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Perspective

Theory of gyroscopic effects for rotating objects

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ABSTRACT: Scientists began to study gyroscopic effects at the time of the Industrial Revolution. Famous mathematician L. Euler described only one gyroscopic effect, which is the precession torque that does not explain other ones. Since those times, scientists could not explain the physics of gyroscopic effects, Recent studies and the method of causal investigatory dependency demonstrated, that the nature of gyroscopic effects turned out that be more sophisticated than contemplated by researchers. The external torque acting on the spinning objects generates the system of the eight inertial torques and their interrelated motions around axes presented in the 3D coordinate system. The interrelated torques and motions of the spinning disc were described by mathematical models, and validated by practical tests that explain the physics of the gyroscopic effects based on the kinetic energy conservation law. The inertial torques generated by the centrifugal, and Coriolis forces, the change in the angular momentum, and the dependent motions of the spinning object around axes constitute the fundamental principles of the gyroscope theory. The derived gyroscopic theory opened a new chapter in the dynamics of rotating objects of classical mechanics that should be presented in all word encyclopedias.

KEYWORDS: gyroscopic effects; inertial torques; kinetic energy

The unusual properties of the movable rotating objects were interesting to mankind of different civilizations from ancient times. Beginning from the Industrial Revolution, physicists and mathematicians began to study the properties of gyroscopic devices. From those times, Euler derived only one fundamental principle of the gyroscope theory which is the change in the angular momentum. Other gyroscopic effects could not be described mathematically in principle because the formulation of basic physical laws was finished at the beginning of the 20th century. From this time, researchers published tons of manuscripts and several dozens of gyroscope theories that did not validate practically[1–4]. The physics of gyroscopic effects turned out that more sophisticated than represented in known theories.

The gyroscope theory is a very important aspect of the dynamics of rotating objects. Gyroscopic effects are relayed in many engineering calculations of rotating parts, and deriving a complete gyroscope theory is a crucial challenge. Gyroscopic effects remain unsolved until recent times. Recent studies show rotating masses of the movable object produce gyroscopic effects that are manifested by the action of the system of the eight inertial torques generated by the centrifugal, and Coriolis forces, and the change in the angular momentum[5–7]. The action of the inertial torques and rotations of the spinning disc are described in detail in the theory of gyroscopic effects and shown at the moveable 3D Cartesian coordinates in **Figure 1**. The method of causal investigatory dependency formulated the mathematical models for the inertial torques gendered by the rotating objects.

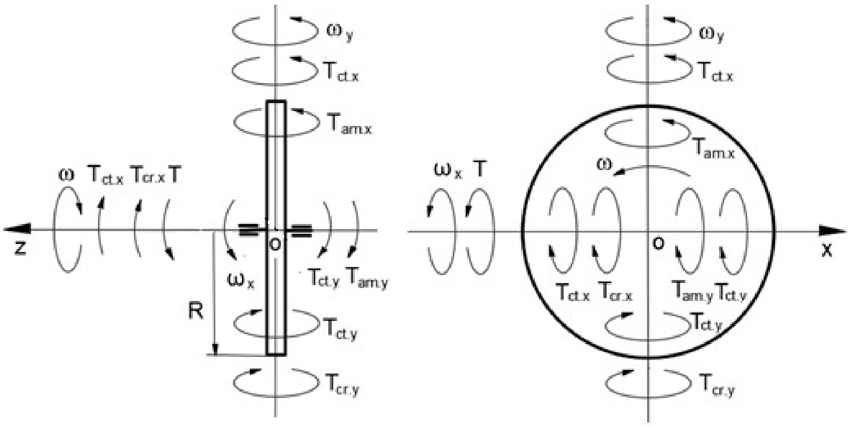

Figure 1. Schematic of the action of the inertial torques on the spinning disc and its motions.

Figure 1 contains the following information. The action of the external *T* generates the initial resistance inertial torques of the centrifugal $T_{ct.x}$, Coriolis $T_{cr.x}$ forces acing around axis *ox*, and the precession torques of the centrifugal $T_{ct.x}$, and the change in the angular momentum $T_{am,x}$ and acting around axis *oy*. The initial inertial torques acting around axis *oy* generate the resistance inertial torques of the centrifugal $T_{ct.y}$, and the change in the angular momentum $T_{am,y}$ acting around axis *oy*. The resulting toque of axis *oy* generates the precession torques of the centrifugal $T_{ct.y}$, and the change in the angular momentum $T_{am,y}$ acting around axis *ox*, which is added to the initial resistance inertial torques of the centrifugal $T_{ct.x}$, Coriolis $T_{cr.x}$ forces. The resulting torque of axis *ox* generates the corrected precession torques of the centrifugal $T_{ct.x}$, and the change in the angular momentum $T_{am,x}$ acting around axis *oy*.

Figure 1 shows inertial torques acting around axes with feedback loops. Any change in the inertial toques about one axis leads to a change in the inertial toques about the other axis. The action of the inertial toques expresses the interrelated motions of the spinning disc around the two axes. It means the kinetic energies of inertial toques about two axes are equal and based on the principle of mechanical energy conservation that constitutes the fundamental principles of the gyroscopic theory (**Table 1**). **Table 1** shows the value of Euler's change in the angular momentum which is less than 10% among the value of other inertial torques, whose action significantly (over 91%) prevails in the manifestation of gyroscopic effects. Nevertheless, the textbooks of classical mechanics continue to solve gyroscopic effects by Euler's principle of the change in the angular momentum[8–10].

Gyroscopic effects are described by mathematical models of the inertial torques, explained in their physics, and validated by practical tests. The mathematical modeling of gyroscopic effects is holistic and should be used for computing the inertial toques and the interrelated motions of the spinning objects for any design of rotating parts in engineering.

Table 1 contains the following: *J* is the moment of inertia of the disc; ω, ω_y, and ω_x are the angular velocities of spinning disc rotations about their axis, *oy*, and *ox*, respectively, other components are described above.

The theory of gyroscopic effects opens a new direction in the dynamics of rotating objects in engineering mechanics and is based on mathematical models, which describe the forces and motions acting by fundamental principles of classical mechanics. Intensification of the work engineering mechanisms and machines with rotating components requires reliable decisions for computing their dynamic parameters. Numerous rotating objects movable in space still do not have engineering methods for computing dynamic parameters and the effective solution for inertial torques of rotating objects with complex geometries.

Table 1. Fundamental principles of the gyroscope theory.

Fundamental principles of the gyroscope theory		**Action**	**Equation**	**Percent of action**
Inertial torques generated by	centrifugal forces	Resistance	$T_{ct.i} = (4\pi^2/9)J\omega\omega_i$	41,141
		Precession		41,141
	Coriolis forces	Resistance	$T_{cr.i} = (8/9)J\omega\omega_i$	8337
	change in angular momentum	Precession	$T_{am,i} = J\omega\omega_i$	9372
Mechanical energy conservation	Dependency of angular velocities of the spinning disc axes of horizontal disposition: $\omega_y = (8\pi^2 + 17)\omega_x$			

Conflict of interest

The author declares no conflict of interest.

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