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Article

The Jacobsthal-Collatz-Terras model of conjecture the natural numbers in $\kappa q+1$ problems

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Abstract: In the work, the unity of the model in both directions of the change of the power of two of the conjecture of natural numbers structured in the form of a set parametrized by a set of odd θ sequences $\theta\times 2^n$ is justified for the first time. It is shown that the graphs of the direct $n(tst)\rightarrow\infty$ and reverse $n\rightarrow 0$ conjecture of numbers are correctly displayed by the branching diagram of the sequences oriented along the time axis of the full stop of Terrase. The distance between neighbouring nodes is shown to correlate with the Collatz function. The distance $\delta_{m(p),\kappa}=\alpha_\kappa C_{\kappa q\pm1}$ between adjacent nodes is shown to be correlated with the Collatz function. The obtained formula for calculating the period $T_\kappa=ln_2(1+\alpha_\kappa\kappa)$ according to the degree of formation of powers n. Based on the analysis of regularities of recurrent Jacobsthal numbers and Terras complete stop time, it is shown that the Collatz problem is a partial case of the general Jacobsthal-Collattz-Terrase model of the conjecture of numbers $\mathbb{N}$ in both directions of the change of the power of two. Based on this model, the formation of $_{tst}\{q\}$ sequences was established for numbers with the same lengths as the Collatz sequence CS_q.

Keywords: recurrence Jacobsthal number; Terrase total stopping time; Collatz conjecture; natural numbers

2020 Mathematics Subject Classification: 37P99; 11Y16; 11A51; 11-xx; 11Y50

1. Introduction

The Collatz conjecture is an arithmetic problem that involves the transformation of the set of natural numbers $q\in\mathbb{N}$ by the following algorithm:

$$C_{3q+1} = if \quad q\equiv 0 \mod 2 \quad then \quad \frac{q}{2} \quad else \quad C_{3q+1}=3q+1 \tag{1}$$

In Equation (1), the set of even numbers $q_{even}(q_e)\in\mathbb{N}_{even}(\mathbb{N}_e)$ is conjectured by the function $C_{q/2}=\frac{q_e}{2}$ and the set odd numbers $q_{odd}(q_o)\in\mathbb{N}_{odd}(\mathbb{N}_o)$ is conjectured by the function $C_{3q+1}=3q+1$. According to the Collatz hypothesis [1], conjectures (1) of natural numbers terminate at one [2].

Not many problems in arithmetic have undergone such intensive research as the Collatz problem. Their list of works is a huge information array, the analysis of the results of which was first done by Lagaria [1], and the results obtained relatively recently were professionally done by the authors [3–6]. A wide range of others, incluing probabilitistic stuies of the Collatz problem, have been made in articles [7–14].

In works [15–17], studies of the role of recurrent Jacobsthal numbers in the Collatz problem have been started. It was shown [18] that their incorrect application can lead to a false conclusion. For the first time, the author [19] drew attention to Jacobsthal numbers in the Collatz problem, and recently the partial results of the work [15] were repeated by the author [20].

Jacobsthal numbers are numbers $J_n = \frac{1}{3}(2^n - (-1)^n)$ $(n \in \mathbb{N} = \mathbb{N}_o \cup \mathbb{N}_e \cup \{0\})$ with initial conditions, $J_{0_\kappa} = 0$, $J_1 = 1$ [19] as Jacobsthal numbers, and with initial conditions $J_0 = 2$, $J_1 = 1$, known as Jacobsthal-Luke numbers [21]. Jacobsthal numbers include Mersenne numbers of the type $M_{-,n} = 2^n - 1$ and also numbers of the type $M_{+,n} = 2^k + 1$. The numbers include the Fermat numbers $2^k + 1$ at $k = 2^n$. The role of Mersenne numbers in the Collatz problem was studied in works [22,23].

Despite its simplicity, Collatz's problem is still relevant today for many branches of mathematics, such as number theory, dynamical systems, ergodic theory, mathematical logic and algorithm theory, random processes and probability theory. In this paper, based on the regularities of Jacobsthal recurrence numbers and the Terrass complete stop time (tst) [24], a general graph-analytic model of the conjecture of numbers $q \in \mathbb{N}$ is developed:

$$\begin{aligned} C_{\kappa q \pm 1} \quad = \quad if \quad q \equiv 0 \quad \text{mod2} \quad then \quad \frac{q}{2} \quad else \\ C_{\kappa q \pm 1} = \kappa q \pm 1, \quad \kappa = 1,\, 3,\, 5,\, \ldots \in \mathbb{N} \end{aligned} \tag{2}$$

in the forward $n \to +\infty$ and reverse $n \to 0$ directions of the change in powers of two 2^n.

