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Article

Elimination of oscillation causing Hopf bifurcations in engineering problems

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Abstract: Bifurcation analysis was performed on various engineering process problems that exhibit undesirable oscillation causing Hopf bifurcations. Hopf bifurcations result in oscillatory behavior which is problematic for optimization and control tasks. Additionally, the presence of oscillations causes a reduction in product quality and in some cases causes equipment damage. The hyperbolic tangent function activation factor is normally used in neural networks and optimal control problems to eliminate spikes in optimum profiles. Spikes are similar to oscillatory profiles and this is the motivation to investigate whether the hyperbolic tangent function activation factor can eliminate the oscillation causing Hopf bifurcations. The results of this paper show that the hyperbolic tangent function activation factor eliminates the Hopf bifurcations. Bifurcation analysis is performed using The MATLAB software MATCONT. Five examples involving problems that exhibit Hopf bifurcations are presented.

Keywords: bifurcation; Hopf; fermentation; nanoparticles

1. Introduction

Activation factors have been recently used in problems involving neural networks and to stifle control profile spikes in optimal control problems. This work aims to study the effect of such activation factors on oscillation causing Hopf bifurcation problems. It is demonstrated that the hyperbolic tangent function is very successful in eliminating unwanted Hopf bifurcations. Most of the work involving Hopf bifurcations demonstrate the existence of the Hopf bifurcations in various engineering problems. The novelty of this work is that a generic strategy is provided to eliminate the problematic oscillation causing Hopf bifurcations. This research is very significant as it provides an easy to implement strategy to eliminate the unwanted oscillatory behavior in several engineering problems.

Spikes occur in optimal control problems and the hyperbolic tangent function activation factor is used to eliminate the spikes in optimum profiles of the control variable. Spikes are similar to oscillations and this is the motivation to determine the effect of the hyperbolic tangent function activation factor can eliminate the oscillation causing Hopf bifurcations

In this paper, the background section (involving activation factors and bifurcation analysis are first presented). This is followed by a description of the examples where the hyperbolic tangent activation function was used to eliminate the Hopf bifurcation points. The results are then presented followed by the conclusions.

2. Background

2.1 Activation factor

The tanh activation factor is used in neural networks [1–3] and in optimal control problems to eliminate spikes in the optimal control profile [4–7]. Sridhar [8] found a correlation between singular points (limit and branch points) and multi-objective Optimal Control. However, so far, the integration of activation functions with bifurcation analysis has never been done. Hopf bifurcations cause periodic oscillatory behavior. In chemical processes, oscillatory behavior is detrimental to product quality and also causes equipment damage. This work uses the tanh activation function to eliminate oscillatory-causing Hopf bifurcations. The bifurcation parameter (control variable) ξ is replaced by by $\frac{\xi\, tanh(\xi)}{\varepsilon}$ where ε is an arbitrary constant. The tanh function is a a standard function which supresses spikes by forcing the function to (0,1).

2.2. Motivation and comparison with a linear piecewise function

The main motivation for this work is to provide an easy-to-use strategy without affecting the nonlinearity of the process to eliminate oscillatons that cause equipment damage and reduce product quality. A piecewise linear function is not as effective because it affects the nonlinearity and reduces the problem's complexity. Additionally, the use of the piecewise linear function does not eliminate the Hopf bifurcations sometimes. Furthermore, the tanh activation function is easier to implement.

2.3. Bifurcation analysis

The three types of bifurcations are limit points, branch points and Hopf bifurcation points.

