

# Research on the strategic choice behavior of green building interest subjects based on evolutionary game

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**Abstract:** As a mainstay industry of national economy, construction brings a country huge benefit, et along with significant amount of pollution to environment. In the age of sustainable development, green building (GB) can greatly reduce pollution caused by the construction industry. To study the evolution of stakeholders engaged in China's green building implementation, this paper designed a three-party game model including government, developer, and consumer, analyzed the stability of the model and obtained the evolutionary stability strategy. This paper also used green building data in China to conduct numerical simulation, including sensitive analysis to explore key factors affecting the game subjects, and phase diagrams and bifurcation diagrams to analyze influence of parameter change to the evolutionary stabilization strategy (ESS). The results show that (1) in the long term, the government will choose the regulatory strategy when the cost of government regulation is below one-third of the financial subsidy; (2) the probability of developers and consumers choosing the green building strategy is negatively correlated to the cost and positively correlated to the benefit; (3) the primary determinant behind customers' decision to purchase a green building revolves around the enhanced quality of life that such buildings offer.

**Keywords:** green building; evolutionary game; stability analysis; sensitivity analysis; evolutionary stabilization strategy

## 1. Introduction

As an industry that enriches people, the construction industry plays an important role in promoting world economic growth, solving unemployment problems, and improving people's living environment. However, the construction industry is also one of the major sources of environmental pollution and a large consumer of energy worldwide [1,2]. In 2007, the World Business Council for Sustainable Development reported that the construction industry accounted for 40 percent of total energy consumption in building materials. Disturbingly, predictions indicate that global carbon emissions from construction activities will surge to 42.4 billion tons by 2035, reflecting a staggering 43 percent increase from the levels recorded in 2007 [3]. Notably, China possesses the largest building market and subsequently generates substantial annual energy demands. The China Energy Consumption Study (2020) shows that the total life-cycle carbon emissions from buildings amount to 4.93 billion tonnes, accounting for 51.3% of the total national carbon emissions. In 2021, the total carbon emissions from the whole process of housing construction in the country will be 4.07 billion t CO<sub>2</sub>, accounting for 38.2% of the national energy-related carbon emissions. Among them: 1.70 billion t CO<sub>2</sub>: in the production stage of building

materials, accounting for 16.0% of the national energy-related carbon emissions, or 41.8% of the total energy emissions; 0.07 billion t CO<sub>2</sub>: in the construction stage of the building, accounting for 0.6% of the national energy-related carbon emissions, or 1.6% of the total energy emissions; and 2.30 billion t CO<sub>2</sub>: in the operation stage of the building, accounting for 2.3 billion t CO<sub>2</sub>: of the total energy-related carbon emissions, or 1.6% of the total energy emissions. The carbon emission from the operation phase of buildings is 2.30 billion t CO<sub>2</sub>:, accounting for 21.6% of the national total energy-related carbon emission and 56.6% of the whole process energy carbon emission [4]. China, at the 75th session of the United Nations General Assembly, solemnly pledged its commitment to achieving carbon peaking by 2030 and carbon neutrality by 2060 [5]. However, embracing green building practices stands out as a crucial approach for tackling the prevalent issue of high energy consumption within the construction sector [6]. In order to implement green reforms within the building sector, it is imperative to minimize energy losses and carbon emissions.

As a novel design concept, green building embraces the dual objective of satisfying people's longing for a connection with nature while conserving resources. The design of green buildings aims to create sustainable living and working spaces that cater to people's needs [7]. As the Green Building Evaluation Standards released by the Ministry of Urban and Rural Development in 2019, green buildings are defined as high-quality buildings that conserve resources, protect the environment, reduce pollution, and provide people with healthy, suitable, and efficient spaces during the whole life cycle, maximizing the harmonious coexistence between humans and nature [8]. The implementation of green building involves various stakeholders, including government entities, consumers, developers, contractors, suppliers, the public, and other parties, each driven by their respective interests [9]. As ecology has been considered as an important issue, environmental preservation and sustainability have gained heightened significance within the construction industry, therefore, green buildings are now highly respected worldwide [10]. In a perfect market environment, as green buildings present positive externalities and public goods attributes in the market, developers developing green buildings can present problems such as high costs and technical uncertainties, requiring government measures such as financial subsidies as well as tax penalties to alleviate the problems [1,11,12]. Governments play a pivotal role in guiding and regulating the market. Developers are serving as suppliers within the green building market, and acting as the driving force behind the conception, construction, and implementation of green building projects. They make critical decisions regarding design, construction, and application. However, the positive externalities associated with green building development and the information asymmetry prevalent in the green building market often result in lengthy building cycles and high costs by developers. Consequently, some developers may prioritize only the initial design stages without adequate funds to complete the final implementation, with the aim of maximizing profits. Consumers play a crucial role as demanders in the green building market, and their preferences and needs directly influence the development and adoption of green buildings. However, the existence of factors such as insufficient supporting facilities, low energy conversion and poor living conditions will gradually erode consumers' willingness to buy such properties.

Contractors and suppliers decide whether to participate in green building projects based on market demand. Given these considerations, it is needed to explore the primary stakeholders: government, developers, and consumers, analyze the change the decision-making process among the three.

The implementation of Green Building is a dynamic process of multi-interested subjects interacting with each other. Different multi-interest subjects make corresponding decisions based on maximizing their own interests, which leads to the conflicting decision choices made [13]. And the evolutionary game theory requires participants to be imperfectly rational and can analyze the dynamic process of the interaction of different interests in the green building market. Therefore, evolutionary game theory is a more effective analytical tool to study the conflict and cooperation relationship between interest subjects in the process of green building implementation [14]. In recent years, scholars have increasingly utilized evolutionary game theory to investigate effective strategies for promoting green building implementation. (1) Research on the impact of government incentive policies on the development of green buildings. For example, Qun et al. [2] showcased that utilizing government subsidies for construction units, as outlined within an evolutionary game model, can facilitate the advancement of green building progress. However, providing direct subsidies to consumers might not automatically enhance their uptake of green buildings. Ke et al. [15] constructed an evolutionary game model to study the interactions between the government and consumers, exploring the impact of different types of incentives on developers' decisions. Meng et al. [12] explored the government's ability to facilitate green building development and investigated the mechanism of construction unit selection under government control using a three-party game model. (2) The conditions for cooperation between game parties in the process of green building development were studied. For example, Chen et al. [16] utilized game theory to elucidate the obstacles to reducing greenhouse gas emissions in Israel. They also created a model for evaluating the government's ability to collaborate with consumers and builders in order to surmount these barriers. Ze and Cao [17] developed an evolutionary game model to examine the interactions between technology, knowledge transfer, and firm behavior in innovation networks. Government grants and financial support are essential to promote the development of green building products. (3) Research on problems encountered in the operational phase of green buildings. For example, Liu [18] tackled the challenges of owner-property requirements and conflicts in the operational phase of green buildings using an evolutionary game model. Zhang and Kong [19] studied the influence of government behavior on the behavior of the main body of green building operation and management and analyzed the behavior of the main body of green building operation and management. They constructed an evolutionary game model of the owner side and the property service enterprise from the interest motivation of the various parties involved to find out the focus of interest of both parties.

Previous domestic green building behavioural game studies were mainly based on the theoretical assumptions of classical games, or only considered the evolutionary game behaviours of two stakeholders and rarely discussed the key factors that promote the choice of green building by the game parties when the three-party game subjects influence each other [20–22]. In this paper, by establishing a three-party asymmetric

green building evolutionary game model, the asymmetry between the government, developers and consumers, including the difference in interests, housing pricing and other factors, is fully considered, thus revealing more accurately the behavioural strategies of each party and the impact on the development of green buildings. In addition, some studies have considered the stakeholder relationships related to green buildings, but have not given conclusions and recommendations in the context of green building development in China, and fewer have given objective parameter values for numerical simulation in the numerical simulation part in combination with the current national policies, this paper assumes parameters objectively and adds sensitivity analysis in combination with the values of parameter numbers mentioned in the current national policies, Bifurcation diagram and chaotic attractor analysis and other dynamics methods, an indepth analysis of consumer purchasing behaviour has been carried out to find out the key factors affecting consumers' purchase of green buildings [23]. The government can formulate corresponding policies and measures based on these key factors to promote consumers' motivation to purchase green buildings. The use of real data makes the key influencing factors more convincing, and the comprehensive use of dynamics to analyse the green building market not only helps to comprehensively assess the effectiveness of policies and predict the stability of the green building system, but also identifies the potential risks and thus promotes the stable development of the green building market. This paper aims to construct a three-party asymmetric evolutionary game model of the government, the developer, and the consumer. The primary explores how the government's reward and punishment mechanisms simultaneously influence the decision-making conduct of both factions engaged in the game, specifically the developer and the consumer. Additionally, the study examines the key factors that shape consumers' decisions. By changing the reward and punishment mechanism, the evolutionary path, and the evolutionary equilibrium state of the evolutionary game model of development cost and purchase price, the relationship between the government, developers, and consumers is explored, and the key conditions affecting consumers' strategy choice are found by applying sensitivity analysis. To provide further insights, numerical simulations are employed, analyzing the evolutionary relationship between strategy selection and variations in key influencing factors under different cost conditions. This analysis is visualized using phase diagrams, bifurcation diagrams, and chaotic attractors.

This paper focuses on the following questions: (1) How should the green building evolutionary game model (GB model) among developers, consumers, and government be established? What is the evolutionary process of the green building evolution game model? What is the evolutionary stabilization strategy of the final green building evolutionary game model? (2) What regulatory steps can the government implement to stimulate growth in the green building market? How might changes in government incentives and penalties affect consumers and developers respectively? (3) What factors play a pivotal role in shaping consumers' choices? In what scenarios would consumers opt for purchasing green buildings? The structure of this paper is as follows: section 2, the fundamental assumptions of the model are introduced; section 3 establishes the three-party evolutionary game model, involving the government, developers, and consumers. This section also examines the asymptotic stability of the evolutionary game model's dynamics; section 4 analyzes the sensitivity of consumer

parameters to find the key conditions that influence the choice of consumer strategies; section 5 employs numerical simulations to assess the impact of parameters on the stabilizing evolutionary strategy, using bifurcation and phase diagrams. Finally, section 6 presents conclusions and recommendations.

## 2. Model assumptions

### 2.1. Description of model-related symbols

All assumed parameters are shown in **Table 1**.

