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Original Research Article

(ϵ)-Kenmotsu manifold admitting Schouten-van Kampen connection

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ABSTRACT: The objective of this paper is to study some properties of quasi-conformal and concircular tensors on the (ϵ)-Kenmotsu manifold admitting the Schouten-van Kampen connection. Expressions of the curvature tensor, Ricci tensor, and scalar curvature admitting Schouten-van Kampen connection have been obtained. Locally symmetric (ϵ)-Kenmotsu manifold admitting the Schouten-van Kampen connection and quasicon formally flat as well as quasi-conformally semisymmetric (ϵ)-Kenmotsu manifolds admitting Schouten-van Kampen connection are studied.

KEYWORDS: (ϵ)-Kenmotsu manifold; quasi-conformal curvature tensor; concircular curvature tensor; Schouten-van Kampen connection
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1. Introduction

De and Sarkar[1] introduced the concept of indefinite metrics on Kenmotsu manifolds, which are called (ϵ)-Kenmotsu manifolds. They studied conformally flat, Weyl semisymmetric, ϕ-recurrent (ϵ)-Kenmotsu manifolds. The Schouten-van Kampen connection has been introduced for studying non-holomorphic manifolds. It preserves, by parallelism, a pair of complementary distributions on a differentiable manifold endowed with an affine connection (see Bejancu and Farran[2], Ianus[3], Schouten and van Kampen[4]). Then, Olszak[5] studied the Schouten-van Kampen connection to adapt it to an almost contact metric structure. He characterized some classes of almost contact metric manifolds with the Schouten-van Kampen connection and established certain curvature properties with respect to this connection. Recently, Ghosh[6] and Yildiz[7] have studied the Schouten-van Kampen connection in Sasakian manifolds and f-Kenmotsu manifolds, respectively. Some related developments can be found in many other works[8–33].

This paper is structured as follows: Section 2 gives a brief review of (ϵ)-Kenmotsu manifolds. In Section 3, we obtain the expressions of the curvature tensor, Ricci tensor, and scalar curvature, admitting the Schouten-van Kampen connection. In Section 4, we study locally symmetric (ϵ)-Kenmotsu manifold admitting Schouten-van Kampen connection. In Sections 5, we study quasiconformally flat and quasi-conformally semisymmetric (ϵ)-Kenmotsu manifolds admitting Schouten-van Kampen connection. In Section 6, we prove (ϵ) Kenmotsu manifolds admitting Schouten-van Kampen connection satisfying $\bar{Z}(X, Y.\bar{S}(U, W)) = 0$ is an η-Einstein manifold.

2. Preliminaries

An almost contact structure on a differentiable manifold M^n is a triple (ϕ, ξ, η), where ϕ is a tensor field of type (1,1), η is a 1-form and ξ is a vector field such that

$$\phi^2 = X_1 + \eta(X_1)\xi, \tag{1}$$

$$\eta(\xi) = 1,\ \phi\xi = 0,\ \eta\phi = 0 \tag{2}$$

A differential manifold with an almost contact structure is called an almost contact manifold. An almost contact metric manifold is an almost contact manifold endowed with a compatible metric g. An almost contact metric manifold M is said to be an (ϵ)-almost contact metric manifold, if

$$g(\xi, \xi) = \pm 1 = \epsilon, \tag{3}$$

$$\eta(X) = \epsilon g(X, \xi), rank(\phi) = n - 1, \tag{4}$$

$$g(\phi X, \phi Y) = g(X, Y) - \epsilon\eta(X)\eta(Y), \forall X, Y \in \Gamma(TM), \tag{5}$$

where ξ is space-like or time-like but it is never a light like vector field. We say that $(\phi,\ \xi,\ \eta,\ g)$ is an (ϵ)-contact metric structure, if

$$d\eta(X, Y) = g(X, \phi Y) \tag{6}$$

In such case, M is an (ϵ)-contact metric manifold. An (ϵ)-contact metric manifold is called an(ϵ)-Kenmotsu manifold[1], if

$$\nabla X\phi Y = g(X, \phi Y)\xi - \epsilon\eta(Y)\phi X, \tag{7}$$

where ∇ is the Riemannian connection of g. An (ϵ)-almost contact metric manifold is an (ϵ)-Kenmotsu manifold if and only if

