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Article

Schwarzschild black hole spacetimes with viscous hot unmagnetized plasma Milky Way optically thin disc

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Abstract: The Schwarzschild spacetimes with hot viscous rarefied unmagnetized plasma are investigated under adiabatic perturbations of the 4-velocity of the plasma of the slim disc. The r-component of the 4-velocity and the ϕ-component of the 4-velocity are analytically written. The ϕ component of the 4-velocity is found not to depend on the 4-position. Indeed, the functional dependence of the canonical energy of the perturbation on the component u_ϕ of the 4-velocity is studied: it is defined to be unvaried for a vanishing value of u_ϕ and for a constant non-vanishing value of u_ϕ; differently, it varies with different characterizations of u_ϕ. The results are a comparison with the current understanding of the central region of the Milky Way and of the further regions. The position of the outer boundary conditions is newly discussed. The speed of sound in the disc is newly found to be dependent on the radial position, the accretion rate of the black hole object and the variation of the gravitational potential of the gravitating disc. The position of the outer boundary conditions is therefore newly discussed according to the transonic behavior of the disc and to the determination of the sonic points.

Keywords: Schwarzschild spacetime; hot viscous rarefied unmagnetized plasma; slim disc; Milky Way

1. Introduction

From [1], the Galaxy is understood to contain a slim disc of hot, rarefied plasma whose extension ranges from the central region to the further regions.

The qualities of the spherically symmetric, non-rotating black hole object spacetimes with perfect fluid solutions are studied in [2].

The model investigated in the present paper is compatible with the current understanding of the behavior of the central region of the Milky Way and that of the further regions, as individuated in [1]. The model which is here analyzed is motivated after the explanations from [3,4].

In the work of [5,6], after [7,8], the 'thin-disc' model is obtained from a cylinder, for which the vertical direction is integrated for the qualifying equations. Differently, in the present work, the Einstein equations are imposed for the disc configuration.

Through the disc configuration, the initial conditions are imposed; the paradigm outlined above is one for which the initial conditions have to be imposed within the integration procedure with respect to the vertical direction.

The aim of the paper is the General-Relativistic analytical characterization of the optically thin disc of the Milky Way (as one described in [1]), i.e., at a temperature of 8 keV extending for hundreds of kpc. The adiabatic perturbations of the slim disc of Schwarzschild spacetimes are here newly studied.

The expression of the r component of the 4-velocity is newly analytically written. The ϕ-direction of the 4-velocity is considered as non-vanishing in the case of a viscous medium.

The energy density is found to depend on the unperturbed ϕ component of the 4-velocity, on the opportune components of the metric tensor and of their derivatives.

The behaviors of the medium are compared with the phenomenon of the trapping of the shock waves found in the case of the gaseous slim disc in the region $3r_S \leq r \leq 4r_S$.

The position of the outer boundary conditions is newly discussed, differently from the mechanisms individuated in [9, 10]. More in detail, the speed of sound is newly analytically calculated. Consequently, the outer boundary conditions have to be imposed after the calculation of the sonic points (which will be addressed elsewhere).

The heating mechanisms of the hot viscous plasma in the Schwarzschild spacetimes were investigated in [1]; the analysis therefore poses the long-standing problem of the descriptions of the configurations of the hot viscous plasma.

The heating mechanisms are found to modify the energy density and the perturbation of the r-direction of the 4-velocity and the energy density; the ϕ-direction component is found to remain constant during the heating mechanisms, and it is not perturbed after these mechanisms.

One of the results of the paper is therefore the understanding of the hot unmagnetized plasma paradigms in Schwarzschild spacetimes under adiabatic perturbations, which can be further enforced after the comparison with the results from [11]: the ϕ component of the 4-velocity does not depend on the 4-position under adiabatic perturbations both in the case of hot inviscid unmagnetized plasma and in that of hot viscous magnetized plasma.

In the present paper, the choice of the Schwarzschild spacetime is dictated after the necessity to outline the characterization of the adiabatic perturbations and to separate them from the forces originating from the Christoffel symbols in the Kerr spacetimes. The choice of analyzing the hot unmagnetized plasma is derived from the need not to overlap the results from Magnetohydrodynamics to those arising from perturbation theory; furthermore, the studies about General-Relativistic Magnetohydrodynamics available at the present stage of the development of the literature are based on numerical simulations, and do not contain analytical results to be compared with.

