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Original Research Article

Statistical analysis of melody lengths, note probabilities and note transitions of a *bandish* in raga *Bihag*

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ABSTRACT: Melody refers to a succession of musical notes that can be regarded as complete in a musical sense. For example, a line of a song or a raga *bandish* is a melody as it is complete. The number of notes in a melody is called its length. Unfortunately, the analysis of melody lengths is a neglected area in music research. Given that the significance of a melody in monophonic music (single melody line), such as Indian classical music is quantified by the product of the length of the melody and its frequency (number of occurrences in the musical piece), the novelty of our research lies in analysing the statistical features of lengths of a well-known *bandish* in raga *Bihag*. Additionally, the probability distribution of the notes is presented and a count of rising and falling note transitions in the *sthayi* and *antara* of the *bandish* is also taken. The experimental results are encouraging.

KEYWORDS: raga; melody; length; probability; chi-square goodness of fit test

1. Introduction

Indian classical music (ICM) can be broadly divided into North Indian or Hindustani classical music and South Indian or Carnatic classical music, with the raga being the central focus in either form. *A raga may be defined as a melodic structure with fixed notes and a set of rules that characterize a particular mood conveyed by performance*[1]. The emotional content of a raga is called its *rasa.* Melody refers to a succession of musical notes (the idea can be extended to beats in a percussion instrument) that can be regarded as complete in a musical sense. For example, a line of a song or a raga *bandish* is a melody as it is complete. In the case of a percussion instrument like a tabla or a drum, we may consider a complete cycle of beats to be a melody (e.g., *Dha-Dhin-Na, Na-Tin-Na* in the *dadra* taal). Note that an incomplete succession of notes (or beats) is called a segment. The number of notes (or beats) in a melody is called its length. A melody need not be a complete musical sentence but can also be a complete musical phrase (such as Sa-Sa-Re-Re-Ga-Ga-Ma-Ma-Pa in raga *Kafi*). Given that the significance of a melody in monophonic music (single melody line) such as Indian classical music is quantified by the product of the length of the melody and its frequency (number of occurrences in the musical piece), the novelty of our research lies in analysing the statistical features of the lengths of a well-known *bandish* in raga *Bihag*[2]. Unfortunately, the analysis of melody lengths is a neglected area in music research. Our analysis is important both in an absolute sense, as each *bandish* is unique in style as it pertains to the composer as well as the *gharana* (a musical school of thought), and in a comparative sense, as such a study will encourage other music researchers to incorporate similar studies on other *bandishes* based on the same raga composed by different composers belonging to the same and other *gharanas*. A *bandish* is a song-like structure in a raga that is bound to maintain the raga rules and elaborates the raga mood in brief. A *bandish* is different from *a song based on a raga* that is not obliged to follow the raga rules strictly (e.g., consider a Bollywood song based on the raga *Bihag*, such as the popular number *Zindagi ke safar mein guzaar jate hain jo makam, woh phir nahi aate*, rendered by Kishore Kumar).

In the first study, we computed some statistical measures based on the data obtained from the *Bihag bandish*. In the second study, we verify whether or not the melody lengths are uniformly distributed over the lines of the *bandish* using the chi-square goodness of fit test. The third study involves finding the probability distribution of the notes in the *bandish*. Finally, in the fourth study, a count of rising and falling note transitions in the *sthayi* and *antara* of the *bandish* is taken.

2. Literature review

For sound literature on Indian classical music, especially Hindustani classical music, see the books by Jairazbhoy[3] and Gautam[4]. As there is no unique way of describing a musical structure or a performance, statistics, and probability have a definite role to play in music analysis[5]. Moreover, music will always have patterns, with musical data being numerical in character, and hence, statistics, being *the science of exploring and studying patterns in numerical data*[1] is a very handy tool for the scientific analysis of music. The book by Beran[6] gives an account of the use of statistics in musicology in the context of Western art music (WAM), while the book by Chakraborty et al.[1] and the recently published book by Tewari and Chakraborty[7] reflect the counterpart in Hindustani classical music. Those interested in musical signal processing in the context of WAM are referred to the books by Klapuri and Davy[8] and Roads et al.[9] while the book by Datta et al.[10] deals with signal analysis in Hindustani classical music.