$$\nabla X\xi = \epsilon(X - \eta(X)\xi). \tag{8}$$

The following conditions hold in an (ϵ)-Kenmotsu manifold[1]:

$$(\nabla_X\eta)(Y) = g(X, Y) - \epsilon\eta(X)\eta(Y), \tag{9}$$

$$\eta(R(X, Y, Z)) = \epsilon(g(X, Z)Y - g(Y, Z)X), \tag{10}$$

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X, R(\xi, X)Y = \eta(Y)X - \epsilon g(X, Y)\xi, \tag{11}$$

$$S(X, \xi) = -(n - 1)\eta(X), Q\xi = -\epsilon(n - 1)\xi, \tag{12}$$

$$S(\phi X, \phi Y) = S(X, Y) + \epsilon(n - 1)\eta(X)\eta(Y). \tag{13}$$

3. (ϵ)-Kenmotsu manifolds admitting Schouten-van Kampen connection

The Schouten-van Kampen connection $\bar{\nabla}$ associated to the Levi-Civita connection ∇ is given by

$$\bar{\nabla}_X Y = \nabla_X Y - \eta(Y)\nabla_X\xi + (\nabla_X\eta)(Y)\xi \tag{14}$$

for any vector fields X, Y on M (see Olszak[5]). Using Equations (8) and (9) in the above equation

$$\bar{\nabla}_X Y = \nabla_X Y - \epsilon\eta(Y)X - g(X, Y)\xi + 2\epsilon\eta(X)\eta(Y)\xi \tag{15}$$

Putting $Y = \xi$ and using (8) in (15), we obtain

$$\bar{\nabla}_X\xi = 0 \tag{16}$$

Let R and $\bar{R}$ denote the curvature tensor ∇ and $\bar{\nabla}$ respectively. Then

$$\bar{R}(X, Y)Z = \bar{\nabla}_X\bar{\nabla}_Y Z - \bar{\nabla}_Y\bar{\nabla}_X Z - \bar{\nabla}_{[XY]}Z \tag{17}$$

Using Equation (15) in Equation (17), we obtain

$$\begin{aligned}\bar{R}(X, Y)Z = \ & R(X, Y)Z + \epsilon g(Y, Z)X - \epsilon g(X, Z)Y + \\ & (1 - \epsilon)\eta(X)g(Y, Z)\xi - (1 - \epsilon)\eta(Y)g(X, Z)\xi\end{aligned} \tag{18}$$

Putting $Z = \xi$ and using (11) in (18), we obtain

$$\bar{R}(X,Y)\xi = 0 \tag{19}$$

On contracting (18), we obtain the Ricci tensor $\bar{S}$ of an (ϵ)-Kenmotsu manifold admitting Schouten-Van Kampen connection $\bar{\nabla}$ as

$$\bar{S}(Y,Z) = S(Y,Z) + (\epsilon n - 2\epsilon + 1)g(Y,Z) - \epsilon(1-\epsilon)\eta(Y)\eta(Z) \tag{20}$$

This gives

$$\bar{Q}Y = QY + (\epsilon n - 2\epsilon + 1)Y - (1-\epsilon)\eta(Y)\xi \tag{21}$$

Contracting with respect to Y and Z in (20), we obtain

$$\bar{r} = r + n(\epsilon n - 2\epsilon + 1) - (1-\epsilon), \tag{22}$$

where $\bar{r}$ and r are the scalar curvatures admitting Schouten-van Kampen connection $\bar{\nabla}$ and the Levi-Civita connection ∇, respectively. From the above discussions we state the following:

Theorem 1. *The curvature tensor $\bar{R}$, the Ricci tensor $\bar{S}$ and the scalar curvature $\bar{r}$ of an (ϵ)-Kenmotsu manifold M with respect to the Schouten-van Kampen connection $\bar{\nabla}$ are given by the Equations (18), (20), (21) and (22) respectively. Further, the curvature tensor $\bar{R}$ of $\bar{\nabla}$ satisfies the following:*

(i) $\bar{R}(X,Y)Z = -\bar{R}(Y,X)Z$,

(ii) $\bar{R}(X,Y,Z,W) + \bar{R}(Y,X,Z,W) = 0$,

(iii) $\bar{R}(X,Y,Z,W) + \bar{R}(X,Y,W,Z) = 0$,

(iv) $\bar{R}(X,Y)Z + \bar{R}(Y,Z)X + \bar{R}(Z,X)Y = 0$,

(v) $\bar{S}$ is symmetric.