**Table 1.** The meaning of each parameter.

Parameter	Meanings
$C_1$	Additional cost to the government when choosing to regulate.
$R_0$	Hidden benefits when governments choose to regulate.
$L_1$	Hidden losses when the government chooses not to regulate.
$R_1$	Benefits for developers when they choose to develop conventional buildings and for consumers when they choose to buy green buildings.
$R_2$	Benefits to developers when they choose to develop green buildings and to consumers when they choose to buy green buildings.
$R_3$	Benefits for developers when they choose to develop green buildings and for consumers when they choose to buy conventional buildings.
$R_4$	Benefits for developers when they choose to develop conventional buildings and for consumers when they choose to buy conventional buildings.
$C_2$	Additional Costs for Developers When Choosing to Develop Green Buildings.
$J_1$	Government incentives when developers choose to develop green buildings.
$L_2$	Penalties for developers who choose to develop conventional buildings.
$J_2$	Incentives for consumers to choose to buy green buildings when the government chooses to regulate them.
$P_1$	Developers develop conventional buildings consumers choose to buy green buildings purchase price.
$P_2$	Developers develop green buildings Consumers choose to buy green buildings at purchase price.
$P_3$	Developers develop conventional buildings consumers choose to buy conventional buildings purchase price.
$P_4$	Developers develop green buildings consumers choose to buy conventional buildings purchase price.
$W_1$	Benefits to consumers when they choose to purchase green buildings.
$W_2$	Losses are suffered by consumers when they choose to purchase conventional buildings.

### 2.2. Basic assumptions

Constructing green building evolutionary game model to analyze the strategy changes and stability of equilibrium points of the game parties, the ensuing assumptions are posited [24]:

(1) In the process of the green building game, the main body of the game, cannot know, the other side of the decision-making, and the need to constantly learn and adjust the final choice of their own strategy, therefore, we assume that the main body

of the three-way game is limited rationality. In the process of the game, the game parties will constantly make corresponding decisions, and over time, all individuals will make decisions toward the strategy with greater self-interest. As well as the existence of incomplete information symmetry between the government, real estate developers, and consumers [25–27];

(2) In the evolutionary game, each participating subject choose a strategy with a certain probability. It is posited that the government's strategy set is  $\alpha = \{\alpha_1, \alpha_2\}$ , the government chooses to regulate  $\alpha_1$  with probability of  $x$ , choose not to supervise  $\alpha_2$  with probability  $1 - x$ ,  $x \in [0, 1]$ . Assume that the real estate developer's strategy set is  $\beta = \{\beta_1, \beta_2\}$ , the developer chooses to develop a green building  $\beta_1$  with probability  $y$ , and chooses to develop conventional buildings  $\beta_2$  with probability  $1 - y$ ,  $y \in [0, 1]$ . Assume that the consumer's strategy set is  $\gamma = \{\gamma_1, \gamma_2\}$ , consumers choose to buy green buildings  $\gamma_1$  with probability  $z$ , choose to buy conventional buildings  $\gamma_2$  with probability  $1 - z$ ,  $z \in [0, 1]$ ;

(3) Assuming that the government chooses a regulatory strategy, it will have to pay additional regulatory costs of  $C_1$ , financial subsidies to developers and consumers of  $J_1$  and  $J_2$ , but the government will receive hidden gain of  $R_0$  (social praise and people's feedback) and tax revenue  $L_2$  from undeveloped green building enterprises, if the government chooses a non-regulatory strategy, it will not need to pay additional costs, it will suffer hidden losses of  $L_1$ ;

(4) It is assumed that when the developer opts for the strategy of developing green buildings, it adds additional development costs of  $C_2$ . When a developer selects the strategy of developing conventional buildings, it does not receive financial subsidies from the government;

(5) It is assumed that when consumers choose to purchase the green building strategy, they will pay a price  $P_2$ , and will receive a gain of  $W_1$  (improved quality of life and energy savings) and a financial incentive of  $J_2$  when the government regulates them [28]. When the consumer chooses to purchase a conventional building strategy, he or she pays a price  $P_3$  and suffers a loss of  $W_2$  (personal quality of life and energy savings are not met);

(6) Government: Assume that the hidden gain when the government chooses to regulate  $R_0$  must be greater than the additional cost of choosing a regulatory strategy  $C_1$ , and the additional regulatory cost paid  $C_1$  must be less than the hidden loss when it does not regulate  $L_1$ . If the hidden loss suffered by the government when it chooses not to regulate  $L_1$  must be greater than the reward to the other parties to the game  $J_1 + J_2$ ;

(7) Developers: Assume that the amount of tax penalty from the government when the developer chooses the conventional building strategy  $L_2$  is greater than the additional cost  $C_2$ , when developing a green building, and the financial incentive from the government when the developer chooses the green building strategy  $J_1$  is greater than the additional cost  $C_2$  for developing a green building. Assuming that the developer's revenue from developing green buildings is greater than the revenue from developing conventional buildings ( $R_2 > R_4$ ,  $R_3 > R_1$ ), the developer will be able to obtain greater revenue when the consumer demand is the same as the developer's supply ( $R_2 > R_3 > R_4 > R_1$ );

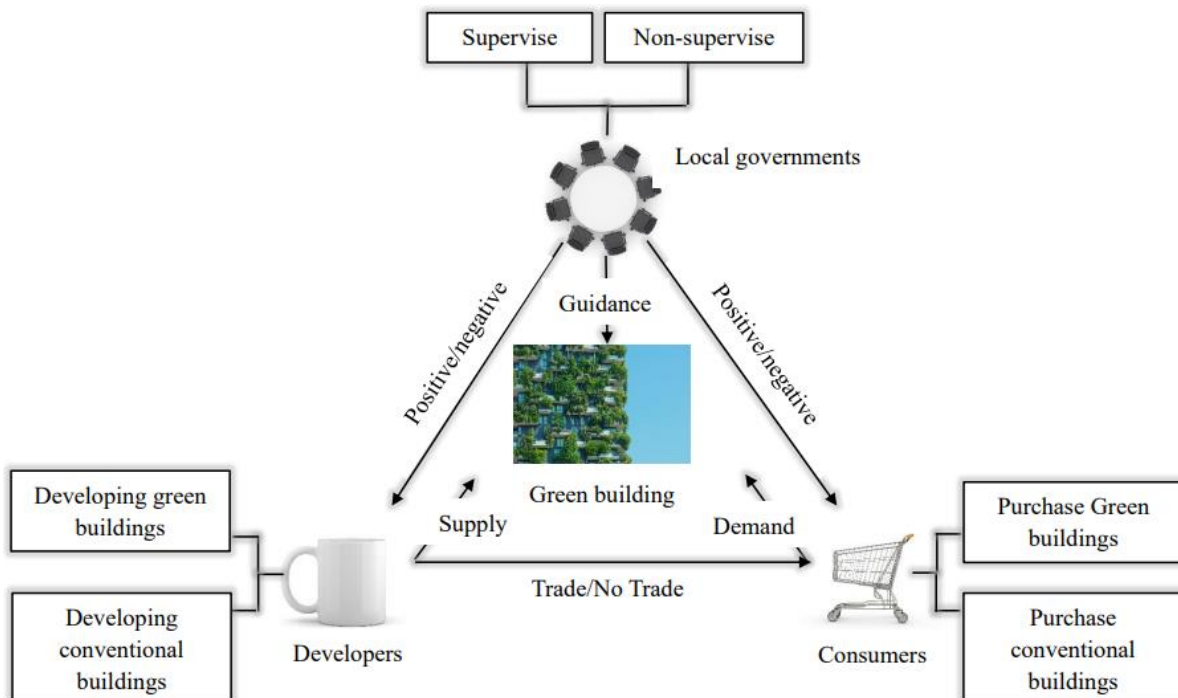
(8) Consumers: Assume that consumers gain more when they choose the green building strategy  $W1$  than they lose when they choose the conventional building strategy  $W2$ , and that consumers receive more financial incentives  $J2$  than the additional amount when they buy a green building  $P1 - P2, P4 - P3$ . Assuming that the price of conventional buildings is lower than the price of green buildings ( $P1 > P4, P2 > P3$ ), the consumer's demand is the same as the developer's supply, and the consumer's purchase price is lower ( $P1 > P2 > P4 > P3$ );

### 3. Construction and stability analysis of the model

#### 3.1. Model construction

Evolutionary stability theory focuses on the process of individuals gradually adjusting their strategies through learning and imitation in a dynamic environment. At its core, it analyses the stability and feasibility of strategies through dynamic mechanisms. In the green building market, evolutionary stability theory can explain why the green building market needs a long promotion period and how different strategies evolve into stable strategies. The evolutionary game model can be used to assess how different strategies adopted by the players in the green building market will affect the green building market. Therefore, we construct a GB model to study the green building market.

The framework diagram illustrating the implementation of the model based on the above assumptions is presented in **Figure 1**.



**Figure 1.** Implementation framework diagram.

Utilizing the model assumptions and parameter explanations provided above, we derive the payoff matrix for the GB model, displayed in **Table 2**.

**Table 2.** GB model payoff.

Player		Governments		
		Regulation $x$	No-regulation $1 - x$	
Developers	Developing green buildings $y$	Consumers Purchase green buildings $z$	$R_2 - C_2 + J_1$	$R_2 - C_2$
			$-P_2 + W_1 + J_2$	$-P_2 + W_1$
	Consumers Purchase conventional buildings $1 - z$	$-C_1 + R_0 - J_1 - J_2$	$-L_1$	
		$R_3 - C_2 + J_1$	$R_3 - C_2$	
Developing conventional buildings $1 - y$	Consumers Purchase green buildings $z$	$-P_4 - W_2$	$-P_4 - W_2$	
		$-C_1 + R_0 - J_1$	$-L_1$	
	Consumers Purchase conventional buildings $1 - z$	$R_1 - L_2$	$R_1 - L_2$	
		$-P_1 + W_1 + J_2$	$-P_1 + W_1$	
		$-C_1 + R_0 + L_2 - J_2$	$-L_1 + L_2$	
		$R_4 - L_2$	$R_4 - L_2$	
		$-P_3 - W_2$	$-P_3 - W_2$	
		$-C_1 + R_0 + L_2$	$-L_1 + L_2$	

Based on the model construction, the model tripartite payoff matrix is obtained and the replication dynamic equations for the three parties are given below.

