Building Engineering 2025, 3(2), 1703.
https://doi.org/10.59400/be1703

Article

Impact of activity time stochasticity on critical paths and their completion probabilities in construction projects

Saurabh Gupta[1], Riya Catherine George[2], Deepu Philip[3], Syam Nair[1,*]

[1] Department of Civil Engineering, Indian Institute of Technology Kanpur, Kanpur 208016, India
[2] Department of Civil and Environmental Engineering, Hiroshima University, Hiroshima 739-8511, Japan
[3] Department of Industrial and Management Engineering, Indian Institute of Technology Kanpur, Kanpur 208016, India
*** Corresponding author:** Syam Nair, syamnair@iitk.ac.in

CITATION

Gupta S, George RC, Philip D, Nair S. Impact of activity time stochasticity on critical paths and their completion probabilities in construction projects. Building Engineering. 2025; 3(2): 1703. https://doi.org/10.59400/be1703

ARTICLE INFO

Received: 11 September 2024
Accepted: 7 January 2025
Available online: 9 April 2025

Abstract: Accurate planning and project control activities are essential in ensuring the timely completion of construction projects. Even though the classical scheduling techniques like Gantt charts, critical path method (CPM), program evaluation and review technique (PERT), etc., are well-designed and can effectively support construction scheduling functions, they do not account for the variability in activity times arising out of the random/stochastic nature of the activities involved. The study utilizes a simulation-based approach to identify the impact of variability in activity time durations on critical paths and on the probability of timely completion of construction projects. The effectiveness of four different probability distribution functions, namely uniform, triangular, bound exponential, and unbound exponential, in capturing the stochasticity of activity times in construction projects was also evaluated as part of the study. A MATLAB-based simulation framework was developed to sample random durations of project activities and compute various project scheduling parameters based on PERT assumptions. Activity time variability was introduced in the network by extending pessimistic time (t_p) by up to 50%. Observation in the study suggests that for projects that experience delays during execution, allocating additional resources to the preassigned critical path may not always be effective in addressing these delays. Instead, dynamic corrective measures (time extensions, resource allocation, etc.) that consider all possible scenarios of project completion and can account for any possible changes in critical path due to the incurred delay may be adopted to ensure a higher probability of project completion within the revised schedule. Exponential bounded distribution was found to provide a more realistic estimate of project completion time with a high probability of timely completion of construction projects. The study also suggests exponential unbound distribution to be effective in simulating worst-case scenarios encountered in construction projects. Future research could validate this approach on real projects and use machine learning to automate scheduling adjustments for delays.

Keywords: activity delays; advanced simulation tool; construction project; critical path; project planning; PERT; robust schedules

1. Introduction

Every construction project is a unique endeavor with a specified start and end time where individual tasks are executed in a logical sequence to meet target timelines for project completion [1]. Proper planning and project control activities are essential in ensuring the timely completion of these construction projects. This essentially involves creating accurate planning horizons (initial project completion time estimates), continuous monitoring of progress, generating and evaluating alternate schedules in case of delays, and implementing corrective actions when required [2].

oFor this, the construction industry relies on classical scheduling techniques like Gantt charts, critical path method (CPM), program evaluation and review technique (PERT), etc., for which software packages like Microsoft Project, Asta Power Project, and Primavera are readily available [3,4]. However, the dynamic nature of construction projects often poses challenges in accurately estimating activity time durations, which makes the planning horizon estimates using current methodologies ineffective [5–7]. Even though the available programs are well-designed and can effectively support construction scheduling functions, they do not account for the variability in activity times arising out of the random/stochastic nature of the activities involved [8]. They typically use deterministic approaches where a fixed activity time duration is assigned to a given task without accounting for variability associated with these works. While CPM relies totally on fixed durations for project activities, PERT acknowledges the variability in activity durations and treats the duration of each activity as a random variable with an associated probability density function (PDF). However, PERT also relies on deterministic time estimates for activity durations obtained from these PDFs using preassigned milestones like optimistic, pessimistic, and most likely time of completion, which often results in a near 50% probability of project completion within the specified duration. These techniques also assume a fixed critical path to solve the network, without considering the possibility for changes in the critical path due to changes in individual activity times and their combinations. Prior research has shown that such an approach can result in unrealistic schedules and a lower probability of timely project completion as the critical path itself is likely to change depending on changes incurred in activity time combinations during project execution [9]. This paper illustrates the shortcomings of PERT, where a project's critical path may change due to the ongoing variability of project activities during execution. Such variation can result in unrealistic schedules and a lower probability of project completion. Furthermore, this research shows that the critical path is likely to change regardless of the project network or the PDF used to estimate activity durations. A mathematical program was developed in MATLAB to sample random durations of project activities and compute various project scheduling parameters based on PERT assumptions.

The second section overviews the PERT technique and its relevant literature, while the third section summarizes the research methodology, including the four project networks used to simulate various construction scenarios against four distinct probability density functions (PDFs), viz., (i) Uniform; (ii) Triangular; (iii) Bounded Exponential; and (iv) Exponential. The fourth section presents the results, focusing on the impact of the construction activities' stochastic and dynamic characteristics on the critical path, project duration, and probability of timely completion.

2. Background of PERT approach and literature review

While scheduling projects using PERT, the network is divided into a set of sub-tasks called activities, which are represented either by nodes or arrows. In accordance with the central limit theorem, it is assumed that the critical path in the network follows a normal distribution. The completion time of the project is taken as the sum of the duration of all activities along the critical path, and the variance of the network is the sum of the variance of individual activities along this path [10]. The PERT approach assumes activity times to follow the beta distribution and uses three-point time estimates, i.e., optimistic time (t_o), most likely time (t_m), and pessimistic time (t_p), in order to calculate the expected time (t_e) and variance (V) as shown in Equations (1) and (2).

$$t_e = \frac{t_o + 4t_m + t_p}{6} \tag{1}$$

$$\sigma_X^2 = (\frac{t_p - t_o}{6})^2 \tag{2}$$

$$\sigma_{Pr\,ojectDuration}^2 = \sum_{x \in C\max} \sigma_X^2 \tag{3}$$

$$Z = \frac{X - \mu}{\sigma} \tag{4}$$

However, the assumptions used in the PERT model have been criticized heavily from the very beginning. Sculli challenged the assumption of beta distribution and showed that the activity times can follow different probability distributions and are not strictly beta all the time [11]. Studies have also shown that due to delays incurred in construction, certain paths can emerge to be the longest during the execution despite the scheduled duration of the path being less than the critical path at the time of initial planning [12,13]. Further, the assumption that the critical path follows normal distribution is too simplistic for large construction projects, as it follows the standard approach of adopting the critical path mean as the mean of the entire project network. The project completion time depends on the PDF of individual activity durations, the total number of activities in the path, the precedence relationship between the activities, the number of paths, and the interrelationship among paths [12,13]. This supports the observation that PERT schedules can be unrealistic due to complex interrelationships between construction project activities [14,15].

Many approaches have been proposed to overcome the inaccuracies because of the assumptions estimating activity durations [16–18]. The consensus was to use various probability distribution functions (PDFs) to estimate the expected time in place of the conventional approach [9,15,19–23]. This includes using lognormal distribution [24], triangular distribution [25,26], tilted beta distribution [16], compound Poisson distribution [27], Weibull distribution [28], beta rectangular distribution [29], modified beta distribution [16,30], etc., to obtain more realistic estimates of activity times used as input parameters for PERT.

Lee used a beta distribution to find out the probability of completion of a project network in their study using stochastic simulations [20,31,32]. Efforts to determine correction factors for activity durations to enhance PERT output robustness by employing beta distribution for mean and variance calculations were also attempted [33]. However, recent studies have suggested beta distribution to be selectively applicable to construction projects, prompting exploration of alternative distributions such as the gamma distribution [21]. Dodin proposed using extreme value distribution to estimate the project duration for accounting events with very low probability when there are multiple dominating paths [12,13]. However, Alagheband and Soukhakian [34] suggested that Dodin's approach overestimates the project completion time. Kleindorfer [35] compared the project completion time obtained using normal, uniform, and exponential distribution in activity times. Mehrotra et al. [36] investigated the interdependencies of network paths and project completion times using a simulation approach on 10 standard project networks identified from literature. Normal and exponential distributions were assumed for sampling activity times for network simulations. The results showed that mixture distribution provides a better fit over normal distribution in calculating project completion times. Thompson et al. [15] explored the use of the Tracy-Widom distribution as underlying activity durations in search of a more suitable time estimation. Irrespective of all these efforts, these studies revert to conventional methods utilizing a single critical path after activity time estimation, which overlooks the potential changes in critical paths due to the stochastic nature of activity times.

