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Article

Octagonal-square tessellation model for masting GSM network: A case study of MTN Kumasi-East, Ghana

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Abstract: Masting in GSM network design is one of the most challenging problems in cell planning. The effect of uniform design pattern has been proven geographically to be hexagonal using uniform cell range. In this paper, we present a new uniform greedy semi-regular tessellation model called the octagonal square tessellation model (OSTM) to address the problem of global minimum overlap difference and area. Data from MTN Kumasi-East Ghana was collected and analyzed using the developed model. The original layout for the 0.6 km cell range accounted for an overlap difference of 937.66 m and a total area of 21.41 km^2 for 50 GSM mosts whereas the OSTM model accounted for an overlap difference of 1316.95 m with an area of 34.23 km^2. This is a 59.87% reduction of the original total area. Our solution is shown to be optimal in overlap difference and area for non-uniform cell range.

Keywords: GSM Network; telecommunication; tessellation model; minimum overlap difference; masting

1. Introduction

According to the study by authors in [1, 2] GSM is the fastest growing and the most challenging telecommunication application of all time having over a billionth subscribers connected between January to March of 2004, which is little over a decade after GSM network was first lunched.

Since there is an increase in the number of subscribers on telecommunication networks, players in telecommunication industries are facing challenges in gearing up to face this task [1].

The challenge revolute around system designs. For this situation, the quantity of stations required to cover similar interest points that are secured by a single non-grid station is given. We call attention to that facility location issue, for instance, those that represent our convincing application are all around analysed issues in Discrete Mathematics, Operation Research, and Computer Science, be that as it may, they furthermore address reflections and therefore, an improvement of certifiable issues [2].

Truth be told, GSM radio receiving wires and base stations cannot be set wherever on the plane, their potential territory can be affected by a combination of the physical and money related matters. Along these lines, only a couple of urban regions (or parts of urban territories) appear as though a standard plane that can be exploited by as a potential location of a facility. Because of economic rivalries in telecommunication providers, the interest for a wide transmissions system has increased [1,2].

This has set off the spate of masting of GSM towers in an excessive number of spots. While trying to offer the most extreme system transmission, these communication networks need to raise fewer masts with specific cell range at certain positions within a given area; as the number of masts raised directly influences the cost [2].

The challenge of supporting quickly developing quantities of mobile subscribers, while constrained by limited radio spectrum, is while compelled by restricted radio range, is being looked by cell operators globally [3].

According to Franeshetti et al. [4], the high expectations of execution and accessibility for wireless system has introduced extraordinary difficulties in the modeling and design of shortcoming lenient remote system.

Elements are always points, and the sets are geometric objects such as polygons and disks in the Geometric Set Cover problem. Many algorithms and theorems have been developed in combinatorial geometry which has prompted the family of approximation algorithms for covering problems [4].

The idea of minimum geometric disk cover problem has another vital practical implications on the designs of wireless network in addition to its hypothetical and theoretical interest. In communication wireless networks, each Base Station (BS) is equipped with a Radio Frequency (RF) transceivers to provide stable and reliable transmission inside a circular range, referred to as a disk, within some bounded distance [5].

To find the number of BS to cover all subscribers in location, geometric problems are naturally considered. Here the transmitters can be modelled as disk $\mathcal{D}$ and the subscribers as points $\mathcal{P}$ [6].

Now, if we are to find the minimum number of disks to cover a given set of n points in the plane, it becomes an NP-complete problem [7]. For a collection $\mathcal{S}$ of subset of a universe $\mathbb{U}$ and $\mathbb{M} \subset \mathbb{U}$, the cover set problem is to find the smallest sub collection $\mathcal{C} \subset \mathcal{S}$ that covers $\mathbb{M}$ [8].

Covering is a fundamental technique in a wireless communication network. Numerous research has been conducted to find the best method of covering a set of nodes n in an infinitely large area with soft handovers. Current and previous studies have considered using tessellate polygons such as triangle, square, and hexagon to optimally enhance disks covering with small overlapping radius [9]. Given a fixed location, points can be covered using unit disk according to [10].

According to Funke et al. [11], the transfer of information without the use of wires over a distance is literally known as wireless communication, which over the years, has gained tremendous applications. Wireless communication since its invention has not only open doors for the invention of gadgets that uses higher bandwidth but has also initiated the replacement of wired installations.

For a successful application, the wireless network must cover a specific or an entire operating area. In view of this, how to position these wireless transceivers to provide connected coverage becomes a critical research issue [12].

Xu and Song [13] studied several and different restricted coverage problems of which the first problem was about K-coverage. This problem was about how one can deploy wireless nodes such that each target is covered by at least K wireless nodes. Researchers by Yang et al. [14] deployed a minimum number of Relay Stations to satisfy all data rate requests from subscriber stations through cooperative communications.

The union of the closed set is the whole plane such that there exists a disjoint interior for two of these sets. The term tessellation is more modern that is used mostly for special tiling. Tessellation can be achieved by using a single convex polygon (regular tessellation) or several convex polygons (semi-regular tessellation) [15].

According to Grünbaum and Shephard et al. [16], there exist eight semi-regular tessellations that is (Two involving triangles and squares; triangles and hexagons; one involving octagons and squares; triangles and dodecagons; squares, hexagons and dodecagons; and triangles, squares and hexagons) and three regular tessellations which is (equilateral triangle, squares and hexagon).

However, in a study conducted by Pellicer and Williams [17], study of less constrained polytopes is largely small. So they determined the minimal regular covers of 3 of the 8 Archimedean tilings. In a related work conducted by the same authors on minimal covers of Archimedean tilings, they employed a new technique in presenting minimal regular covers of certain periodic abstract polytopes [18].

In each Archimedean tiling, sequence of numbers are usually used to represent each of the 8 tessellation. This sequence is constructed by taken into consideration the faces around each vertex of the covers and writing down the number of edges of each face as shown in **Figures 1–3** [19].

