a light clock on a circular platform with arbitrary orientation of its light beam.

Conflict of interest: The author declares no conflict of interest.

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