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Article

Wronskian representations of the solutions to the Burgers' equation

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Abstract: A representation of the solutions to the Burgers' equation by the Wronskiens is given. For this, we use particular polynomials and we obtain a very efficient method to construct solutions to this equation. We deduce rational solutions from the latter equation. We explicitly build solutions for first orders.

Keywords: Wronskians; rational solutions; determinants

1. Introduction

The following Burgers' equation:

$$u_t + u_{xx} + uu_x = 0 \tag{1}$$

is considered.

This equation was introduced in 1915 by Bateman [1] as formulated in Equation (1). The Equation (1) is used in different areas of physics. The article [2] studies the different methods of statistical analysis and statistical mechanics related to the problem of turbulent fluid motion. In the paper [3], it is treated of problems of initial value for the Equation (1). The article [4] gives an algebraic method for solving partial differential equations including Equation (1) using infinitesimal transformations. In book [5], the author reports his results about fluid turbulence from 1939 to 1954. In [6], simple examples have been developed to illustrate some general characteristics of the interaction between non-linearity and viscosity. The book [7] covers all major ideas well established in differential equations, but at the same time emphasizes non-linear theory from the beginning and introduces the very active research areas in this field.

In 1915 Bateman [1] proposed a first resolution of Equation (1). Different types of methods have been used to solve this equation. Using the exp-function method [8], exact solutions in particular for the Burgers' equation are obtained. In the work [9], in particular solutions to the Burgers' equation are constructed using the tanh-coth method and the Cole-Hopf transformation. The group actions on coset bundles are used in [10] to study families of Burgers' equations. The Cole-Hopf method is used in the works [11–13]. In [14], the homotopic perturbation method, the adomian decomposition method and the differential transformation method are used to obtain solutions of the Burgers' equation.

Some recent results in connection with this study have been given in the following works. The work [15] proposes analytical solutions for the two-dimensional and three-dimensional Burgers' equation. In the paper [16], the method of the local fractional differential equation of Riccati is used to study a family of Burgers-type equations. The Burgers' equation is considered in [17], in dimensions (2 + 1), (3 + 1) and (4 + 1) where explicit exact solutions are given. In [18] a new semi-analytic method is given for analytic and bounded series solutions of the Burgers' equation. In the paper [19] an initial boundary value problem for the Burgers' equation on the positive quarter-plane is investigated. Recent developments of the mathematical modeling of the Burgers' equation are discussed in detail in [20]. A new approach for the study of the Burgers' equation is given in [21], describing the asymptotic behavior of the solution in the cauchy problem for a viscous equation with small parameters. A modified Burgers' type equation with a quadratically cubic nonlinear term is studied in [22] as a new model of perfectly soluble mathematical physics. The Hopf-Cole transformation is used in the article [23] to transform the Burgers' equation into a heat equation and the Fourier transformation then allows to obtain an exact solution of the Burgers equation.

Recently, deep learning methods [24] especially physics-informed neural networks, have emerged as a new approach to solving, in particular, the hierarchy of Burgers' including the Burgers' equation. More generally, the bilinear residual network method [25] can be proposed to solve non-linear evolution equations.

Using some particular polynomials, we get a new representation of these solutions.

The solutions to the Burgers' equation are given by means of Wronskians. With this method, we can construct very easily and efficiently some solutions for the first orders.

2. Solutions to the Burger's equation by means of Wronskian

Polynomials expressed as

$$p_{2k}(x,t) = \sum_{l=0}^{n} \frac{x^{2l}}{(2l)!}\frac{t^{k-l}}{(k-l)!}, \text{ for } k \geq 0$$

$$p_{2k+1}(x,t) = \sum_{l=0}^{n} \frac{x^{2l+1}}{(2l+1)!}\frac{t^{k-l}}{(k-l)!}, \text{ for } k \geq 0$$

$$p_n(x,t) = 0 \text{ for } n < 0 \tag{2}$$

are considered.