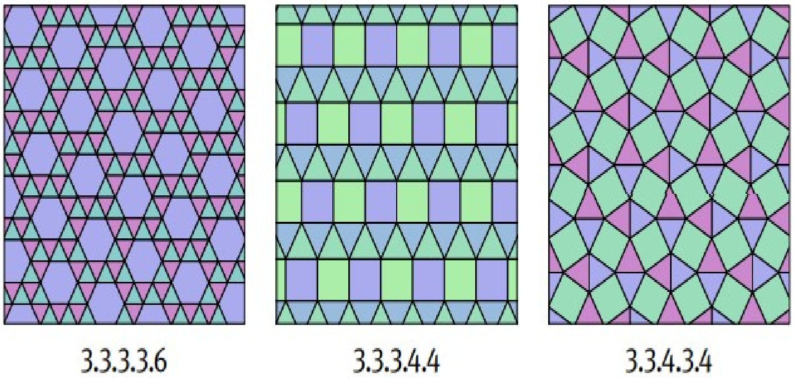


Figure 1. Triangles and hexagons; two triangles and squares.

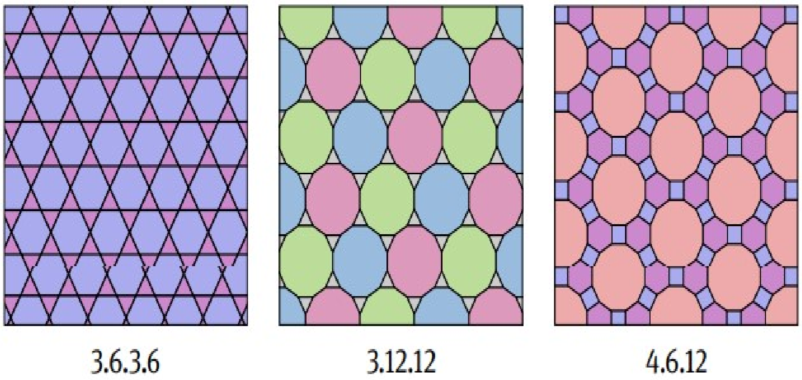


Figure 2. Triangles and hexagons; triangles and dodecagons; squares, hexagons and dodecagons.

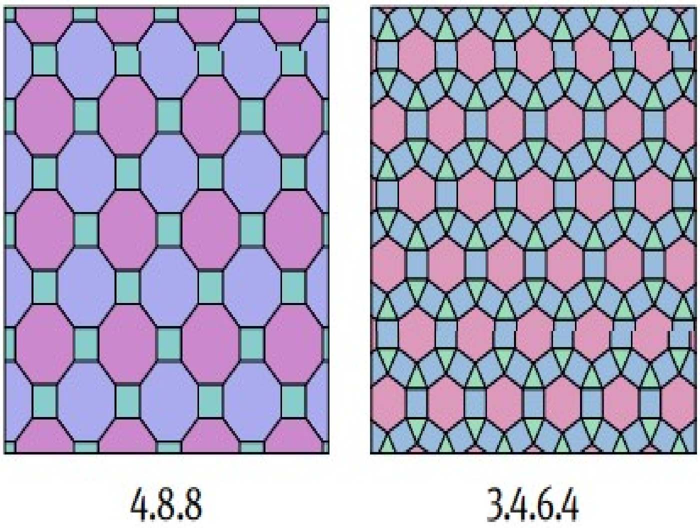
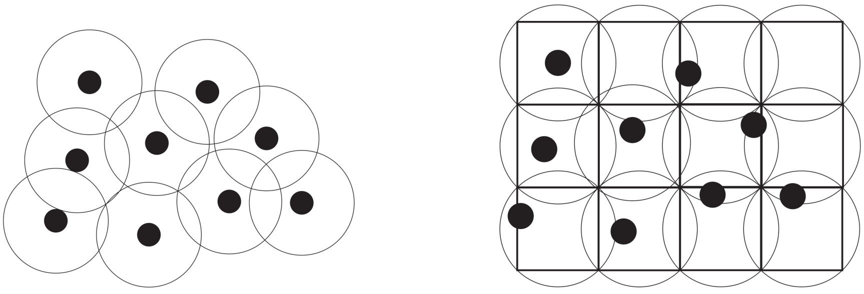


Figure 3. Octagons and squares; triangles, squares and hexagons.

The Set Cover problem is inevitably one of the most important and best-known covering problems there is in geometry. Given a set of elements and a collection of sets, the Set Cover problem is to find the smallest sub-collection to cover all the elements. The Geometric Disk Cover problem represent a geometric version of the general cover set problem which is to check whether all points $\mathcal{P}$ is covered by at least one disk $\mathcal{D}$ with the minimum cardinality $\mathcal{D}^* \subseteq \mathcal{D}$ [20].

It can be seen in **Figure 4** that all points $\mathcal{P}$ is covered with at least one disk $\mathcal{D}$.



(a) Disks covering points.

(b) Square guided covering points.

Figure 4. Cover set problem.

This study as stated earlier only considers the electromagnetic mode of transmission. The graph X = (A, M) is a subgraph of Y = (B, N) if $A \subseteq B$ and $M \subseteq N$

The degree noted as $\deg(\rho)$ of a vertex ρ is the number of its neighbours or simply the number of edges incident on the vertex. Given E as the number of edges and V as a vertex,

$$\sum_{\rho \in V} \deg \rho = 2|E| \tag{1}$$

Equation (1) is know as the Handshaking Lemma.

2. Data

Table 1 shows 50 local grid coordinates of MTN GSM masts for Kumasi-East (Ghana) in zone 30N provided by ATC (Ghana). Eye4software Hydromagic was used in the conversion of geographical and grid coordinate system. **Figure 5** shows the original location of MTN network in the Ashanti region of Ghana.