The Matlab software MATCONT [9,10] is used to detect Limit points (LP), branch points (BP) and Hopf bifurcation points (HB). Consider an ODE system

$$\frac{dx}{dt} = f(x, \beta) \tag{1}$$

where the tangent function at any point x be $[v_1, v_2, v_3, v_4, \ldots. v_{n+1}]$. Let Matrix A be defined as

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} \cdots \frac{\partial f_1}{\partial x_n} & \frac{\partial f_1}{\partial \beta} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} \cdots \frac{\partial f_2}{\partial x_n} & \frac{\partial f_2}{\partial \beta} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \frac{\partial f_n}{\partial x_3} & \frac{\partial f_n}{\partial x_4} \cdots \frac{\partial f_n}{\partial x_n} & \frac{\partial f_n}{\partial \beta} \end{bmatrix} \tag{2}$$

where the bifurcation parameter is β. A can be expressed as

$$A = [B| \quad \frac{\partial f}{\partial \beta}] \tag{3}$$

The tangent function must satisfy

$$Av = 0 \tag{4}$$

For a limit point (LP) the n + 1 th component of the tangent vector v_{n+1} must be 0 and for a branch point (BP) $\begin{bmatrix} A \\ v^T \end{bmatrix}$ must have a determinant of 0. For a Hopf bifurcation, the function $det\,(2f_x(x,\beta)@I_n)$ should be zero. @ indicates the bialternate product while I_n is the n-square identity matrix. Kuznetsov [11,12] and Govaerts [13] provide a detailed explanation for these conditions. Sridhar [14] used Matcont to perform bifurcation analysis on chemical engineering problems.

3. Main objective of this work

Spikes formed in the control profiles are eliminated by the use of the tanh activation function. Hopf bifurcation points cause oscillatory behavior which is similar to spikes and this motivates the question of how the tanh activation factor would perform in eliminating the Hopf bifurcation points. Five problems that exhibit Hopf bifurcations are presented. In all these problems the effect of replacement of the bifurcation variable ξ with $\frac{\xi\, tanh(\xi)}{\varepsilon}$ where ε is an arbitrary constant is studied. Details of these problems are now presented followed by the results.

4. Examples of problems that exhibit Hopf bifurcations

1) Catalytic oscillator problem

In the catalytic oscillator problem [9] three differential equations in the mathematical problem are

$$\frac{dx}{dt} = 2q_1z^2 - 2q_5x^2 - q_3xy \tag{5}$$

$$\frac{dy}{dt} = q_2z - q_6y - q_3xy \tag{6}$$

$$\frac{dz}{dt} = q_4z - kq_4(1 - x - y - z) \tag{7}$$

2) Adaptive control was used in a feedback control system

In the problem involving adaptive control used in a feedback control system [9] the differential equations involved are

$$\frac{dx}{dt} = y \tag{8}$$

$$\frac{dy}{dt} = z \tag{9}$$

$$\frac{dz}{dt} = x^2 - \alpha z - \beta y - x \tag{10}$$

3) Phytoplankton-zooplankton model

In this model [15], For this problem, p and z are the respective population densities of phytoplankton and zooplankton species,n_p the nanoparticle density. The birthrate of the phytoplankton is *r* the carrying capacity is *k*. k_1, k_2 represent the handling time and the magnitude of the interference from the zooplankton. γ

represent the conversion efficiency and the zooplankton death rate is given by μ. The contact rate of the nanoparticles with the phytoplankton and the strength of toxicit are given by β, β_1. The input rate of nanoparticles and the natural depletion of the nanoparticles are given by A and e. The equations representing this model are

$$\frac{dp}{dt} = \frac{rp}{1 + \beta\beta_1 p n_p}(1 - \frac{p}{k}) - \frac{\omega pz}{1 + k_1 p + k_2 z + k_1 k_2 pz} \tag{11}$$

$$\frac{dz}{dt} = \gamma \frac{\omega pz}{1 + k_1 p + k_2 z + k_1 k_2 pz} - \mu z \tag{12}$$

$$\frac{dn_p}{dt} = A - \beta p n_p - e n_p \tag{13}$$

The parameter values are $r = 1; \beta_1 = 0.2; \omega = 4; k_1 = 3.09; k_2 = 0.35; \gamma = 1; \mu = 0.5; A = 2; e = 0.5$

In this model the Hopf bifurcation point was obtained by Kumar and Pramanick [15] but no strategy was presented to eliminate the Hopf bifurcation point. In this paper an activation factor is used to eliminate the Hopf bifurcation point.