Based on the payoff matrix in **Table 2**, the anticipated gain  $E_{x1}$  for the government when selecting the regulatory strategy can be described as follows:

$$E_{x1} = yz(-C_1 + R_0 - J_1 - J_2) + y(1 - z)(-C_1 + R_0 - J_1) + (1 - y)z(-C_1 + R_0 + L_2 - J_2) + (1 - y)(1 - z)(-C_1 + R_0 + L_2) \quad (1)$$

The expected benefits  $E_{x2}$  of the government choosing a no-regulation strategy are:

$$E_{x2} = yz(-L_1) + y(1 - z)(-L_1) + (1 - y)z(L_2 - L_1) + (1 - y)(1 - z)(L_2 - L_1) \quad (2)$$

The average revenue  $\bar{E}_x$  of the government is:

$$\bar{E}_x = xE_{x1} + (1 - x)E_{x2} \quad (3)$$

The replication dynamics equation for the government is:

$$\begin{aligned} F(x, y, z) &= x(1 - x)(E_{x1} - E_{x2}) \\ &= x(1 - x)(-C_1 + R_0 - J_2z + L_1 - J_1y) \end{aligned} \quad (4)$$

Based on the payoff matrix in **Table 2**, the anticipated gain  $E_{y1}$  of developers when selecting the develope green buildings strategy can be described as follows:

$$E_{y1} = xz(R_2 - C_2 + J_1) + (1 - x)z(R_2 - C_2) + (1 - z)x(R_3 - C_2 + J_1) + (1 - z)(1 - x)(R_3 - C_2) \quad (5)$$

The expected benefits  $E_{y2}$  of the developers choosing conventional buildings development strategy are:

$$E_{y2} = xz(R_1 - L_2) + (1 - x)z(R_1 - L_2) + (1 - z)x(R_4 - L_2) + (1 - z)(1 - x)(R_4 - L_2) \quad (6)$$

The average revenue  $\bar{E}_y$  of the developer is:



$$\overline{E}_y = yE_{y1} + (1 - y)E_{y2} \quad (7)$$

The replication dynamic function for the developer is:

$$G(x, y, z) = y(1 - y)[z(R_2 - R_1 - R_3 + R_4) + R_3 - C_2 + xJ_1 - R_4 + L_2] \quad (8)$$

Based on the payoff matrix in **Table 2**, the anticipated gain  $E_{z1}$  of consumers when selecting the purchase green buildings strategy can be described as follows:

$$E_{z1} = xy(-P_2 + W_1 + J_2) + (1 - x)y(-P_2 + W_1) + x(1 - y)(-P_3 - W_2) + (1 - x)(1 - y)(-P_3 - W_2) \quad (9)$$

The expected benefit  $E_{z2}$  of consumers choosing to purchase the conventional building strategy is:

$$E_{z2} = xy(-P_4 - W_2) + (1 - x)y(-P_4 - W_2) + x(1 - y)(-P_3 - W_2) + (1 - x)(1 - y)(-P_3 - W_2) \quad (10)$$

The average revenue  $\overline{E}_z$  of the consumers is:

$$\overline{E}_z = zE_{z1} + (1 - z)E_{z2} \quad (11)$$

The replication dynamic function for the consumer is:

$$H(x, y, z) = z(1 - z)[xJ_2 + y(-P_2 + P_1 - P_3 + P_4) - P_1 + W_1 + P_3 + W_2] \quad (12)$$

Based on the preceding analysis, we obtain the replicated dynamic equations of the green building evolutionary game model:

$$\begin{aligned} \dot{x} &= F(x, y, z), \\ \dot{y} &= G(x, y, z), \\ \dot{z} &= H(x, y, z). \end{aligned}$$

among which

$$\begin{aligned} F(x, y, z) &= x(1 - x)(-C_1 + R_0 - J_2z + L_1 - J_1y), \\ G(x, y, z) &= y(1 - y)[z(R_2 - R_1 - R_3 + R_4) + R_3 - C_2 + xJ_1 - R_4 + L_2], \\ H(x, y, z) &= z(1 - z)[xJ_2 + y(-P_2 + P_1 - P_3 + P_4) - P_1 + W_1 + P_3 + W_2]. \end{aligned}$$

### 3.2. Stability analysis

Next we will replicate the dynamics of the dynamic equations for the:

Since the government's replication dynamic equation is:

$$\begin{aligned} F(x, y, z) &= x(1 - x)(E_{x1} - E_{x2}) \\ &= x(1 - x)(-C_1 + R_0 - J_2z + L_1 - J_1y) \end{aligned} \quad (13)$$

If  $z = z_0 = \frac{-C_1 + R_0 + L_1 - J_1y}{J_2}$ , for any  $x$ ,  $F(x, y, z) = 0$ , shows that any value of  $x$  is an evolutionary steady state.

If  $z \neq z_0 = \frac{-C_1 + R_0 + L_1 - J_1y}{J_2}$ , let  $F(x, y, z) = 0$ , the two stable points  $(0, y, z)$ ,  $(1, y, z)$  can be obtained, and the derivative of  $F(x, y, z)$  can be obtained as:

$$\frac{\partial F(x, y, z)}{\partial x} = (1 - 2x)(-C_1 + R_0 - J_2z + L_1 - J_1y) \quad (14)$$

Case1:

If  $0 < z < \frac{-C_1+R_0+L_1-J_1y}{J_2}$ , then  $\left. \frac{\partial F(x,y,z)}{\partial x} \right|_{(0,y,z)} > 0$  and  $\left. \frac{\partial F(x,y,z)}{\partial x} \right|_{(1,y,z)} < 0$ ,  $(1, y, z)$  is the evolutionary dynamic equilibrium point, when the probability of consumers choosing to purchase a green building is less than  $\frac{-C_1+R_0+L_1-J_1y}{J_2}$ , the government will choose regulate the strategy.

Case2:

If  $\frac{-C_1+R_0+L_1-J_1y}{J_2} < z < 1$ , then  $\left. \frac{\partial F(x,y,z)}{\partial x} \right|_{(0,y,z)} < 0$  and  $\left. \frac{\partial F(x,y,z)}{\partial x} \right|_{(1,y,z)} > 0$ ,  $(1, y, z)$  is the evolutionary dynamic equilibrium point; when the probability of consumers choosing to purchase a green building is greater than  $\frac{-C_1+R_0+L_1-J_1y}{J_2}$ , the government will choose no-regulate the strategy.

Since the developer's replication dynamic equation is:

$$G(x, y, z) = y(1 - y)[E_{y1} - E_{y2}] \quad (15)$$

$$= y(1 - y)[z(R_2 - R_1 - R_3 + R_4) + R_3 - C_2 + xJ_1 - R_4 + L_2]$$

If  $z = z_1 = \frac{-xJ_1-R_3+R_4+C_2-L_2}{R_2-R_1-R_3+R_4}$ , for any  $y$ ,  $G(x, y, z) = 0$ , shows that any value of  $y$  is an evolutionary steady state.

If  $z \neq z_0 = \frac{-xJ_1-R_3+R_4+C_2-L_2}{R_2-R_1-R_3+R_4}$ , let  $G(x, y, z) = 0$ , the two stable points  $(x, 0, z)$ ,  $(x, 1, z)$  can be obtained, and the derivative of  $G(x, y, z)$  can be obtained as:

$$\frac{\partial G(x, y, z)}{\partial y} = (1 - 2y)[z(R_2 - R_1 - R_3 + R_4) + J_1x + R_3 - R_4 - C_2 + L_2] \quad (16)$$

Case1:

If  $0 < z < \frac{-xJ_1-R_3+R_4+C_2-L_2}{R_2-R_1-R_3+R_4}$ , then  $\left. \frac{\partial G(x,y,z)}{\partial y} \right|_{(x,0,z)} < 0$  and  $\left. \frac{\partial G(x,y,z)}{\partial y} \right|_{(x,1,z)} > 0$ ,  $(x, 0, z)$  is the evolutionary dynamic equilibrium point; when the probability of consumers choosing to purchase a green building is less than  $\frac{-xJ_1-R_3+R_4+C_2-L_2}{R_2-R_1-R_3+R_4}$ , the developers will choose to develop conventional building strategy;

Case2:

If  $\frac{-xJ_1-R_3+R_4+C_2-L_2}{R_2-R_1-R_3+R_4} < z < 1$ , then  $\left. \frac{\partial G(x,y,z)}{\partial y} \right|_{(x,0,z)} > 0$  and  $\left. \frac{\partial G(x,y,z)}{\partial y} \right|_{(x,1,z)} < 0$ ,  $(x, 1, z)$  is the evolutionary dynamic equilibrium point; when the probability of consumers choosing to purchase a green building is greater than  $\frac{-xJ_1-R_3+R_4+C_2-L_2}{R_2-R_1-R_3+R_4}$ , the developers will choose to develop green building strategy.

Since the consumer's replication dynamic equation is:

$$H(x, y, z) = z(1 - z)(E_{z1} - E_{z2}) \quad (17)$$

$$= z(1 - z)[xJ_2 + y(-P_2 + P_1 - P_3 + P_4) - P_1 + W_1 + P_3 + W_2]$$

If  $y = y_0 = \frac{P_1-P_3-xJ_2-W_1-W_2}{-P_2-P_3+P_1+P_4}$ , for any  $z$ ,  $H(x, y, z) = 0$ , shows that any value of  $z$  is an evolutionary steady state.

If  $y \neq y_0 = \frac{P_1 - P_3 - xJ_2 - W_1 - W_2}{-P_2 - P_3 + P_1 + P_4}$ , let  $H(x, y, z) = 0$ , the two stable points  $(x, y, 0)$ ,  $(x, y, 1)$  can be obtained, and the derivative of  $H(x, y, z)$  can be obtained as:

$$\frac{\partial H(x, y, z)}{\partial z} = (1 - 2z)[y(-P_2 - P_3 + P_1 + P_4) - P_1 + P_3 + xJ_2 + W_1 + W_2] \quad (18)$$

Case1:

If  $0 < y < \frac{P_1 - P_3 - xJ_2 - W_1 - W_2}{-P_2 - P_3 + P_1 + P_4}$ , then  $\frac{\partial H(x, y, z)}{\partial z} \Big|_{(x, y, 0)} < 0$  and  $\frac{\partial H(x, y, z)}{\partial z} \Big|_{(x, y, 1)} > 0$ ,  $(x, y, 0)$  is the evolutionary dynamic equilibrium point, when the probability of consumers choosing to purchase a green building is less than  $\frac{P_1 - P_3 - xJ_2 - W_1 - W_2}{-P_2 - P_3 + P_1 + P_4}$ , the consumers will choose to buy conventional building strategy;

Case2:

If  $\frac{P_1 - P_3 - xJ_2 - W_1 - W_2}{-P_2 - P_3 + P_1 + P_4} < y < 1$ , then  $\frac{\partial H(x, y, z)}{\partial z} \Big|_{(x, y, 0)} > 0$  and  $\frac{\partial H(x, y, z)}{\partial z} \Big|_{(x, y, 1)} < 0$ ,  $(x, y, 1)$  is the evolutionary dynamic equilibrium point; when the probability of consumers choosing to purchase a green building is greater than  $\frac{P_1 - P_3 - xJ_2 - W_1 - W_2}{-P_2 - P_3 + P_1 + P_4}$ , the consumers will choose to buy green building strategy.