We use the classical notation $W(f_1, \ldots, f_n)$ for the Wronskian of the functions $f_1, \ldots, f_n$ defined by

$$\det\left((\partial_x^{j-1} f_i)_{j\in[1,n]\, i\in[1,n]}\right)$$

the notation $\partial_x^0 f_i$ meaning f_i.

Then we have the statement:

Theorem 1. *v_n expressed as*

$$v_n(x,t) = 2\partial_x\left(\ln W(p_n, \ldots, p_1)\right) \tag{3}$$

p_n *being given by Equation* (2)*, is a solution to Equation* (1).

$$u_t + u_{xx} + uu_x = 0$$

Remark 1. *We will call* v_n*, the solution of order* n *to the Burgers' Equation* (1).
Remark 2. *For example, we give the first expressions of these polynomials for* $n = 0$ *to* 10.

$$p_0(x,t) = 1$$

$$p_1(x,t) = x$$

$$p_2(x,t) = \frac{1}{2}x^2 + t$$

$$p_3(x,t) = \frac{1}{6}x^3 + tx$$

$$p_4(x,t) = \frac{1}{24}x^4 + \frac{1}{2}tx^2 + \frac{1}{2}t^2$$

$$p_5(x,t) = \frac{1}{120}x^5 + \frac{1}{6}tx^3 + \frac{1}{2}t^2x$$

$$p_6(x,t) = \frac{1}{720}x^6 + \frac{1}{24}tx^4 + \frac{1}{4}t^2x^2 + \frac{1}{6}t^3$$

$$p_7(x,t) = \frac{1}{5040}x^7 + \frac{1}{120}tx^5 + \frac{1}{12}t^2x^3 + \frac{1}{6}t^3x$$

$$p_8(x,t) = \frac{1}{40320}x^8 + \frac{1}{720}tx^6 + \frac{1}{48}t^2x^4 + \frac{1}{12}t^3x^2 + \frac{1}{24}t^4$$

$$p_9(x,t) = \frac{1}{362880}x^9 + \tfrac{1}{5040}tx^7 + \tfrac{1}{240}t^2x^5 + \frac{1}{36}t^3x^3 + \frac{1}{24}t^4x$$

$$p_{10}(x,t) = \frac{1}{3628800}x^{10} + \frac{1}{40320}tx^8 + \frac{1}{1440}t^2x^6 + \frac{1}{144}t^3x^4 + \frac{1}{48}t^4x^2 + \frac{1}{120}t^5$$

Proof. For simplicity, we denote W the Wronskian $W(p_n, \ldots, p_1)$.

The function:

$$v_n(x,t) = 2\partial_x\left(\ln W(p_n, \ldots, p_1)\right)$$

is a solution to Equation (1) if

$$A = 2(\ln W)_{xt} + 2(\ln W)_{3x} + 4(\ln W)_{2x}\ln W)_x = 0$$

or if

$$A = (\ln W)_t + (\ln W)_{2x} + (\ln W)_x)^2 = 0$$

This can be written as:

$$A = \frac{W_t}{W} + \frac{W_{2x}W - W_x^2}{W^2} + \frac{W_x^2}{W^2}$$

Thus, the equality $A = 0$ is obtained if

$$W_t + W_{2x} = 0$$

Taking into account that

$$(p_n)_x = p_{n-1}$$

and

$$(p_n)_t = p_{n-2}$$

we can write

$$W_t = W(p_n, \ldots, p_3, p_0, p_1)$$

and

$$W_{2x} = (W(p_n, \ldots, p_3, p_2, p_0))_x = W(p_n, \ldots, p_3, p_1, p_0) = -W(p_n, \ldots, p_3, p_0, p_1) = -W_t$$

Thus

$$W_t + W_{2x} = 0$$

which give $A = 0$ and the result. □

3. First order solutions

These rational solutions are all singular. In the following, we see the appearance of curves of singularities. We observe the patterns of singularities. We get lines or horseshoe type depending on the order of the solution (as presented in the following **Figures 1–20**).