4) Zymomonas Fermentation process

In the Zymomonas Mobilis fermentation problem [14,16] involving substrate (*S*), the key compound (*e*), microorganism or biomass (*X*), and product (*P*) are given by the following equations.

$$\frac{dC_e}{dt} = \bar{D}(C_{e0} - C_e) + [k_1 - k_2 C_P + k_3 C_P^2](\frac{C_S C_e}{K_S + C_S}) \tag{14}$$

$$\frac{dC_X}{dt} = \bar{D}(C_{X0} - C_X) + P(\frac{C_S C_e}{K_S + C_S}) \tag{15}$$

$$\frac{dC_S}{dt} = -m_S C_X + \bar{D}(C_{e0} - C_e) - P(\frac{1}{Y_{SX}})(\frac{C_S C_e}{K_S + C_S}) \tag{16}$$

$$\frac{dC_P}{dt} = m_P C_X + \bar{D}(C_{P0} - C_P) + P(\frac{1}{Y_{PX}})(\frac{C_S C_e}{K_S + C_S}) \tag{17}$$

D is the dilution rate. The values of K_1, K_2, K_3, Y_{px}, K_s, P, ms, mp, Ysx are 16,0.497, 0.00383, 0.0526315,0.5,0.1283,2.16,1.1 and 0.02444498.

5) Saccharomyces cerevisiae fermentation process

The Saccharomyces Cerevisiae fermentation process problem was discussed by Jones and Kompala [17], Simpson et al. [18], and Sridhar [14]. The equations are

$$u_i = \frac{r_i}{\sum_j r_j} \tag{18}$$

$$v_i = \frac{r_i}{max_j\, r_j} \tag{19}$$

while the expressions r_i are given by

$$r_1 = \mu_1 e_1 \frac{G}{K_1 + G} \tag{20}$$

$$r_2 = \mu_2 e_2 (\frac{E}{K_2 + E})(\frac{O}{K_{O_2} + O}) \tag{21}$$

$$r_3 = \mu_3 e_3 (\frac{G}{K_3 + G})(\frac{O}{K_{O_3} + O}) \tag{22}$$

The dynamic equations are given by

$$\frac{dX}{dt} = (\sum_i r_i v_i - D)X \tag{23}$$

$$\frac{dG}{dt} = (G_0 - G)D - (\frac{r_1 v_1}{Y_1} - \frac{r_2 v_2}{Y_2})X - (C\frac{dX}{dt} + X\frac{dc}{dt})\phi_4 \tag{24}$$

$$\frac{dE}{dt} = -DE + (\phi_1 \frac{r_1 v_1}{Y_1} - \frac{r_2 v_2}{Y_2})X \tag{25}$$

$$\frac{dO}{dt} = k_L a(O^* - O) - (\phi_2 \frac{r_2 v_2}{Y_2} + \phi_3 \frac{r_3 v_3}{Y_3})X \tag{26}$$

$$\frac{de_i}{dt} = \alpha u_i \frac{S_i}{K_i + S_i} - (\sum_j r_j\, v_j + \beta)e_i + \alpha^* \tag{27}$$

$$\frac{dC}{dt} = \gamma_3 r_3 v_3 - (\gamma_1 r_1 v_1 + \gamma_2 r_2 v_2)C - \sum_i (r_i v_i)C \tag{28}$$

G, E, O. are the glucose, ethanol and dissolved oxygen and μ_i is the modified growth rate constant. The concentrations of glucose, ethanol and dissolved oxygen are given by G, E, O. μ_i represents the modified growth rate constant. and K_i represent the saturation constant for each pathway. G_0 represents the inlet glucose feed concentration; and X the cell mass concentration. the dissolved oxygen mass transfer coefficient is $k_L a$.$\alpha$and β represent the enzyme synthesis and decay rate constants the yield coefficient. The stoichiometric coefficients for the intercellular storage carbohydrate synthesis and consumption are given by ϕ_i and γ_i. The values of G_0, Y_1, Y_2, Y_3, ϕ_1,ϕ_2, ϕ_3,ϕ_4, $O^*K_{O_2}$,K_{O_3},$\gamma_i(i = 1,2,3)$, $\mu_i^{max^*{}_{123}}$ are (10, 0.16, 0.75, 0.6, 0.403, 2000, 1000, 0.95, 7.5, 0.01, 2.2, 10, 10, 0.8, 0.44, 0.19, 0.36, 0.3, 0.1, 0.7, 0.05, 0.01, 0.001)