Bihag raga is known for its cheerful, romantic, and captivating mood. It is often performed during the evening and is associated with the spring season. The *Vadi* (most important note) of raga *Bihag* is G (the third note), and the *Samvadi* (second-most important note) is N (the seventh note). *Raga Bihag* is characterized by its joyful and playful mood. It has the ability to evoke feelings of love, happiness, and celebration. The interplay of *Komal Gandhar* and *Shuddh Nishad* adds a unique touch to its melodic structure and emotional depth. The defining phrase, or *Pakad*, of this raga is often rendered as “N S, G M P, G M G”.

Apart from classical performances, this raga is often used in semi-classical forms like *Thumri, Dadra*, and *Ghazals*. *Raga Bihag* is characterized by a serene and tranquil mood. It evokes a sense of longing, beauty, and contemplation. The romantic elements of the raga make it a popular choice for expressing emotions related to love and affection. *Raga Bihag*, with its melodious and appealing nature, has also made it popular in light music forms, including film songs and various regional music traditions[11–15].

3. Methodology

Average, dispersion, skewness, and kurtosis are the fundamental statistical features that we look for in the data. While average gives a one-figure summary of the data, dispersion gives the amount of variation or scatteredness of the observations around the average. Skewness measures the lack of symmetry, while kurtosis measures the peakedness of the distribution. The unconditional probability of a musical note is obtained by dividing the number of occurrences of the particular note by the total number of notes in the musical piece. The conditional probability of a note Y, given that the previous note is X, denoted by P(Y/X), is obtained as a ratio, with the denominator giving the number of occurrences of X in the musical piece and the numerator giving the number of occasions in which the note X is followed by Y. However, if the last note is X in the concerned musical piece, we have to subtract 1 from the denominator as we do not have information about the next note for the last note!

Note that probability theory does not directly explain the decision process of the artist, as music is always planned and not random. However, what is deterministic from an artist's point of view may be taken as the outcome of a stochastic process from the analyst's or the listener's point of view[1].

3.1. Chi-square goodness of fit test

The chi-square goodness of fit test[16] tests the significance of the difference between observed frequencies O_i and expected frequencies E_i, where the latter are calculated assuming a particular probability model whose goodness is to be tested after fitting the model to the given data. This test is carried out by computing $\sum(O_i - E_i)^2/E_i$, where the summation is over $i = 1$ to k, k being the number of classes, which follows a chi-square distribution with $(k - 1)$ degrees of freedom. 1 degree of freedom is lost because of the linear restriction $\sum O_i = \sum E_i$. In cases where the class frequency is small (< 5), adjacent classes will have to be pooled, resulting in a loss of degrees of freedom. Pooling is necessary because chi-square is a continuous distribution, and it cannot maintain its character of continuity if any theoretical cell frequency is less than 5. Also, the total frequency should be at least 50.

3.2. Comparison between Indian and Western notation

Table 1 provides a comparison between Indian and Western notation.

Table 1. Comparison between Indian and Western notation.

Western notation	C	Db	D	Eb	E	F	F#	G	Ab	A	Bb	B
Indian notation	S	r	R	g	G	M	m	P	d	D	n	N

In the Indian notation, a capital letter indicates a natural note and small letter indicates sharp or flat as the case may be. Thus, S = Sa; r = Komal Re; R = Shudh Re; g = Komal Ga; G = Shudh Ga; M = Shudh Ma; m = Tibra Ma; P = Pa; d = Shudh Dha; n = Komal Ni; N = Shudh Ni.

Further italics will imply the note is in the lower octave. The normal font will indicate the note to be in the middle octave, while bold letters are reserved to reflect that the note is in the higher octave.