The choice not to add angular perturbation is studied after [12], in which it is proven that the same canonical energy associated with the perturbation is shared after both a vanishing component $u_\phi = 0$ and a constant component $u_\phi = const$ of the ϕ component of the 4-velocity vector, as the canonical energy of the perturbation depends on dL_z/dt and not on L_z itself.

The boundary conditions are studied for the Einstein Field Equations. For this reason, the gravitational torque is not imposed on the radius of the fictitious singularity r_S of the Schwarzschild spacetime; rather, the region $3r_S \leq r \leq 4r_S$ is studied because it contains the trapped shock waves [13]: the forces that rule the dynamics of Relativistic plasmas and that cause the aforementioned trapping are starting to be investigated in [14].

The Navier-Stokes equations for the matter content of the Einstein Field Equation are here not used because they are not needed to solve the EFE's, which are completely determined after the geometrical objects. Indeed, as confirmed from [15], the Navier-Stokes equations are to be implemented in the case of turbulence: in the present paper, the plasma is assumed to be rarefied enough for the vorticity and for the turbulence to be absent: it is proven that, in the configuration chosen in the present paper, the vorticity and the turbulence are absent [16] because the configuration is isoentropic and the differential enthalpy is vanishing.

The paper is organized as follows.

In Section 3, the accretion around a Schwarzschild black hole with hot viscous unmagnetized plasma is demonstrated.

In Section 4, the adiabatic perturbations of the hot viscous unmagnetized plasma are presented; the ϕ-component of the 4-velocity vector is calculated.

In Section 5, the behaviors of the radial component of the four-velocity are analyzed.

The results are discussed in Section 6.

2. Description of the Galactic disc

2.1. The hot-accretion paradigm

The 'hot-accretion' mechanism to which there undergoes the Galactic disc is reviewed, i.e., in [17].

The hot accretion is 'virially hot' and 'optically thin' (i.e., where the latter request is done in [1]).

The mass-accretion rate of the black hole object is lower with respect to that found for the other accretion protocols.

The advection flow is described as an 'advection-dominated' flow and a 'luminous' flow.

The energy advection causes the 'radiative' efficiency to be smaller than that found for the other disc-accretion protocols.

At the decrease of the mass-accretion rate, the efficiency decreases.

The hot accretion flow is commented to be theorized as accompanied by jets.

The advection-dominated accretion flow has been described since [7] as one for which the accretion energy dissipated in viscosity can render the temperature of the disc higher. The paradigm of [7] is one in which the temperature is stable.

In the analyses of [5, 6, 8], the thin disc is obtained from a 'thin cylinder'; the qualifying equations are obtained after integration with respect to the vertical direction of the cylinder. For hot accretion the fraction δ is studied, which accounts for the proportion of energy dissipated after viscosity that heats electrons directly. For the two-temperature plasma, the mechanism was described in [8], to which the analysis of [5] and [6] allows one to take the parameter δ to split the energy equation into two coupled equations.

The two-temperature plasma description is therefore achieved; the electrons and the ions are at two different temperatures; the energy distribution of the electrons is interrogated about in [17], i.e., the present understanding of the experimental data does not allow for one to discriminate between the thermal energy distribution or a non-thermal one.

The two-temperature plasma reviewed in [17] can be compared with that of [18] as far as the specifications needed for the description of the Milky Way disc are concerned.

The outer boundary conditions for the integrated-cylinder model were discussed in [9,10].

From [9], a small amount of flows such as jets and winds can be produced from the inner part of the cylindrical model, but no jets are produced from ibidem at the outer boundaries. This phenomenon is ibidem explained as due to the circumstance that in the inner region the viscosity parameter is approximatively calculated as close to unity, and the flow is transonic.

In [10], the Paczýski-Wiita potential was used for the position of the outer boundary condition of the thin-disc problem; in the present paper, the gravitational potential of the gravitating thin disc is determined after the General-Relativistic properties of the disc configuration only.