To study the strategic choices of the game parties in the green building evolution game model, this section determines the stability point of the evolutionary game model by replicating the dynamic equations for stability analysis [29].

Let  $F(x, y, z) = 0$ ,  $G(x, y, z) = 0$ ,  $H(x, y, z) = 0$ , the equilibrium points of the evolutionary game model are obtained as follows, respectively:  $(0, 0, 0)$ ,  $(0, 0, 1)$ ,  $(1, 0, 0)$ ,  $(0, 1, 0)$ ,  $(1, 1, 0)$ ,  $(0, 1, 1)$ ,  $(1, 0, 1)$ ,  $(1, 1, 1)$ ,  $(\alpha^*, \beta^*, \gamma^*)$ , where  $(\alpha^*, \beta^*, \gamma^*)$  for mixed strategy. Since in asymmetric games, when the equilibrium evolving in the game is an evolutionarily stable strategy, it must also be a strict Nash equilibrium, which in turn is a pure strategy equilibrium. In other words, the mixed strategy equilibrium in asymmetric game dynamics must not be an evolutionary stable equilibrium [26]. Therefore, we only discuss the eight pure strategy equilibria in the three-party game model of green building.

To analyze the local asymptotic stability of the equilibrium point, the following Jacobi matrix J is established:

$$J = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

where:

$$a_{11} = \frac{\partial F(x, y, z)}{\partial x} = (1 - 2x)(-C_1 + R_0 - J_2z + L_1 - yJ_1)$$

$$a_{12} = \frac{\partial F(x, y, z)}{\partial y} = -x(1 - x)J_1$$

$$a_{13} = \frac{\partial F(x, y, z)}{\partial z} = -x(1 - x)J_2$$

$$a_{21} = \frac{\partial G(x, y, z)}{\partial x} = y(1 - y)J_1$$

$$a_{22} = \frac{\partial G(x, y, z)}{\partial y} = (1 - 2y)[z(R_2 - R_1 - R_3 + R_4) + xJ_1 - C_2 + R_3 - R_4 + L_2]$$

$$a_{23} = \frac{\partial G(x, y, z)}{\partial z} = y(1 - y)(R_2 - R_1 - R_3 + R_4)$$

$$a_{31} = \frac{\partial H(x, y, z)}{\partial x} = z(1 - z)J_2$$

$$a_{32} = \frac{\partial H(x, y, z)}{\partial y} = -z(1 - z)(P_2 + P_3 - P_1 - P_4)$$

$$a_{33} = \frac{\partial H(x, y, z)}{\partial z} = (1 - 2z)[xJ_2 + y(-P_2 - P_3 + P_1 + P_4) - P_1 + W_1 + P_3 + W_2]$$

Using Lyapunov’s first method, it is judged that if the equilibrium point in the system satisfies the Jacobi matrix and makes all eigenvalues negative, it indicates that this equilibrium point is an evolutionary stable strategy (ESS). On the contrary, when all eigenvalues are positive, it indicates that this equilibrium point is an unstable point, and when there are positive and negative eigenvalues, it indicates that this eigenvalue is a saddle point. The outcomes are illustrated in **Table 3**:

When the equilibrium point is (0, 0, 0), the eigenvalue size is judged according to the following assumptions:

$$\lambda_1 = -C_1 + R_0 + L_1 > 0$$

$$\lambda_2 = -C_2 + R_3 - R_4 + L_2 > 0$$

$$\lambda_3 = -P_3 + W_1 + P_4 + W_2 > 0$$

The eigenvalues of other equilibrium points are calculated in the same way as when the equilibrium point is (0, 0, 0), and the results are presented in **Table 3**.

**Table 3.** Local stability analysis.

Equilibrium points	Eigenvalue $\lambda_1$	Eigenvalue $\lambda_2$	Eigenvalue $\lambda_3$	Local stability
(0, 0, 0)	> 0	> 0	> 0	Unstable
(0, 0, 1)	> 0	> 0	< 0	Saddle
(1, 0, 0)	< 0	> 0	> 0	Saddle
(0, 1, 0)	> 0	< 0	> 0	Saddle
(0, 1, 1)	> 0	< 0	< 0	Saddle
(1, 1, 0)	< 0	< 0	> 0	Saddle
(1, 0, 1)	> 0	> 0	< 0	Saddle
(1, 1, 1)	< 0	< 0	< 0	Stable

From **Table 3**, it is evident that the equilibrium point (1, 1, 1) is the asymptotic evolutionary stability point when the assumptions on the parameters are satisfied. At this equilibrium point, the government chooses the regulatory strategy, the developer chooses the strategy of developing green buildings, and the consumer chooses the strategy of purchasing green buildings. In this scenario, the game parties will make decisions in the direction of the greatest benefit. The government chooses the regulatory strategy, improves the incentive mechanism for developers and consumers

to choose green buildings, and increases the financial subsidy  $J_1$  for developers engaging in green building projects. This in turn elevates the additional profit gained by developers who choose green building development. Additionally, the government can increase the tax  $L_2$  for developers to develop conventional buildings, thus increasing their cost to develop conventional buildings. In addition, the government can enhance the price subsidy  $J_2$  provided to consumers for procuring green buildings. This adjustment serves to lower the purchase cost for consumers opting for green buildings. When developers and consumers choose green buildings, developers bring more price concessions to consumers and consumers bring more benefits to developers. Eventually, the evolutionary game model will tend to approach the asymptotic evolutionary stability point (1, 1, 1).

## 4. Numerical simulation

To evaluate the model's analysis accuracy, it makes numerical assumptions in conjunction with China's green building policy. It uses Python as well as MATLAB R2018a for numerical simulation to clearly demonstrate the evolution of government, developer, and consumer decisions in the model.

### 4.1. Numerical assumptions

China's green building market has experienced an exceptionally rapid expansion, and according to the Ministry of Housing and Urban-Rural Development, China's green building area accounted for 90% of new buildings as of the first half of 2022, having grown from 4 million square meters in 2012 to 2 billion square meters. We selected the 2012 "Implementation Opinions on Accelerating the Development of China's Building Group" jointly issued by the Ministry of Finance and the Ministry of Housing and Construction and the 2012 data on the incremental cost of green buildings for 148 projects reviewed by China Science Research Council and the 2020 China Green Building Development Report on government incentives, incremental costs, and incremental benefits of secondary green buildings for our study. In order to ensure a comprehensive examination of green buildings and their surrounding infrastructure, we have chosen to focus on the concept of a "5-minute living circle residential area." This concept is defined in the 2018 Urban Residential Area Planning and Design Standards. Within this context, we assume that the land area of a community within a 5-minute living circle residential area falls between 10,000 and 25,000 square meters, and reasonable numerical assumptions are made for the convenience of calculation. We assumed that the initial value of the parameter to be  $C_1 = 2$ ,  $R_0 = 4$ ,  $L_1 = 5$ ,  $R_1 = 200$ ,  $R_2 = 240$ ,  $C_2 = 8$ ,  $J_1 = 4$ ,  $L_2 = 2$ ,  $P_2 = 12$ ,  $P_3 = 10$ ,  $P_4 = 11$ ,  $W_1 = 1$ ,  $W_2 = 1$ ,  $J_2 = 2$ , the unit is 100,000 RMB. In this paper, by changing the value of the five-minute living circle of green buildings, we find out the impact on the green building market when different players adopt different strategies through numerical simulation, and we find out the critical values of government subsidies, developers' and consumers' development costs and selling prices of green buildings, so as to find a stable strategy that can make the main players of the game choose green buildings [30–32]. The detailed code can be found in the Appendix.

## 4.2. Sensitivity analysis

Consumer demand for housing in the market can influence the strategies made by developers and even impact the choice of green buildings in the entire market. To assess the extent to which changes in different consumer parameters affect the evolutionary game model of green building, this section conducts a sensitivity analysis of three parameters in this paper: the price  $P_1$  of consumers buying green buildings, the price  $P_4$  of consumers buying conventional buildings and the consumer's gain  $W_1$ . This paper will conduct a sensitivity analysis.

Assuming the nominal values of the parameters  $W_1, P_4, P_2$  are  $P_2 = 12, P_4 = 11, W_1 = 1$ , then the nominal system is:

$$\dot{x} = F(x, y, z)|_A = x(1 - x)(7 - 2z - 4y) \quad (19)$$

$$\dot{y} = G(x, y, z)|_A = y(1 - y)(14 + 20z + 4x) \quad (20)$$

$$\dot{z} = H(x, y, z)|_A = z(1 - z)(2x + 2y - 1) \quad (21)$$

Jacobi matrix  $\left. \frac{\partial F(x,y,z)}{\partial(x,y,z)} \right|_{nominal}$  nominal and  $\left. \frac{\partial F(F,G,H)}{\partial(P_2,P_4,W_1)} \right|_{nominal}$  nominal respectively:

$$\left. \frac{\partial F(x,y,z)}{\partial(x,y,z)} \right|_{nominal} = \begin{bmatrix} (1-2x)(7-2z-4y) & -4x(1-x) & -2x(1-x) \\ 4y(1-y) & (1-2y)(20z+4x+14) & 20y(1-y) \\ 2z(1-z) & 2z(1-z) & (1-2z)(2x+2y-1) \end{bmatrix} \quad (22)$$

$$\left. \frac{\partial F(F,G,H)}{\partial(P_2,P_4,W_1)} \right|_{nominal} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -z(1-z)y & z(1-z)y & z(1-z) \end{bmatrix} \quad (23)$$

Set up

$$s = \left[ \begin{array}{ccc} x_4 & x_7 & x_{10} \\ x_5 & x_8 & x_{11} \\ x_6 & x_9 & x_{12} \end{array} \right] \Bigg|_{nominal} = \left[ \begin{array}{ccc} \frac{\partial x_1}{\partial P_2} & \frac{\partial x_1}{\partial P_4} & \frac{\partial x_1}{\partial W_1} \\ \frac{\partial x_2}{\partial P_2} & \frac{\partial x_2}{\partial P_4} & \frac{\partial x_2}{\partial W_1} \\ \frac{\partial x_3}{\partial P_2} & \frac{\partial x_3}{\partial P_4} & \frac{\partial x_3}{\partial W_1} \end{array} \right] \Bigg|_{nominal} \quad (24)$$