From Equation (20), the following result is immediate.

Theorem 2. *An (ϵ)-Kenmotsu manifold M^n admitting the Schouten-van Kampen connection is Ricci flat admitting Schouten-van Kampen connection if and only if M^n is an η-Einstein manifold with respect to Levi-Civita connection.*

Now, if $\bar{R}(X,Y)Z = 0$, then Equation (18) becomes

$$R(X,Y)Z + \epsilon(g(Y,Z)X - g(X,Z)Y) + (1-\epsilon)(\eta(X)g(Y,Z)\xi - \eta(Y)g(X,Z)\xi) = 0 \tag{23}$$

Thus, we have the following theorem.

Theorem 3. *Let M^n be a (ϵ)-Kenmotsu manifold admitting the Schouten-van Kampen connection. The curvature tensor of M admitting Schouten-van Kampen connection vanishes if and only if M with respect to the Levi-Civita connection is isomorphic to the hyperbolic space $H^n(-1)$.*

4. Locally symmetric (ϵ)-Kenmotsu manifold admitting Schouten-van Kampen connection

Theorem 4. *A locally symmetric (ϵ-Kenmotsu manifold M^n admitting Schouten-van Kampen connection $\bar{\nabla}$ is an η-Einstein manifold.*

Proof. Let M^n be a locally symmetric (ϵ)-Kenmotsu manifold admitting Schouten-van Kampen connection $\bar{\nabla}$. Then $(\bar{\nabla}_X R)(Y,Z)W = 0$. By contraction of the equation, we get

$$(\bar{\nabla}_X \bar{S})(Z,W) = \bar{\nabla}_X \bar{S}(Z,W) - \bar{S}(\bar{\nabla}_X Z,W) - \bar{S}(Z,\bar{\nabla}_X W) = 0 \tag{24}$$

Putting $W = \xi$ in (24), we have

$$\bar{\nabla}_X \bar{S}(Z,\xi) - \bar{S}(\bar{\nabla}_X Z,\xi) - \bar{S}(Z,\bar{\nabla}_X \xi) = 0 \tag{25}$$

Using (15) and (20) in (25), we obtain

$$S(X,Z) = Ag(X,Z) + B\eta(X)\eta(Y) \tag{26}$$

where $A=-\epsilon(n-2)+1$ and $B=-\epsilon(n-2)+n$. □

5. Quasi-Conformally flat (ϵ)-Kenmotsu manifold admitting Schouten van-Kampen connection

Theorem 5. *A quasi-conformally flat* (ϵ)*-Kenmotsu manifold admitting Schouten van-Kampen connection is an* η*-Einstein manifold.*

Proof. An (ϵ)-Kenmotsu manifold admitting a Schouten van-Kampen connection is said to be quasi-conformally flat if

$$\bar{C}(X,Y)Z=0 \tag{27}$$

The quasi-conformal curvature tensor $\bar{C}$ admitting a Schouten van-Kampen connection is (see Yano[34]):

$$\begin{aligned}\bar{C}(X,Y)Z= \quad & a\bar{R}(X,Y)Z+b(\bar{S}(Y,Z)X-\bar{S}(X,Z)Y+g(Y,Z)\bar{Q}X \\ & -g(X,Y)\bar{Q}Y)-\frac{\bar{r}}{n}\left(\frac{a}{n-1}+2b\right)(g(Y,Z)X-g(X,Z)Y).\end{aligned} \tag{28}$$

In view of Equations (27) and (28), we have

$$\begin{aligned}a\bar{R}(X,Y)Z=b(\bar{S}(X,Z)Y-\bar{S}(Y,Z)X+g(X,Z)\bar{Q}Y-g(Y,Z)\bar{Q}X)+\frac{\bar{r}}{n}\left(\frac{a}{n-1}+2b\right)(g(Y,Z)X \\ -g(X,Z)Y)\end{aligned} \tag{29}$$