To address this issue and to enhance project completion probability, Soroush [37,38] proposed the concept of the most critical path (MCP), which corresponds to the path with the highest variance among all converging paths. Lee used stochastic simulations on historically available data to generate numerous schedules to arrive at a realistic project schedule [21,31,39,40] and also identified the existence of alternate dominant paths alongside the critical path that has the potential to become critical due to changes in activity times during execution. Williams [22] observed that near-critical paths can become critical when activity times are modeled using beta and triangular distributions. Gupta et al. [9] showed the impact of activity time variabilities on the existence of dominant and near-critical paths and indicated the possibility of changes in critical path dominance during project execution. The objective of the current study is to quantify the influence of the stochastic nature of construction project activities on the critical path, project completion time, and the probability of timely completion. The research also attempts to identify the effectiveness of four commonly used probability distribution functions, namely uniform, triangular, bound exponential, and unbound exponential, in capturing the stochasticity of activity times in construction projects.

3. Methodology

The following sections outline the methodology used to investigate the relationship between PDFs and variability in activity time and the average critical path duration of four different networks using project simulations. A flowchart demonstrating the details is given in **Figure 1**.

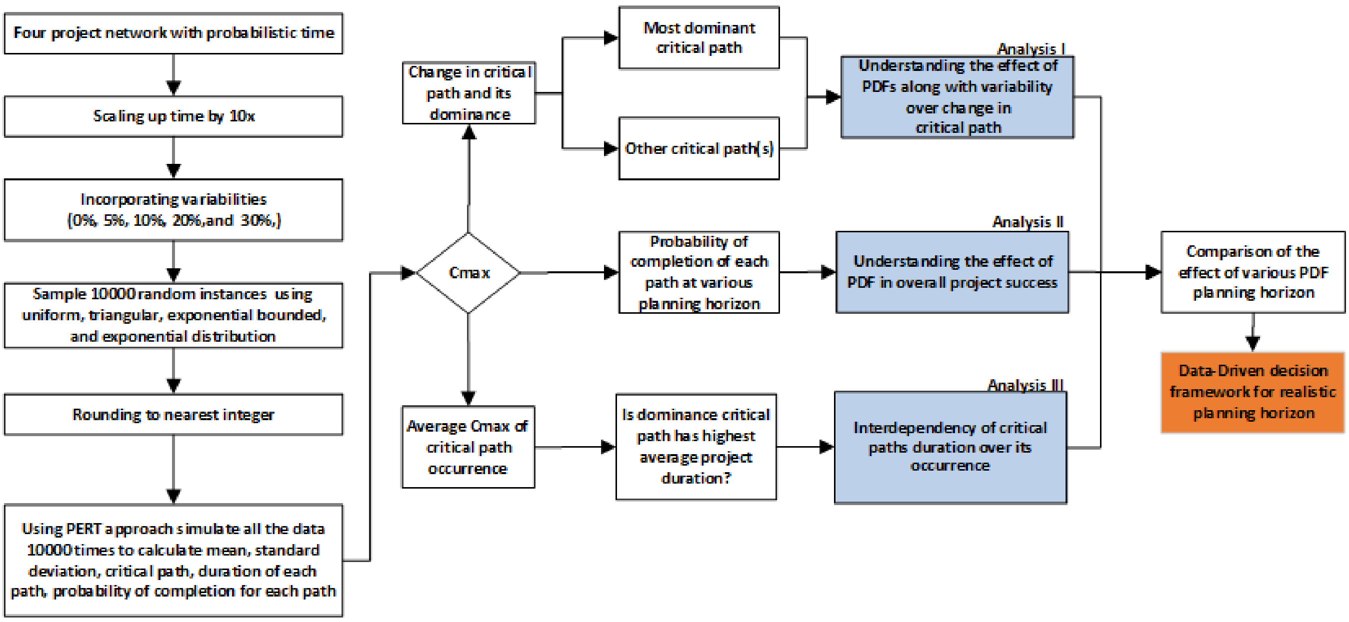
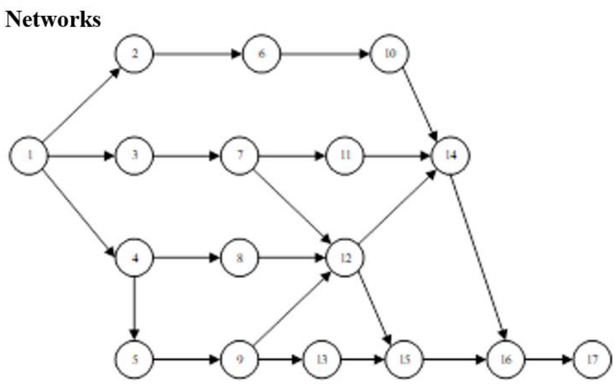


Figure 1. Steps of Methodology used in the current study.

3.1. Networks

Two categories of networks with one start and one end point were chosen for the study: (i) Skewed networks with one path having high variance and (ii) balanced networks. Two networks with different numbers of activities, paths, and precedence relationships were chosen in each category (a) Network one (N_1) adopted from Ahuja et al. [41] represents activity on arrows and contains 22 activities and 9 paths (**Figure 2a**,**b**). (b) Network two (N_2) has 14 activities and 6 paths (**Figure 2c**,**d**) adopted from Hillier and Lieberman [42] are the two skewed networks used in the study. The balanced networks (c) Network three (N_3) was adopted from Pinedo [43] and has 14 activities and 9 paths (**Figure 2e**,**f**), whereas (d) Network four (N_4) selected from Winston [44] was a smaller network with 9 activities and 4 paths (**Figure 2g**,**h**). Networks N_2, N_3, and N_4 represent activity on nodes.



(a) N_1

Paths
1-2-6-10-14-16-17
1-3-7-11-14-16-17
1-4-5-9-12-14-16-17
1-4-8-12-14-16-17
1-3-7-12-14-16-17
1-4-5-9-13-15-16-17
1-4-5-9-12-15-16-17
1-4-8-12-15-16-17
1-3-7-12-15-16-17

(b)

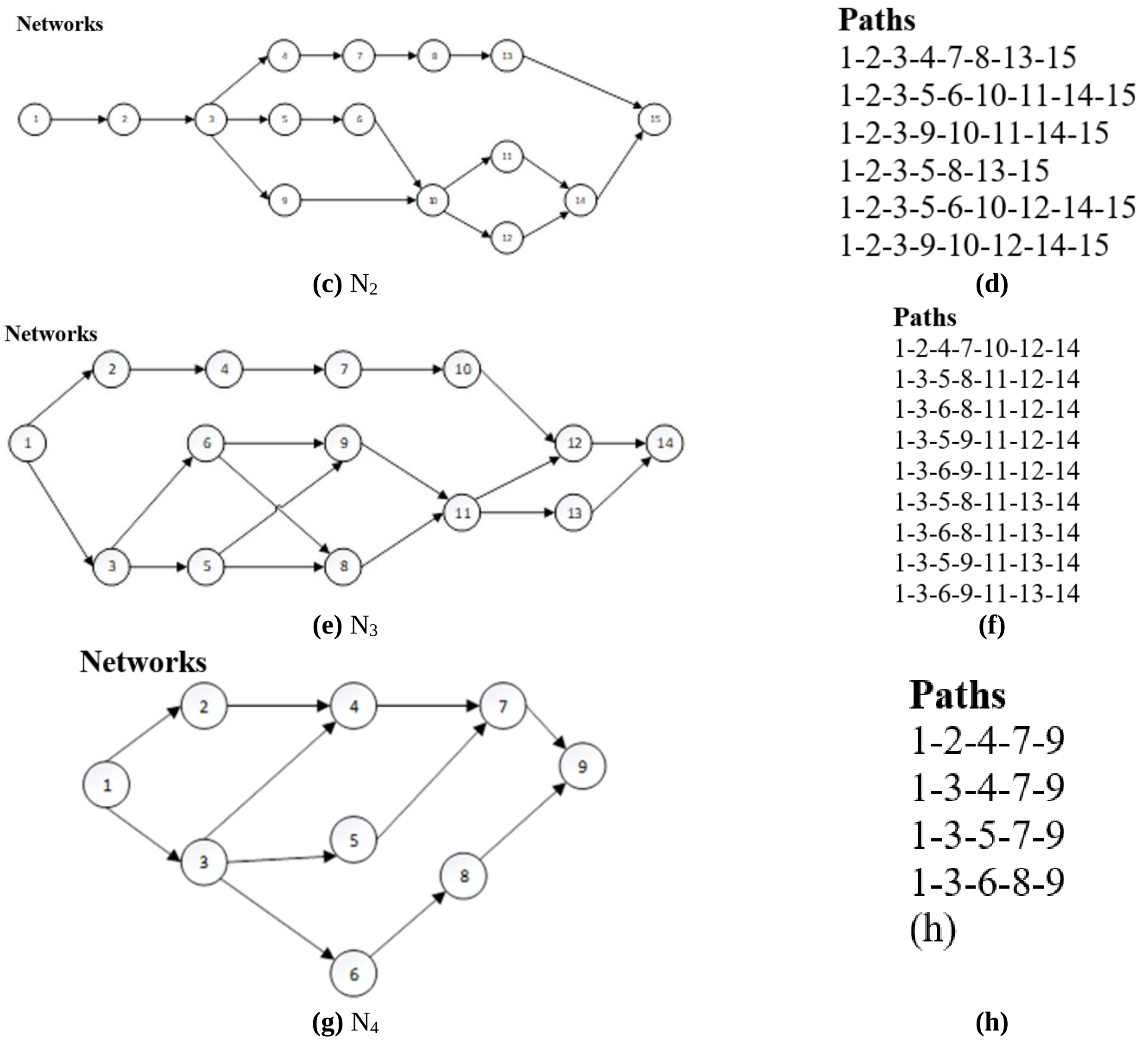


Figure 2. Project network adapted for this study, **(a)** Ahuja et al., 1994, [41]; **(c)** Hillier and Lieberman, 2021, [42]; **(e)** Pinedo, 2000, [43]; **(g)** Winston, 2022 [44]; and **(b)**, **(d)**, **(f)**, **(h)** show the possible paths in the network respectively.