By

$$\dot{s} = \left[ \frac{\partial f(P_2, P_4, W_1)}{\partial x} \right] \Bigg|_{nominal} s + \left[ \frac{\partial f(P_2, P_4, W_1)}{\partial \lambda} \right] \Bigg|_{nominal} \quad (25)$$

It can be concluded that:

$$\dot{x}_1 = x(1 - x)(6 - 2z - 4y), \quad x_1(0) = x_{10}$$

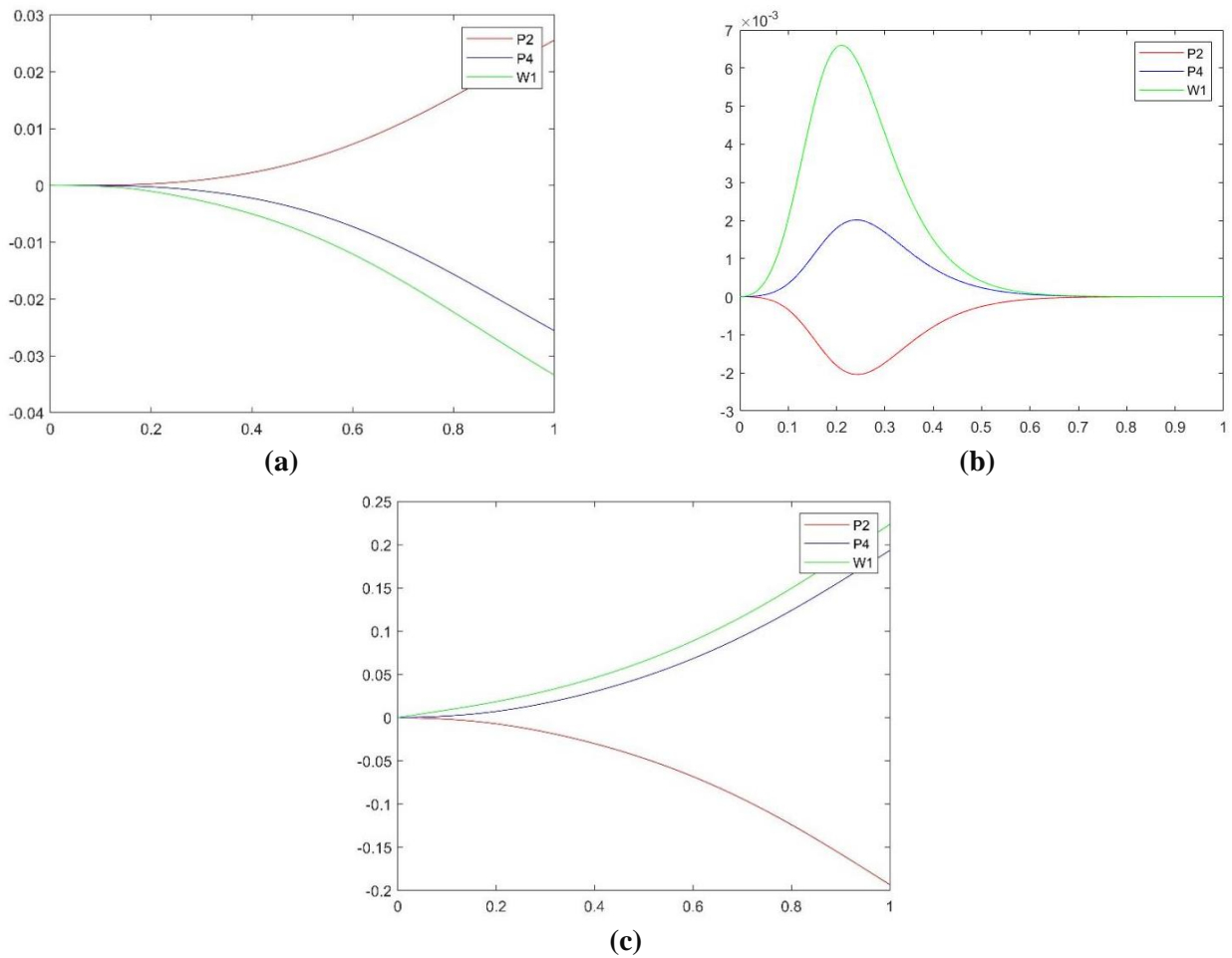
$$\dot{x}_2 = y(1 - y)(14 + 20z + 4x), \quad x_2(0) = x_{20}$$

$$\dot{x}_3 = z(1 - z)(2x + 2y - 1), \quad x_3(0) = x_{30}$$

$$\dot{x}_4 = (1 - 2x)(6 - 2z - 4y)x_4 - 4x(1 - x)x_5 - 2x(1 - x)x_6, \quad x_4(0) = 0$$

$$\begin{aligned} \dot{x}_5 &= 4y(1-y)x_4 + (1-2y)(14+20z+4x)x_5 + 20y(1-y)x_6, \quad x_5(0) = 0 \\ \dot{x}_6 &= 2z(1-z)x_4 + 2z(1-z)x_5 + (1-2z)(2x+2y-1)x_6 - z(1-z)y, \quad x_6(0) = 0 \\ \dot{x}_7 &= (1-2x)(6-2z-4y)x_7 - 4x(1-x)x_8 - 2x(1-x)x_9, \quad x_7(0) = 0 \\ \dot{x}_8 &= 4y(1-y)x_7 + (1-2y)(14+20z+4x)x_8 + 20y(1-y)x_9, \quad x_8(0) = 0 \\ \dot{x}_9 &= 2z(1-z)x_7 + 2z(1-z)x_8 + (1-2z)(-1+2y+2x)x_9 + z(1-z)y, \quad x_9(0) = 0 \\ \dot{x}_{10} &= (1-2x)(6-2z-4y)x_{10} - 4x(1-x)x_{11} - 2x(1-x)x_{12}, \quad x_{10}(0) = 0 \\ \dot{x}_{11} &= 4y(1-y)x_{10} + (1-2y)(14+20z+4x)x_{11} + 20y(1-y)x_{12}, \quad x_{11}(0) = 0 \\ \dot{x}_{12} &= 2z(1-z)x_{10} + 2z(1-z)x_{11} + (1-2z)(-1+2x+2y)x_{12} + z(1-z), \quad x_{12}(0) = 0 \end{aligned}$$

We assume that the initial state is:  $x = y = z = 0.1$ ,  $x_4(0) = x_5(0) = x_6(0) = x_7(0) = x_8(0) = x_9(0) = x_{10}(0) = x_{11}(0) = x_{12}(0) = 0$ . Obtained by MATLAB R2018a in **Figure 2a** is shown the sensitivity of  $x$  to  $P_2, P_4, W_1$ . The sensitivity of  $y$  to  $P_2, P_4, W$  is shown in **Figure 2b** and the sensitivity of  $z$  to  $P_2, P_4, W_1$  is shown in the third panel of **Figure 2c** [33].



**Figure 2.** Sensitivity analysis of parameters  $P_2, P_4, W_1$ .

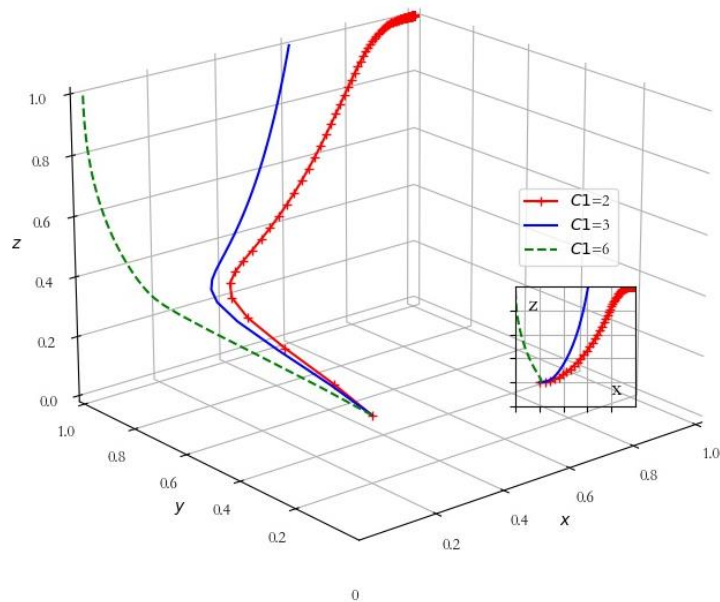
Based on **Figure 2**, it can be seen that the benefits obtained  $W_1$  by consumers from purchasing green buildings have the highest sensitivity to the overall model.

Based on the above analysis, it is evident that improving the benefits of green buildings to consumers, such as enhancing their personal quality of life and energy-saving benefits, will drive consumers to prefer green buildings. Therefore, developers who focus on developing green buildings should prioritize improving the quality of life of consumers by enhancing supporting facilities, increasing green areas around buildings, improving energy efficiency, and implementing strict environmental monitoring of houses where consumers reside. Moreover, developers should use cost-effective and low-polluting decorative materials that prioritize the quality of life of consumers and energy-saving features.

### 4.3. Evolutionary tests of stabilization strategies

#### 4.3.1. The impact of regulatory costs on government strategy

This section explores how the regulatory cost affects the regulatory strategy chosen by the government. The regulatory cost  $C_1$  is compared at 2, 3, and 6, respectively, to observe how the government's strategy selection will be impacted. Our analysis is based on the findings presented in **Figure 3**, which were generated using Python.



**Figure 3.** The influence of government regulatory costs on the adoption of regulatory strategies.

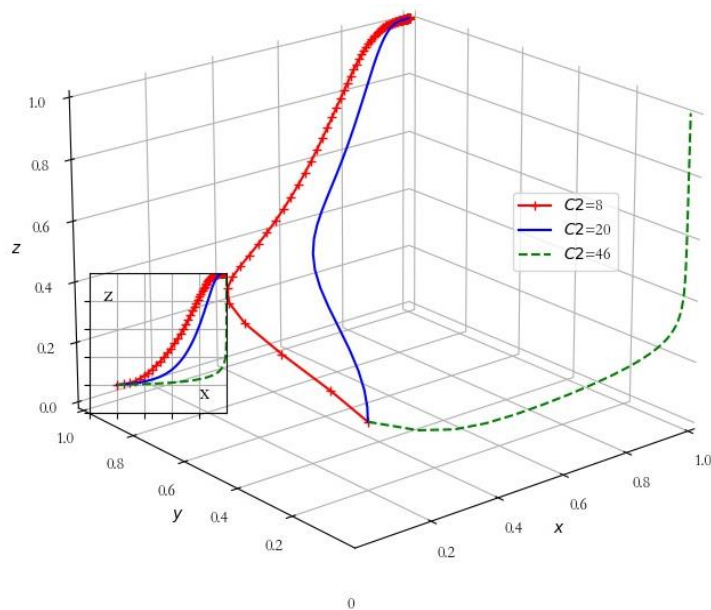
Based on the numerical simulation results in **Figure 3**, the influence of regulatory costs on the government's decision regarding the choice of regulatory strategy is highlighted. As the regulatory cost  $C_1$  increases, the government's willingness  $\lambda$  to choose a regulatory strategy decreases and tends towards 0. The results indicate that the cost  $C_1$  of regulation affects the government's decision-making process. In this study, the regulatory cost  $C_1$  of the government should be less than 1/3 of the financial subsidies (incentives for developers  $J_1$  and  $J_2$  incentives for consumers). When the regulatory cost  $C_1$  exceeds 1/3 of the financial subsidies, the government is no longer willing to choose the regulatory strategy. Therefore, it is recommended to decrease the



regulatory cost  $C_1$  to encourage the government's adoption of a reward and penalty system. This approach can help tackle the market challenges tied to the elevated costs and technical uncertainties of opting for green buildings.