2. Basic results and discussion

We formulate the necessary definitions for the rest of the paper.

Definition 1. *Nodes are points* $\theta \times 2^n$ *in sequences* $\{\theta \times 2^n\}_{n=0}^{n=\infty}$ *of Jacobsthal numbers:*

$$m(p)_{\kappa,\theta,n} = \frac{1}{\kappa}\Big[\theta \times 2^n \mp 1\Big], \quad \theta, \kappa = 1,\, 3,\, 5,\, \ldots \in \mathbb{N}_o \tag{3}$$

where equalities hold:

$$\theta \times 2^n = \kappa m(p)_{\kappa,\theta,n} \pm 1 \tag{4}$$

Definition 2. *Nodes from which sequences* $\{\theta \times 2^n\}_{n=0}^{n=\infty}$ *with branching points are generated are active; otherwise, they are inactive.*

Definition 3. *The* T_κ *period is the distance in powers* n *of two* 2^n *between two adjacent nodes* $m_{\kappa,\theta,n}$ *of the same type for the* $C_{\kappa q+1} = \kappa q + 1$ *conjecture and* $p_{\kappa,\theta,n+T_\kappa}$ *for* $C_{\kappa q-1} = \kappa q - 1$.

For example, the period between two adjacent nodes with the numbers $m_{3,1,4} = 5$ and $m_{3,1,6} = 21$ is equal to $T_\kappa = 6 - 4 = 2$.

Definition 4. *The point attractor* (PA) *is the smallest odd number in the trivial periodic cycle of the completion of the Collatz process. According to Collatz's hypothesis, the conjecture (1) of an arbitrary natural number ends with a trivial cycle* $\ldots \to 4 \to 2 \to 1 \to \ldots$*, where the minimum odd number is equal to one, therefore* $PA = 1$.

Definition 5. *The oriented Jacobsthal-Terrase diagram* $(OJTD\,)$ *is a graph of* $\{\theta \times 2^n\}_{n=0}^{n=\infty}$ *sequences branching in the direction* $tst(n) \to +\infty$.

Problems of type Equation (2) are based on two types of arithmetic operations: halving $q_e \to q_e/2$ an even q_e number to an odd value and converting $q_o \to q_e$ an odd q_o number into an even number. For their implementation, sequences $\{\theta \times 2^n\}_{n=0}^{n=\infty}$ on a binary basis, among which the root $\{1 \times 2^n\}_{n=0}^{n=\infty}$ is used to achieve a single value by the Collatz sequence (CS_q).

The first element of the sequences $\{\theta \times 2^n\}_{n=0}^{n=\infty}$ is odd, and the others are even. Therefore, the set $q \in \mathbb{N}$ can be represented as their set. As shown in **Figure 1**, then the task of this work will be reduced to establishing the rules of superposition of sequences $\{q_0 \times 2^n\}_{n=0}^{n=\infty}$ at nodes (points $\theta \times 2^n$) with recurrent numbers Equation (3) [15–17].

Numbers $m_{\kappa,\theta,n} = \dfrac{1}{\kappa}[\theta \times 2^n - 1]$ form nodes at points $\theta \times 2^n$ of sequences $\{\theta \times 2^n\}_{n=0}^{n=\infty}$ for problems $C_{\kappa q+1} = \kappa q + 1$, and numbers $p_{\kappa,\theta,n} = \dfrac{1}{\kappa}[\theta \times 2^n + 1]$, respectively, for problems $C_{\kappa q-1} = \kappa q - 1$. Combined set

$$\{m_{\kappa,\theta,n}\}_{n=0}^{n=\infty} \cup \{p_{\kappa,\theta,n}\}_{n=0}^{n=\infty} = \{J^{\pm}_{\kappa,\theta,n}\}_{n=0}^{n=\infty} \tag{5}$$

forms the set of Jacobsthal numbers $J^{\mp}_{\kappa,\theta,n}$ calculated by the Binet formula

$$J^{\mp}_{\kappa,\theta,n} = \frac{1}{\kappa}[\theta \times 2^n \mp (-1)^n]. \tag{6}$$