The activity times (t_o, t_p and t_m) used in the study were directly adopted from the respective references and scaled by 10 times to mimic time units similar to construction projects and given in **Table 1**.

3.2. Selection of probability distribution function

A random sampling of activity times (t_o, t_p and t_m) from different probability distributions was used in modeling to simulate the uncertainties associated with individual activities. For every simulation, a random duration is sampled for each activity. Chosen activity time and their combinations along paths were used to estimate the path and critical path (C_{max}) durations during each simulation. Even though the activities in a network can follow any distribution depending on the nature of the activity and variables involved, only four distributions were considered in the present study.

Table 1. Activity time for all 4 networks adopted from the original book problem.

	N_1			N_2			N_3			N_4		
Activities	t_o	t_m	t_p	t_o	t_m	t_p	t_o	t_m	t_p	t_o	t_m	t_p
1	40	210	310	10	20	30	40	50	60	40	60	80
2	20	50	80	20	35	80	40	60	80	20	40	80
3	20	110	160	60	90	180	80	80	140	10	30	70
4	20	70	80	40	55	100	100	110	180	60	90	120
5	30	50	70	10	45	50	60	70	80	50	100	150
6	40	50	100	40	40	100	120	120	120	70	120	180
7	20	60	90	50	65	110	40	110	120	50	90	120
8	20	40	80	50	80	170	50	60	70	10	20	30
9	20	30	80	30	75	90	100	100	100	20	30	60
10	10	30	60	30	90	90	70	80	150	100	150	200
11	20	50	90	40	40	40	60	70	80	60	90	110
12	10	30	60	10	55	70	60	80	100			
13	30	50	70	10	20	30	70	70	70			
14	10	30	50	50	55	90	20	50	80			
15	10	80	100	1	1	1						
16	30	50	70									
17	10	30	70									
18	30	50	80									
19	20	60	90									
20	20	60	80									
21	20	40	70									
22	10	40	90									

a) Uniform distribution was chosen to give an equal likelihood of occurrence of activity duration within t_o–t_p range [45,46] and is relevant in modeling uncertainty in activities that lack historical data on activity duration or have high variability in completion times.

$$p(x) = \frac{1}{(t_o - t_p)} \tag{5}$$

b) Triangular distributions are typically used when there is some information or data available on the range and most likely value of the activity's duration [47]. The triangular distribution is characterized by three parameters: the minimum value (t_o), the maximum value (t_p), and the mode or most likely value (t_m).

$$p(x) = \begin{cases} \dfrac{2(x - t_o)}{(t_m - t_o)(t_p - t_o)} & t_o < x < t_m \\ \dfrac{2(t_p - x)}{(t_p - t_o)(t_p - t_m)} & t_m < x < t_p \end{cases} \tag{6}$$

c) Exponential distributions are effective in modeling the duration of activities or events that occur randomly and independently over time [21]. Two scenarios are considered in this study where all sampled values are (a) within pessimistic time (exponential bounded distribution) and (b) values can fall beyond pessimistic time (exponential unbounded distribution).

$$p(x) = \lambda\, e^{-\lambda x}\ 0 \leq x\ \leq t_p \tag{7}$$

$$p(x) = \lambda\, e^{-\lambda x}\ x\ \geq\ 0 \tag{8}$$

3.3. Simulation modelling

The methodology is a MATLAB-based simulation framework that allows for sampling random variates of activity durations and evaluates various combinations of such durations to identify potential critical paths that can drastically alter the probability of timely project completion (Algorithm 1). Any randomly sampled value from the probability distribution having t_o and t_p as lower and upper limits, respectively, represented as UNIFORM [t_o, t_p], TRIANGULAR [t_o, t_p], EXPONENTIAL [t_o, t_p], and EXPONENTIAL_BOUNDED [t_o, t_p], for each activity, is a feasible instance of activity duration [48,49]. The duration of the critical path (C_{max}) was obtained using different sampling times (between t_o and t_p) using CPM approaches. Completion probabilities of individual paths for each simulation instance were calculated as per Equation (4) using standard deviation and expected time (t_e) calculated from Equations (1) and (2). To accommodate project delays in simulation, scenarios were included with constant $\boldsymbol{t_o}$ values and extended t_p values to simulate various levels of activity time variability (5%, 10%, 15%, 20%, 40%, and 50%) in all networks and distributions.

For all the activities in the network N_1, N_2, N_3, N_4, one-time value (t) using each distribution is simulated from UNIFORM [t_o, t_p], TRIANGULAR [t_o, t_p], EXPONENTIAL [t_o, t_p], and EXPONENTIAL_BOUNDED [t_o, t_p]. The sampled values were then ceiled to the nearest integer, and the network was solved using the CPM method with t values as individual activity durations. The resulting completion time of the critical path, i.e., $C_{\max}$, was used to determine the probability of completion (POC) in accordance with Equation (5). $C_{\mathbf{max\_}}t_e^0$ (estimated from deterministic time values in **Table 1**) was used as the benchmark for comparison while estimating the POC.

Further, to capture the impact of delays in individual activity times on POC, the simulation framework was rerun by extending the pessimistic time (t_p) up to 50% (time extensions of 0%, 5%, 10%, 15%, 20%, 30%, 40%, and 50%) and keeping optimistic time (t_o) unchanged. Time extensions were capped at 50% based on available data on various construction projects [43–45], as extensions beyond 50% can result in disproportionately large increases in both cost and time. Further, random sampling within the extended time windows is conducted separately for each activity, which ensures that not all activities in the network will encounter delays in any given simulation run [50–53]. This approach closely mirrors real-life scenarios in construction projects, where only specific activities within the project network are subject to time delays during the execution phase. The time extensions considered above resulted in eight different scenarios, which are consecutively labelled from V_0 to V_7 and discussed in the following sections. The sampling process remained the same for all eight scenarios.

Simulation-based Approach

Algorithm 1 Simulation approach in the present study

```
Input:  Example project network
Output: 10000 Simulation PERT V0, V1, V2, V3, V4, V5, V6, V7 cases
Require: MATLAB
   procedure
      scale up activity 10x times [to, tp]
      incorporate 5% (V1), 10% (V2), 20% (V3), 30% (V4), 40% (V5), 50% (V6) variability to tp
for each activity in the project network do
repeat
repeat
             repeat
       generate 1-time estimate using UNIF [to, tp]
        ciel to next integer
calculate Cmax using CPM.
     if change in critical path
          document dominance of critical path
          end if
       calculate the POC at Cmax_te
      until 10000
   until V1, V2, V3, V4, V5, V6, V7
       until TRIANGULAR [to, tp], EXPONENTIAL [to, tp], and EXPONENTIAL_BOUNDED [to, tp]
     end for
end procedure
```

Since the simulation framework involves random sampling of activity times, their combinations may lead to multiple paths becoming critical for each set (10,000 nos.) of simulations. Hence, the steady state was taken as the number of simulations required where the percentage of the emergence of a specific path as a critical path remains unchanged irrespective of the number of simulations (Law of large numbers) [53–55]. Hence, the number of sampling repetitions required per network per scenario was estimated based on trial runs to identify the minimum number at which the system starts behaving steadily. A minimum of 10,000 simulations was selected as a baseline for this study to accommodate safety margins, which also agrees well with prior studies in this domain [9,14,21].

4. Results and discussion

To estimate the impact of activity time variations during the execution phase on possible changes in the critical path, a base case (V_0) was estimated for each network for the respective PDF as per data given in **Table 1**. For every network and PDF, 10,000 simulations are performed where individual activities are sampled between t_o and t_p (using data given in **Table 1**) to estimate the number of times different paths became critical when there is zero extension in t_p. These are identified as ($U_{N1}V_0$, $U_{N2}V_0$, $U_{N3}V_0$, $U_{N4}V_0$), ($T_{N1}V_0$, $T_{N2}V_0$, $T_{N3}V_0$, $T_{N4}V_0$), ($EB_{N1}V_0$, $EB_{N2}V_0$, $EB_{N3}V_0$, $EB_{N4}V_0$), and ($E_{N1}V_0$, $E_{N2}V_0$, $E_{N3}V_0$, $E_{N4}V_0$) for uniform, triangular, exponential bound, and exponential, respectively. The results obtained were then compared with the respective cases where there was variability in activity times.