For both the Zymomonas mobilis and Saccharomyces Cerevisiae fermentation process problems the Hopf bifurcation points were eliminated using some experimental strategies by Sridhar [14]. In this article, an elegant and easy to use activation factor that eliminates the Hopf bifurcation point is presented.

5. Results and discussion

Bifurcation analysis using MATCONT was performed on the five problems with and without the tanh activation function. For problem 1, MATCONT resulted in two Hopf bifurcation points and two limit points. These Hopf bifurcations occur at (0.016357 0.523973 0.328336 1.051558) and x = (0.077929 0.233063 0.492149 1.040991). (**Figure 1**). These values represent the variabeles (x, y, z) and the bifurcation parameter k. When the parameter k was modified to ktanh (k) the Hopf bifurcations disappear (**Figure 2**).

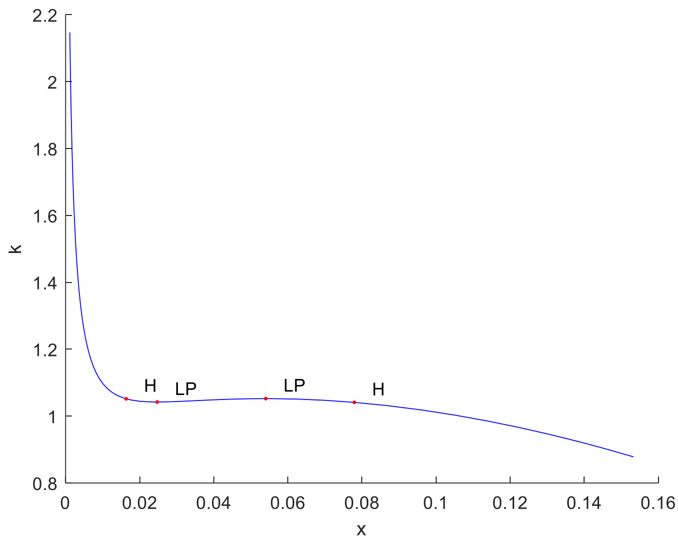

Figure 1. Hopf bifurcation for catalytic oscillator problem.

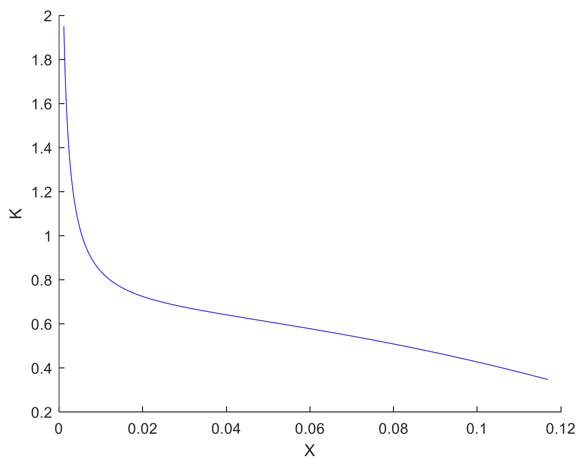

Figure 2. Hopf Bifurcation for catalytic oscillator problem eliminated by activation factor.

For problem, Hopf bifurcation point was found at, x = (0.000000 0.000000 0.000000 1.000002), (**Figure 3**) and when α, β were modified to $\alpha \tanh(\alpha)$, $\beta \tanh(\beta)$ the Hopf bifurcation point dissappears (**Figure 4**).