2.2. The Galactic disc

From [3], a description of the features of the Galaxy as far as the Galactic ridge is concerned is provided. Along the Galactic ridge, an intense emission is evidenced, whose characteristics are commented to be as compatible with Fe of He-like ions. The presence of 'optically-thin' plasma is outlined, in which thermal bremsstrahlung takes place. The analysis of the temperature reveals a range of 5–10 keV4.

The intensity is studied as compatible with a 'disc-shaped emission region' within length scales of 100–300 pc and a radius of 10 kpc.

The presence of an 'excess emission' with respect to the thermal bremsstrahlung was already described in [19–23]. The Galactic diffuse component of the extension of several kpc was pointed out in [24].

From [1], the presence of plasma of 8 keV for hundreds of kpc has not found an explanation until now. Furthermore, the power estimated in order to make the plasma refrain from escaping from the region under the effect(s) of the escape force(s) is not attributed to any known source. The protocol according to which the plasma is heated to the evaluated temperature is not known as well.

As highlighted in [1] from [25], the temperature and the density of the plasma hot phase are such that the Hi is described as of 'weakly collisional' ions: the ions can leave the disc under the escape force(s) without 'dragging' the other elements; differently, the He is heavier, and undergoes the gravitational interaction. For He, the energy loss is mostly due to radiation [4] at longer timescales.

From [18], the plasma can be described as one in which the electrons and the ions are at different temperatures.

From [25], the high temperature of the plasma is explained as due to viscous friction on molecular clouds that flow towards the Galactic center.

Moreover, the phenomenological interpretation is proposed in [1], that there exists a possibility for a mechanism that 'taps' gravitational energy from molecular clouds, as the 8 keV plasma is ibidem commented to be viscous; accordingly, the viscous friction of molecular clouds is apt for energy dissipation in the gas and to heat the gaseous material with the flow within the hot phase.

The study of the diffuse 8 keV plasma inhabiting for several kpc's the center of the Galactic region was confirmed in [25] for the x-ray observation Chandra. From [26], no demonstration was found that the 'hard X-ray spectrum' can be ascribed to the interaction between a single-temperature soft plasma (i.e., one with $kT = 0.3$ keV and non-thermal electrons, i.e., at least two species of the plasma are newly inferred to be needed. The spectra are hypothesized to be more pertinent to a thermal origin.

Some plasma-heating mechanisms were previously hypothesized in [27], but the two protocols were not apt to explain the previous heating and the expansion along the vertical direction of the magnetic field.

The found plasma is described to be placed with the usual component of the intergalactic medium, i.e., cold molecular clouds and material from supernova remnants. The origin of this plasma is declared as of uncertain origin from [25].

The possibility of a Hi plasma was formulated and the hypothesis was tested from calculating the space velocity of sound; if so, its gravitational potential can be considered as negligible. The energy necessary to increase the temperature of this plasma before the plasma exits the region under the effect of newly studied Relativistic escape forces is here newly estimated and requested as very high. Furthermore, no Astrophysical protocol to be applied was scrutinized in [25] in order for such energy to be created.

The plasma was hypothesized ibidem to be a He plasma as well, specified as a 'gravitationally confined' plasma.

The present aim is to further study the plasma as one with gravitational potential as follows.

From [25], the hot He plasma is described as a form of ambipolar separation of elements.

The thermal velocity $\bar{v}_{th}$ is defined as

$$\bar{v}_{th} \equiv \frac{\sqrt{K_B T}}{\mu m_p} < v_{esc} \tag{1}$$

with K_B the Boltzmann constant, being v_{esc} the escape velocity, from which the condition is implied, that the thermal velocity prevents the particles from undergoing the Relativistic escape forces.

The molecular-weight method is considered for the calculation in Equation (1), where the molecular weight μ is defined as

$$\mu \equiv \frac{n_i m_i + n_e m_e}{n_i m_p + n_e m_p} \tag{2}$$

The molecular weight μ in Equation (2) accounts for the number of protons and for the number of electrons of the perfect fluid.

A magnetic field can possibly be introduced in order to modify the particle dynamics; in the presented protocol, nevertheless, the vertical motion of particles is studied, whose direction is parallel to that of the proposed magnetic field.

From [25], the properties of the plasma are scrutinized according to the possibilities of the analyses of the experimental data.

In [25], the difficulty is highlighted in the detection of a Hi-He plasma, as the plasma would be fully ionized, and there would be no lines to be observed.