4.1. Impact of activity time variation on the dominance of critical path

Since the critical path has zero slack, any delay in activities on the critical path can directly influence the overall project duration. Hence the network was simulated with various possible changes in activity times to identify the critical path's dominance and the existence of near-critical and alternate critical paths within the network. **Figures 3–6** show the frequency of occurrence (out of 10,000) of various paths as critical when activity times follow the four different PDFs. The *X*-axis in **Figures 3–6** represents the percentage variability introduced in t_p (0%–50%), and the *Y*-axis shows the number of simulations when each path became critical in each case.

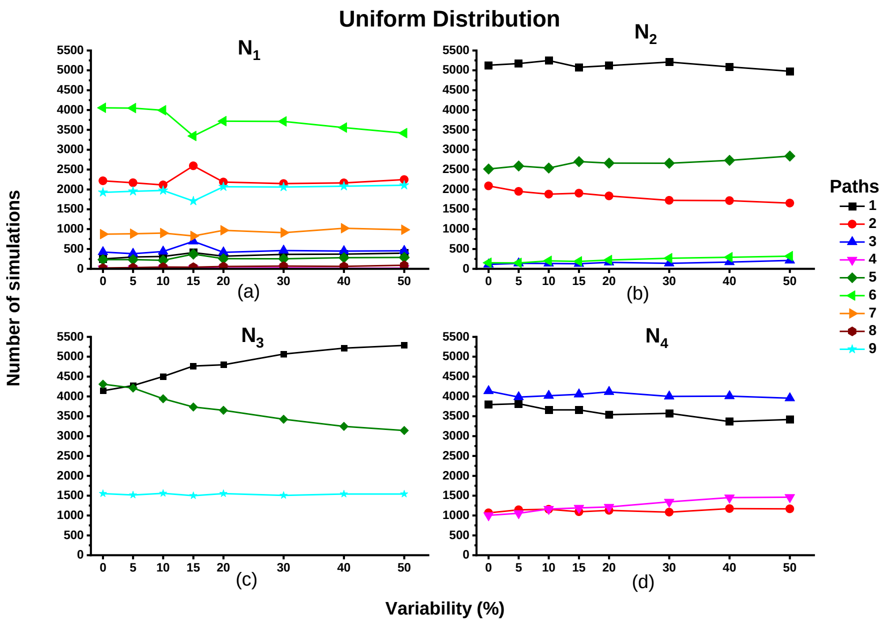


Figure 3. Critical paths and their frequency of occurrence in cases with and without allowances pessimistic times (t_p) when time is sampled using a uniform distribution.

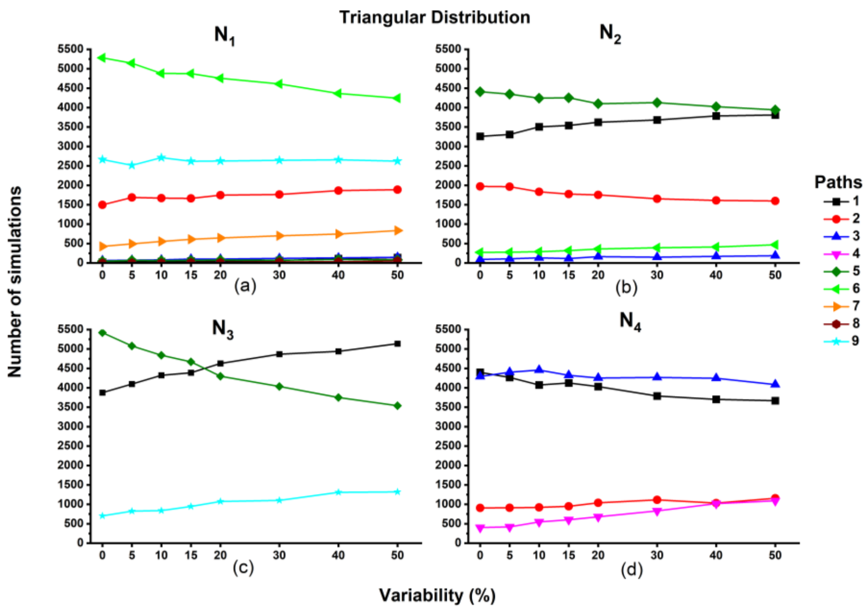


Figure 4. Critical paths and their frequency of occurrence in cases with and without allowances pessimistic times (t_p) when time is sampled using a uniform distribution.

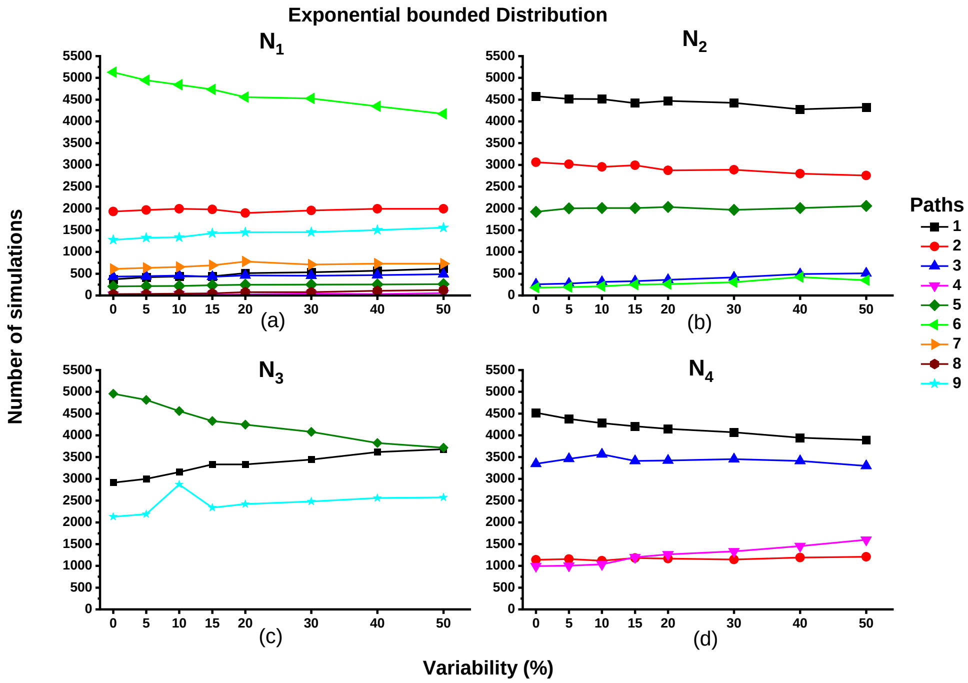


Figure 5. Critical paths and their frequency of occurrence in cases with and without allowances pessimistic times (t_p) when time is sampled using an exponentially bounded distribution.

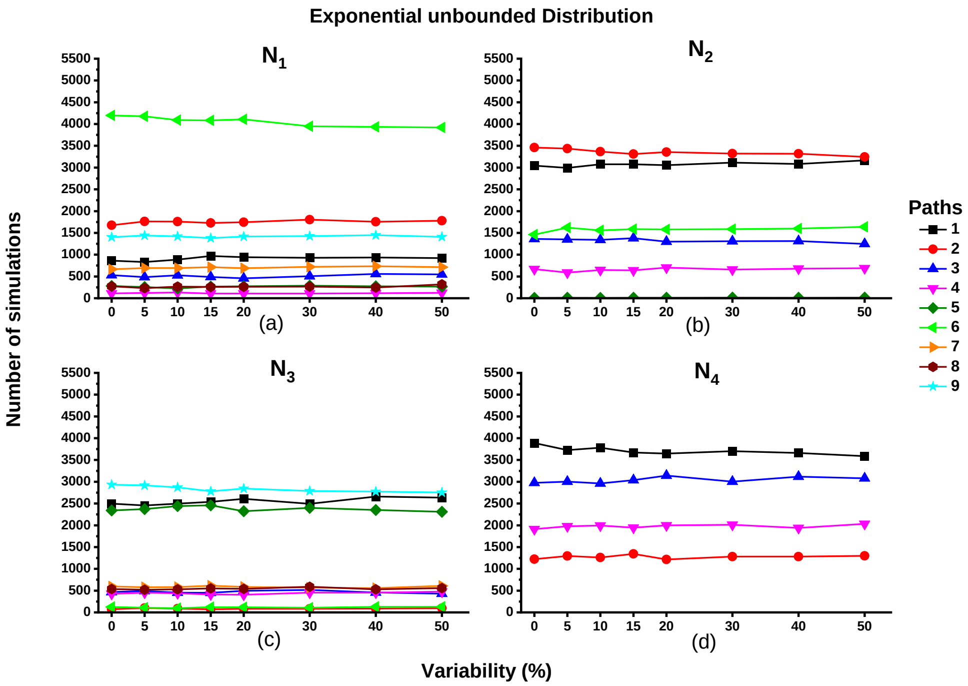


Figure 6. Critical paths and their frequency of occurrence in cases with and without allowances pessimistic times (t_p) when time is sampled using an exponential distribution.