3.1. Case of order 1

Proposition 1. v_1 *expressed as*

$$v_1(x,t) = \frac{2}{x} \tag{4}$$

is a solution to Equation (1).

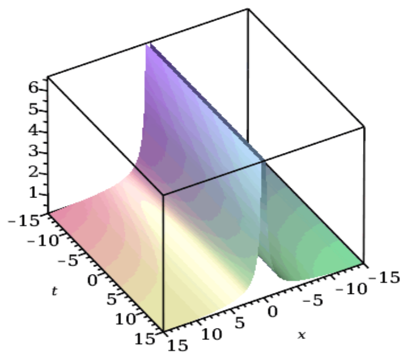

Figure 1. Modulus of v_1.

3.2. Case of second order

Proposition 2. *v_2 expressed as*

$$v_2(x,t) = \frac{-4x}{-x^2 + 2\,t} \tag{5}$$

is a solution to Equation (1).

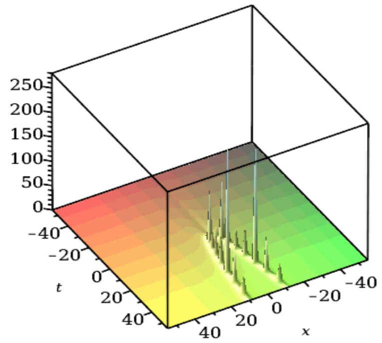

Figure 2. Modulus of v_2.

3.3. Case of third order

Proposition 3. *v_3 expressed as*

$$v_3(x,t) = 6\,\frac{-x^2 + 2\,t}{x\,(-x^2 + 6\,t)} \tag{6}$$

is a solution to Equation (1).

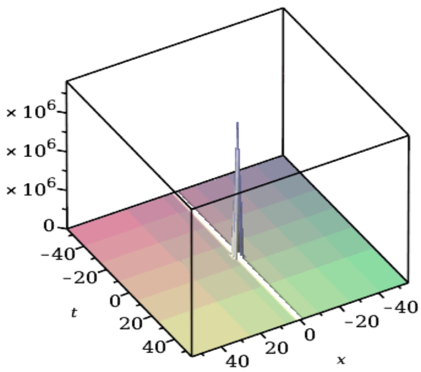

Figure 3. Modulus of v_3.

3.4. Case of fourth order

Proposition 4. *v_4 expressed as*

$$v_4(x,t) = -8\,\frac{x\left(-x^2 + 6\,t\right)}{x^4 - 12\,x^2 t + 12\,t^2} \tag{7}$$

is a solution to Equation (1).

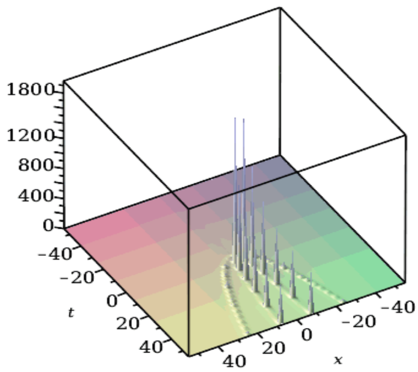

Figure 4. Modulus of v_4.

3.5. Case of fifth order

Proposition 5. v_5 *expressed as*

$$v_5(x,t) = 10\,\frac{x^4 - 12\,x^2t + 12\,t^2}{x\,(x^4 - 20\,x^2t + 60\,t^2)} \tag{8}$$

is a solution to Equation (1).

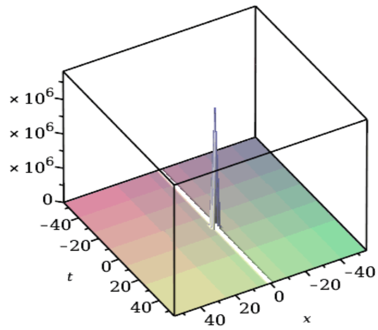

Figure 5. Modulus of v_5.