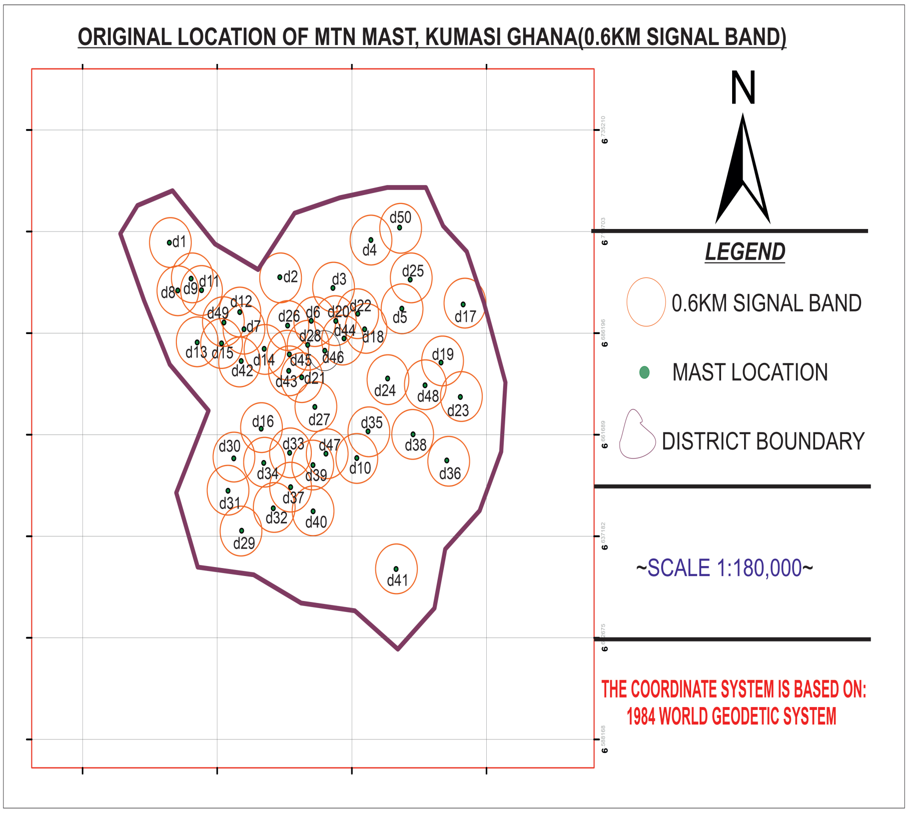


Figure 5. Original layout of GSM Masts, MTN Kumasi East, Ghana.

Table 1. WGS-84 coordinates of MTN masts of Kumasi-Ghana.

MTN Masts with Node Points (WGS-84)				
	Geographical Coordinates		**Grid Coordinates (UTM)**	
Location	**LAT**	**LONG**	**Xm**	**Ym**
OFORIKROM	1°35′31.308″ W	6°40′51.780″ N	655,630.2732	738,712.7884
ASAFO	1°36′38.988″ W	6°41′19.428″ N	653,549.4121	739,556.1153
MARKET 1	1°37′26.799″ W	6°41′47.299″ N	652,078.7343	740,408.0943
MARKET 3	1°37′12.972″ W	6°41′57.516″ N	652,502.4958	740,723.0772
ASAFO MOSQUE	1°36′41.472″ W	6°41′1.356″ N	653,474.6962	739,000.8039
STADIUM	1°36′21.132″ W	6°40′45.984″ N	654,100.6726	738,530.4053
AYIGYA	1°34′46.164″ W	6°41′49.416″ N	657,011.5483	740,487.1305
OFORIKROM	1°35′8.592″ W	6°41′20.904″ N	656,325.3178	739,609.3644
RAILWAYS	1°37′6.744″ W	6°41′2.148″ N	652,698.5293	739,022.9469
MARKET 4	1°37′2.172″ W	6°41′47.508″ N	652,835.0163	740,416.6069
ABOABO	1°35′41.172″ W	6°41′58.776″ N	655,321.466	740,769.7737
KUM ACADEMY	1°33′37.152″ W	6°42′41.868″ N	659,126.1498	742,104.442
SOCIAL CLUB	1°37′35.508″ W	6°42′28.800″ N	651,807.9997	741,682.0457
KASSE	1°36′28.800″ W	6°39′21.456″ N	653,872.5124	735,933.3918
ASOKWA 2	1°36′18.252″ W	6°41′13.668″ N	654,186.7081	739,380.9942
ATONSU 2	1°35′30.984″ W	6°39′26.280″ N	655,647.7204	736,086.5958
ATONSU 1	1°35′29.976″ W	6°38′56.400″ N	655,681.2921	735,168.8894
GYINYASI	1°34′9.804″ W	6°39′44.892″ N	658,139.2574	736,665.4495
AYIDUASI	1°33′49.536″ W	6°40′30.828″ N	658,757.6153	738,078.2453
KNUST	1°34′34.680″ W	6°41′5.568″ N	657,368.1159	739,141.3048
EMENA	1°32′34.260″ W	6°40′14.772″ N	661,070.8858	737,591.8449
ATONSU 3	1°35′6.648″ W	6°38′35.448″ N	656,399.5888	734,527.3699
KASSE 2	1°36′34.812″ W	6°38′53.268″ N	653,690.3083	735,067.0554
GYINYASI 2	1°34′53.364″ W	6°39′25.524″ N	656,803.1661	736,066.679
AHINSAN 2	1°36′0.432″ W	6°39′47.232″ N	654,741.4979	736,727.5861
KENTINKRONO	1°33′26.136″ W	6°41′56.652″ N	659,468.5224	740,716.5581
KENTINKRONO 2	1°33′34.800″ W	6°41′31.499″ N	659,204.719	739,943.1956
AMAKOM	1°35′57.336″ W	6°40′56.568″ N	654,830.5288	738,857.5765
ANLOGA	1°35′31.920″ W	6°40′37.416″ N	655,612.7397	738,271.5294
OLD AHINSAN	1°35′18.420″ W	6°40′31.872″ N	656,027.82	738,102.4257
OFORIKROM 3	1°35′33.108″ W	6°41′16.764″ N	655,572.7999	739,480.04
AYIDUASE 2	1°32′54.240″ W	6°40′44.652″ N	660,454.566	738,507.8476
KNUST 2	1°35′4.884″ W	6°40′6.132″ N	656,445.7896	737,312.9848
KOTEI	1°33′23.184″ W	6°39′42.300″ N	659,571.2753	736,589.9997
KAASE 3	1°36′20.700″ W	6°38′18.600″ N	654,126.7163	734,003.4187
BOMSO	1°34′54.552″ W	6°40′54.912″ N	656,758.7857	738,812.231
GYINYASI 3	1°34′21.432″ W	6°39′21.744″ N	657,784.1961	735,953.3956
ODUOM	1°32′31.344″ W	6°41′35.088″ N	661,153.1346	740,059.1525
ATONSO 4	1°35′6.792″ W	6°39′15.588″ N	656,391.6405	735,760.3015
AYIGYA 2	1°34′20.424″ W	6°41′27.096″ N	657,804.0023	739,803.834
KAASE 4	1°35′57.804″ W	6°39′17.460″ N	654,824.7999	735,813.3404
KNUST AYIGYA	1°34′13.296″ W	6°41′13.560″ N	658,024.1114	739,388.6937
ASOKORE MAMPG	1°34′6.800″ W	6°42′31.100″ N	658,216.6627	741,771.0215
KTI	1°36′22.608″ W	6°41′28.500″ N	654,051.6454	739,836.1929
DECABIN	1°35′12.228″ W	6°40′59.952″ N	656,215.5055	738,965.4792
AYEDUASE 3	1°33′10.800″ W	6°40′24.888″ N	659,947.7756	737,899.2691
DEDUAKO	1°32′48.264″ W	6°39′19.620″ N	660,645.7882	735,896.4927
ATONSU 4	1°35′47.940″ W	6°38′38.040″ N	655,131.1762	734,603.3768
AYIGYA 3	1°34′43.140″ W	6°41′20.724″ N	657,106.9623	739,606.0889
ESERESO	1°33′40.824″ W	6°37′45.300″ N	659,039.9622	732,994.599