4.1.1. Uniform distribution

Uniform distribution is more of a conservative approach where all values between maximum and minimum have equal probability of occurrence (applicable when there is no prior information about activity durations within the specified range). When unbalanced networks (N_1 and N_2) are simulated assuming a uniform distribution of activity times, almost all paths showed the possibility to emerge as critical but with varying frequencies. Even though no change in the dominant critical path was observed in these networks, the dominance level (frequency of occurrence) of individual paths was found to change with an increase in variability. For network N_1, the deterministic approach of scheduling (using activity time t_e) suggests paths P_6, P_9, and P_2 to be the top three dominant critical paths in the respective order. However, a simulation-based approach suggests the frequency of occurrence of P_2 as the dominant path to be higher than P_9 (**Figure 3**). A comparison of path variances of P_9 (3144 days) and P_2 (3030 days) suggests no appreciable difference in their variabilities and suggests a likelihood of one replacing the other depending on changes/delays incurred in individual activities during construction. For network N_1, the frequency of emergence of path P_6 as a critical path decreased from 40.54% to 34.18% (**Figure 3a**) when activity time variability was increased from zero ($U_{N1}V_0$) to 50% ($U_{N1}V_7$). Similarly, path P_2 and path P_9 showed the possibility of becoming critical between 21%–22.5% and 17%–28% respectively when the variability increased from zero to 50%. Other paths P_7, P_1, and P_5 also showed a gradual increase in the potential to be critical as the variability increased to 50%.

Network N_2 also showed a similar trend as N_1, where path P_1 remained as the dominant path in all cases (V_0–V_7) with the frequencies of occurrence decreasing with increasing variability (**Figure 3b**). However, the chances of the second dominant critical path P_5 becoming critical increase from 25.15% to 28.40% with an increase in variability. However, for network N_2, the deterministic approach suggests paths P_5, P_2 and P_6 to be the top three dominant paths in the respective order, whereas the simulation approach suggests the hierarchy of occurrence to be in the order $P_1 > P_5 > P_2$. A higher path variance of P_1 (1122 days) when compared to P_5 (967 days) and P_2 (800 days) is leading to the emergence of P_1 as the dominant critical path when various possible scenarios encountered in construction are accounted for in the simulation approach. However, P_6 with a higher path variance of 856 days emerged to be less dominant when compared to P_2 in the simulation study. A comparison of activities in P_6 and P_2 suggests the presence of one additional activity (no. 11) in P_2 with zero variability and a 40-day duration is causing the observed difference (**Table 1**). Since it is less likely to have activities with zero variance in construction networks, the dominance of the low variance path (P_2) can be considered inconsequential.

For the balanced network N_3, critical path estimation using deterministic activity times suggests path P_5 to be the dominant critical path (550 days) for the network. However, the simulation approach suggests a change in the dominance of the critical path from P_5 to P_1 once the activity time variability increases beyond 5% (**Figure 3c**). Frequency of occurrence of P_5 decreased from nearly 43.07% to below 31.41% once the variability increased to 50%, whereas the occurrence of the P_1 increased from 41.43% to 52.84%. Observation clearly suggests that the conventional approach of allocating resources to path P_5 to address minor delays incurred during construction may not be effective, as the critical path itself has shifted to P_1 due to the delay. Even though path P_9 also has a duration equal to P_1 when estimated using a deterministic approach (550 days), the former has a lower dominance in comparison to P_1 when different possible scenarios of activity time changes were simulated. The difference in path variances of P_1 (733 days), P_5 (267 days), and P_9 (222 days) is influencing the observed changes in the dominance of these paths. For network N_4, the deterministic activity time approach showed both P_1 and P_3 to have the same path duration (315 days). However, a higher variance of P_3 (603 days) when compared to P_1 (425 days) resulted in path P_3 emerging as the dominant critical path in simulation studies. Similar to networks N_1 and N_2, the dominance of the top two longest paths (P_1 and P_3) of N_4 was not found to change with an increase in variability. Furthermore, the frequency of emergence of P_4 as the dominant critical path increases with an increase in variability, even though the deterministic approach showed a lower path duration for P_4 (268 days) when compared to P_2 (305 days).

Among all four networks considered in this study, the occurrence of a high-variance activity, either on or outside of a critical path, was found to increase the frequency of emergence of that path as a critical path if the activity time changes due to delays incurred during construction. The comparisons depicted in **Figure 3** clearly indicate that the conventional strategy of allocating supplementary resources to a predefined critical path to address schedule delays does not ensure the timely completion of construction projects within the revised timeline. Hence, it is imperative to consider alternate critical paths and their relative dominance during the project planning phase and at various stages of project execution to ensure the timely completion of construction projects.

4.1.2. Triangular distribution

The section investigates the effect of assuming a triangular distribution for project activity time during project scheduling. Both balanced networks (N_3 and N_4) showed a reversal in critical path dominance with an increase in variability. However, the frequency of occurrence as the dominant path was observed to be different when compared to uniform distribution. The hierarchy of dominance was also found to change with distribution in all networks other than N_3.

The differences observed above are closely tied to the properties of triangular distribution, where a high percentage of sampled times falls near the mean value chosen for individual activity. A similar hierarchy of critical paths was obtained using t_m values as path duration (N_1: $P_6 > P_2 > P_9$, N_2: $P_5 > P_1 > P_2$, N_3: $P_5 > P_1 > P_9$, N_4: $P_3 > P_1 > P_2$) suggest triangular distribution to be strongly influenced by t_m values assigned for each activity. Hence, the use of a simplistic PDF-like triangular distribution may not be effective in capturing the stochasticity of activity times in a construction project network. For network N_4, both dominant paths P_3 and P_1 (**Figure 4d**) were found to have the same path duration of 310 days when t_m values were used in estimating critical path duration. However, a higher path variance ratio of 1.4 (P_3/P_1 = 602/425) is contributing to P_3 becoming dominant once the variability increases to 5% and beyond. A similar trend was observed in the case of N_3 with a variance ratio of 2.75 (P_1/P_5 = 733/267), causing P_1 to emerge as the dominant critical path beyond 15% variability in activity times. No change of critical path dominance in N_1 and N_2 with a lower path variance ratio (1.17 and 1.26, respectively) also supports the need for accounting activity variances in decision-making process.

4.1.3. Exponential bounded distribution

Results of bounded exponential distribution where activity time value sampling is restricted between t_o and t_p are given in the following section. As against uniform and triangular distributions, the reversal of dominance of critical paths P_5 and P_1 of network N_3 (**Figures 3** and **4**) occurs only when the variability in activity times was increased to 50%. The simulations show that the impact of path variances on changes in the dominance of critical paths is not very high for the selected networks as the probability of selection of activity times decreases towards the upper bound values due to the skewness of the exponential distribution.

Figure 5 also shows that even though the frequency of emergence of critical paths is different, the dominant critical paths remain the same in the majority of cases for all three distributions (uniform, triangular, and bound exponential). Being a negative exponential distribution, the effect of variability (t_p time extensions) in activity times is not as pronounced in exponential when compared to Unform and Triangular distributions. However, considering that not all activities in the network experience delays at the same time and that the chances of activity durations extending to and beyond the pessimistic values (t_p) occur in limited cases only (worst-case scenario), the results can be considered reasonable given the nature of construction projects. Hence, exponential distribution can be considered a better option in simulating activity durations in construction projects as the time extensions in individual activities of construction projects are random and independent of each other. However, the changes in critical path durations may also be looked into to assess the efficacy of using exponential distribution in sampling activity times.

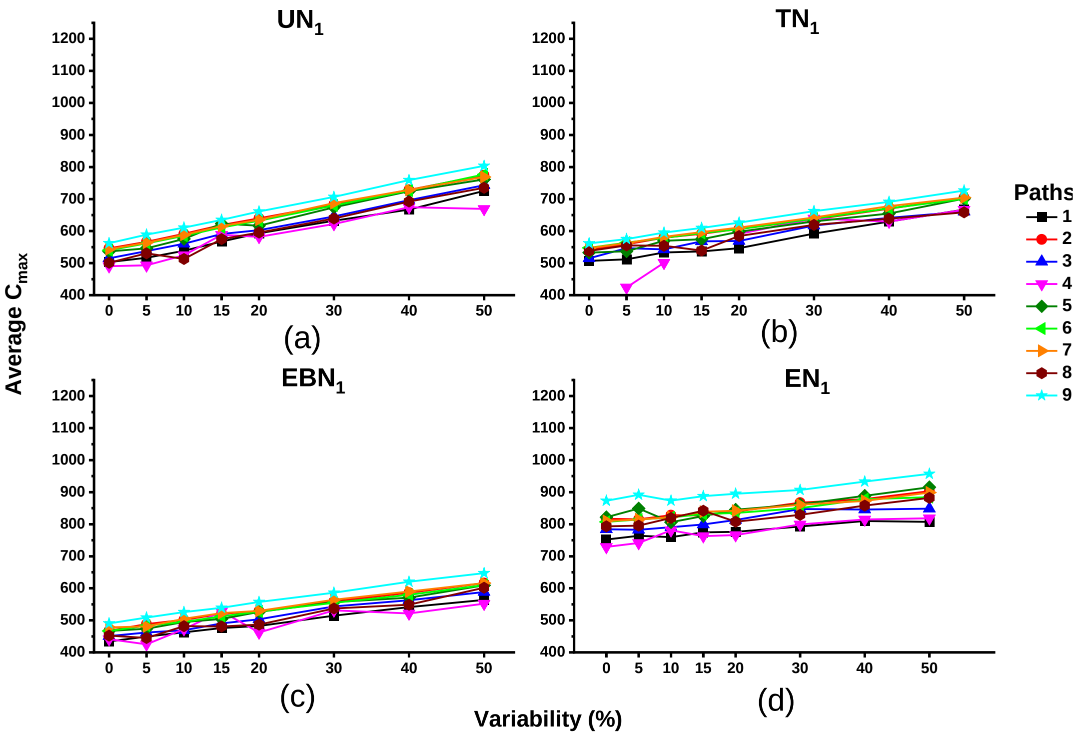
4.1.4. Exponential unbounded distribution

Exponential unbound distribution was selected to represent the worst-case scenario where certain activities in the project get extended indefinitely beyond the pessimistic time t_p causing unacceptable extensions on project completion times. Apart from N_1, the dominant critical path and their hierarchies were found to be different in all networks from those obtained during simulations using uniform, triangular, and bound exponential distributions.