4. Experimental results

The *bandish* in raga *Bihag* being studied has four lines (four melodies)[2].

Line 1: *N* S G M P N S N D P P G M G G (15 notes) [*Kyon Tum Ruth Gaye ManMohan*].

Line 2: S N D P P P P P m P G M G R S (14 notes) [*Kaun Si Bhool Bhai Hai Mo Se*].

Line 3: G G G M P P N N S S S N R S (14 notes) [*Vinati karat Tori Paiyan Parat Hoon*].

Line 4: N S N S R S N D P P S N D P G M G G (18 notes) [*Aake Darash Dikha Ja Mohan*].

Study 1

The statistical measures that we computed are given in **Table 2**.

Table 2. Statistical measures of melody lengths of the *Bihag bandish*.

Statistical measure of melody lengths	Value
Mean	15.25
Standard error of the mean	0.846
Standard deviation (SD)	1.893
Coefficient of variation = SD/mean	0.1241
Min	14
Max	18
Range=max − min	4
Median	14.5
Q1 = first quartile	14
Q3 = third quartile	17.25
IQR (inter quartile range) = Q3 − Q1	3.25
Mode	14
Karl Pearson's coefficient of Skewness = (mean − mode)/SD	0.66
Kurtosis	2.62

Note: The mean melody length is 15.25 with a standard deviation of 1.893. As mean > mode, the distribution of melody lengths is positively skewed. Further, as the kurtosis measure is 2.62 < 3, hence the distribution of melody lengths is platykurtic.

Study 2

We shall test the null hypothesis.

H_0: Melody lengths are uniformly distributed over the song lines, against the alternative hypothesis.

H_1: Melody lengths are not uniformly distributed over the song lines, at a level of significance α: 5%.

As there are four lines with 61 notes in all, expected notes per line = 61/4 = 15.25.

The observed four melody lengths are 15, 14, 14, and 18. Writing O for observed frequency and E for expected frequency of a class, our chi-square test statistic = $\sum(O - E)^2/E$ which follows the chi-square distribution with 4 − 1 = 3 degrees of freedom in our case.

Calculated chi-square = $(15 - 15.25)^2/15.25 + (14 - 15.25)^2/15.25 + (14 - 15.25)^2/15.25 + (1815.25)^2/15.25 = 0.0041 + 0.1025 + 0.1025 + 0.4960 = 0.7051$

As the calculated chi-square is less than the table chi-square, H0 may be accepted at a 5% level of significance. In other words, the melody lengths are uniformly distributed over the lines of the *Bihag bandish*.

Study 3

Taking the tonic Sa at natural C, and using the numbers 0, 1, 2, ..., 11 to represent the 12 notes in the octave, and using **Table 1**, the probability distribution of the *Bihag bandish* is presented in **Table 3**.

Table 3. Probability distribution of the notes of *Bihag bandish*.

Note	S	R	G	M	m	P	D	N
X	0	2	4	5	6	7	9	11

Probability P(X = x): 12/61, 3/61, 12/61, 5/61, 1/61, 13/61, 4/61, 11/61

Mean = E(X) = $\sum xP(X = x) = 0 \times 12/61 + 2 \times 3/61 + 4 \times 12/61 + 5 \times 5/61 + 6 \times 1/61 + 7 \times 13/61 + 9 \times 4/61 + 11 \times 11/61 = 0 + 6/61 + 48/61 + 25/61 + 6/61 + 91/61 + 36/61 + 121/61 = 333/61 = 5.46$

$E(X^2) = \sum x^2P(X = x) = 0 \times 12/61 + 4 \times 3/61 + 16 \times 12/61 + 25 \times 5/61 + 36 \times 1/61 + 49 \times 13/61 + 81 \times 4/61 + 121 \times 11/61 = 0 + 12/61 + 192/61 + 125/61 + 36/61 + 637/61 + 324/61 + 1331/61 = 2657/61 = 43.56$

Variance = $E(X^2) - [E(X)]^2 = 43.56 - (5.46)^2 = 13.75$

We observe that the note Pa has the highest probability, while the tivra Ma has the lowest probability.