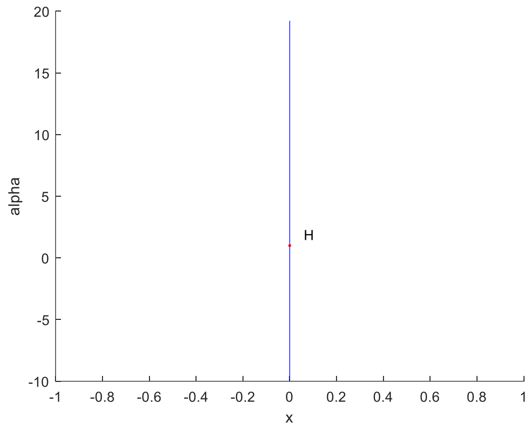

Figure 3. Hopf Bifurcation for adaptive control problem.

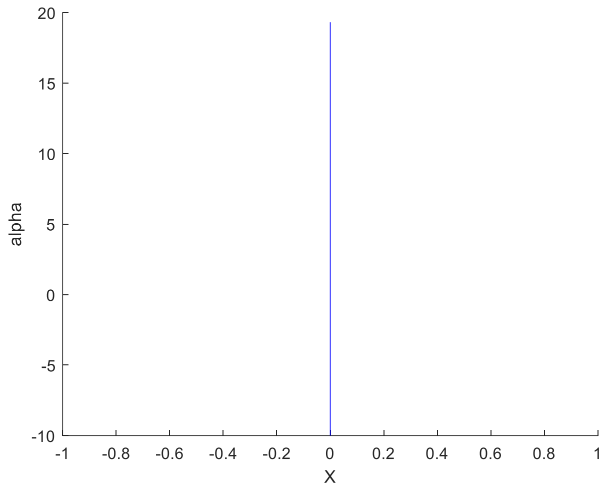

Figure 4. Hopf Bifurcation for Adaptive control Eliminated by Activation factor.

In the Crowley Martin phytoplankton-zooplankton model, a Hopf bifurcation was found at label = H, x = (0.249760 0.364966 3.731642 0.143967) (**Figure 5**).

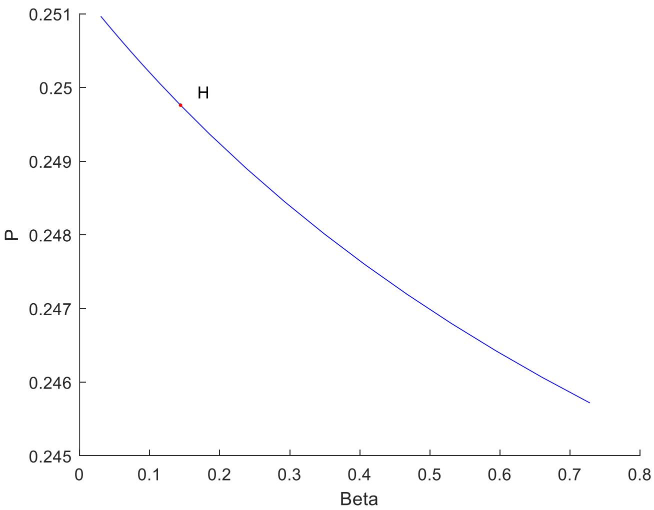

Figure 5. Hopf Bifurcation for Phytoplankton-zooplankton Problem.

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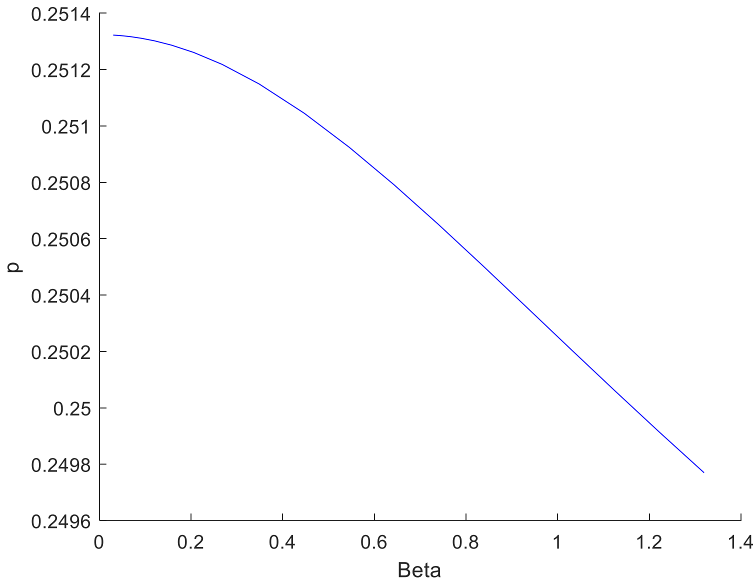