Further, path (P_6) of network N_2 and path (P_4) of network N_4 emerged as the third dominant critical path by replacing P_5 and P_2, respectively, when activity times were sampled using exponential unbound distribution. Further, all nine paths in network N_3 showed possibilities of emerging as critical paths as against observations made in **Figures 3**–**5**. The observed change can be assigned to the skewness of the distribution and possible sampling of activity times beyond *tp* (unbound higher-end) values. The simulations also confirm that the unbound distribution may be more effective in modelling worst-case scenarios during construction. The trends observed in **Figures 3**–**6** confirm the network's sensitivity to changes in activity times and the possibility for changes in critical path during execution. Hence, the conventional strategy of focusing resources solely on a predefined critical path may not be effective for dynamic construction networks. The results also warrant the need for continuous monitoring of all network paths during project execution, especially while allocating additional resources to compensate for delays incurred during construction.

4.2. Impact of activity time variation on the duration of critical paths (C_{max})

Figure 7 shows the average duration of all critical paths identified in the four networks during simulations conducted as part of the study. The *x*-axis of the graph shows the percentage variability introduced on activity times (extension of t_p), and the *y*-axis represents the average of critical path durations obtained. This analysis clearly shows that irrespective of the distribution followed, the average C_{max} value of all networks increases with activity time variability. The hierarchy in average critical path durations does not follow the order of dominance of critical paths (frequency of emergence of critical paths) identified in **Figures 3–6**. For the network N_1, paths P_9 and P_2, having the largest variances in the respective order, had the highest average critical path durations for all distributions attempted in the study. Despite a higher average duration of critical paths, the frequency of emergence of these paths as dominant critical paths is less than path P_6, which has a lower path variance. Hence, it is reasonable to assume that the cost implications of project completion time extensions can be much higher if the critical path shifts from P_6 to P_9 or P_2, which cannot be identified in the conventional scheduling approach followed by the construction industry.



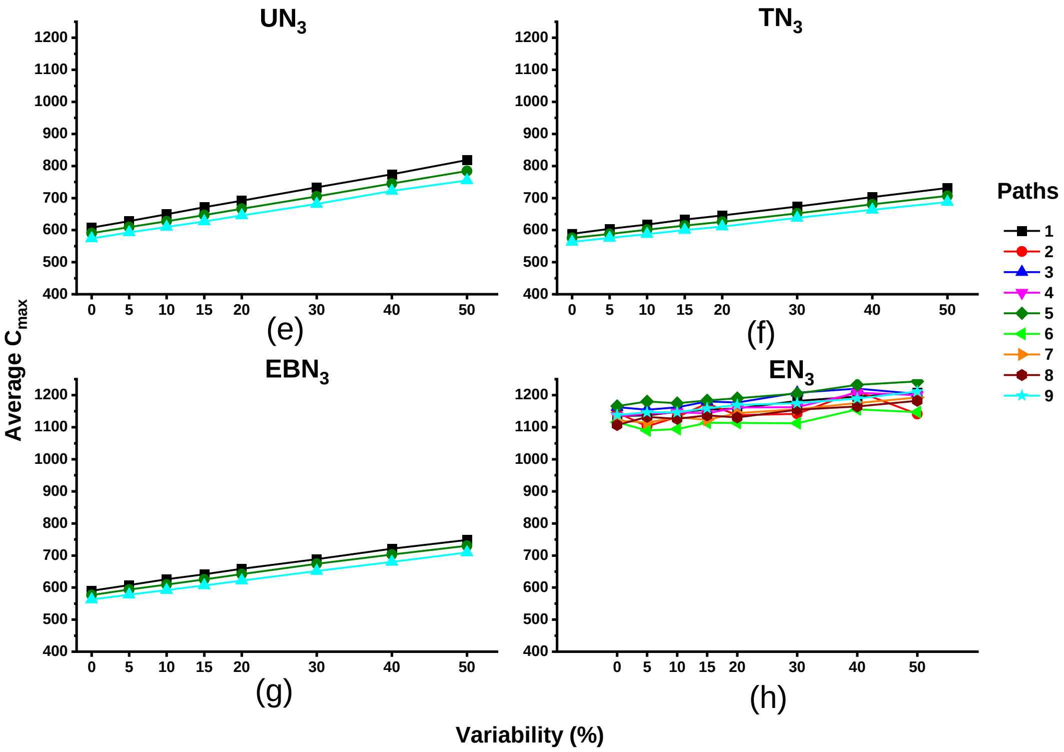


Figure 7. Average critical path durations (C_{max}) with and without variability (extension of pessimistic time, t_p) obtained during simulation of networks N_1 and N_3.

For the balanced network N_3, P_1, with the highest path variance and identified as the second dominant critical path in all distributions, showed the highest average critical path duration when activity times were sampled using uniform, triangular, and exponential bounded distributions. However, P_5 was found to have the highest average critical path duration when the worst-case scenario was modeled using an exponential unbound distribution. Moreover, the average path duration of P_3, which showed less than 5% chance of emerging as a critical path (**Figure 6**), was found to have the second highest average critical path duration. Simulation in **Figure 7** also shows that the rate of change in C_{max} with variability is highest for uniform distribution (44.12% time extension for N_1 and 34.69% for N_3 when variability increased from zero to 50%). Exponential bound distribution showed a time extension of approximately 25% for N_1 and N_3, which appears to be more reasonable based on field experiences in construction projects [50–52,56,57]. Exponential unbound distribution showed the highest average critical path duration for both networks, indicating their effectiveness in simulating worst-case scenarios in construction project scheduling. Similarly, a trend is observed in N_2 and N_4, as shown in Supplementary **Figures S1** and **S2**, respectively.

Based on the results in **Figure 7**, it is reasonable to conclude that exponentially bound distribution may be a preferred option to sample activity times for construction project simulations as it can account for uncertainties in activity times and give reasonable estimates of project completion time in case of project delays. Further, the simulation reflects the implications of changes in individual activity times during execution. It demonstrates the need to account for activity time variability in project control and monitoring. Results also suggest that the use of a dominant critical path identified during initial planning may not guarantee the timely completion of construction projects. Hence, instead of retaining a conventional single critical path approach, dynamic corrective measures need to be adopted in a case-by-case manner after accounting for the changes in critical path and project completion times due to the delays incurred during construction. Analysis of average critical path durations of networks N_2 and N_4 supports the above inferences.

4.3. Effect of various distributions on Probability of completion (POC)

Figure 8 shows the distribution of the probability of completion (POC) of paths in networks N_1 with different levels of variability in activity times (V_0 to V_7). The rectangular box displays the distribution of POC within the first and third quartiles, with the box's spread capturing the inter-quartile range (IQR). The horizontal line within the box denotes the median POC value. POC is obtained using Equation (5), where the X is taken as Cmax_te^0, which is calculated using initial t_o, t_m, and t_p of each variability case; μ is the critical path duration of the unique simulation instances, and σ is the standard deviation of the respective critical path. The base case value is the expected C_{max} value for the network using original time estimates considered during the project planning phase, which is compared with all other cases shown in **Figure 8**. Each box in **Figure 8** represents the POC of the paths that become critical at some point in time in 10,000 simulations.