Using Equations (18), (20), and (21) and taking inner product with ξ in Equation (29) we obtain

$$\begin{aligned}& a(g(R(X,Y)Z,\xi)+\epsilon g((Y,Z)g(X,\xi)-\epsilon g(X,Z)g(Y,\xi)+ \\ & (1-\epsilon)\eta(X)g(Y,Z)g(\xi,\xi)-(1-\epsilon)\eta(Y)g(X,Z)g(\xi,\xi))) \\ & =b((S(X,Z)g(Y,\xi)+(\epsilon n-2\epsilon+1)g(X,Z)g(Y,\xi))-\epsilon(1-\epsilon)\eta(X)\eta(Z)g(Y,\xi) \\ & -(S(Y,Z)g(X,\xi)+(\epsilon n-2\epsilon+1)g(Y,Z)g(X,\xi)+\epsilon(1-\epsilon)\eta(Y)\eta(Z)g(X,\xi)) \\ & +g(X,Z)(g(QY,\xi)+(\epsilon n-2\epsilon+1)g(Y,\xi)+(1-\epsilon)\eta(Y)g(\xi,\xi)) \\ & -g(Y,Z)(g(QX,\xi)+(\epsilon n-2\epsilon+1)g(X,\xi)+(1-\epsilon)\eta(X)g(\xi,\xi) \\ & +\frac{\bar{r}}{n}\left(\frac{a}{n-1}+2b\right)(g(Y,Z)g(X,\xi)-g(X,Z)g(Y,\xi))\end{aligned} \tag{30}$$

Putting $X=\xi$ and using Equations (3), (4), (11) in Equation (30) we obtain

$$S(Y,Z)=Cg(Y,Z)+D\eta(Y)\eta(X), \tag{31}$$

where

$$C=\frac{1}{\epsilon b}\left((1-\epsilon)(a+2b)+\epsilon\frac{\bar{r}}{n}\left(\frac{a}{b(n-1)}+2b\right)\right)$$

and

$$D=\frac{1}{\epsilon b}\left((1-\epsilon)(a+2b)-\frac{\bar{r}}{n}\left(\frac{a}{b(n-1)}+2b\right)\right). \ \square$$

6. Quasi-Conformally semisymmetric (ϵ)-Kenmotsu manifold admitting Schouten van-Kampen connection

Theorem 6. *A quasi-Conformally semisymmetric* (ϵ)*-Kenmotsu manifold admitting Schouten van-Kampen connection is an*η*-Einstein manifold.*

Proof. An (ϵ)-Kenmotsu manifold admitting a Schouten van-Kampen connection is said to be quasi-conformally semisymmetric if

$$\bar{R}(\xi,Y)\cdot\bar{C}(U,V)W=0 \tag{32}$$

which implies that

$$\bar{R}(\xi,Y)\bar{C}(U,V)W-\bar{C}(\bar{R}(\xi,Y)U,V)W-\bar{C}(U,\bar{R}(\xi,Y)V)W-\bar{C}(U,V)\bar{R}(\xi,Y)W=0 \tag{33}$$

In view of Equation (18) in Equation (33), we have

$$\begin{aligned}&(1-\epsilon)g(Y,\bar{C}(U,V)W)\xi-\epsilon(1-\epsilon)\eta(Y)\eta(\bar{C}(U,V)W)\xi\\&-(1-\epsilon)g(Y,U)\bar{C}(\xi,V)W+\epsilon(1-\epsilon)\eta(Y)\eta(U)\eta(\bar{C}(\xi,V)W\\&-(1-\epsilon)g(Y,V)\bar{C}(U,\xi)W+\epsilon(1-\epsilon)\eta(Y)\eta(V)\eta(\bar{C}(U,\xi)W\\&-(1-\epsilon)g(Y,W)\bar{C}(U,V)\xi+\epsilon(1-\epsilon)\eta(Y)\eta(W)\bar{C}(U,V)\xi=0\end{aligned} \tag{34}$$

Replacing $Y=U$ and taking inner product with ξ in (34), we have

$$\begin{aligned}&\epsilon g(U,\bar{C}(U,V)W)-\eta(U)\eta(\bar{C}(U,V)W)-(g(U,U)-\epsilon\eta(U)\eta(U))\eta(\bar{C}(\xi,V)W)\\&-(g(U,V)+\epsilon\eta(U)\eta(V)\bar{C}(U,\xi)W)-(g(U,W)-\epsilon\eta(U)\eta(W))\eta(\bar{C}(U,V)\xi)=0\end{aligned} \tag{35}$$

provided $(1-\epsilon)\neq 0$.