The conditional probabilities are represented by the following transition probability matrix (**Table 4**) with the row indicating the current note and the column the next note. The cells give the probability of the next note, given the current note (we are assuming a first-order Markov chain, i.e., the next note depends only on the current note).

Table 4. Transition probability matrix depicting the conditional probabilities of the *Bihag bandish*.

Row = current note Column = next note	**S**	**R**	**G**	**M**	**m**	**P**	**D**	**N**
S	2/12	1/12	2/12	0	0	0	0	7/12
R	3/3	0	0	0	0	0	0	0
G	1/11	1/11	4/11	5/11	0	0	0	0
M	0	0	3/5	0	0	2/5	0	0
m	0	0	0	0	0	1/1	0	0
P	1/13	0	3/13	0	1/13	6/13	0	2/13
D	0	0	0	0	0	4/4	0	0
N	5/11	1/11	0	0	0	0	4/11	1/11

Caution: Note that in non-zero probabilities, it is better to keep the ratio as it is rather than simplify it. This is because, although P(S/S) = 2/12, we do not write it as 1/6 as musically the meaning changes. The ratio 2/12 means that on 12 occurrences of Sa, the next note is also Sa in 2 cases. The ratio 1/6 (although it mathematically equals 2/12) means that Sa appears only on 6 occasions in the musical piece, and only on one occasion is it followed by Sa. This is obviously not true for the *Bihag bandish* and hence musically wrong. We can, of course, write the numerical value of 2/12, but then it is not clear how many times Sa is followed by Sa out of the total occurrences of Sa in the musical piece. In fact, it is good practice to write even the zero conditional probabilities in this manner. For example, the four zero probabilities in the first row are each 0/12.

Study 4

Using MS-Excel, **Figures 1–4** are obtained, which give the pitch profiles of the individual four melodies 1, 2, 3 and 4, which correspond to lines 1, 2, 3 and 4, respectively while **Figure 5** provides the overall pitch profile of the *Bihag bandish*.

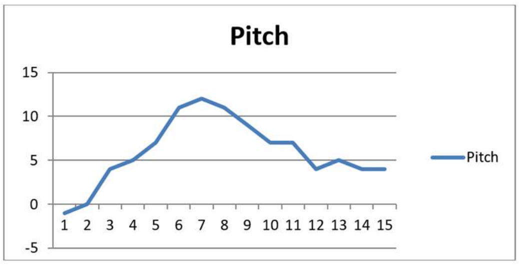

Figure 1. Pitch profile of melody 1 (line 1).

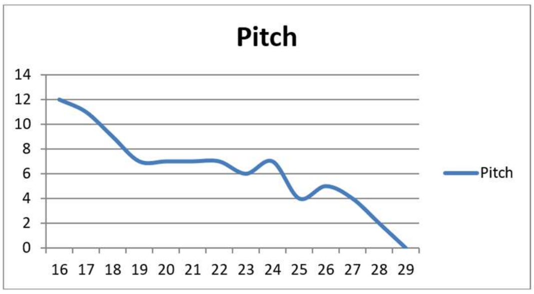

Figure 2. Pitch profile of melody 2 (line 2).

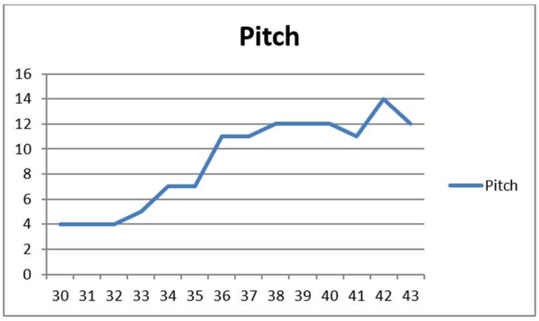

Figure 3. Pitch profile of melody 3 (line 3).