#### 4.3.2. Impact of additional costs on developers' strategies

This section explores how the additional cost affects the developer's strategy chosen development. The additional cost  $C_2$  is compared at 8, 20, and 46, respectively, to observe how the developer's strategy selection will be impacted. Our analysis is based on the findings presented in **Figure 4**, which were generated using Python.

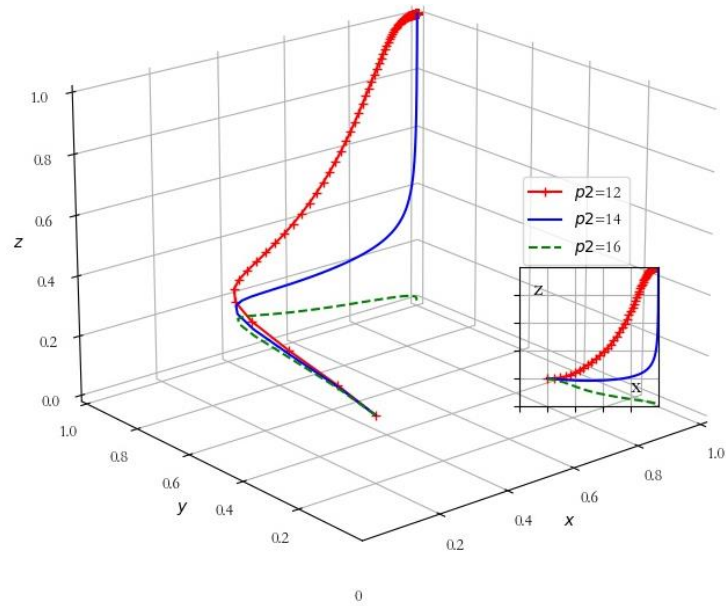


**Figure 4.** The impact of additional costs for developers on their choice of strategies for developing green buildings.

Based on the numerical simulation results in **Figure 4**, we can see the influence of different extra costs  $C_2$  on the developer's choice of developing green building strategy, as the extra costs  $C_2$  increase, the developer's willingness  $y$  to choose the development of green building strategy becomes less and less, and gradually tends to 0. The results show that the increase of extra costs  $C_2$  will have an impact on the developer's strategy. In the set parameters, when both developers and consumers opt for green building, the development cost  $C_2$  should remain less than  $3/16$  of the developer's revenue, and when the development cost  $C_2$  is higher than  $3/16$  of the revenue developers will no longer choose to develop green building. When the developer chooses to develop green building and the consumer chooses to buy conventional building, the development cost  $C_2$  of the developer should be less than  $9/46$  of its benefit, and when the development cost  $C_2$  is higher than  $9/46$  of the benefit the developer will no longer choose to develop green building. Therefore, the reward and punishment mechanism of the government should reward developers who develop green buildings accordingly, so that the incremental cost of development is lower than the incremental profit of developers of green buildings and solve the problem of high cost and high risk caused by the long period of developers in the process of developing green buildings and promote developers to develop green buildings.

### 4.3.3. Impact of the purchase price of green building on consumer strategies

This section examines how the purchase price of green buildings influences a consumer's decision to purchase green buildings. To investigate this, we compare the purchase price  $P_2$  of green building for three different scenarios: 11, 14, and 16, respectively, and observe how these price variations impact a consumer's purchasing behavior. Our analysis is based on the findings presented in **Figure 5**, which were generated using Python.



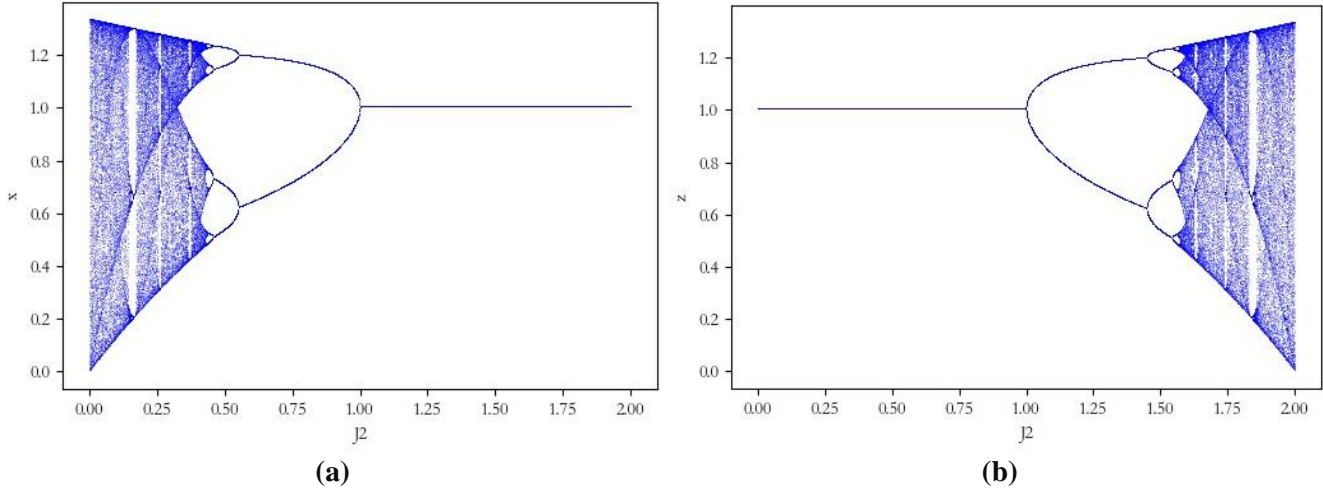
**Figure 5.** The impact of additional costs for developers on their choice of strategies for developing green buildings.

Based on the data simulation results in **Figure 5**, we can observe the impact of different purchase prices  $P_2$  on a consumer's decision to buy green buildings. As the purchase price  $P_2$  increases, the consumer's willingness  $z$  to buy green buildings decreases and eventually approaches 0. These results indicate that increasing the purchase price  $P_2$  can yield an influence on a consumer's decision-making process. Within the existing parameter configuration, when consumers choose to acquire green buildings, the price of green buildings  $P_2$  must not surpass five times the summation of the government reward  $J_2$  and the personal benefits  $W_1$  accruing to consumers. If the purchase price  $P_2$  exceeds 5 times of the benefits consumers receive, consumers will no longer choose to purchase green buildings, therefore, the price of consumers purchasing green buildings should be reduced in the market, the living conditions of consumers should be improved, and the supporting facilities around green buildings should be improved. To promote consumers' willingness to purchase green buildings.

### 4.4. Dynamic behaviors

In real life, when a system becomes unstable, game parties may be unable to adapt their strategies through learning and imitation. This can be detrimental to the overall construction market. To investigate the stability of the model, we analyze its dynamic behavior with different parameter variations [34].

We assume that the parameters  $J_1 = 3$ ,  $R_0 = 4$ ,  $L_1 = 3$ ,  $C_1 = 2$ ,  $L_2 = 2$ ,  $C_2 = 26$ ,  $R_4 = 210$ ,  $R_3 = 230$ ,  $R_2 = 240$ ,  $R_1 = 200$ ,  $P_4 = 11$ ,  $P_1 = 13$ ,  $P_2 = 132$ ,  $P_3 = 10$ ,  $W_1 = 1$ ,  $W_2 = 1$ , the effect of a change in the value of the parameter  $J_2$  (government incentives to consumers) on the stability of the system is explored. As shown in **Figure 6**.



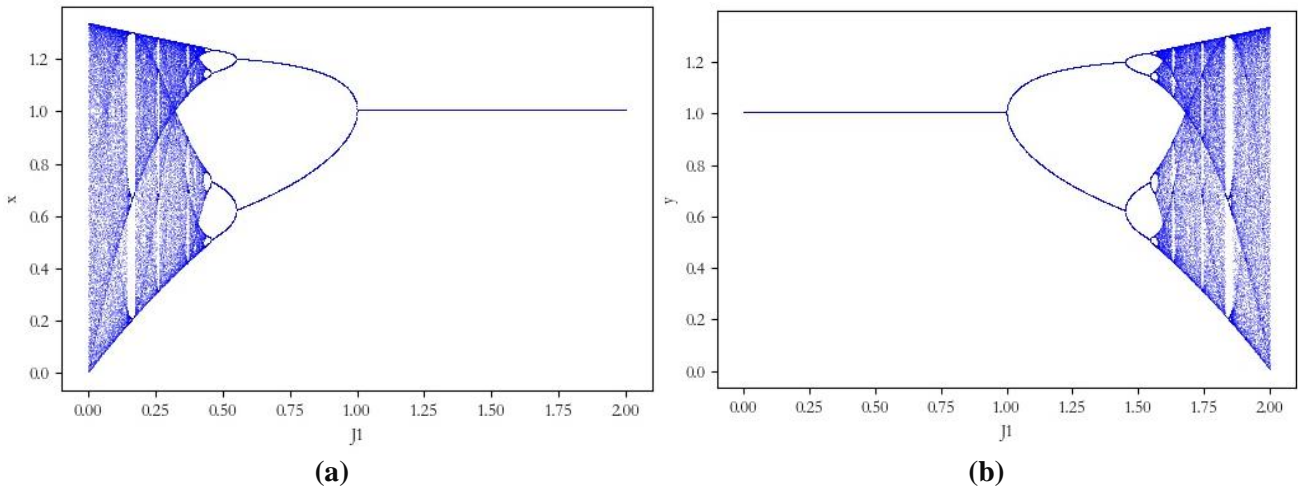
**Figure 6.** (a) Bifurcation diagram of  $x$  with respect to the variation of parameter  $J_2$ ; (b) Bifurcation diagram of  $z$  with respect to the variation of parameter  $J_2$ .