3.6. Case of sixth order

Proposition 6. v_6 *expressed as*

$$v_6(x,t) = -12\,\frac{x\left(x^4 - 20\,x^2t + 60\,t^2\right)}{-x^6 + 30\,x^4t - 180\,x^2t^2 + 120\,t^3} \tag{9}$$

is a solution to Equation (1).

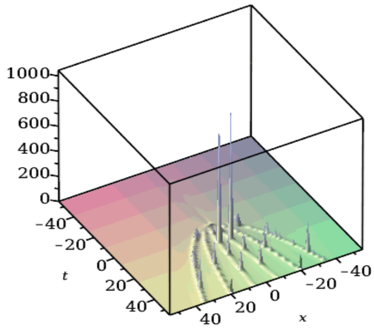

Figure 6. Modulus of v_6.

3.7. Case of seventh order

Proposition 7. v_7 *expressed as*

$$v_7(x,t) = 14\,\frac{-x^6 + 30\,x^4 t - 180\,x^2 t^2 + 120\,t^3}{x\left(-x^6 + 42\,x^4 t - 420\,x^2 t^2 + 840\,t^3\right)} \tag{10}$$

is a solution to Equation (1).

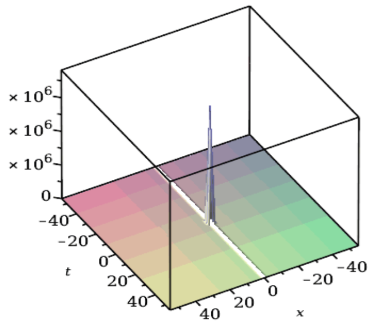

Figure 7. Modulus of v_7.

3.8. Case of eighth order

Proposition 8. v_8 *expressed as*

$$v_8(x,t) = -16\,\frac{x\left(-x^6 + 42\,x^4 t - 420\,x^2 t^2 + 840\,t^3\right)}{x^8 - 56\,x^6 t + 840\,x^4 t^2 - 3360\,x^2 t^3 + 1680\,t^4} \tag{11}$$

is a solution to Equation (1).

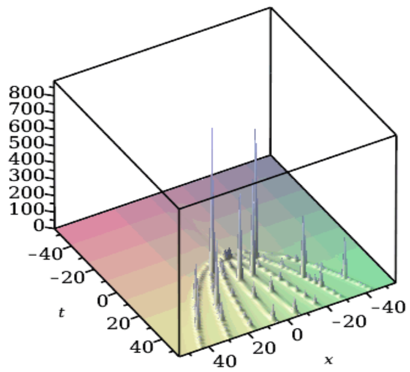

Figure 8. Modulus of v_8.

3.9. Case of ninth order

Proposition 9. v_9 *expressed as*

$$v_9(x,t) = 18\,\frac{x^8 - 56\,x^6 t + 840\,x^4 t^2 - 3360\,x^2 t^3 + 1680\,t^4}{x\left(x^8 - 72\,x^6 t + 1512\,x^4 t^2 - 10080\,x^2 t^3 + 15120\,t^4\right)} \tag{12}$$

is a solution to Equation (1).

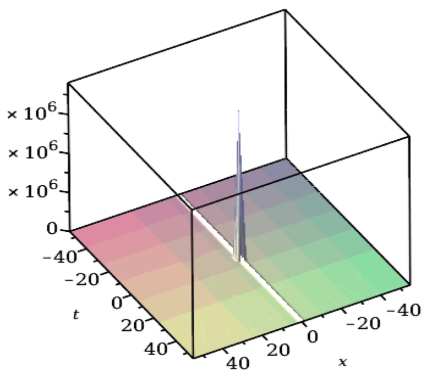

Figure 9. Modulus of v_9.

3.10. Case of tenth order

Proposition 10. v_{10} *expressed as*

$$v_{10}(x,t) = 20\,\frac{x\left(x^8 - 72\,x^6 t + 1512\,x^4 t^2 - 10080\,x^2 t^3 + 15120\,t^4\right)}{-x^{10} + 90\,x^8 t - 2520\,x^6 t^2 + 25200\,x^4 t^3 - 75600\,x^2 t^4 + 30240\,t^5} \tag{13}$$

is a solution to Equation (1).