n	$1\cdot 2^n$	$3\cdot 2^n$	$5\cdot 2^n$	$7\cdot 2^n$	$9\cdot 2^n$	$11\cdot 2^n$	$13\cdot 2^n$
0	$1\cdot 2^{0}$	$3\cdot 2^{0}$	$5\cdot 2^{0}$	$7\cdot 2^{0}$	$9\cdot 2^{0}$	$11\cdot 2^{0}$	$13\cdot 2^{0}$
1	$1\cdot 2^{1}$	$3\cdot 2^{1}$	$5\cdot 2^{1}$	$7\cdot 2^{1}$	$9\cdot 2^{1}$	$11\cdot 2^{1}$	$13\cdot 2^{1}$
2	$1\cdot 2^{2}$	$3\cdot 2^{2}$	$5\cdot 2^{2}$	$7\cdot 2^{2}$	$9\cdot 2^{2}$	$11\cdot 2^{2}$	$13\cdot 2^{2}$
3	$1\cdot 2^{3}$	$3\cdot 2^{3}$	$5\cdot 2^{3}$	$7\cdot 2^{3}$	$9\cdot 2^{3}$	$11\cdot 2^{3}$	$13\cdot 2^{3}$
4	$1\cdot 2^{4}$	$3\cdot 2^{4}$	$5\cdot 2^{4}$	$7\cdot 2^{4}$	$9\cdot 2^{4}$	$11\cdot 2^{4}$	$13\cdot 2^{4}$
5	$1\cdot 2^{5}$	$3\cdot 2^{5}$	$5\cdot 2^{5}$	$7\cdot 2^{5}$	$9\cdot 2^{5}$	$11\cdot 2^{5}$	$13\cdot 2^{5}$
6	$1\cdot 2^{6}$	$3\cdot 2^{6}$	$5\cdot 2^{6}$	$7\cdot 2^{6}$	$9\cdot 2^{6}$	$11\cdot 2^{6}$	$13\cdot 2^{6}$
7	$1\cdot 2^{7}$	$3\cdot 2^{7}$	$5\cdot 2^{7}$	$7\cdot 2^{7}$	$9\cdot 2^{7}$	$11\cdot 2^{7}$	$13\cdot 2^{7}$
8	$1\cdot 2^{8}$	$3\cdot 2^{8}$	$5\cdot 2^{8}$	$7\cdot 2^{8}$	$9\cdot 2^{8}$	$11\cdot 2^{8}$	$13\cdot 2^{8}$
9	$1\cdot 2^{9}$	$3\cdot 2^{9}$	$5\cdot 2^{9}$	$7\cdot 2^{9}$	$9\cdot 2^{9}$	$11\cdot 2^{9}$	$13\cdot 2^{9}$
10	$1\cdot 2^{10}$	$3\cdot 2^{10}$	$5\cdot 2^{10}$	$7\cdot 2^{10}$	$9\cdot 2^{10}$	$11\cdot 2^{10}$	$13\cdot 2^{10}$
11	$1\cdot 2^{11}$	$3\cdot 2^{11}$	$5\cdot 2^{11}$	$7\cdot 2^{11}$	$9\cdot 2^{11}$	$11\cdot 2^{11}$	$13\cdot 2^{11}$
12	$1\cdot 2^{12}$	$3\cdot 2^{12}$	$5\cdot 2^{12}$	$7\cdot 2^{12}$	$9\cdot 2^{12}$	$11\cdot 2^{12}$	$13\cdot 2^{12}$

Figure 1. Illustration of the structuring of the set $q \in \mathbb{N}$ as a set of parameterized θ sequences $\{\theta \times 2^n\}_{n=0}^{n=\infty}$.

According to Equaion (4), in the $n \to +\infty$ direction of increasing n powers of two 2^n an arbitrary number $q = \theta \times 2^k$, $k = 0, 1, 2, 3, \ldots, n, \ldots$ doubles until the branching condition is fulfilled

$$\frac{\theta \times 2^n \mp 1}{\kappa} = m(p)_{\kappa,\tau,0}. \tag{7}$$

This is how a directed graph branches, known as a Jacobsthal tree [15]. Therefore, points Equation (7) of sequences $\{\tau \times 2^n\}_{n=0}^{n=\infty}$ with numbers $m(p)_{\kappa,\tau,n}$ are nodes from which other sequences $\{m(p)_{\kappa,\tau,0} \times 2^n\}_{n=0}^{n=\infty}$ are generated.