3. Computational experience

3.1. Overlap difference for octagonal-square disks

In **Figure 6** there exist two overlaps (A) and (B) with different dimensions. One of the overlaps occurs as two identical cells intercept at two points while the other occurs with the intersection of two different radii cells at two points.

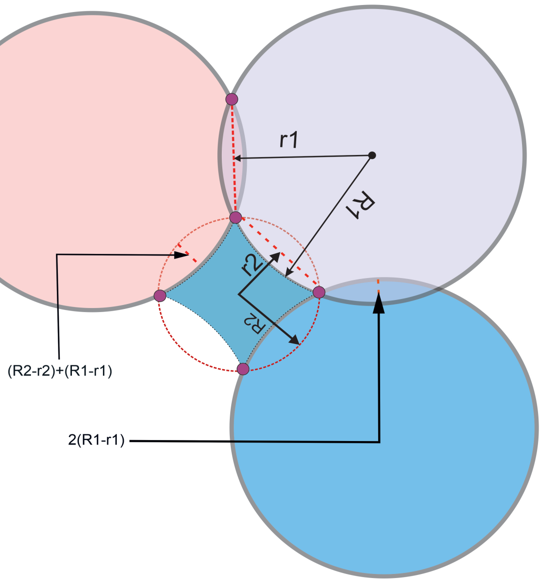


Figure 6. Overlap difference in GSM cell.

Figure 6 shows two overlappings. The first overlap difference is $2(R_1 - r_1)$ which occurs at the intersection of two identical octagnal cells and the other overlap happens as two different radii cells intersect, the second overlap difference is therefore $(R_1 - r_1) + (R_2 - r_2)$, such that $R_1 > R_2$ and $(R_2 - r_2) + (R_1 - r_1) > 2(R_1 - r_1)$.

3.1.1. Octagon-octagon cells

It has been established that to cover a given plane or point sets with disks of radius R_1 and Octagonal apothem r_1, we require an overlap difference of $2(R_1 - r_1)$ as soon in **Figure 6**. We shall, however, deduce formulas for calculating the width of any octagonal disks covering in terms of the apothem (r_1) or the radius of the disks (R_1) or the height of the overlap area (H). Consider two intersecting uniform disks shown in **Figure 7**.

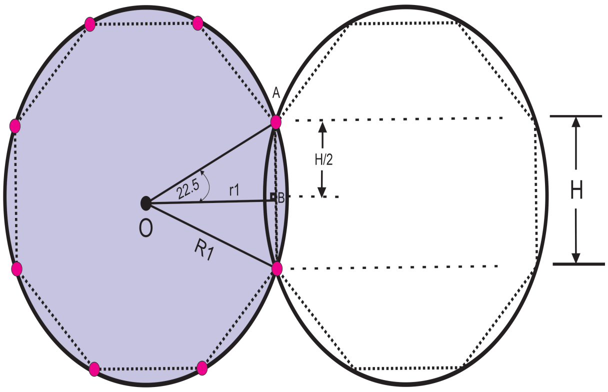


Figure 7. Overlap width for uniform disks (cell radius).

Consider triangle AOB in **Figure 7**.

$$\sin(\frac{\pi}{8}) = \frac{AB}{OA}$$

$$\sin(\frac{\pi}{8}) = \frac{H/2}{R_1}$$

$$R_1 = \frac{H}{2\sin(\frac{\pi}{8})} \tag{2}$$

$$R_1 = 1.3065H$$

$$\text{Width} = 2(R_1 - r_1)$$

$$\therefore \text{Width} = 2(1.3065H - r_1)$$

3.1.2. Octagon-square cells

Consider triangle AOB in **Figure 8**.