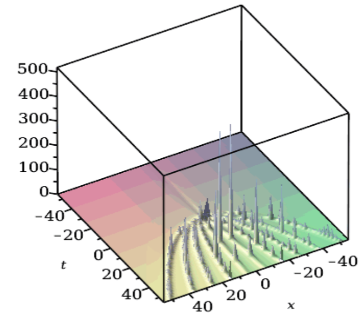

Figure 10. Modulus of v_{10}.

3.11. Case of eleventh order

Proposition 11. *v_{11} expressed as*

$$v_{11}(x,t) = 22\,\frac{-x^{10} + 90\,x^8 t - 2520\,x^6 t^2 + 25200\,x^4 t^3 - 75600\,x^2 t^4 + 30240\,t^5}{x\left(-x^{10} + 110\,x^8 t - 3960\,x^6 t^2 + 55440\,x^4 t^3 - 277200\,x^2 t^4 + 332640\,t^5\right)} \quad (14)$$

is a solution to Equation (1).

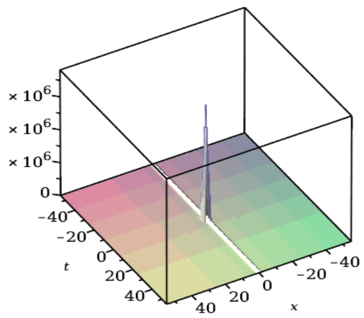

Figure 11. Modulus of v_{11}.

3.12. Case of twelfth order

Proposition 12. *v_{12} expressed as* $v_{12}(x,t) = \dfrac{n_{12}(x,t)}{d_{12}(x,t)}$,

$$n_{12}(x,t) = -24x(-x^{10} + 110\,x^8 t - 3960\,x^6 t^2 + 55440\,x^4 t^3 - 277200\,x^2 t^4 + 332640\,t^5)$$

$$d_{12}(x,t) = x^{12} - 132\,t x^{10} + 5940\,t^2 x^8 - 110880\,t^3 x^6 + 831600\,t^4 x^4 - 1995840\,t^5 x^2 + 665280\,t^6$$

is a solution to Equation (1).

3.13. Case of thirteenth order

Proposition 13. *v_{13} expressed as* $v_{13}(x,t) = \dfrac{n_{13}(x,t)}{d_{13}(x,t)}$,

$$n_{13}(x,t) = 26(x^{12} - 132\,t x^{10} + 5940\,t^2 x^8 - 110880\,t^3 x^6 + 831600\,t^4 x^4 - 1995840\,t^5 x^2 + 665280\,t^6)$$

$$d_{13}(x,t) = x(x^{12} - 156\,t x^{10} + 8580\,t^2 x^8 - 205920\,t^3 x^6 + 2162160\,t^4 x^4 - 8648640\,t^5 x^2 + 8648640\,t^6)$$

is a solution to Equation (1).

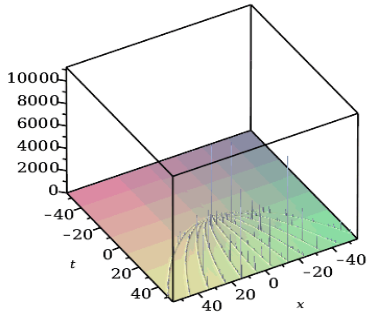

Figure 12. Modulus of v_{12}.

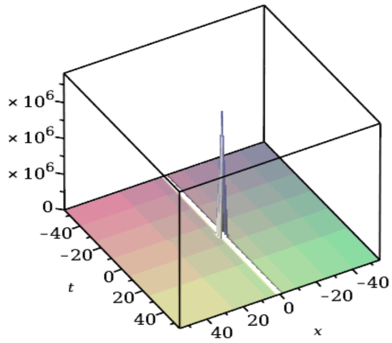

Figure 13. Modulus of v_{13}.