In the reverse $n \to 0$ direction, in active nodes with numbers according to the rule.

$$\kappa m(p)_{\kappa,\tau,0} \pm 1 = \theta \times 2^n \tag{8}$$

the sequences $\{\kappa m(p)_{\kappa,\tau,0} \times 2^n\}_{n=0}^{n=\infty}$ merge, forming the well-known Collatz sequences CS_q, which either go to point attractors (PA), or grow infinitely, as a sequence of conjecture of a unit by a function $C_{9q+1} = 9q+1$ [15] and a number $q = 7$ by a function $C_{5q+1} = 5q+1$ [25], or are isolated from the root $\{1 \times 2^n\}_{n=0}^{n=\infty}$, as a sequences of conjectures $C_{7q-1} = 7q-1$ [26].

Let's investigate the regularities of the formation of numbers $m(p)_{\kappa,\theta,n}$ of nodes. Knowing the numbers $m(p)_{1,\theta,n}$, in addition to Equation (3), the numbers $m(p)_{\kappa>1,\theta,n}$ can also be calculated by the selection method

$$m(p)_{\kappa>1,\theta,n} = \frac{m(p)_{1,\theta,n}}{\kappa} \tag{9}$$

as shown in **Figure 2** for the interval $\kappa = 3 \div 7$.

$p_{7,1,n}$																
$p_{5,1,n}$			1				13				205				3277	
$p_{3,1,n}$		1		3		11		43		171		683		2731		10923
$p_{1,1,n}$	2	3	5	9	17	33	65	129	257	513	1025	2049	4097	8193	16385	32769
2^n	2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}	2^{11}	2^{12}	2^{13}	2^{14}	2^{15}
$m_{1,1,n}$	0	1	3	7	15	31	63	127	255	511	1023	2047	4095	8191	16383	32767
$m_{3,1,n}$	0		1		5		21		85		341		1365		5457	
$m_{5,1,n}$	0				3				51				819			
$m_{7,1,n}$	0			1			9			73			585			4681

Figure 2. Different colors represent different periods of recurrent number formation $m(p)_{3\div7,1,n}$.

The numbers $m(p)_{3\div7,1,n}$ that form the nodes at the points of the root sequence $\{1 \times 2^n\}_{n=0}^{n=\infty}$ according to the rules Equation (4) are highlighted in color. Empty cells correspond to numbers with fractional values. Therefore, we substantiate the statement for the number of nodes.

Satement 1. ***If the*** θ ***parameter of number*** $m(p)_{\kappa,\theta,n} = \frac{1}{\kappa}[\theta \times 2^n \mp 1]$ ***(***$\theta, \kappa = 1, 3, 5, \ldots \in \mathbb{N}_{odd}$***) is a multiple*** κ ***(***$\theta_\kappa = Integer \times \kappa = I \times \kappa$***), then the numbers*** $m(p)_{\kappa,\theta,n}$ ***are fractional.***

Proof. If $\theta_\kappa = I \times \kappa$ then

$$m(p)_{\kappa,\theta_\kappa,n} = \frac{1}{\kappa}[\kappa \times 2^n \mp 1] = 2^n \mp \frac{1}{\kappa} \neq I. \tag{10}$$

The statement is well-founded. This statement is relevant for the cases $\kappa > 1$, since if $\kappa = 1$, then the numbers $m(p)_{1,1_\kappa,n} = 2^n \mp 1 = M_{\mp,n}$ are integers. □

The numbers $m_{\kappa,\theta,n}$ and $p_{\kappa,\theta,n}$ are the numbers of nodes at the points $\theta \times 2^n$ of the sequences $\{\theta \times 2^n\}_{n=0}^{n=\infty}$ for the transformations $C_{\kappa q+1} = \kappa q + 1$ and $C_{\kappa q-1} = \kappa q - 1$. From nodes with multiples κ of the parameter θ, other sequences are branched (merged) on a binary basic. Therefore, such nodes can be considered active. If the number $m(p)_{\kappa,\theta,n}$ of nodes is a multiple of κ, then there are no points with active nodes in the sequences $\{m(p)_{\kappa,\theta,0} \times 2^n\}_{n=0}^{n=\infty}$. For example, at the points of the sequence $\{5 \times 2^n\}_{n=0}^{n=\infty}$, for the model $\kappa = 3$, the numbers $m(p)_{\kappa,\theta,n}$ are equal to: $m(p)_{3,5,n}$: 2(3), 7(13), 27(53), 107(213), 427(853), ... Therefore, the nodes in the sequence $\{5 \times 2^n\}_{n=0}^{n=\infty}$ points with numbers $p_{3,5,n}$: 7, 27, 107, 427, . . . are active for the conjecture C_{3q-1}, and the nodes with numbers $m_{3,5,n}$: 3, 13, 53, 213, 853, . . . are active for the conjecture C_{3q+1}.