In **Figure 6a,b**, the parameter  $J_2$  on the stability of the system, when the game player is the government, when  $J_2 < 1$ , the government will steadily choose a regulatory strategy. When  $J_2 = 1$ , generating period-doubling bifurcation. The four-period bifurcation is generated when  $J_2 = 1.45$ , until  $J_2$  increases to 1.54 to generate eight-period bifurcation, when  $1.56 < J_2 < 2$  gradually enters the chaotic state. When the game player is the consumers, when  $J_2 > 1$ , consumers steadily choose to purchase green buildings strategy. When  $J_2 = 1$ , generate period-doubling bifurcation, when  $0.46 < J_2 < 0.55$ , the four-periods bifurcation is generated, until  $J_2$  increases to 0.46 to generate eight-periods bifurcation, when  $0 < J_2 < 0.44$  gradually enters the chaotic state. The analysis reveals that when the government offers an excessively high incentive to consumers, the regulatory strategy becomes unstable and transitions to a chaotic state. However, as the government's incentive amount gradually increases, consumers steadily begin to choose the green building strategy.

We assume that the parameters  $J_2 = 3$ ,  $R_0 = 4$ ,  $L_1 = 3$ ,  $C_1 = 2$ ,  $L_2 = 2$ ,  $C_2 = 41$ ,  $R_4 = 210$ ,  $R_2 = 240$ ,  $R_1 = 200$ ,  $P_4 = 11$ ,  $P_1 = 13$ ,  $P_2 = 12$ ,  $P_3 = 10$ ,  $W_1 = 1$ ,  $W_2 = 1$ , the effect of a change in the value of the parameter  $J_1$  (government incentive to the developer) on the stability of the system are explored.

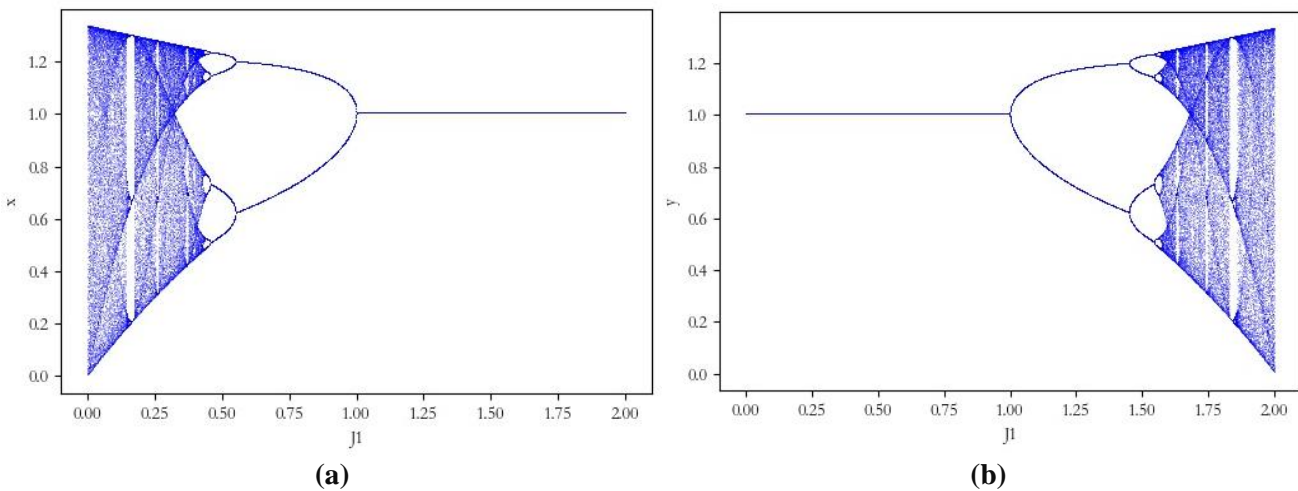
As shown in **Figure 7**. In **Figure 7a,b**, the parameter  $J_1$  on the stability of the system, when the game player is the government, when  $J_1 < 1$  the government will steadily choose a regulatory strategy. When  $J_1 = 1$ , generating period-doubling bifurcation. The four-period bifurcation is generated when  $J_1 = 1.45$ , until  $J_1$  increases to 1.54 to generate eight-period bifurcation, when  $J_1 > 1.56$  gradually enters the chaotic state. When the game player is the developers. When  $J_1 > 1$ , developers steadily choose strategies for developing green buildings. When  $J_1 = 1$ , generate

period-doubling bifurcation, when  $0.46 < J_1 < 0.55$ , the four-periods bifurcation is generated, until  $J_1$  increases to 0.46 to generate eight-periods bifurcation, when  $0 < J_1 < 0.44$ , gradually enters the chaotic state. The analysis shows that when the government provides excessive incentives to developers, the regulatory strategy becomes unstable and transitions to a chaotic state. However, as the amount of government incentives gradually increases, developers steadily begin to choose the green building strategy.



**Figure 7.** (a) Bifurcation diagram of  $x$  with respect to the variation of parameter  $J_1$ ; (b) Bifurcation diagram of  $y$  with respect to the variation of parameter  $J_1$ .

We assume that the parameters  $J_2 = 2, J_1 = 4, R_0 = 4, L_1 = 5, C_1 = 8, L_2 = 5, C_2 = 8, R_4 = 210, R_3 = 230, R_2 = 240, R_1 = 200, P_4 = 11, P_1 = 13, P_3 = 12, W_1 = 1, W_2 = 1$  the effect of the change in the value of the parameter  $P_2$  (the price consumers pay for purchasing green buildings) on the stability of the system is explored. As shown in **Figure 8**.

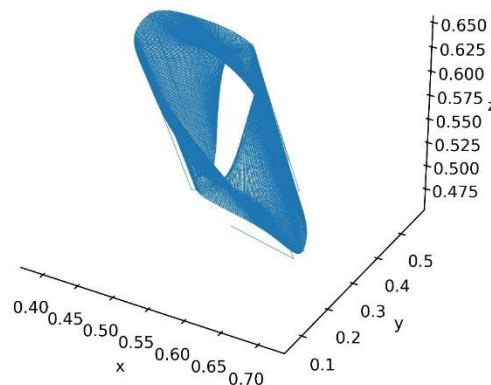


**Figure 8.** (a) Bifurcation diagram of  $z$  with respect to the variation of parameter  $P_2$ ; (b) Bifurcation diagram of  $y$  with respect to the variation of parameter  $P_2$ .

In **Figure 8a,b**, the parameter  $P_2$  on the stability of the system, when the game player is the consumer, when  $P_2 < 14$ , the consumer will steadily choose buying green buildings strategy, when  $14 < P_2 < 16$ , the consumer will steadily choose buying conventional buildings strategy. When  $P_2 = 16$ , generating period-doubling bifurcation. The four-period bifurcation is generated when  $P_2 = 16.45$ , when  $16.54 < P_2 < 17$ , the system generates an eight-period bifurcation and gradually enters a chaotic state.

When the game player is the developer. When  $P_2 > 15.539$ , developers steadily choose strategies for developing green buildings. When  $15.385 < P_2 < 15.539$ , developers steadily choose a strategy for developing conventional buildings. When  $P_2 = 15.385$ , generating period-doubling bifurcation, when  $15.31 < P_2 < 15.385$ , generating a four-period bifurcation system, and then generating an eight-period bifurcation gradually enters a chaotic state. The analysis reveals that with an excessive increase in the purchase price of green buildings, consumers transition from steadily choosing the green building strategy to a chaotic state. However, as the purchase price gradually increases, developers steadily begin to choose the green building strategy. The phase diagram and bifurcation diagram analysis reveal that, based on our hypothetical parameter values, consumers will stop choosing the green building strategy when the purchase price exceeds 1.4 million yuan. To encourage consumer purchases of green buildings, the purchase subsidy provided by the government should ensure that the price is less than 5 times the gain obtained by the consumer (government's reward  $J_2$ , their gain  $W_1$ ). If the purchase price exceeds 5 times the gain, consumers will no longer buy green buildings.

When the parameters are adjusted to  $C_1 = 1$ ,  $C_2 = 18.4$ ,  $W_1 = 2$ , the initial value  $x, y, z$  are 0.5, 0.4, 0.4, respectively, and the developer's gain is adjusted to  $R_1 = 200$ ,  $R_2 = 213$ ,  $R_3 = 210$ ,  $R_4 = 205$ . When we adjust the parameter values with the initial values, the system gradually changes from a stable state to a chaotic state. At this point, the system is in a chaotic state, generating chaotic attractors as shown in **Figure 9**.



**Figure 9.** Chaos attractor.

Based on the outcomes, it becomes evident that as the initial values and parameters undergo modifications, the system progressively shifts from a stable state to a chaotic state. The system gradually diverges from its original stable immobile

point, which makes the system have chaotic characteristics to produce chaotic attractors, and the sensitivity of the chaotic system to initial values can be seen.

When the revenue for developers engaging in green building projects declines while their costs remain constant, the GB model enters a state of chaos. Specifically, if the developers' costs surpass  $9/46$  of their revenue, the system becomes chaotic, which will cause developers to gradually be reluctant to choose to develop green buildings. Government incentives should be strategically aligned to ensure that the cost to developers of developing green buildings remains below  $9/46$ ths of their revenue. This targeted approach aims to mitigate the challenges posed by the long delivery cycles and high risks associated with green building development. Through such measures, developers could be incentivized to adopt green building strategies, resulting in a more organic growth trajectory for the green building market.

## **5. Conclusion**

The implementation of a sustainable development strategy has positioned green building as a critical area of focus within the construction industry. Examining the influence of changing interests among game stakeholders on decision making is a valuable research endeavor in the realm of green building. This paper investigates the relationship among government, developers, and consumers through a constructed three-party game model. The study analyzes the equilibrium points within the model and identifies the evolutionary stabilization strategies of the system. Furthermore, it explores the main factors affecting the game subjects, and validates the accuracy of the findings through data simulation.

(1) When the government chooses a regulatory strategy, it can utilize financial subsidies to reduce the costs of opting for green buildings for developers and consumers. Conversely, tax penalties can elevate the expenses tied to conventional building development, thereby incentivizing developers and consumers to prioritize green building strategies.

(2) When the cost of government regulation should be less than  $1/3$  of the financial subsidies to prompt the government to choose the regulation strategy in the long run.