3.14. Case of fourteenth order

Proposition 14. v_{14} *expressed as* $v_{14}(x,t) = \dfrac{n_{14}(x,t)}{d_{14}(x,t)}$,

$$
\begin{aligned}
n_{14}(x,t) &= -28x(x^{12} - 156\,tx^{10} + 8580\,t^2x^8 - 205920\,t^3x^6 + 2162160\,t^4x^4 \\
&\quad - 8648640\,t^5x^2 + 8648640\,t^6) \\
d_{14}(x,t) &= -x^{14} + 182\,tx^{12} - 12012\,t^2x^{10} + 360360\,t^3x^8 - 5045040\,t^4x^6 \\
&\quad + 30270240\,t^5x^4 - 60540480\,t^6x^2 + 17297280\,t^7
\end{aligned}
$$

is a solution to Equation (1).

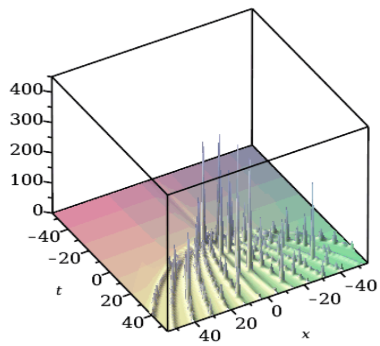

Figure 14. Modulus of v_{14}.

3.15. Case of fifteenth order

Proposition 15. v_{15} *expressed as* $v_{15}(x,t) = \dfrac{n_{15}(x,t)}{d_{15}(x,t)}$,

$$\begin{aligned} n_{15}(x,t) &= 30(-x^{14} + 182\,tx^{12} - 12012\,t^2x^{10} + 360360\,t^3x^8 - 5045040\,t^4x^6 \\ &\quad + 30270240\,t^5x^4 - 60540480\,t^6x^2 + 17297280\,t^7) \\ d_{15}(x,t) &= x(-x^{14} + 210\,tx^{12} - 16380\,t^2x^{10} + 600600\,t^3x^8 - 10810800\,t^4x^6 \\ &\quad + 90810720\,t^5x^4 - 302702400\,t^6x^2 + 259459200\,t^7) \end{aligned}$$

is a solution to Equation (1).

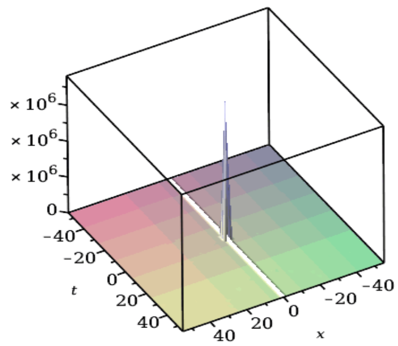

Figure 15. Modulus of v_{15}.

3.16. Case of sixteenth order

Proposition 16. v_{16} *expressed as* $v_{16}(x,t) = \dfrac{n_{16}(x,t)}{d_{16}(x,t)}$,

$$\begin{aligned} n_{16}(x,t) &= -32x(-x^{14} + 210\,tx^{12} - 16380\,t^2x^{10} + 600600\,t^3x^8 - 10810800\,t^4x^6 \\ &\quad + 90810720\,t^5x^4 - 302702400\,t^6x^2 + 259459200\,t^7) \\ d_{16}(x,t) &= x^{16} - 240\,tx^{14} + 21840\,t^2x^{12} - 960960\,t^3x^{10} + 21621600\,t^4x^8 \\ &\quad - 242161920\,t^5x^6 + 1210809600\,t^6x^4 - 2075673600\,t^7x^2 + 518918400\,t^8 \end{aligned}$$

is a solution to Equation (1).