It is known [24] that Terras developed a theoretical model of the so-called complete stopping time (tst), which determines the length of the sequence CS_q and the number of iterations N during which it reaches the point attractor PA. Therefore, let's analyze the patterns of direct and reverse number conversion problems, for which we will build an oriented diagram $OJTD$ branching in the direction of tst.

Equality is fulfilled in the nodes of the root sequence $\{1 \times 2^n\}_{n=0}^{n=\infty}$.

$$tst = n \tag{11}$$

that is, time tst coincides with the power of two 2^n. Therefore, we will construct $OJTD$ for one of the two pairs $C_{3q\pm1} = 3q \pm 1$ or $C_{5q\mp1} = 5q \mp 1$ conjectures with a point attractor $PA = 1$. For a pair $C_{3q\pm1} = 3q \pm 1$, the fragment $OJTD$ has the form shown on **Figure 3**. $m(p)_{3,1,n}$ are also highlighted in yellow, from which sequences $\{\theta \times 2^n\}_{n=0}^{n=\infty}$ are generated, one of which is shown in blue for the number $q = 11$.

												768
											384	120
										192		118
									96		60	115
$3q-1$								48		30	59	704
						12	24		15	176	352	688
					6		22	44	88	172	344	684
		1		3		11		43	86	171	342	683
$PA = 1$	2	4	8	16	32	64	128	256	512	1024	2046	4096
	0		1		5		21		85		341	682
						10		42	84	170	340	680
							20		80	168	336	113
$3q+1$							3	40	13	160	320	672
								6		26	53	640
									12		52	106
										24		104
											48	17
												96
tst	1	2	3	4	5	6	7	8	9	10	11	12

Figure 3. $OJTD$ model in the branching direction $n \to +\infty$.

As we can see from **Figure 3**, the diagram $OJTD$ structures the set $q \in \mathbb{N}$ by the parameter tst. In each column, parametrized by tst sequences CS_q of the same length are formed, one of which for $q = 104$ over time $tst = 12$ is shown in red. We see that sequences CS_q of numbers $\{17, 96, 104, 106, 113, 640, 672, 680, 682, 4096\}_{tst=1}^{tst=\infty}$ have the same length. In general, the established regularities $OJTD$ appear for C_{3q-1}

with other point attractors $PA = 5$ and $PA = 17$, as well as with C_{5q+1} with $P = 1,\ 13,\ 17$. Recall that the sequence CS_q with $PA \neq 1$ is isolated from the root $\{1 \times 2^n\}_{n=0}^{n=\infty}$ sequence [15,17,25,26]. For isolated sequences CS_q, the starting point is the value of the corresponding attractor PA.

So, in the direction $n \to +\infty$ of the diagram $OJTD$, they form a set of trajectories for sequences CS_q an illustration of which is given in **Figure 4** for a set of numbers $_{tst=7}\{3,\ 29,\ 21,\ 128\}$. We see that during the same time $tst = 7$, they all reach a single point attractor $PA = 1$ in a trivial cycle

$$cycle_{1\to 4\to 1}^{3q+1} = \{\ldots \to 1 \to 4 \to 2 \to 1 \to \ldots\}. \tag{12}$$

A similar regularity is true for other models of number transformation, including with other completion attractors. In other words, type $OJTD$ diagrams reflect patterns of structuring of the set $q \in \mathbb{N}$ according to the parameter. The number of numbers in the columns increases according to the power law, therefore, the construction of a graph of the type in **Figure 4** [27] requires correction $q \to \langle q \rangle$ by the average $\langle q \rangle$ values of the numbers q within each sequence $_{tst}\{q\}$.

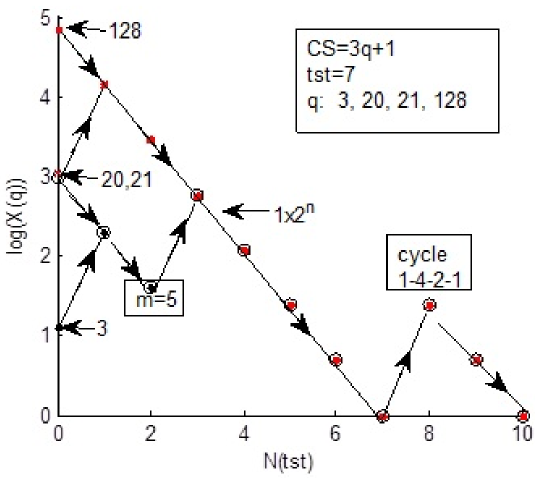

Figure 4. Illustration of the formation of a set of sequences CS_q of the same length for a set of numbers $_{tst=7}\{3,\ 29,\ 21,\ 128\}$ over time $tst = 7$.