Figure 6. Hopf Bifurcation for Phytoplankton-zooplankton Problem Eliminated by Activation Factor.

The Hopf bifurcation disappears (**Figure 6**) when the parameter β was modified to $\beta \tanh\ (\beta)/8$.

In problem 4, a Hopf bifurcation point was found at label = H, *x* = (1.749993 1.458091 0.519342 48.147753 0.078453) (**Figure 7**). When the dilution rate D was modified to Dtanh (D) the Hopf bifurcation point disappears (**Figure 8**).

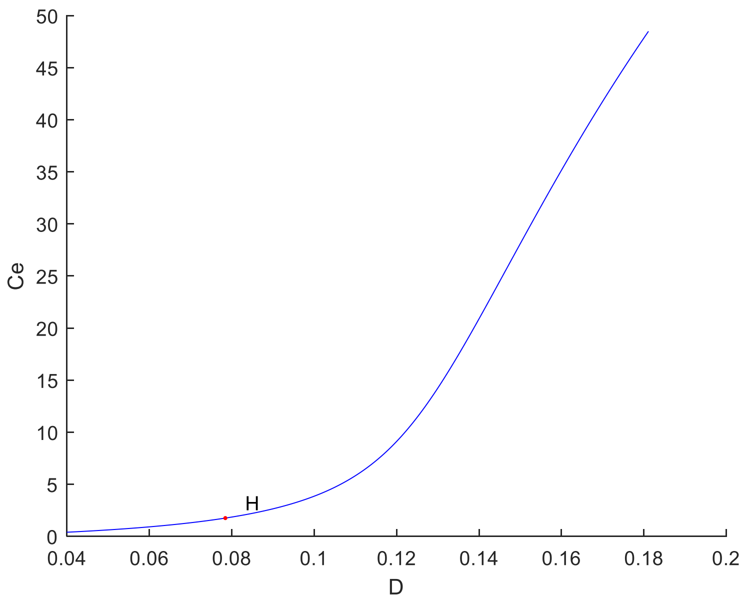

Figure 7. Hopf Bifurcation for Zymomonas Fermentation process Problem.

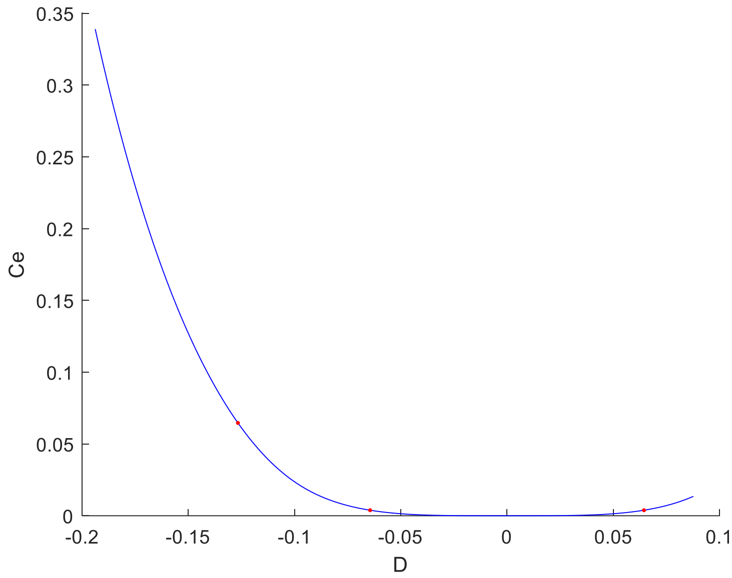

Figure 8. Hopf Bifurcation for Zymomonas Fermentation process Problem Eliminated by activation factor.