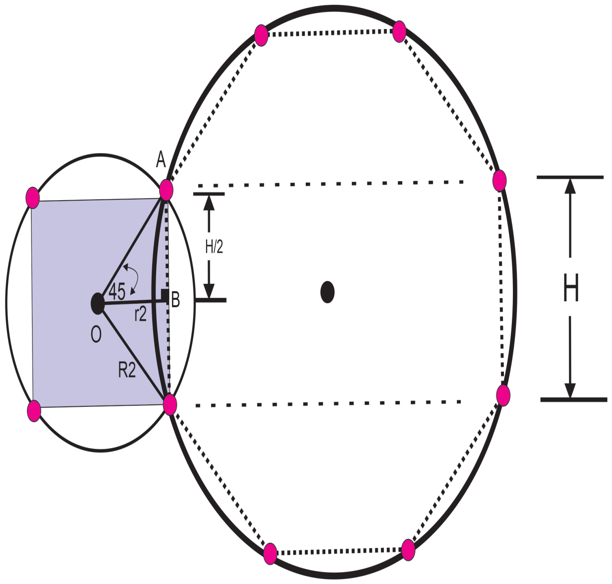


Figure 8. Overlap width for non-uniform disks (cell radius).

Case I:

$$\sin(\frac{\pi}{4}) = \frac{AB}{OA}$$

$$\sin(\frac{\pi}{4}) = \frac{H/2}{R_2}$$

$$R_2 = \frac{H}{2\left(\frac{\sqrt{2}}{2}\right)}$$

$$R_2 = \frac{\sqrt{2}}{2}H$$

$$\text{Width} = (R_1 - r_1) + (R_2 - r_2) \tag{3}$$

$$\text{From Equation (2)} \;\; R_1 = \frac{1}{2\sin(\frac{\pi}{8})}H$$

$$\therefore \text{Width} = (1.3065H - r_1) + \left(\frac{\sqrt{2}}{2}H - r_2\right)$$

$$= \left(1.3065 + \frac{\sqrt{2}}{2}\right)H - (r_1 + r_2)$$

$$\begin{aligned} A &= A(R, d_1) + A(r, d_2) \\ &= R^2 \cos[-1]\left(\frac{d_1}{R}\right) + r^2 \cos[-1]\left(\frac{d_2}{r}\right) - d'\sqrt{R'^2 - d'^2} \\ A &= R^2 \cos[-1]\left(\frac{d_1}{R}\right) + r^2 \cos[-1]\left(\frac{d_2}{r}\right) \\ &- \frac{1}{2}\sqrt{(-d + r - R)(-d - r + R)(-d + r + R)(d + r + R)} \end{aligned} \tag{4}$$

Equation (4) is used to compute the overlap area for the original masts location where $d_m - d_n$ is the distance between the mast with radius R.

Table 2. Overlap area.

SE	$d(m)$	Value	A_d	SE	$d(m)$	Value	A_d
1.	$d_1 - d_8$	17.10	27,455.19	2.	$d_1 - d_9$	15.88	98,225.35
3.	$d_{38} - d_{36}$	17.90	1023.72	4.	$d_8 - d_9$	7.26	1,030,800.81
5.	$d_8 - d_{11}$	10.37	637,646.16	6.	$d_9 - d_{11}$	6.14	1,183,663.97
7.	$d_{11} - d_{49}$	14.49	206,716.95	8.	$d_{49} - d_{12}$	7.84	955,374.08
9.	$d_{49} - d_{13}$	13.56	291,648.17	10.	$d_{49} - d_7$	9.32	765,771.33
11.	$d_{49} - d_{15}$	7.25	1,033,469.10	12.	$d_{13} - d_{15}$	10.58	612,747.05
13.	$d_{12} - d_7$	11.07	555,672.33	14.	$d_{15} - d_7$	11.51	505,711.55
15.	$d_{15} - d_{42}$	10.51	621,018.31	16.	$d_{42} - d_{14}$	11.04	559,124.19
17.	$d_{14} - d_7$	11.36	522,601.26	18.	$d_{14} - d_{26}$	12.8	367,089.63
19.	$d_{14} - d_{43}$	13.29	317,869.90	20.	$d_{14} - d_{45}$	11.57	498,997.92
21.	$d_2 - d_{26}$	17.27	20,084.90	22 .	$d_{26} - d_6$	10.86	579,953.35
23.	$d_{26} - d_{28}$	8.25	901,900.77	24.	$d_{26} - d_{45}$	9.78	708,924.13
25.	$d_{45} - d_{28}$	8.20	908,385.19	26.	$d_{45} - d_{21}$	10.01	680,911.98
27.	$d_{45} - d_{43}$	5.29	1,301,168.11	28 .	$d_{21} - d_{27}$	11.38	520,340.66
29.	$d_{27} - d_{43}$	12.35	414,029.02	30.	$d_{43} - d_{21}$	6.11	1,187,776.95
31.	$d_{28} - d_{21}$	12.19	431,097.06	32.	$d_6 - d_{20}$	11.43	514,700.74
33.	$d_{28} - d_6$	8.28	898,015.13	34.	$d_6 - d_{44}$	15.83	101,676.61
35.	$d_{44} - d_{28}$	15.84	100,983.35	36.	$d_{44} - d_{18}$	9.98	684,549.68
37.	$d_{44} - d_{22}$	10.93	571,829.24	38 .	$d_{44} - d_{20}$	7.48	1,002,870.98
39.	$d_3 - d_6$	14.87	214,546.15	40.	$d_3 - d_{20}$	11.83	470,191.09
41.	$d_3 - d_{22}$	14.40	214,546.15	42.	$d_{20} - d_{22}$	10.19	659,188.95
43 .	$d_{22} - d_{18}$	6.19	1,176,814.88	44.	$d_{22} - d_{44}$	10.42	631,694.68
45.	$d_{44} - d_{46}$	9.61	729,807.53	46.	$d_{46} - d_{18}$	2.80	1,654,611.37
47.	$d_{18} - d_5$	17.15	25,210.01	48.	$d_5 - d_{25}$	10.42	631,694.68
49.	$d_{50} - d_4$	13.46	301,282.68	50.	$d_{27} - d_{21}$	11.43	514,700.74
51.	$d_{27} - d_{47}$	17.62	7565.55	52.	$d_{16} - d_{30}$	16.14	80,903.21
53.	$d_{16} - d_{34}$	12.25	424,673.93	54.	$d_{16} - d_{33}$	15.44	129,842.26
55.	$d_{30} - d_{34}$	13.10	336,711.53	56.	$d_{30} - d_{31}$	12.41	407,678.68
57.	$d_{31} - d_{29}$	15.70	110,823.70	58.	$d_{29} - d_{32}$	16.01	89,431.04
59.	$d_{32} - d_{34}$	16.92	36,035.70	60.	$d_{32} - d_{37}$	10.18	660,391.10
61.	$d_{32} - d_{40}$	17.25	20,912.40	62.	$d_{40} - d_{37}$	13.24	322,797.59
63.	$d_{37} - d_{39}$	12.93	353,833.16	64.	$d_{37} - d_{34}$	14.06	244,900.36
65.	$d_{33} - d_{39}$	11.03	560,276.07	66.	$d_{33} - d_{47}$	16.20	77,060.14
67.	$d_{47} - d_{10}$	14.56	200,689.04	68.	$d_{10} - d_{35}$	10.22	655,585.84
69.	$d_{24} - d_{48}$	17.16	24,768.53	70.	$d_{48} - d_{23}$	16.71	46,957.50
71.	$d_{48} - d_{19}$	10.80	586,940.79	72.	$d_{19} - d_{23}$	14.48	207,582.49