Differently, the analysis of the He plasma is of more direct experimental verification—such a hypothesis is apt for the description of the X-ray spectra from the Galactic center region.

Data analyses [4] allow one to define the He number density $n_{He} = 0.04$ cm^{-3}, which is lower than the Hi model, i.e., one for which the Hi number density is $n_{Hi} = 0.1$ cm^{-3}.

The data analysis allows one to discriminate about the presence of Fe.

The bound He plasma is characterized after a short lifetime, which is due to the radiation losses, the escape forces and further processes, such as evaporation from high latitudes.

These features are nevertheless precisely enough delineated in order to discriminate about the effects of the energies that can be detected after supernovae (which are not proven to produce this phase [4,28]).

The description of unmagnetized plasma can be followed from [29,30].

Form [29], the Knudsen number is used to ensure species continuity. The drift velocities are calculated in the comoving reference frame. The local flux is characterized after viscosity.

From [30], the unmagnetized plasma with multiple ion species is described. The perturbation theory is studied, which relies on the smallness of the Knudsen number for both electrons and ions.

Because ions can be described as 'at rest' with respect to electrons, the ion velocity distribution function is not necessitated for the kinetic equation of this type of fluid. The ion systems are very variable for the rates of thermalization, that of friction and that of heat flux. All the ion species are coupled: the equilibrium rates qualify the relaxation modes of the system, and not of the species.

The purpose of the present study is to analytically describe the 8 keV disc of viscous plasma extending for hundreds of kpc's according to the pertinent radial momentum equation in General Relativity.

3. Accretion around a Schwarzschild black hole with hot viscous unmagnetized plasma

The analysis [31] is motivated in order to depict the behavior of viscous plasma accretion flows around a Kerr black hole.

The viscous plasma is a macroscopic solution of the Einstein field equations, whose stress-energy tensor $T^{\mu\nu}_{pf}$ is written as

$$T^{\mu\nu}_{pf} = (E + P)u^\mu u^\nu + pg_{\mu\nu} + \pi^{\mu\nu} \tag{3}$$

with $\pi^{\mu\nu} = -2\eta\sigma^{\mu\nu}$, where η is the viscosity coefficient, and $\sigma^{\mu\nu}$ is the shear tensor (where the latter quantity is explained in the work of Peitz [32]). As spelled out in [33], the radial momentum equation and the continuity equation are prepared from [31].

The radial momentum equation is here written in the specific way for a spherically symmetric, non-rotating black hole object

$$u^r {u^r}_r + \frac{1}{2} g_{rr} \frac{g_{tt,r}}{g_{tt}} + \frac{1}{2} u^u u^r \left[\frac{g_{tt,r}}{g_{tt}} + g^{rr} g_{rr,r} \right] + \frac{1}{2} u^\phi u^\phi \left[g^{rr} \frac{g_{tt,r}}{g_{tt}} - g_{\phi\phi,r} \right] + \frac{g^{rr} + u^r u^r}{E+P} P, r = 0 \tag{4}$$

with E the energy density and P the pressure. In Equation (4), a non-rotating black hole object is specified for vanishing shear viscosity from the work of Dihingia et al. [31].

The continuity equation is here reported for a spherically symmetric, non-rotating black hole as

$$\dot{M} = -4\pi r u^r \rho \tilde{H} \tag{5}$$

with $\tilde{H}$ defined as

$$\tilde{H} = \sqrt{\frac{P r^3}{\rho \tilde{f}}} \tag{6}$$

where $\tilde{f}$ is taken from ibidem.

The radial momentum equation Equation (4) is prepared from the work of Das et al. [33].

Equation (5) is derived from the conservation equation of the particle number

$$(\rho u^\nu)_{;\nu} \tag{7}$$

as explained in the work of Dihingia et al. [31].

The main steps of the derivation of Equation (6) are sketched from the work of Riffert et al. [34] and from the work of Peitz [32]; more in detail, the vertical quantity H is obtained as a function of the pressure P, of the density ρ, and of the.

The results have to be compared with the trapping region of the epicyclic frequency of gaseous discs around black holes [13].