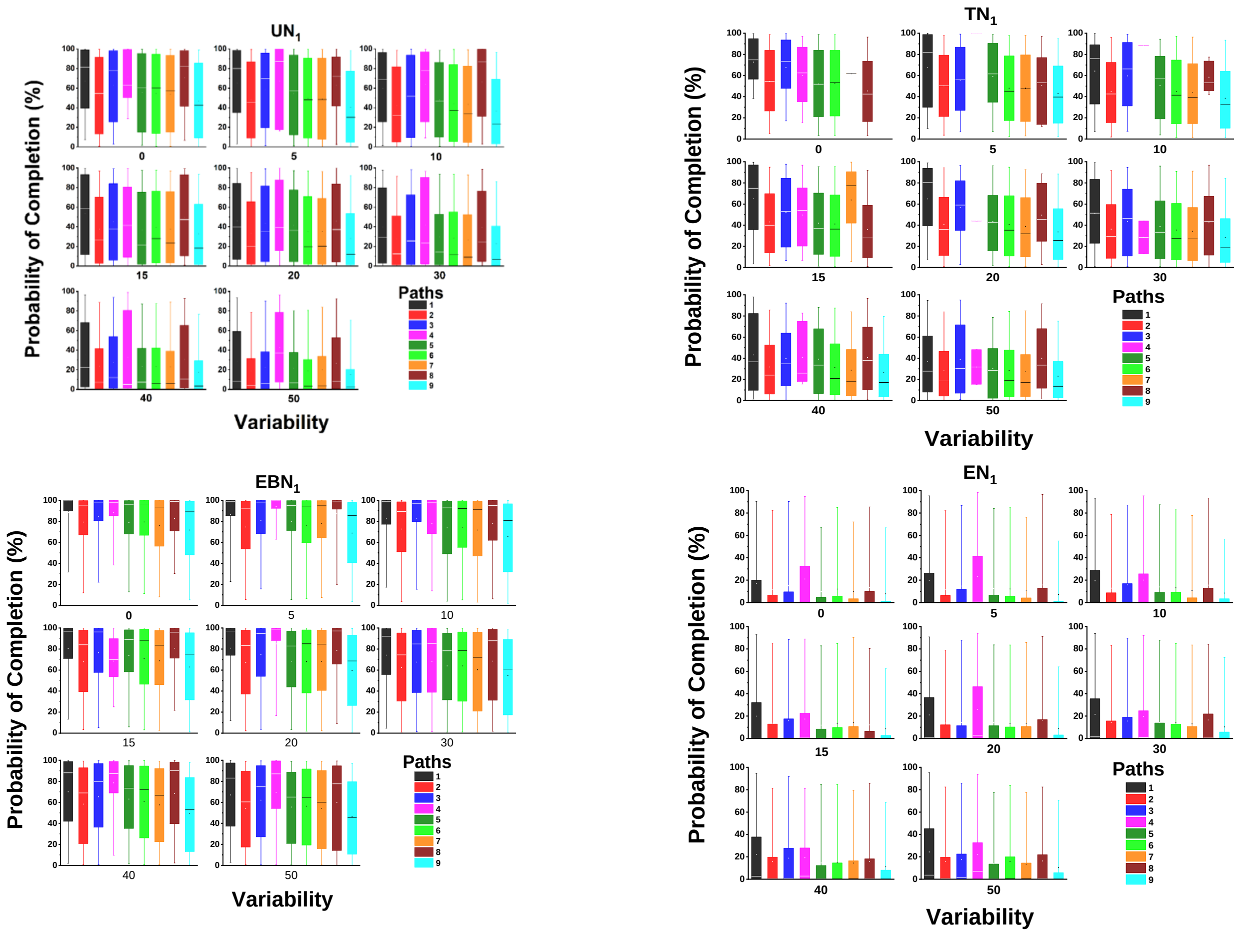


Figure 8. Probability of completion (POC) of all paths of N_1 using Cmax_te^0 as a benchmark when activity time is sampled using different distributions.

The results show that irrespective of the distribution followed in sampling activity times, the median POC of paths decreases as the variability increases from V_0 to V_7. The observed decline in POC with increasing variability highlights the uniform distribution's sensitivity to changes in the range of possible activity durations. The V_0 case, with no external variability, shows a median POC of all paths ranging between 80%–40%, indicating the possibility of the project not completing on time with an initial C_{max} value (planning horizon completion time estimate) of 553.3 days. However, as variability increases to 50%, the median POC of critical and near-critical paths reduces to nearly 10%, suggesting that it is highly unlikely for the project to finish in the planning horizon estimate used in initial scheduling. Assuming triangular distribution for activity times (**Figure 8b**) also supports the above. Even though the zero-variability case (V_0) shows a similar range for median POC values as that uniform distribution, a comparison shows the values to be less affected by an increase in variability. A lower rate of change in average critical path duration in triangular (**Figure 7b**) may be contributing to this observation.

The median POC values obtained using exponential bounded distribution show that the majority of paths have POC values near 90% when the network has zero variability. The results also support the effectiveness of the current practice followed by the construction industry, which assumes an ideal case scenario where activities do not have any variability. However, the median POC values reduce to below 60% for multiple paths as the variability increases to 50%. Average POC values for paths also decrease as the variability is introduced in activity times. This agrees well with field practices where large construction projects are found to experience time extensions beyond 25% when planning horizons (initial critical path duration) are estimated using deterministic time values [50,52,56,57]. The results also support the earlier observation that bounded exponential distribution may provide a more realistic estimate of project completion times while ensuring a high probability of timely completion. The median POC obtained using exponential unbound distribution (**Figure 8d**) reiterates the effectiveness of the distribution in modeling worst-case scenarios. The POC for other networks N_2, N_3, and N_4 are shown in Supplementary **Figures S3-S5**, respectively, where a similar trend is observed.

5. Conclusion

A simulation-based approach that accounts for variability in activity times was found to be an effective tool for estimating a realistic completion time during project planning. Assigning different levels (percentages) of variability to individual activities based on field experiences and site conditions can help to develop customized planning horizons for projects. Critical path estimation based on deterministic activity times can change due to delays incurred during project execution. Hence, the current industry practice of focusing on the initial critical path can lead to intolerable project extensions. For projects that are delayed during execution, allocating additional resources to the initial critical path may be ineffective in addressing these delays. Instead, dynamic corrective measures (time extensions, resource allocation, etc.) that can account for any possible changes in the critical path due to delays may ensure a higher probability of project completion within the revised schedule. The study demonstrated that changes in the duration of selected activities can reduce the overall probability of timely completion. Considering the variances of all paths within the network along with path durations is advisable in determining the planning horizon. The presence of a high-variance activity on or outside of the critical path can change the critical path due to delays during execution. While the exponentially bounded distribution provided a more realistic estimate of project completion time, the exponentially unbounded distribution effectively simulated the worst-case scenarios encountered in construction projects.

A limitation of this study is that the approach was only tested on a simulated project network designed to resemble construction project conditions. Testing on actual project data would provide more accurate insights into its practical applicability and effectiveness. Future studies could apply this simulation-based approach to scenarios where activities in the projects get delayed differently, and individual activity durations follow different distributions.

Author contributions: Conceptualization, DP, SG and SN; methodology, SG, DP and SN; software, SG; validation, SG, SN, RCG and DP; formal analysis, SG; investigation, SG; resources, SG; data curation, SG; writing—original draft preparation, SG; writing—review and editing, SG, DP and SN; visualization, SG; supervision, DP and SN.

Acknowledgments: The authors acknowledge the laboratory support from the Indian Institute of Technology, Kanpur, for conducting the research.

Institutional review board statement: Not applicable.

Informed consent statement: Not applicable.

Data: The model code, scripts, and the dataset prepared are available upon request to the corresponding author.

Conflict of interest: The authors declare no conflict of interest.

References

1. Rose KH. A Guide to the Project Management Body of Knowledge (PMBOK Guide) Fifth Edition. Project Management Journal. 2013; 44(3). doi: 10.1002/pmj.21345
2. Kerzner H. Project Management: A systems approach to planning, scheduling and control, 10th Ed. Industrial Engineering/Project Management; 2010.
3. Lester EIA. Project Management. Project Management, Planning and Control. Butterworth-Heinemann; 2014.
4. Nicholas JM, Steyn H. Project Management for Engineering, Business and Technology. Routledge; 2020.
5. González-Cruz M-C, Ballesteros-Pérez P, Lucko G, et al. Critical Duration Index: Anticipating Project Delays from Deterministic Schedule Information. Journal of Construction Engineering and Management. 2022; 148(11). doi: 10.1061/(ASCE)CO.1943-7862.0002387
6. Khamooshi H, Cioffi DF. Uncertainty in Task Duration and Cost Estimates: Fusion of Probabilistic Forecasts and Deterministic Scheduling. Journal of Construction Engineering and Management. 2013; 139(5): 488-497. doi: 10.1061/(ASCE)CO.1943-7862.0000616
7. Zhang S, Song X, Shen L, et al. Complicated Time-Constrained Project Scheduling Problems in Water Conservancy Construction. Processes. 2023; 11(4): 1110. doi: 10.3390/pr11041110
8. Olawale YA, Sun M. Cost and time control of construction projects: inhibiting factors and mitigating measures in practice. Construction Management and Economics. 2010; 28(5): 509-526. doi: 10.1080/01446191003674519