Putting $U=\xi$ and using Equations (28), (18), (20) and (21), we obtain

$$S(V,W)=Eg(V,W)+F\eta(V)\eta(W) \tag{36}$$

where

$$E=-\frac{1}{b}\left[a(1-3\epsilon)+b(\epsilon n-2\epsilon+1)+2b(1-\epsilon)-\frac{\bar{r}}{n}\left(\frac{a}{n-1}+2b\right)\right]$$

and

$$F=-\frac{1}{b}\left[-a\epsilon(1-\epsilon)+b(n-1)-b\epsilon(\epsilon n-2\epsilon+1)-b(1-\epsilon)-\frac{\bar{r}}{n}\left(\frac{a}{n-1}+2b\right)\right]\ \square$$

7. (ϵ)-Kenmotsu manifold admitting Schouten van-Kampen connection satisfying

$$\bar{Z}(X,Y\cdot\bar{S}(U,W))=0$$

Theorem 7. *An* (ϵ)- *Kenmotsu manifold admitting Schouten van-Kampen connection satisfying* $\bar{Z}(X,Y\cdot\bar{S}(U,W))=0$ *is an* η*-Einstein manifold.*

Proof. An (ϵ)-Kenmotsu manifold admitting Schouten van-Kampen connection satisfies.

$$\bar{Z}(X,Y.\bar{S}(U,W))=0 \tag{37}$$

which implies that

$$\bar{S}(\bar{Z}(\xi,Y)U,W)+\bar{S}(U,\bar{Z}(\xi,Y)W)=0 \tag{38}$$

The concircular curvature tensor $\bar{Z}$ admitting Schouten van-Kampen connection is given by (see Yano[35])

$$\bar{Z}(X,Y)Z=\bar{R}(X,Y)Z-\left(\frac{\bar{r}}{n(n-1)}(g(Y,Z)X-g(X,Z)Y)\right) \tag{39}$$

In view of Equations (3), (4), (18), (38), and (39), we have

$$\begin{aligned}&\bar{S}R(\xi,Y)U,W)+\epsilon g(Y,U)\bar{S}(\xi,W)-\eta(U)\bar{S}(Y,W)+(1-\epsilon)g(Y,U)\bar{S}(\xi,W)\\&-\epsilon(1-\epsilon)\eta(Y)\eta(U)\bar{S}(\xi,W)-\left(\frac{\bar{r}}{n(n-1)}\right)g(Y,U)\bar{S}(\xi,W)+\epsilon\left(\frac{\bar{r}}{n(n-1)}\right)\eta(U)\bar{S}(Y,W)\\&+\bar{S}(U,R(\xi,Y)W)+\epsilon g(Y,W)\bar{S}(\xi,U)-\eta(W)\bar{S}(Y,U)+(1-\epsilon)g(Y,W)\bar{S}(\xi,U)\\&-\epsilon(1-\epsilon)\eta(Y)\eta(W)\bar{S}(\xi,U)-\left(\frac{\bar{r}}{n(n-1)}\right)g(Y,W)\bar{S}(\xi,U)+\epsilon\left(\frac{\bar{r}}{n(n-1)}\right)\eta(W)\bar{S}(Y,U)=0.\end{aligned} \tag{40}$$

Using Equation (20) and putting $U=\xi$ in Equation (40), we obtain

$$S(Y,W)=Gg(Y,W)+H\eta(Y)\eta(W), \tag{41}$$

where

$$G = -\frac{1}{2}\left(\epsilon(n-1) + (1-\epsilon) + (\epsilon n - 2\epsilon + 1)\left(\frac{\bar{r}\epsilon}{n(n-1)} - 1\right)\right),$$

and

$$H = -\frac{1}{2}\left(-\epsilon(1-\epsilon) - \epsilon(1-\epsilon)\left(\frac{\bar{r}\epsilon}{n(n-1)} - 1\right)\right) \square$$

Author contributions

Conceptualization, SGB, RR, PSKR and NP; methodology, SGB, RR and PSKR; validation, SGB, RR, PSKR and NP; formal analysis, SGB and NP; investigation, SGB, RR, PSKR and NP; resources, SGB, RR and PSKR; data curation, SGB, RR and PSKR; writing—original draft preparation, SGB, RR and PSKR; writing—review and editing, RR and PSKR; visualization, RR and PSKR; supervision, RR and PSKR. All authors have read and agreed to the published version of the manuscript.