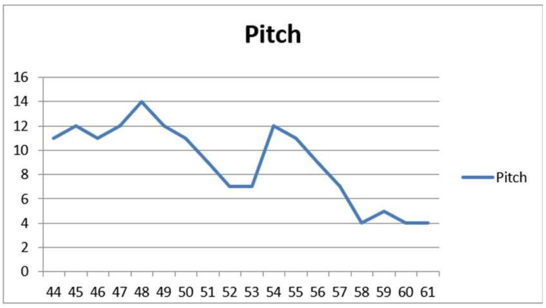

Figure 4. Pitch profile of melody 4 (line 4).

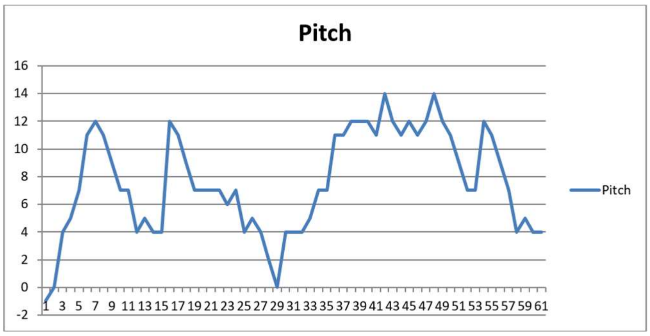

Figure 5. Pitch profile of the entire Bihag bandish (lines 1–4 taken together).

We observe that in line 1, there are 7 rising transitions and 5 falling transitions as there are 7 and 5 occasions respectively in which the next note is higher or lower than the previous note. There are 2 cases of no transition as there are two occasions in which the next note is the same as the previous note. In contrast, there are only 2 rising transitions, 8 falling transitions, and 3 cases of no transition in line 2. In line 3, there are 5 rising transitions, 2 falling transitions, and a good number of 6 cases of no transition. Finally, in line 4, the rising transitions are 4, the falling transitions are 10 and there are again a sizeable 7 cases of no transition.

5. Concluding remarks

The mean melody length is 15.25, with a standard deviation of 1.893. As mean > mode, the distribution of melody lengths is positively skewed. Further, as the kurtosis measure is 2.62 < 3, hence the distribution of melody lengths is platykurtic. From the chi-square goodness of fit test, the melody lengths may be taken to be uniformly distributed over the lines of the *Bihag bandish*. We have also obtained the probability distribution of the notes, with a mean of 43.56 and a variance of 13.75. The note Pa has the highest probability of 13/61, while the tivra Ma has the lowest probability of 1/61. An interesting aspect of the melodies of the *bandish* is that the rising transitions are more than the falling transitions in lines 1 and 3 (which are the opening lines of the *sthayi* and the *antara*, respectively) while the falling transitions are more than the rising transitions in lines 2 and 4 (which are the closing lines of the *sthayi* and the *antara*, respectively). Another interesting aspect of this *bandish* is that the cases of no transition are more in lines 3 and 4, i.e., in the *antara*, as compared to lines 1 and 2, which represent the *sthayi*.

This work is part of an ongoing research project at our institute. The title of the project is *Hindustani raga analysis using statistical musicology with therapeutic applications for stress management*. The raga *Bihag* being studied here has been found to be effective in insomnia (sleep disturbance) and hence a potential raga for music therapy intervention in stress management. A review of music therapy in western and non-western (Indian) music can be found in the survey paper by Singh et al.[17].

Author contributions

Conceptualization, SC; methodology, SC; software, PS; validation, SC and PS; formal analysis, SC and PS; investigation, PS; resources, PS; data curation, PS; writing—original draft preparation, SC and PS; writing—review and editing, SC; visualization, SC; supervision, SC; project administration, SC; funding acquisition, SC. All authors have read and agreed to the published version of the manuscript.

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Conflict of interest

The authors hereby declare that they have no conflict of interest.

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