4. Adiabatic perturbations of the hot viscous plasma

On the equatorial plane, the case of adiabatic perturbations of the 4-velocity is studied after implementing the perturbed expressions.

$$u^r = 0 + \delta u^r \tag{8}$$

and

$$u^\phi = U_0^\phi + \delta u_\phi \tag{9}$$

The definition of Equation (8) is implied after the definition of adiabatic perturbations of the velocities; more in detail, the adiabatic perturbation of the velocities implies a vanishing leading-order term for the radial velocity, and its perturbation (which is next order).

The radial momentum equations are therefore split according to the orders from Equations (8) and (9). In the case of rarefied plasma, the following split holds:

$$\frac{1}{2}U_0^{\phi}U_0^{\phi}\left[g^{rr}\frac{g_{tt,r}}{g_{tt}}-g_{\phi\phi,r}\right]+\frac{g^{rr}+u^{r}u^{r}}{E+P}P,r=0 \tag{10}$$

The case of the adiabatic perturbations lets the following integration hold:

$$\begin{aligned}\ln(\mathcal{E}+P)=&-\frac{1}{6}U_0^{\phi}U_0^{\phi}r_S-\frac{1}{4}U_0^{\phi}U_0^{\phi}r_S^2r^2+\frac{1}{2}U_0^{\phi}U_0^{\phi}r^2-\frac{1}{2}U_0^{\phi}U_0^{\phi}r_S^3r\\&-\frac{1}{2}U_0^{\phi}U_0^{\phi}r_S^4\ln(r-r_S)-\frac{1}{2}\frac{r_S}{r-r_S}-\frac{1}{4}\frac{R_S^2}{(r-r_S)^2}\end{aligned} \tag{11}$$

The very new behavior of the energy density $\mathcal{E}$ is outlined from Equation (11) in the trend dictated after the addend $-\frac{1}{2}U_0^{\phi}U_0^{\phi}r_S^4\ln(r-r_S)$, which is not described in the case of inviscid hot plasma under adiabatic perturbation, as analyzed in [11].

5. Behaviors of the radial component of the four-velocity

The implications of Equation (8) can now be analyzed.

Two possible protocols can be outlined, i.e., about whether the perturbation δu^r is changing slowly or not with respect to the radial position 4-vector.

5.1. Radial component of the four-velocity weakly dependent on the radial position

In the case of the radial component of the velocity being weakly dependent on the radial position, the derivative of the perturbation δu^r from Equation (8) is much smaller than δu^r itself, i.e.

$$\delta u^r_{,r}=\mathcal{O}(\delta u^r) \tag{12}$$

Therefore, at the given order, δu^r is approximated as a constant.

5.2. Radial component of the four-velocity strongly dependent on the radial position

In the case the radial component of the four-velocity is strongly dependent on the radial position, the perturbation δu^r is found to obey the differential equation

$$\delta u^r\delta u^r_{,r}+\frac{1}{2}\delta u^r\delta u^r\left(\frac{g_{tt,r}}{g_{tt}}+g^{rr}g_{rr,r}\right)=0 \tag{13}$$

In the case of a Schwarzschild spacetime, Equation (13) is reconducted to

$$\frac{d}{dr}\delta u^r=-\delta u^r\frac{r_S}{r(r-r_S)} \tag{14}$$

Equation (14) is eventually integrated as

$$\delta u^r=-\frac{1}{C_0}\frac{r-r_S}{r} \tag{15}$$

where C_0 is the suitable integration constant; the integration constant C_0 can be fixed, i.e., after the comparison with Equation (11).

From the velocity Equation (15), one newly finds that the corresponding acceleration which defines the force(s) which act on the radial component of the motion of the flow does not correspond to that calculated from the Paczýnski-Wiita potential [35] used in [10] to calculate the outer boundary conditions because the Paczýnski-Wiita potential is one for supercritical luminosities, while the Galactic flow is one with a different accretion rate, as newly calculated and as newly explained in the present paper; furthermore, one newly determines that the value of the integration constant C_0 defines the effect of the gravitational potential of the gravitating disc: a $C_0 < 0$ implies infall, while a $C_0 > 0$ implies radial escape of the matter. A change in $\dot{M}$ allows one to newly find the gravitational potential available for X-ray emission.