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Conflict of interest

The authors declare no conflict of interest.

References

1. De UC, Sarkar A. On (ϵ)-Kenmotsu manifold. *Hadronic Journal* 2009; 32(2): 231–242.
2. Bejancu A, Farran HR. *Foliations and Geometric Structures*. Springer; 2006.
3. Ianus S. Some almost product structures on manifolds with linear connection. *Kodai Mathematical Seminar Reports* 1971; 23(3): 305–310. doi: 10.2996/kmj/1138846369
4. Schouten JA, van Kampen ER. On the embedding and curvature theory of nonholonomic entitie (German) *Mathematische Annalen* 1930; 103: 752–783. doi: 10.1007/BF01455718
5. Olszak Z. The Schouten-van Kampen affine connection adapted to an almost (para) contact metric structure. *arXiv* 2013; arXiv:1402.5796. doi: 10.2298/PIM1308031O
6. Ghosh G. On Schouten-van Kampen connection in Sasakian manifolds. *Boletim da Sociedade Paranaense de Matematica* 2018; 36(4): 171–182. doi: 10.5269/bspm.v36i4.33105
7. Yildiz A. *f*-Kenmotsu manifolds with the Schouten–van Kampen connection. *Publications de L'institut Mathématique* 2017; 102(116): 93–105. doi: 10.2298/PIM1716093Y
8. Alghamdi MA, Bahadir O. Some characterizations in Kenmotsu manifolds with a new connection. *arXiv* 2020; arXiv:1804.10020. doi: 10.48550/arXiv.1804.10020
9. Angadi PG, Shivaprasanna GS, Somashekhara G, Reddy PSK. Ricci-Yamabe solitons on submanifolds of some indefinite almost contact manifolds. *Advances in Mathematics: Scientific Journal* 2020; 9(11): 10067–10080. doi: 10.37418/amsj.9.11
10. Angadi PG, Shivaprasanna GS, Somashekhara G, Reddy PSK. Ricci solitons on (*LCS*)-manifolds under *D*-homothetic deformation. *Italian Journal of Pure & Applied Mathematics* 2021; 46: 672–683.
11. Angadi PG, Reddy PSK, Shivaprasanna GS, Somashekhara G. On weakly symmetric generalized (k, μ)-space forms. *Proceedings of the Jangjeon Mathematical Society* 2022; 25(2): 133–144.
12. Mert T, Atceken M, Uygun P. Semi-symmetric generalized Sasakian space forms on some special curvature tensors. *Konuralp Journal of Mathematics* 2022; 10(2): 240–249.
13. Mert T, Atceken M, Uygun P. Semi-symmetric almost C(α)-manifold on some curvature tensors. *Gulf Journal of Mathematics* 2023; 14(2): 101–114. doi: 10.56947/gjom.v14i2.1179
14. Mert T, Atceken M, Uygun P. Normal paracontact metric space form on w_0- curvature tensor. *Journal of Amasya University the Institute of Sciences and Technology* 2023; 4(1): 53–64. doi: 10.54559/jauist.1312242
15. Murthy BP, Naveen Kumar RT, Reddy PSK, Venkatesha. On *N(k)*-contact metric manifold endowed with pseudo-quasi-conformal curvature tensor. *Advances in Mathematics: Scientific Journal* 2021; 10(4): 1969–1982. doi: 10.37418/amsj.10.4.11