Pairs of conjectures $C_{\kappa q \pm 1} = \kappa q \pm 1$ have the peculiarity that the union of sets of numbers $\{m(p)_{\kappa,\theta,n}\}_{n=0}^{n=\infty}$ forms a single set Equation (5) of recurring Jacobsthal numbers Equation (6). Therefore, we systematize recurrence relations for numbers $m(p)_{\kappa,\theta,n}$ expressing the correlation between conjectures of the type $C_{\kappa q \pm 1} = \kappa q \pm 1$:

The Equation (12) agree with data on **Figure 2**. If the formula $m_{3,\theta,n+T_3} = 4m_{3,\theta,n} + 1$ is known [28], then the rest of the formulas are obtained for the first time. Ratio Equation (**Table 1**) can be summarized as:

$$m(p)_{\kappa,\theta,n+T_\kappa} = \beta_\kappa m(p)_{\kappa,\theta,n} \pm \alpha_\kappa. \tag{13}$$

Table 1. Formulas $m(p)_{\kappa,\theta,n+T_m}$ for the first $\kappa = 1 \div 23$ transformation models

$\boldsymbol{\kappa = 1:}$	$\boldsymbol{m(p)_{1,\theta,n+T_1} = 2m(p)_{1,\theta,n} \pm 1}$	$\Rightarrow$ $\Rightarrow$	$\boldsymbol{\delta_{m(p),1} = m(p)_{1,\theta,n} \pm 1}$ $\boldsymbol{\alpha_1 = 1}$
$\boldsymbol{\kappa = 3:}$	$m(p)_{3,\theta,n+T_3} = 2^2 m(p)_{3,\theta,n} \pm 1$	$\Rightarrow$ $\Rightarrow$	$\delta_{m(p),3} = 3m(p)_{3,\theta,n} \pm 1$ $\alpha_3 = 1$
$\boldsymbol{\kappa = 5:}$	$m(p)_{5,\theta,n+T_5} = 2^4 m(p)_{5,\theta,n} \pm 3$	$\Rightarrow$ $\Rightarrow$	$\delta_{m(p),5} = 3(5m(p)_{5,\theta,n} \pm 1)$ $\alpha_5 = 3$
$\boldsymbol{\kappa = 7:}$	$m_{7,\theta,n+T_7} = 2^3 m_{7,\theta,n} + 1$	$\Rightarrow$ $\Rightarrow$	$\delta_{m,7} = 9(7m_{7,\theta,n} + 1)$ $\alpha_7 = 9$
$\boldsymbol{\kappa = 9:}$	$m(p)_{9,\theta,n+T_9} = 2^6 m(p)_{9,\theta,n} \pm 7$	$\Rightarrow$ $\Rightarrow$	$\delta_{m(p),9} = 7(9m(p)_{9,\theta,n} \pm 1)$ $\alpha_9 = 7$
$\boldsymbol{\kappa = 11:}$	$m(p)_{11,\theta,n+T_{11}} = 2^{10} m(p)_{11,\theta,n} \pm 93$	$\Rightarrow$ $\Rightarrow$	$\delta_{m(p),11} = 93(11m(p)_{11,\theta,n} \pm 1)$ $\alpha_{11} = 93$
$\boldsymbol{\kappa = 13:}$	$m(p)_{13,\theta,n+T_{13}} = 2^{12} m(p)_{13,\theta,n} \pm 315$	$\Rightarrow$ $\Rightarrow$	$\delta_{m(p),13} = 315(13m(p)_{13,\theta,n} \pm 1)$ $\alpha_{13} = 315$
$\boldsymbol{\kappa = 15:}$	$m_{15,\theta,n+T_{15}} = 2^4 m_{15,\theta,n} + 1$	$\Rightarrow$ $\Rightarrow$	$\delta_{m,15} = 15m_{15,\theta,n} + 1$ $\alpha_{15} = 1$
$\boldsymbol{\kappa = 17:}$	$m(p)_{17,\theta,n+T_{17}} = 2^8 m(p)_{17,\theta,n} \pm 15$	$\Rightarrow$ $\Rightarrow$	$\delta_{m(p),17} = 15(17m(p)_{17,\theta,n} \pm 1)$ $\alpha_{17} = 15$
$\boldsymbol{\kappa = 19:}$	$m(p)_{19,\theta,n+T_{19}} = 2^{18} m(p)_{19,\theta,n} \pm 13797$	$\Rightarrow$ $\Rightarrow$	$\delta_{m(p),19} = 13797(19m(p)_{19,\theta,n} \pm 1)$ $\alpha_{19} = 13797$
$\boldsymbol{\kappa = 21:}$	$m_{21,\theta,n+T_{21}} = 2^6 m_{21,\theta,n} + 3$	$\Rightarrow$ $\Rightarrow$	$\delta_{m,21} = 3(21m_{21,\theta,n} + 1)$ $\alpha_{21} = 3$
$\boldsymbol{\kappa = 23:}$	$m_{23,\theta,n+T_{23}} = 2^{11} m_{23,\theta,n} + 89$	$\Rightarrow$ $\Rightarrow$	$\delta_{m,23} = 89(23m_{23,\theta,n} + 1)$ $\alpha_{23} = 89$