Table 2 shows the total overlap area as computed by summing all the individual overlap areas.

The total overlap area is computed by summing all the individual overlap areas; $\sum A_d$= $35132474.34\ m^2$.

3.1.3. Octagon-octagon cells

We will however settle on an equation for ascertaining the zone of a couple of cover for the ideal covering of cells. Area of single overlap = 2×(area of sector OXY − area of $\triangle OXY$).

$$\begin{aligned} A_s &= 2 \times \left(\frac{1}{2}R_1^2\theta - \frac{1}{2}R_1^2 \sin(\theta)\right) \\ &= R_1^2(\theta - \sin(\theta)) \end{aligned} \tag{5}$$

The total overlaps for n overlaps will be

$$A_T = nR_1^2(\theta - \sin(\theta)) \tag{6}$$

For Octagon, $\theta = \dfrac{\pi}{4}$ so a single area will be,

$$\begin{aligned} A_s &= R_1^2\left(\frac{\pi}{4} - \sin(\frac{\pi}{4})\right) \\ &= R_1^2\left(\frac{\pi}{4} - \frac{\sqrt{2}}{2}\right) \\ &= \frac{R_1^2}{4}(\pi - 2\sqrt{2}) \end{aligned} \tag{7}$$

3.1.4. Octagon-square cells

Area of single overlap = (area of sector OXY − area of $\triangle OXY$) + (area of sector PAB − area of $\triangle PAB$).

$$\begin{aligned} &= \left(\frac{1}{2}R_1^2\theta_1 - \frac{1}{2}R_1^2\sin(\theta_1)\right) + \left(\frac{1}{2}R_2^2\theta_2\frac{1}{2}R_2^2\sin(\theta_2)\right) \\ A_s &= \frac{1}{2}R_1^2(\theta_1 - \sin(\theta_1)) + \frac{1}{2}R_2^2(\theta_2 - \sin(\theta_2)) \end{aligned} \tag{8}$$

The total overlaps for n overlaps will be

$$A_T = n\left(\frac{1}{2}R_1^2(\theta_1 - \sin(\theta_1)) + \frac{1}{2}R_2^2(\theta_2 - \sin(\theta_2))\right), \tag{9}$$

where μ is the overlaps of the octagon-square

For Octagon and Square, $\theta_1 = \dfrac{\pi}{4}$, $\theta_2 = \dfrac{\pi}{2}$ and so a single area will be,

$$\begin{aligned} A_s &= \frac{1}{2}R_1^2\left(\frac{\pi}{4} - \sin(\frac{\pi}{4})\right) + \frac{1}{2}R_2^2\left(\frac{\pi}{2} - \sin(\frac{\pi}{2})\right) \\ &= \frac{1}{2}R_1^2\left(\frac{\pi}{4} - \frac{\sqrt{2}}{2}\right) + \frac{1}{2}R_2^2\left(\frac{\pi}{2} - 1\right) \\ &= R_1^2\left(\frac{\pi}{8} - \frac{\sqrt{2}}{4}\right) + R_2^2\left(\frac{\pi}{4} - \frac{1}{2}\right) \\ &= \frac{R_1^2}{8}(\pi - 2\sqrt{2}) + \frac{R_2^2}{4}(\pi - 2) \end{aligned} \tag{10}$$

The price lists of GSM masts with full equipments by ATC, Ghana with estimated price for the month of April–August 2019 is shown in **Table 3**.

Table 3. Cell tower costs for various GSM cell range.

Cell Tower Costs Huawei Brand (GSM 900 Mhz)	**Difference in Costs for Cell Range**	**Full Cost of GSM Masts (Huawei DXX-824-960/1710-2170-65/65-17-5/18dbi-M/M)**
0.6 km	$000.00	$142,000–$162,000
0.8 km	$712.04	$141,287.96–$162,712.04
1.2 km	$9369.32	$151,369.32–$171,369.32
1.0 km	-	$146,328.64–$167,040.65
1.6 km	-	$159,669.135–$178,381.175

Source: ATC (Ghana, April–August, 2019).