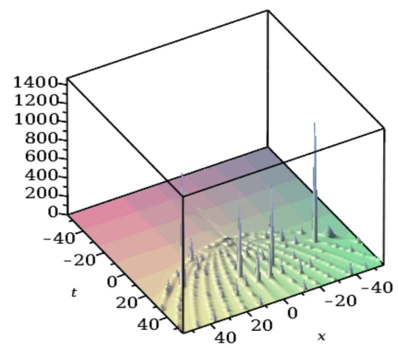

Figure 16. Modulus of v_{16}.

3.17. Case of seventeenth order

Proposition 17. v_{17} *expressed as* $v_{17}(x,t) = \dfrac{n_{17}(x,t)}{d_{17}(x,t)}$,

$$\begin{aligned} n_{17}(x,t) = 34(&x^{16} - 240\,tx^{14} + 21840\,t^2x^{12} - 960960\,t^3x^{10} + 21621600\,t^4x^8 \\ &- 242161920\,t^5x^6 + 1210809600\,t^6x^4 - 2075673600\,t^7x^2 \\ &+ 518918400\,t^8) \\ d_{17}(x,t) = x(&x^{16} - 272\,tx^{14} + 28560\,t^2x^{12} - 1485120\,t^3x^{10} + 40840800\,t^4x^8 \\ &- 588107520\,t^5x^6 + 4116752640\,t^6x^4 - 11762150400\,t^7x^2 \\ &+ 8821612800\,t^8) \end{aligned}$$

is a solution to Equation (1).

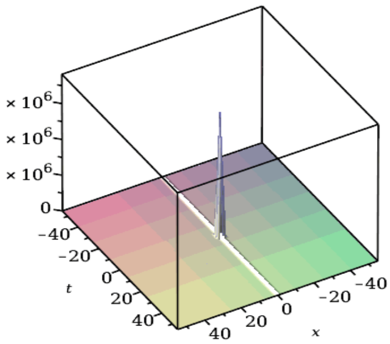

Figure 17. Modulus of v_{17}.

3.18. Case of eighteenth order

Proposition 18. v_{18} *expressed as* $v_{18}(x,t) = \dfrac{n_{18}(x,t)}{d_{18}(x,t)}$,

$$\begin{aligned} n_{18}(x,t) = &-36x(x^{16} - 272\,tx^{14} + 28560\,t^2x^{12} - 1485120\,t^3x^{10} + 40840800\,t^4x^8 \\ &- 588107520\,t^5x^6 + 4116752640\,t^6x^4 - 11762150400\,t^7x^2 \\ &+ 8821612800\,t^8) \\ d_{18}(x,t) = &-x^{18} + 306\,tx^{16} - 36720\,t^2x^{14} + 2227680\,t^3x^{12} - 73513440\,t^4x^{10} \\ &+ 1323241920\,t^5x^8 - 12350257920\,t^6x^6 + 52929676800\,t^7x^4 \\ &- 79394515200\,t^8x^2 + 17643225600\,t^9 \end{aligned}$$

is a solution to Equation (1).

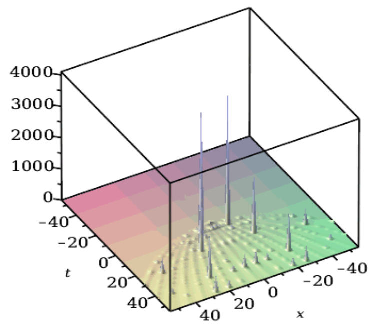

Figure 18. Modulus of v_{18}.

3.19. Case of nineteenth order

Proposition 19. v_{19} *expressed as* $v_{19}(x,t) = \dfrac{n_{19}(x,t)}{d_{19}(x,t)}$,

$$\begin{aligned}
n_{19}(x,t) = {} & 38(-x^{18} + 306\,tx^{16} - 36720\,t^2x^{14} + 2227680\,t^3x^{12} - 73513440\,t^4x^{10} \\
& + 1323241920\,t^5x^8 - 12350257920\,t^6x^6 + 52929676800\,t^7x^4 \\
& - 79394515200\,t^8x^2 + 17643225600\,t^9) \\
d_{19}(x,t) = {} & x(-x^{18} + 342\,tx^{16} - 46512\,t^2x^{14} + 3255840\,t^3x^{12} - 126977760\,t^4x^{10} \\
& + 2793510720\,t^5x^8 - 33522128640\,t^6x^6 + 201132771840\,t^7x^4 \\
& - 502831929600\,t^8x^2 + 335221286400\,t^9)
\end{aligned}$$

is a solution to Equation (1).