In Equation (13), the facto β_κ is even, and the term α_κ is odd (see **Table 1**) and equality holds for them

$$\beta_\kappa = \kappa\alpha_\kappa + 1 \tag{14}$$

which is confirmed by calculations:

Then, substituting Equation (14) into Equation (13), we obtain

$$\alpha_\kappa = \frac{m(p)_{\kappa,\theta,n+T_\kappa} - m(p)_{\kappa,\theta,n}}{\kappa m(p)_{\kappa,\theta,n} \pm 1} = \frac{\delta_{m(p),\kappa}}{\kappa m(p)_{\kappa,\theta,n} \pm 1}, \tag{15}$$

where the calculated $\delta_{m(p),\kappa}$ are given in the **Table 2**. For example, for $\theta = 1, \kappa = 9 : \alpha_9 = \frac{m_{9,1,12} - m_{9,1,6}}{9m_{9,1,6}+1} = \frac{455-7}{7\times9+1} = \frac{448}{64} = 7$, which is consistent with the formula $m(p)_{9,\theta,n+6} = 64m(p)_{9,\theta,n} \pm 7$ in Equation (**Table 1**).

Nodes $m(p)_{\kappa,\theta,n}$ of conjectures $C_{\kappa q\pm1}$ are formed with a period T_κ to the power of two. Therefore, we justify the following statement for the period T_κ:

Satement 2. ***The period*** T_κ ***to the power*** n ***of two*** 2^n ***between nodes*** $m(p)_{\kappa,\theta,n+T_\kappa}$ ***and*** $m(p)_{\kappa,\theta,n}$.

$$T_\kappa = ln_2(1 + \alpha_\kappa\kappa) = ln_2\left(1 + \frac{\kappa m(p)_{\kappa,\theta,n+T_\kappa}}{\kappa m(p)_{\kappa,\theta,n} \pm 1}.\right) \tag{16}$$

Proof. For the points $\theta \times 2^n$ and $\theta \times 2^{n+T_\kappa}$ the relations $\theta \times 2^n = \kappa m(p)_{\kappa,\theta,n}$ and $\theta \times 2^{n+T_\kappa} = \kappa m(p)_{\kappa,\theta,n+T_\kappa} \pm 1$, therefore $\theta \times 2^n(2^{T_\kappa} - 1) = \kappa(m(p)_{\kappa,\theta,n+T_\kappa} - m(p)_{\kappa,\theta,n}) = \kappa\delta_{m(p),\kappa} = \kappa\alpha_\kappa(\kappa m(p)_{\kappa,\theta,n} \pm 1)$, from where $2^{T_\kappa} = 1 + \kappa\alpha_\kappa$. *The* *statement is proved.*

Let's calculate the period using Equation (16) $T_{1\div 25}$.

Data (from **Table 3**) are consistent with **Figure 2**, so Equation (13) can also be written as:

$$m(p)_{\kappa,\theta,n+T_\kappa} = 2^{T_\kappa} m(p)_{\kappa,\theta,n} \pm \alpha_\kappa. \tag{17}$$

Table 2. Values of α_κ, $\beta_\kappa = \kappa\alpha_\kappa + 1$ and formulas for calculating $\delta_{m(p),\kappa}$ in the interval $\kappa = 1 \div 25$.