In problem 5, a Hopf bifurcation point (**Figure 9**) was found at (7.642545 0.018353 0.014161 0.047815 0.174613 0.393904 0.736104 0.001461 0.126028When the dilution rate was modified to D tanh (D)/1.35, the HOPf bifurcation point disappeared (**Figure 10**). In all five cases it is seen that the hyperbolic tangent function was effective in eliminating the Hopf bifurcation point.

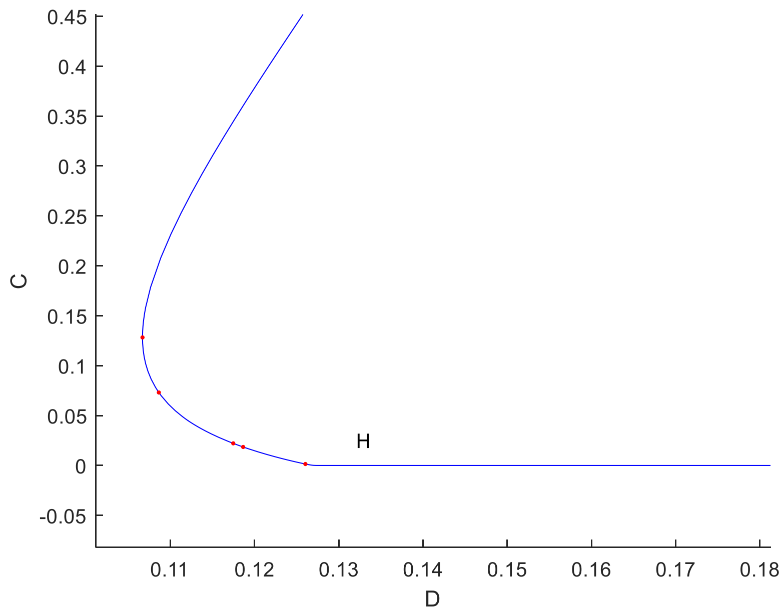

Figure 9. Hopf Bifurcation for Saccharomyces Cerevisiae Fermentation Problem.

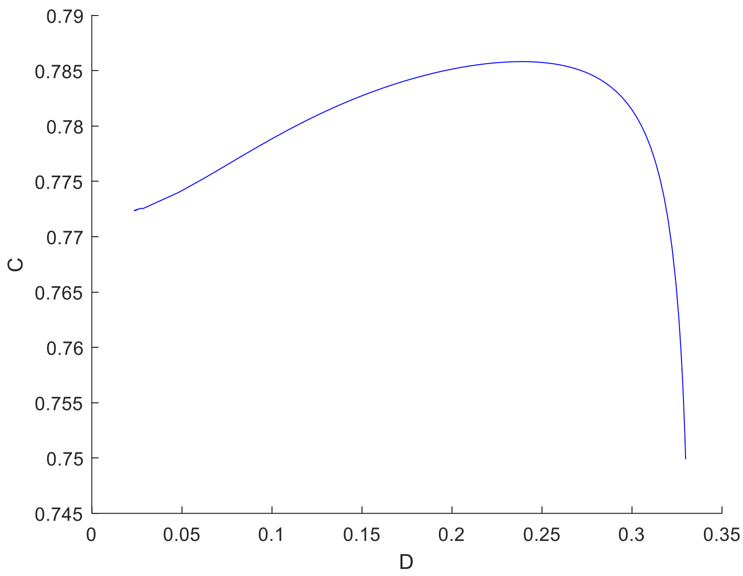

Figure 10. Hopf Bifurcation for Saccharomyces Cerevisiae Fermentation Eliminated by activation factor.