9. Gupta S, George RC, Philip D, et al. Activity Time Variations and Its Influence on Realization of Different Critical Paths in a PERT Network: An Empirical Study Using Simulations. In: Proceedings of the 2nd International Civil Engineering and Architecture Conference; 11-14 March 2022; Singapore.
10. Malcolm DG, Roseboom JH, Clark CE, et al. Application of a Technique for Research and Development Program Evaluation. Operations Research. 1959; 7(5): 646-669. doi: 10.1287/opre.7.5.646
11. Sculli D. A historical note on PERT times. Omega. 1989; 17(2):195-196. doi: 10.1016/0305-0483(89)90011-X
12. Dodin B. Approximating the distribution functions in stochastic networks. Computers & Operations Research. 1985; 12(3): 251-264. doi: 10.1016/0305-0548(85)90024-3
13. Dodin B, Sirvanci M. Stochastic networks and the extreme value distribution. Computers & Operations Research. 1990; 17(4): 397-409. doi: 10.1016/0305-0548(90)90018-3
14. Tesfaye E, Girma K, Berhan E, et al. A simulation of project completion probability using different probability distribution functions. Advances in Intelligent Systems and Computing. 2015; 334: 133–145. doi: 10.1007/978-3-319-13572-4_11
15. Thompson RC, Lucko G, Su Y. Reconsidering an Appropriate Probability Distribution Function for Construction Simulations. Construction Research Congress 2016; 2016.
16. Hahn ED, Martin MDML. Robust project management with the tilted beta distribution. Maryland Shared Open Access Repository; 2015.
17. Lau HS, Somarajan C. A proposal on improved procedures for estimating task-time distributions in PERT. European Journal of Operational Research. 1995; 85(1): 39-52. doi: 10.1016/0377-2217(93)E0213-H
18. Premachandra IM. An approximation of the activity duration distribution in PERT. Computers & Operations Research. 2001; 28(5): 443-452. doi: 10.1016/S0305-0548(99)00129-X
19. Badiru AB. A simulation approach to PERT network analysis. SIMULATION. 1991; 57(4): 245-255. doi: 10.1177/003754979105700409
20. Lee DE, Arditi D, Son CB. The probability distribution of project completion times in simulation-based scheduling. KSCE Journal of Civil Engineering. 2013; 17(4): 638-645. doi: 10.1007/s12205-013-0147-x
21. Salas-Morera L, Arauzo-Azofra A, García-Hernández L, et al. New Approach to the Distribution of Project Completion Time in PERT Networks. Journal of Construction Engineering and Management. 2018; 144(10). doi: 10.1061/(ASCE)CO.1943-7862.0001552
22. Williams FE. PERT Completion Times Revisited. INFORMS Transactions on Education. 2005; 6(1): 21-34. doi: 10.1287/ited.6.1.21
23. Yao MJ, Chu WM. A new approximation algorithm for obtaining the probability distribution function for project completion time. Computers & Mathematics with Applications. 2007; 54(2): 282-295. doi: 10.1016/j.camwa.2007.01.036
24. Mohan S, Gopalakrishnan M, Balasubramanian H, et al. A lognormal approximation of activity duration in PERT using two time estimates. Journal of the Operational Research Society. 2007; 58(6): 827-831. doi: 10.1057/palgrave.jors.2602204
25. Hajdu M, Bokor O. The Effects of Different Activity Distributions on Project Duration in PERT Networks. Procedia - Social and Behavioral Sciences. 2014; 119: 766-775. doi: 10.1016/j.sbspro.2014.03.086
26. Khan IA, Bickel JE, Hammond RK. Determining the Accuracy of the Triangular and PERT Distributions. Decision Analysis. 2023; 20(2): 151-165. doi: 10.1287/deca.2022.0464
27. Parks WH, Ramsing KD. The Use of the Compound Poisson in Pert. Management Science. 1969; 15(8): B-397-B-402. doi: 10.1287/mnsc.15.8.b397
28. Abdelkader YH. Evaluating project completion times when activity times are Weibull distributed. European Journal of Operational Research. 2004; 157(3): 704-715. doi: 10.1016/S0377-2217(03)00269-8
29. Hahn ED. Mixture densities for project management activity times: A robust approach to PERT. European Journal of Operational Research. 2008; 188(2): 450-459. doi: 10.1016/j.ejor.2007.04.032
30. Pérez JG, Martín M del ML, García CG, et al. Project management under uncertainty beyond beta: The generalized bicubic distribution. Operations Research Perspectives. 2016; 3: 67-76. doi: 10.1016/j.orp.2016.09.001
31. Lee D-E. Stochastic Project Scheduling Simulation System (SPSS III). Korean Journal of Construction Engineering and Management. 2005; 6: 73-79
32. Lee D-E, Yi C-Y, Lim T-K, et al. Integrated Simulation System for Construction Operation and Project Scheduling. Journal of Computing in Civil Engineering. 2010; 24(6): 557-569. doi: 10.1061/(ASCE)CP.1943-5487.0000061

33. Golenko-Ginzburg D. On the Distribution of Activity Time in PERT. Journal of the Operational Research Society. 1988; 39(8): 767-771. doi: 10.1057/jors.1988.132
34. Alagheband A. An efficient algorithm for calculating the exact overall time distribution function of a project with uncertain task durations. Indian Journal of Science and Technology. 2012; 5(9): 1-7. doi: 10.17485/ijst/2012/v5i9.20
35. Kleindorfer GB. Bounding Distributions for a Stochastic Acyclic Network. Operations Research. 1971; 19(7): 1586-1601. doi: 10.1287/opre.19.7.1586
36. Mehrotra K, Chai J, Pillutla S. A study of approximating the moments of the job completion time in PERT networks. Journal of Operations Management. 1996; 14: 277-289. doi: 10.1016/0272-6963(96)00002-2
37. Soroush HM. The Most Critical Path in a PERT Network. Journal of the Operational Research Society. 1994; 45(3): 287-300. doi: 10.1057/jors.1994.42
38. Soroush HM. The most critical path in a PERT network: A heuristic approach. European Journal of Operational Research. 1994; 78(1): 93-105. doi: 10.1016/0377-2217(94)90124-4
39. Lee D-E. Probability of Project Completion Using Stochastic Project Scheduling Simulation. Journal of Construction Engineering and Management. 2004; 131(3): 310-318. doi: 10.1061/(ASCE)0733-9364(2005)131:3(310)
40. Lee DE, Bae TH, Arditi D. Advanced Stochastic Schedule Simulation System. Civil Engineering and Environmental Systems; 2011.
41. Ahuja HN, Dozzi SP, AbouRizk SM. Project management: techniques in planning and controlling construction projects. Wiley India; 1994.
42. Hillier FS, Lieberman GJ. Introduction to operations research, ELEVENTH. McGraw Hill; 2021.
43. Pinedo ML. Planning and Scheduling in Manufacturing and Services, 2nd ed. Springer-Verlag New York; 2000.
44. Winston WL. Operations Research: Applications and Algorithms. Cengage Learning; 2022.
45. Hajdu M. Effects of the application of activity calendars on the distribution of project duration in PERT networks. Automation in Construction. 2013; 1(35): 397-404.
46. Zohrehvandi S. A Project-Scheduling and Resource Management Heuristic Algorithm in the Construction of Combined Cycle Power Plant Projects. Computers. 2022; 11(2): 23. doi: 10.3390/computers11020023
47. Hajdu M, Bokor O. Sensitivity analysis in PERT networks: Does activity duration distribution matter? Automation in Construction. 2016; 65: 1-8. doi: 10.1016/j.autcon.2016.01.003
48. Mályusz L, Vattai Z. Project scheduling algorithms for construction projects with stretchable activities. IOP Conference Series: Materials Science and Engineering. 2022; 1218(1): 012043. doi: 10.1088/1757-899x/1218/1/012043
49. Wang J, Liu H, Wang Z. Stochastic Project Scheduling Optimization for Multi-stage Prefabricated Building Construction with Reliability Application. KSCE Journal of Civil Engineering. 2023; 27(6): 2356-2371. doi: 10.1007/s12205-023-2164-8
50. Ansar A, Flyvbjerg B, Budzier A, et al. Does infrastructure investment lead to economic growth or economic fragility? Evidence from China. Oxford Review of Economic Policy. 2016; 32(3): 360-390. doi: 10.1093/oxrep/grw022
51. Cantarelli CC, Flyvbjerg B, Molin EJE, et al. Cost Overruns in Large-scale Transportation Infrastructure Projects: Explanations and Their Theoretical Embeddedness. European Journal of Transport and Infrastructure Research; 2010.
52. Ellis RD, Thomas HR. The root causes of delays in highway construction. In: Proceedings of the 82nd Annual Meeting of the Transportation Research Board; 2002; Washington, D.C.
53. Salhab D, Lindhard SM, Hamzeh F. Simulation-Based Approximation of the Gain from Applying Overlapping Activities. Journal of Construction Engineering and Management. 2024; 150(4). doi: 10.1061/jcemd4.coeng-14166
54. Révész P. The laws of large numbers. Academic Press; 1968.
55. Salhab D, Møller DE, Lindhard SM, et al. Accounting for Variability: Identifying Critical Activities as a Supplement to the Critical Path. Journal of Construction Engineering and Management. 2022; 148(5). doi: 10.1061/(ASCE)CO.1943-7862.0002266
56. MoSPI G of I. FLASH REPORT ON (Rs. 150 crore and above). Government of India Ministry of Statistics and Programme Implementation Infrastructure and Project Monitoring Division; 2020.
57. Gupta S, George RC, Philip D, et al. Data-Driven Decision Framework for Modeling and Quantification of Uncertainty and Variability in Construction Project Scheduling. Journal of Structural Design and Construction Practice ASCE. 2024. doi: 10.1061/JSDCCC/SCENG-1817