κ	α_κ	$\beta_\kappa = \kappa \times \alpha_\kappa + 1$	$\delta_{m(p),\kappa}$
1	± 1	$2 = 1 \times 1 + 1$	$1m(p)_{1,\theta,n} \pm 1$
3	± 1	$4 = 3 \times 1 + 1$	$3m(p)_{3,\theta,n} \pm 1$
5	± 3	$16 = 5 \times 1 + 1$	$3(5m(p)_{5,\theta,n} \pm 1)$
7	$+9$	$64 = 7 \times 9 + 1$	$9(7m_{7,\theta,n} + 1)$
9	± 7	$64 = 9 \times 7 + 1$	$7(9m(p)_{9,\theta,n} \pm 1)$
11	± 93	$1024 = 11 \times 93 + 1$	$93(11m(p)_{11,\theta,n} \pm 1)$
13	± 315	$4096 = 13 \times 315 + 1$	$315(13m(p)_{13,\theta,n} \pm 1)$
15	$+17$	$256 = 15 \times 17 + 1$	$17(15m_{15,\theta,n} + 1)$
17	± 15	$256 = 17 \times 15 + 1$	$15(17m(p)_{17,\theta,n} \pm 1)$
19	± 13797	$262144 = 19 \times 13797 + 1$	$13797(19m(p)_{19,\theta,n} \pm 1)$
21	$+3$	$64 = 21 \times 3 + 1$	$3(21m_{21,\theta,n} + 1)$
23	$+89$	$2048 = 23 \times 89 + 1$	$89(23m_{23,\theta,n} + 1)$
25	± 41943	$1048576 = 25 \times 41943 + 1$	$41943(25m(p)_{25,\theta,n} \pm 1)$

Table 3. The value of the period in the interval $\kappa = 1 \div 25$.

κ	1	3	5	7	9	11	13	15	17	19	21	23	25
α_κ	1	1	3	1	7	93	315	1	15	13797	3	89	41943
T_κ	1	2	4	3	6	10	12	4	8	18	6	11	20

The period T_κ is also a parameter of the time tst periodicity of numbers $m(p)_{\kappa,\theta,n}$. For example, as can be seen from **Table 4**, tst for numbers $m_{5,1,n}$, changes with the period $T_\kappa = 4$:

Table 4. The value of tst for Jacobsthal numbers $m_{5,1,n}$.

$m_{5,1,n}$	0	3	51	819	13,107	209,715
tst	0	5	9	13	17	21

For a given θ, the sequence of numbers $m(p)_{\kappa,\theta,n}$ ends with the same attractor. However, as shown on the left in **Figure 5**, for the number $q_o = 7$ conjectured by the function $C_{5q+1} = 5q + 1$, the point attractor of the increasing sequence CS_q is clearly defined, i.e., it does not have a trivial cycle. On the right in **Figure 5** is a diagram $OJTD$ for the number $q_o = 7$.

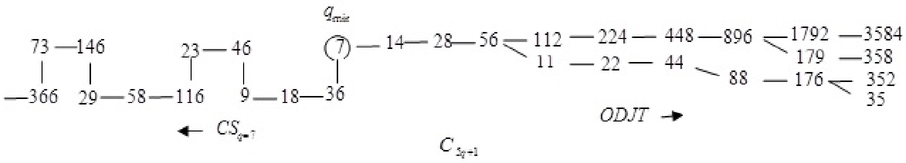

Figure 5. Directions of formation of the Collatz sequence $CS_q = 7$ by the function C_{5q+1} and branching of the sequence $OJTD$ of the number $q = 7$.

In the model C_{5q+1}, in addition to the growing sequence CS_q, three more sequences CS_q with point attractors $PA = 1, 13, 17$. From the point of view of Equation (2), all given types of sequences are equivalent to each other and the corresponding transformations divide the set of numbers $\mathbb{N}$ in the same proportions [15].

3. Conclusion

In the paper, for the first time, the unity of the model of the conjecture of natural numbers structured in the form of a set of parametrized by a set of odd θ sequences $\theta \times 2^n$ in both directions of the change of powers of two is substantiated. It is shown that the graphs of direct $n(tst) \to +\infty$ and reverse $n \to 0$ conjecture of numbers are correctly reflected by the branching diagram of sequences oriented along the time axis of the full stop tst of Terrase. It is shown that the distance $\delta_{m(p),\kappa}$ between neighboring nodes $m(p)_{\kappa,\theta,n}$ correlates with the Collatz function $C_{\kappa q\pm1}$. The obtained formula $T_\kappa = ln_2(1 + \alpha_\kappa \kappa)$ for calculating the period by the powers n of formation of knots. On the basis of the analysis of regularities of recurrent Jacobsthal numbers and Terras complete stop time, it is shown that the Collatz problem is a partial case of the general Jacobsthal-Collatz-Terrase model of the conjecture of numbers $q \in \mathbb{N}$ in both directions of the change of the power of two.

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