16. Murthy BP, Naveen Kumar RT, Reddy PSK, Venkatesha. Extended Pseudo projective curvature tensor on $(LCS)_n$-manifold. *Proceedings of the Jangjeon Mathematical Society* 2022; 25(3): 347–354.
17. Nagaraja HG, Kumari D, Reddy PSK. Submanifold of (κ, μ)- contact metric manifold as a Ricci soliton. *Proceedings of the Jangjeon Mathematical Society* 2021; 24(1): 11–19. doi: 10.17777/pjms2021.24.1.11
18. Naveen Kumar RT, Reddy PSK, Venkatesha, Alloush KAA. $(LCS)_n$-manifold endowed with torse forming vector field and concircular curvature tensor. *International Journal of Mathematical Combinatorics* 2022; 3: 38–47.
19. Siddiqi MD, Bahadir O. η-Ricci solitons on Kenmotsu manifold with generalized symmetric metric connection. *arXiv* 2018; arXiv:1809.00485. doi: 10.22190/FUMI2002295S
20. Somashekhara G, Pavani N, Reddy PSK. Invariant sub-manifolds of *LP*-Sasakian manifolds with semi-symmetric connection. *Bulletin of Mathematical Analysis and Applications* 2020; 12(2): 35–44.
21. Somashekhara G, Babu SG, Reddy PSK. *C*-Bochner curvature tensor under *D*-homothetic deformation in *LP*-Sasakian manifold. *Bulletin of the International Mathematical Virtual Institute* 2021; 11(1): 91–98.doi: 10.7251/BIMVI2101091S
22. Somashekhara G, Babu SG, Reddy PSK. Indefinite Sasakian manifold with quarter-symmetric metric connection. *Proceedings of the Jangjeon Mathematical Society* 2021; 24(1): 91–98.
23. Somashekhara G, Reddy PSK, Pavani N, Manjula GJ. η-Ricci-Yamabe solitons on submanifolds of some indefinite almost contact manifolds. *Journal of Mathematical and Computational Science* 2021; 11(3): 3775–3791.
24. Somashekhara G, Babu SG, Reddy PSK. Conformal Ricci soliton in an indefinite trans-Sasakian manifold. *Vladikavkaz Mathematical Journal* 2021; 23(3): 45–51. doi: 10.46698/t3715-2700-6661-v
25. Somashekhara G, Babu GS, Reddy PSK. Ricci solitons and generalized weak symmetries under *D*-homothetically deformed *LP*-Sasakian manifolds. *Italian Journal of Pure & Applied Mathematics* 2021; (46): 684–695.
26. Somashekhara G, Babu SG, Reddy PSK. Conformal η-Ricci solitons in Lorentzian para-Sasakian manifold admitting semisymmetric metric connection. *Italian Journal of Pure & Applied Mathematics* 2021; 46: 1008–1019.
27. Somashekhara G, Reddy PSK, Shivashankara, Pavani N. Slant sub-manifolds of generalized Sasakian-space-forms. *Proceedings of the Jangjeon Mathematical Society* 2022; 25(1): 83–88.
28. Somashekhara G, Reddy PSK, Pavani N. Semi-invariant submanifolds of generalized Sasakian-space-forms. *International Journal of Mathematical Combinatorics* 2022; 2: 47–55.
29. Somashekhara G, Babu SG, Reddy PSK, Shivashankara. On LP-Sasakian manifolds admitting generalized symmetric metric connection. *Proceedings of the Jangjeon Mathematical Society* 2022; 25(3): 287–296.
30. Somashekhara G, Rajendra R, Shivaprasanna GS, et al. Generalised Sasakian-space-form in submanifolds. *International Journal of Mathematical Combinatorics* 2022; 3: 69–81.
31. Somashekhara G, Babu SG, Reddy PSK. η-Ricci solitonin an indefinite trans-Sasakian manifold admitting semi-symmetric metric connection. *Boletim da SociedadeParanaense de Matemática* 2023; 41: 1–9. doi: 10.5269/bspm.51358
32. Somashekhara G, Rajendra R, Shivaprasanna GS, Reddy PSK. Pseudo parallel and generalized Ricci pseudo parallel invariant submanifolds of a generalized Sasakian space form. *Proceedings of the Jangjeon Mathematical Society* 2023; 26(1): 69–78.
33. Somashekhara P, Naveen Kumar RT, Reddy PSK, et al. Pseudo projective curvature tensor on generalized Sasakian space forms. *Proceedings of the Jangjeon Mathematical Society* 2023; 26(3): 243–251.
34. Yano K, Sawaki S. Riemannian manifolds admitting a conformal transformation group. *Journal of Differential Geometry* 1968; 2(2): 161–184.
35. Yano K. Concircular geometry. I. Concircular transformations. *Proceedings of the Imperial Academy Tokyo* 1940; 16(6): 195–200. doi: 10.3792/pia/1195579139