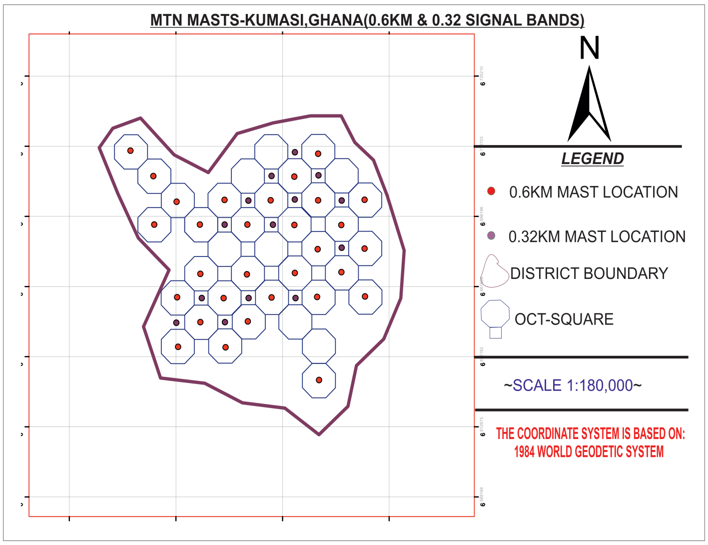


Figure 9. Minimum octagon-square tessellation of GSM mast, MTN Kumasi East, Ghana.

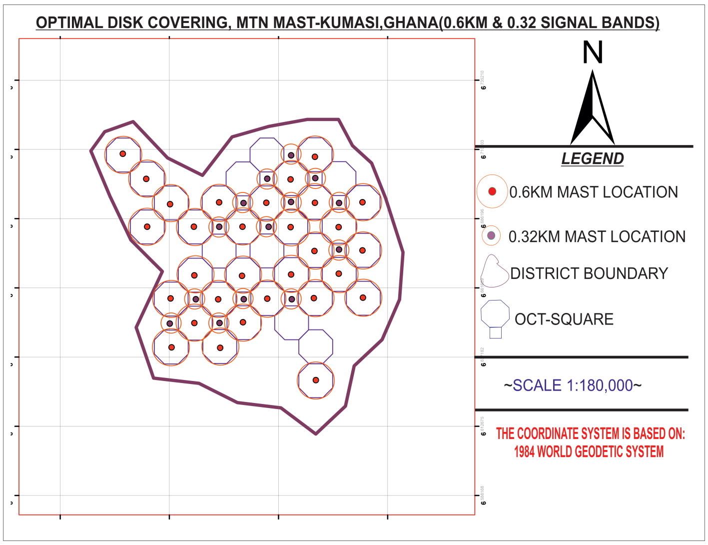


Figure 10. Optimal disk covering, MTN Kumasi East, Ghana.

Figure 9 shows the minimum octagon-square tessellation of GSM mast, MTN Kumasi East, Ghana. **Figure 10** shows the optimal disk covering, MTN Kumasi East, Ghana.

3.2. New location obtained from octagon-square tessellation, MTN-GHANA (WGS-84 Coordinate)

Table 4. WGS-84 coordinates of MTN masts in Kumasi-Ghana obtained from octagon-square tessellation.

MTN Masts with Node Points (WGS-84)				
	Geographical Coordinates		**Grid Coordinates (UTM)**	
Location	**LAT**	**LONG**	**Xm**	**Ym**
0.6 km Band				
N1	−1.63	6.71	651,423.5303	741,902.1243
N2	−1.622	6.7	652,311.0275	740,798.8379
N3	−1.613	6.691	653,308.8012	739,806.4477
N4	−1.62	6.684	652,537.0971	739,030.2344
N5	−1.605	6.684	654,195.4282	739,034.9096
N6	−1.595	6.692	655,298.4612	739,922.6757
N7	−1.612	6.658	653,429.6315	736,157.7245
N8	−1.603	6.666	654,422.182	737,045.1464
N9	−1.587	6.684	656,185.4393	739,040.5865
N10	1.578	6.692	657,177.893	739,928.08
N11	−1.57	6.7	658,059.7612	740,815.2766
N12	−1.561	6.709	659,051.8284	741,813.3976
N13	−1.612	6.641	653,434.9057	734,277.9203
N14	−1.604	6.65	654,316.6211	735,275.5996
N15	−1.596	6.658	655,198.6177	736,162.7228
N16	−1.587	6.666	656,191.1461	737,050.1829
N17	−1.57	6.683	658,065.2293	738,935.4362
N18	−1.561	6.692	659,057.3383	739,933.5493
N19	−1.595	6.641	655,314.5186	734,283.2195
N20	−1.587	6.65	656,196.206	735,280.9364
N21	−1.578	6.658	657,188.7413	736,168.4144
N22	−1.569	6.667	658,180.9244	737,166.4963
N23	−1.561	6.675	659,062.8344	738,053.7022
N24	−1.553	6.683	659,944.7155	738,940.9289
N25	−1.544	6.692	660,936.7974	739,939.0837
N26	−1.561	6.659	659,067.9944	736,284.4355
N27	−1.553	6.667	659,949.9105	737,171.6555
N28	−1.544	6.675	660,942.3584	738,059.2228
N29	−1.544	6.659	660,947.5795	736,289.9431
N30	−1.561	6.63	659,077.3155	733,077.6423
N31	−1.595	6.684	655,300.9881	739,038.0544
		0.32 km Band		
N32	−1.587	6.692	656,182.898	739,925.2108
N33	−1.578	6.701	657,175.0121	740,923.2864
N34	−1.569	6.709	658,167.4125	741,810.8092
N35	−1.613	6.649	653,321.8621	735,162.2238
N36	−1.603	6.658	654,424.6848	736,160.529
N37	−1.577	6.683	657,291.3272	738,933.1934
N38	−1.57	6.692	658,062.3361	739,930.6457
N39	−1.561	6.701	659,054.423	740,928.7629
N40	−1.596	6.65	655,201.13	735,278.103
N41	−1.586	6.658	656,304.24	736,165.8758
N42	−1.522	6.692	663,369.0592	739,946.3424
N43	−1.569	6.658	658,183.8088	736,171.2874
N44	−1.553	6.675	659,947.3146	738,056.2921

Table 4 shows the WGS-84 coordinates of MTN masts in Kumasi-Ghana obtained from octagon-square tessellation.