In the present investigation, it is possible to impose the outer boundary as a further new result from the comments in [9]: the inner region of the disc is characterized as having transonic behavior, the boundary conditions can be imposed on the trapping region(s): in the Schwarzschild case, in Section 6, the behavior is analyzed. In particular, the region $3r_S \leq r \leq 4r_S$ is investigated as from the determinations of [13].

The inner region, characterized by approximately constant temperature, allows one to define the isothermal velocity of sound C_s as $C_s \equiv \sqrt{\frac{P}{\rho}}$; in the present case, one newly finds that the speed of sound depends on the accretion rate $\dot{M}$ and on the radial position r as $c_S(\dot{M})$ as

$$c_s(\dot{M})(r) \equiv \sqrt{P}\dot{M}4\pi r \mid u_r \mid \tilde{H} \tag{16}$$

where the pressure P is taken from the radial momentum equation (Equation (4)), the density ρ from its definition (Equation (5)), M is the mass of the black-hole object, $\mid u_r \mid$ is the absolute value of the radial velocity, and $\tilde{H}$ is from Equation (6). Therefore, as a further new result, one finds that the speed of sound is defined as a function of the variation of the gravitational potential of the gravitating disc.

It is now newly possible to discuss the outer boundary conditions as in the following.

6. Discussion

The Schwarzschild black hole spacetimes surrounded by hot, viscous, rarefied, unmagnetized plasma are investigated in the slim-disc accretion-object scenario. Adiabatic perturbations of the slim disc are analytically studied. The r-components of the 4-velocity and the ϕ-component of the 4-velocity are newly analytically written. The results are compared with those of the trapping region of the gaseous slim disc.

The comparison of the results found in the present paper with those written in [11] proves that the ϕ-component of the 4-velocity does not depend on the 4-position, whatever the temperature-changing paradigms. This feature is compatible with the descriptions of the clouds.

The discussion of the position of the outer boundary conditions is therefore dictated to make the definition of the transonic behavior join that of the speed of sound found for the Milky Way disc as studying the first sonic point of the baryon-number conservation for $c_S(r)$.

As a prospective study, the non-isothermal speed of sound can be studied to better understand the particular case of the isothermal relation, which leads to Equation (16) after taking into account the specific hypothesis of Boltzmann distribution for electrons as studied in [36]—the novel characterization of the shock waves follows directly.

As from [1], the result is very compatible with the X-ray spectra from the Milky Way center region. These spectra are qualified after the features similar to those of a "diffuse, thin, very hot plasma at 8 keV on a scale of hundreds of parsecs". The requested keV plasma in the present paper is provided with. In [1], the description is requested of further details because ibidem no reasons are provided with or taken from the analyzed items of literature about how such plasma is bound on the Galactic plane, and because the mechanism(s) that allow for the observed behavior are questioned. Ibidem, most of the energy is hypothesized to have originated from the damping of Alfvénic perturbations after two different 'effects', one being non-linear aspects of the dynamics, and the other being large-scale-curvature effects.

In [4], the known explanations for the observed 8 keV lines are commented to be not comprehended within the paradigms known from the literature, nor are any new mechanism(s) postulated. The X-ray emission is commented to be of improbable origin from point-like sources. No explanation is produced ibidem about non-thermal mechanisms. Because of the properties of the lines, new questions are asked. The X-ray emission is stated to be incompatible with the properties of the interstellar medium which are reported in the literature analyzed ibidem. In [4], the exhibited spectrum is discussed to be compatible with the HE-like lines and with H-like lines from Si, S, Ar, Ca, and Fe.

In [3], the Galactic ridge is examined. Emissions are commented to be attributed to the presence of iron or to that of Helium(-like) ions. Ibidem, thermal bremsstrahlung is hypothesized to be a mechanism apt to explain the 6.7 keV line from 'optically-thin' plasma; furthermore, good agreement is found with a possibly 'disc-shaped' region of emission at a radius of 10 kpc. Moreover, ibidem, the average luminosity for such an accretion object is estimated.

The properties are here to be studied of the plasma, and the attention is paid to the region $3r_S \leq r \leq 4r_S$ as from the description of [13], in which the shockwaves are demonstrated to be trapped. Furthermore, from [14], the shockwaves are studied for gaseous material surrounding a Schwarzschild black hole object. For this reason, it is expected that the origin of shockwaves in accretion thin discs and in accretion cylinders is a property to be ascribed to the understanding of the variation of the gravitational potential in accretion objects. More in detail, these properties are expected to be originated from the variation of the gravitational potential as far as the radial variable is concerned rather than with respect to angular perturbation [12].