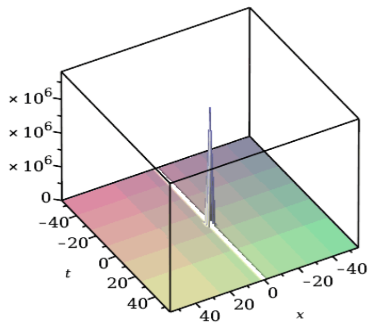

Figure 19. Modulus of v_{19}.

3.20. Case of twentieth order

Proposition 20. v_{20} *expressed as* $v_{20}(x,t) = \frac{n_{20}(x,t)}{d_{20}(x,t)}$,

$$\begin{aligned}
n({}_{20}x,t) = {} & -40x(-x^{18} + 342\,tx^{16} - 46512\,t^2x^{14} + 3255840\,t^3x^{12} \\
& - 126977760\,t^4x^{10} + 2793510720\,t^5x^8 - 33522128640\,t^6x^6 \\
& + 201132771840\,t^7x^4 - 502831929600\,t^8x^2 + 335221286400\,t^9) \\
d_{20}(x,t) = {} & x^{20} - 380\,tx^{18} + 58140\,t^2x^{16} - 4651200\,t^3x^{14} + 211629600\,t^4x^{12} \\
& - 5587021440\,t^5x^{10} + 83805321600\,t^6x^8 - 670442572800\,t^7x^6 \\
& + 2514159648000\,t^8x^4 - 3352212864000\,t^9x^2 + 670442572800\,t^{10}
\end{aligned}$$

is a solution to Equation (1).

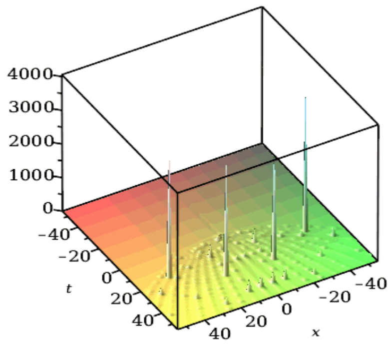

Figure 20. Modulus of v_{20}.

4. Conclusion

We have given a new formulation of rational solutions to the Burgers' equation by means of Wronskians.

Explicit solutions to the Burgers' equation are constructed for the orders $n = 1$ until $n = 20$.

The singularities of these solutions depend on the orders of the solutions.

For odd orders, the singularities of the solutions are always lines $x = 0$. In the case of even order solutions $n = 2p$, the singularities form horseshoe patterns with p branches.

This method easily gives different solutions to the Burgers' equation.

We can compare this method with, for example, the exp-function method. This last one requires performing a change of variable in n dependent on x and t allowing to transform the given equation dependent on x and t into a differential equation depending only on the variable n. A solution in the form of a quotient of finite sums of exponential is sought. This expression is derived and replaced in the different quantities of the differential equation. By identifying the different terms, a system of equations is obtained that allows us to determine the various coefficients of the quotient of the sum of the exponentials. So solutions of the given equation are obtained. But, this method is unfortunately not straightforward and requires a lot of calculation. The advantage of the Wronskian method is that it gives a direct expression to all possible orders and that one single determinant is sufficient to obtain the solution.

Future research should focus on stability analysis and convergence of solutions. This could involve the use of mathematical techniques such as perturbation analysis or numerical simulations to study the behavior of solutions under different conditions.

It will be relevant to construct solutions of this equation depending on some real parameters.

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