4. Proposed location of masts

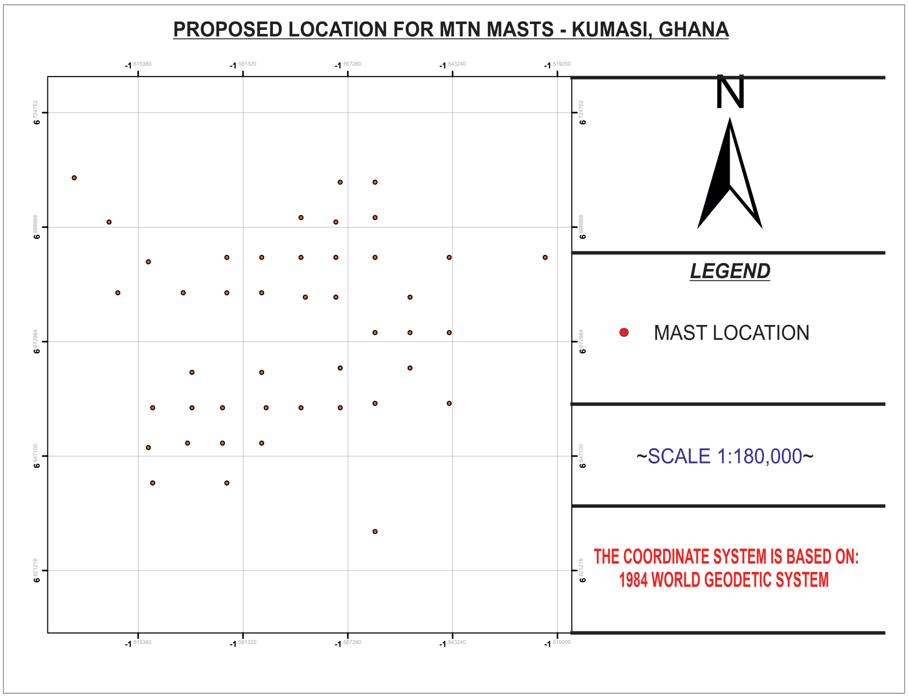


Figure 11. Proposed location of masts in a well spaced manner.

A total of 50 mast with 0.6 km signal band were originally used. After using Octagon-Square tilling with dual signals, 44 masts were used. 31 masts with 0.6 km signal band and 13 masts with 0.32 km signal band. The cost of the masts are respectively \$191,520.00 and \$117,070.00 for 0.6 km and 0.32 km. **Figure 11** shows the proposed location of masts in a well spaced manner.

$$\begin{aligned} d &= (1-0.9239)\frac{n}{2}R_p + \left(1-\frac{\sqrt{2}}{2}\right)\frac{n}{2}R_t \\ &= (1-0.9239)\frac{31}{2}\times 600 + \left(1-\frac{\sqrt{2}}{2}\right)\frac{13}{2}\times 320 \\ &= 707.73 + 609.217 \\ &= 1316.947m \end{aligned} \tag{11}$$

$$\begin{aligned} \text{Original Cost} &= \text{Unit Price} \times \text{Number of Masts} \\ &= \$191,520.00 \times 50 \\ &= \$9,576,000.00 \end{aligned}$$

now with the octagon-square design;

$$\begin{aligned} \text{New Cost} &= \text{Unit Price} \times \text{Number of Masts} \\ &= (31 \times \$191,520.00) + (13 \times \$117,070.09) \\ &= \$7,459,030.17 \end{aligned}$$

5. Conclusions

The data covers 50 MTN original masts in Kumasi-East of Ghana. The overlap area of the original mast was estimated as 35.13 km^2 with total coverage of 56.55 km^2 should there have been no overlap. This indicates a 21.41 km^2 coverage area with 59.87% percentage change as compared to the coverage area of 34.23 km^2 using octagon-square.

After using the dual signals, the overlap area was 2.79 km^2 which also shows a 92% reduction as compared to the original layout. The estimated cost for the original layout was $9, 576,000.00, and a new cost of $7,459,031.17 from the proposed layout. This shows a 22.16% cost reduction.

This work investigated the strategies in designing the structure of GSM system for wireless communication. The development of the optimal minimum disk covering was employed and guided by the concept of Archimedean tessellate. In covering planes application in GSM system, single signals pertaining to the hexagonal tessellation are predominantly used.

Using MTN as a case study, the proposed algorithm was tested and compared to the existing layout. It was found that the octagon-square tessellates indeed have a larger coverage area with smaller overlaps. Again the use of dual masting also renders a least-cost since different signal band ranges are used. The estimated cost for the original layout was $9, 576,000.00, and a new cost of $7,459,031.17 from the proposed layout. This shows a 22.16% cost reduction.

Author contributions: Conceptualization, EKD and JAP; methodology, FM and EKD; validation, OD and SO; formal analysis, EKD and FM; supervision, JAP and EKD; writing original draft preparation, FM and SO; writing review and editing, FM and SO. All authors have read and agreed to the published version of the manuscript.

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Data availability: The data used in the analysis of this manuscript is taken from ATC, Ghana. They usually mount the mast for MTN-Ghana.

Conflict of interest: The authors declare no conflict of interest.

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