The figures show that each Hopf bifurcation point was eliminated without any undesirable side effects demostrating that the use of the activation factor is generic. Normally experimental/practical strategies are needed to eliminnate the Hopf bifurcations. This was especially true in the fermentation problems as shown by Sridhar [14]. These experimental strategies cause reduction in the quantity of the product obtained. The figures clearly demonstrate that the activation factor eliminates the Hopf bifurcations without too many undesirable side effects.

6. Explanation of why the tanh factor eliminates the Hopf bifurcation point

The tanh factor is very effective in eliminating spikes that occur in control profiles. Hopf bifurcation points cause oscillatory behavior which are similar to spikes and the examples described demonstrate the effectiveness of the tanh factor in eliminating the Hopf bifurcation by preventing the occurrence of oscillations. **Figures 11** and **12** demonstrate this fact. **Figure 11** shows sin$x$ vs x and the waves (oscillations) are clearly visible, however a plot of sin x vs tanhx (**Figure 12**) demonstrates an absence of the oscillations. The tanh factor takes all the variables to $(-1,1)$ causing the oscillations to disappear resulting in a non-oscillatory curve and thereby eliminating the Hopf bifurcations.

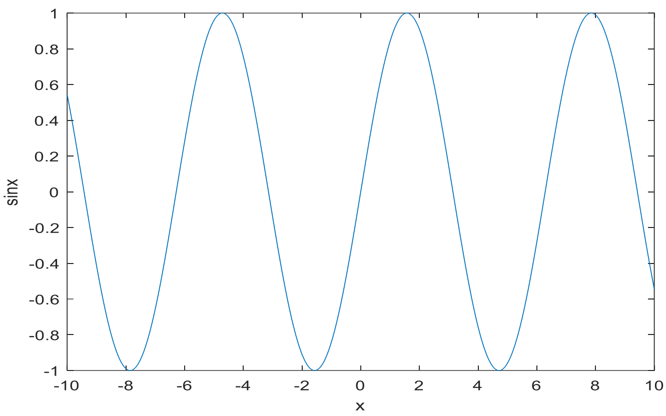

Figure 11. Sin x vs. x showing oscillations.

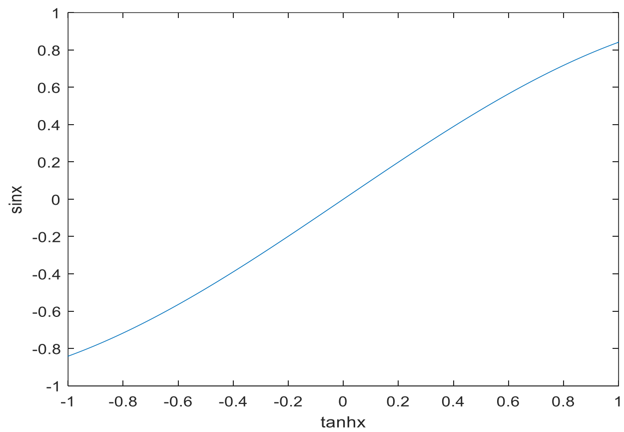

Figure 12. Sin x vs. tanh x oscillations eliminated.

7. Conclusion

Two classic books on Hopf bifurcations [19,20] show several properties of Hopf bifurcation points. They demonstrate that the Hopf bifurcations cause oscillatory behavior and limit cycles. There exist no articles in the open literature that demonstrate techniques to eliminate these problematic Hopf bifurcations. This research is the first attempt to provide a generic strategy to remove these Hopf bifurcations. The results indicate that the hyperbolic tangent activation factor eliminates the Hopf bifurcations without any unwanted side effects. Five different problems that exhibit Hopf bifurcations were considered. It is shown that the tanh activation factor successfully eliminates the undesirable oscillation causing Hopf bifurcations in all these problems.

The strength of this work lies in the elimination of the damaging oscillatory behavior by the use of a reliable activation factor. Future work involves dealing with problems that have oscillations and multiple steady states

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Conflict of interest: The author declares no conflict of interest.

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