(3) The probability of developers and consumers choosing the green building strategy is negatively correlated to the cost and positively correlated to the benefit.

(4) The evolutionary stable strategies of the green building evolutionary game model are as follows: the government chooses the regulatory strategy, developers choose the development green building strategy, and the consumers choose the purchase green building strategy.

(5) The sensitivity analysis shows that the pivotal drivers for consumers to opt for purchasing a green building are the enhancements in their overall quality of life stemming from the building (personal quality of life is improved and energy-saving benefits are improved). Developers should develop green buildings that meet consumers' quality of life needs and augment the ancillary amenities linked to green buildings. These measures are instrumental in motivating consumers to favor green building choices.

## **6. Countermeasures and suggestions**

For the government:

(1) The government should set up a special green building regulatory body with a clear division of responsibilities and a detailed green building standard and assessment system: special inspections should be carried out on a regular basis to ensure that construction companies comply with green building standards.

(2) Impose financial penalties on companies that fail to comply with green building standards: develop a graded penalty mechanism and determine the level of penalty based on the severity and impact of the violation.

(3) Implement tax relief policies for companies that actively carry out green building projects: design differentiated tax incentives for companies of different sizes and introduce a tax return mechanism that rewards companies according to the quality and quantity of green building projects they complete.

For developers:

(1) A detailed introduction of the benefits associated with green buildings should be provided to potential buyers. This will ensure that developers' gains outweigh their costs and incentivize them to pursue green building projects. Moreover, continuous subsidies should be offered to consumers for purchasing green buildings, along with corresponding housing subsidies. By doing so, consumers' willingness and ability to purchase green buildings will increase, leading to a genuine demand for such properties. This mutually beneficial scenario enables developers to generate sustained profits while consumers remain eager to make purchases, thereby fostering the growth of green buildings within the construction market.

(2) Developers should integrate the entire green building industry chain. This integration should aim to create a green building industry chain with scale effects, resulting in a reduction in the long-term average cost for developers. Furthermore, the incremental benefit should outweigh the incremental cost. To achieve this, developers should leverage advanced green building technologies from both domestic and international sources. These technologies should be processed and upgraded to lower the development costs of green buildings while enhancing energy utilization.

(3) Factors influencing consumers' decisions should be a primary focus for developers. When constructing green buildings, greater emphasis should be placed on enhancing the comfort of consumer living. This can be achieved through the use of construction materials that minimize energy loss. Furthermore, developers should strive to enhance the surrounding infrastructure of the buildings, including parking facilities and fitness amenities, to ensure convenience for consumers accessing nearby shopping malls, schools, and hospitals. Such improvements will contribute to the overall enhancement of consumers' quality of life.

For the consumer:

(1) When purchasing a home, prioritise properties with superior energy efficiency and environmental performance, ensuring they have green building certification. Verify the certification details and gather information about the property's energy-saving features from government resources or developers.

(2) Consumers should proactively understand the government's green building subsidy policies and apply for relevant incentives through official channels. They can

research application requirements for subsidies on the government's official website or at community service centres and participate in government-organised promotional activities to gain additional insights and benefits.

(3) When purchasing a home, prioritise green buildings with good ventilation and lighting performance, and select house types that align with personal needs. Consult developers about the use of environmentally friendly materials and avoid renovation materials containing harmful substances. After moving in, use professional testing tools to ensure indoor air quality meets health standards.

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## Appendix

```

import numpy as np
from scipy.integrate import odeint
import matplotlib.pyplot as plt

plt.rcParams['font.sans-serif']=['STSong']
#plt.rcParams['font.sans-serif']=['SimHei']
plt.rcParams['axes.unicode_minus']=False

def lvse(Fx,t,R0,R1,R2,R3,R4,C1,C2,L1,L2,P1,P2,P3,P4,W1,W2,J1,J2):
    x, y, z = Fx.tolist()
    return x*(1-x)*(-C1+R0-J2*z+L1-J1*y), \
y*(1-y)*(z*(R2-R1-R3+R4)+R3-C2+J1*x-R4+L2), \
z*(1-z)*(x*J2-(P2+P3-P1-P4)*y-P1+W1+P3+W2)

plt.close("all")
fig=plt.figure(figsize=(8,8),dpi=600)
ax=fig.gca(projection='3d')
#ax.grid(False)
#ax.w_xaxis.set_pane_color((1.0, 1.0, 1.0, 1.0))
#ax.w_yaxis.set_pane_color((1.0, 1.0, 1.0, 1.0))
#ax.w_zaxis.set_pane_color((1.0, 1.0, 1.0, 1.0))
t = np.arange(0, 50, 0.05)
Rp,Cph,Cpl,Cp,Bt,Fp,Mp,Ct,Ft,Mt,Cg,Tg
args=(4,200,240,230,210,2,8,5,2,12.5,11,10,10.5,1,1,4,2)
track5 = odeint(lvse, (0.2, 0.2, 0.2), t,args )
ax.plot(track5[:,0],track5[:,1],track5[:,2],r+'-')
args=(4,200,240,230,210,3,8,5,2,12.5,11,10,10.5,1,1,4,2)
track5 = odeint(lvse, (0.2, 0.2, 0.2), t,args )
ax.plot(track5[:,0],track5[:,1],track5[:,2],'b-')
args=(4,200,240,230,210,4,8,5,2,12.5,11,10,10.5,1,1,4,2)
track5 = odeint(lvse, (0.2, 0.2, 0.2), t,args )
ax.plot(track5[:,0],track5[:,1],track5[:,2],'g--')
ax.view_init(elev=20, azim=-130)
ax.set_facecolor('w')
ax.set_xlabel(r"$x$",labelpad=0)
ax.set_ylabel(r"$y$",labelpad=0)
ax.set_zlabel(r"$z$",labelpad=0)
ax.set_xlim3d(xmin=0, xmax=1)
ax.set_ylim3d(ymin=0, ymax=1)
ax.set_zlim3d(zmin=0, zmax=1)
plt.legend(labels=('$C_1=2','$C_1=3','$C_1=4'),loc=(0.7,0.5))
#plt.title("The influence of government regulatory costs on the adoption of regulatory strategies. ",x=0.5,y=-
0.1,fontsize=15,fontweight='bold')
plt.xticks(np.arange(0.2,1.01,step=0.2))
plt.yticks(np.arange(0.2,1.01,step=0.2))

```

```

ax.set_zticks(np.arange(0,1.01,step=0.2))
left, bottom, width, height = 0.66,0.35,0.13,0.13
ax1 = fig.add_axes([left,bottom,width,height])
args=(4,200,240,230,210,2,8,5,2,12.5,11,10,10.5,1,1,4,2)
track5 = odeint(lvse, (0.2, 0.2, 0.2), t,args )
ax1.plot(track5[:,0],track5[:,2], 'r+-')
args=(4,200,240,230,210,3,8,5,2,12.5,11,10,10.5,1,1,4,2)
track5 = odeint(lvse, (0.2, 0.2, 0.2), t,args )
ax1.plot(track5[:,0],track5[:,2], 'b-')
args=(4,200,240,230,210,4,8,5,2,12.5,11,10,10.5,1,1,4,2)
track5 = odeint(lvse, (0.2, 0.2, 0.2), t,args )
ax1.plot(track5[:,0],track5[:,2], 'g--')
plt.grid()
ax1.set_facecolor('white')
plt.xticks(np.arange(0,1,step=0.2))
plt.yticks(np.arange(0,1,step=0.2))
ax1.axes.xaxis.set_ticklabels([])
ax1.axes.yaxis.set_ticklabels([])
ax1.set_xlim(0,1)
ax1.set_ylim(0,1)
plt.text(0.8, 0.1, s="x", transform=ax1.transAxes,fontsize=15)
plt.text(0.1, 0.8, s="z", transform=ax1.transAxes,fontsize=15)
plt.text(0.46, 0.02, s="0", transform=ax.transAxes,fontsize=10)
#plt.text(0, 0.8, s=r"$z$", transform=ax.transAxes,fontsize=10)
ax.w_xaxis.set_pane_color((1.0, 1.0, 1.0, 1.0))
ax.w_yaxis.set_pane_color((1.0, 1.0, 1.0, 1.0))
ax.w_zaxis.set_pane_color((1.0, 1.0, 1.0, 1.0))
plt.savefig('1.jpg', transparent=True, dpi=100)
plt.show()

```

```

from tqdm import tqdm
import matplotlib.pyplot as plt
import numpy as np

```

### Bifurcation diagram

```

def LogisticMap():
    j = np.arange(0,3,0.001)
    x=0.1
    y=1
    z=0.1# 初值
    iters = 1000 # 不进行输出的迭代次数
    last = 100 # 最后画出结果的迭代次数
    for i in tqdm(range(iters+last)):
        x =x+(6-j*z-3*y)*(1-x)*x
        y =y+(-4+20*z+3*x)*(1-y)*y

```

```

z = z + (-1 + 2 * y + j * x) * (1 - z) * z
if i >= iters:
    plt.plot(j, x, 'b', alpha=0.25) # alpha 设置透明度

```

```

plt.xlabel("J2")
plt.ylabel("x")
plt.savefig('12.jpg', transparent=True, dpi=100)
plt.show()

```

LogisticMap()

### Chaos attractor

```

import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
xs, ys, zs = [], [], []
def mkPoints():
    c1, c2, w1 = 1, 19.4, 1
    x0, y0, z0 = 0.5, 0.4, 0.4
    for i in range(1000):
        x1 = (6 - 3 * z0 - 3 * y0 - c1) * (1 - x0) * x0
        y1 = (15 + 8 * z0 + 3 * x0 - c2) * (1 - y0) * y0
        z1 = (-1 + 2 * y0 + 3 * x0 + w1) * (1 - z0) * z0
        x0, y0, z0 = x1, y1, z1
        xs.append(x0)
        ys.append(y0)
        zs.append(z0)

if __name__ == "__main__":
    fig = plt.figure()
    ax = Axes3D(fig)
    mkPoints()
    ax.plot(xs, ys, zs, lw=0.3)
    ax.legend()
    ax.grid(False)
    ax.w_xaxis.set_pane_color((1.0, 1.0, 1.0, 1.0))
    ax.w_yaxis.set_pane_color((1.0, 1.0, 1.0, 1.0))
    ax.w_zaxis.set_pane_color((1.0, 1.0, 1.0, 1.0))
    ax.set_xlabel("x ")
    ax.set_ylabel("y ")
    ax.set_zlabel("z ")
    plt.subplots_adjust(bottom=0.1, top=0.9)
    plt.savefig('21.jpg', transparent=True, dpi=600, bbox_inches='tight')
    plt.show()

```