The features of the plasma of the thin disc at the Galactic center are still under investigation, i.e., as from [37]. The new research interests in the plasma accretion disc are dictated after the high-energy X-ray probe [38]: the new research targets are to understand the growth mechanisms of supermassive black hole objects, how they lead the Galaxy evolution, and to start investigating the 'luminous' accreting plasma close to the black hole object which is the main responsible for X-ray emission.

Within the present framework, as from [39], the 'diffuse X-ray emission in the Galactic center' is being newly investigated. The main tools utilized are high-spatial resolution of X-ray imaging and the 'broad spectral coverage'. The investigation of the Galactic center conducted this way. The features of the black hole object to be investigated are the X-ray flares emitted from the black hole object: the particle acceleration and the emission mechanisms are analyzed; the fundamental parameters of the Galactic black hole object, such as spin, the X-ray transient are taken into account; the particle-acceleration mechanisms of the Galactic-X-rays Ridge emission; the track of the past activities of the x-ray outbursts from the Galaxy as far as the X-ray reflection components are induced after 'giant molecular cloud'. The X-ray transient was explained in [40].

From [41], within the project to scan the sample of low-luminosity X-ray sources in the Galactic plane after the X spectra and after the periodic signals in the light curves, the Galactic disc is further characterized.

Within this scenario, the study of the General-Relativistic forces that rule the dynamics of the thin 8 keV disc is of fundamental importance.

From [42], the 'inhomogeneous hot atmosphere' located around the Galactic center, which can be characterized after the 'outflows of mass and energy' form the Galactic-center region is newly introduced.

The diffuse X-ray emission from the Galactic center was newly studied in [43]. The mechanisms can be compared with those analyzed from different sources, i.e., such as [44] (where the plasma temperature is investigated), and in [45] (where the diffuse X-ray emission is found as volume-filling): the studies are for comparisons and discriminations of the emissions spectra. In [46], the behavior of the outflows from galactic nuclei is theorized and compared with the available experimental evidence.

6.1. Prospective studies

In [47], the role of the hot-electron plasma is interrogated about as far as the 'hard' X-ray emission is concerned in active galactic nuclei and in black hole-X-ray-binary objects; ibidem, the cosmological redshift and the geometry of this phase are further questioned about.

In the meanwhile, the Braginsky model [18], [48] for where the viscosity models in [29] and [30] provide applications is further generalized in [49], where the Landau operator is applied to different-species plasma. Furthermore, in [49], a comprehensive review of one-ion electron plasma, i.e., one such as that of the He plasma of the description of the Galactic disc we have recalled, is provided in Section 6 ibidem.

The description of the multi-species plasma is different from the single-species plasma also as far as the heating mechanism, the possible transonic behavior(s) and the shock formations are concerned; the diffusion coefficients were recently calculated in [50].

The optical properties of advection-dominated discs were developed from [5].

The advection-dominated two-temperatures optically-thin discs were further investigated in [51–53].

In [52], the black hole accretion and the jet formation were studied for the Milky Way black hoel object in [52].

The global solutions were sketched in [17]; the transonic properties in the presence of thermal conduction are introduced in [51]. In [53], a global transonic solution in the hypothesis of thermal conduction is proposed. The discussion of the outer boundary conditions is in order.

The numerical-simulation approach to General-Relativistic Magnetohydordynamics of black hole accretion has been recently summarized in [54].

Numerical simulations are available for magnetically arrested plasma behaviors to be studied especially for the Milky Way black hole object, i.e., such as [55–57]. Time-dependent General-Relativistic magnetohydrodynamics simulations were proposed for the Milky Way black hole object in [55].

In the work of Dhruv et al. [54], a review of General-Relativistic Magnetohydrodynamics models is presented.

As already stressed, it is one of the major aims of the present paper to describe the optically thin disc of the Milky Way black hole object within the framework of General Relativity alone; it is the aim of a subsequent paper to prove that the cusp of such black hole object arises as a consequence of the General-Relativist paradigm. Comparison with the experimental evidence will therefore be possible.

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