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Article

Actions with manipulatives support second graders' learning about place-value concepts

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Abstract: The aim of the present study was to examine the impact of conceptual transparency of mathematics manipulatives used in instruction on the learning of place-value concepts in typically developing second graders (n = 88) and those at-risk for mathematics learning disabilities (n = 29). We randomly assigned the children of each classification to three instructional conditions that varied according to the extent to which place-value concepts were made visible in the objects' perceptual features. In one condition, the ten and hundreds denominations were already grouped and the ones were visible in each denomination; in the second condition the denominations were already grouped but the ones were not visible; in the third condition, we provided children with individual beads that could be attached in groups of tens and hundreds. We assessed the accuracy of the children's representations of two- and three-digit numerals using manipulatives and their place-value knowledge on symbolic tasks. Contrary to our expectations, we found that the manipulatives requiring children to construct their own denominations were related to gains in the accuracy of the physical representations, but not to gains on the symbolic measures. We speculate that the actions involved in constructing the denominations provided opportunities for children to encode the materials' salient features in ways that led to the greatest benefits. We suggest that teachers ensure that students encode manipulatives used during instruction in meaningful ways.

Keywords: manipulatives; conceptual transparency; mathematics learning disabilities; place value

1. Introduction

Teachers often use concrete objects, such as blocks and counters, to help children acquire mathematical concepts. Using these objects, or "manipulatives," in mathematics instruction has demonstrated benefits for both typical achievers and children who struggle (Byrne et al., 2023; Carbonneau et al., 2013). Recent research has shown that the conceptual transparency of manipulatives (i.e., the extent to which the target mathematical concepts are visible in the physical configuration of the objects) by themselves can guide and constrain learning in important ways (Lafay et al., 2023). Because manipulatives are so often encouraged in the elementary classroom, more research is needed to understand how their physical characteristics are related to children's learning in the context of instruction. In the present research, we examined the effects of empirically validated instruction with manipulatives on children's base-ten knowledge as a function of the objects' physical structure.

1.1. Base-ten knowledge

Base-ten number concepts are central to children's understanding of multidigit numerals and strategies used in whole-number computation (Byrge et al., 2014; Laski et al., 2016; Mix et al., 2019). Base-ten knowledge relies on place-value concepts, or what Cheung and Ansari (2021) called the principle of position, which dictates a specific syntax that allows a correct interpretation of a numeral's value (Mix et al., 2022). The principle of position is based on two interrelated components (Ross, 1989): (a) the position of each digit in a numeral is associated with a specific unit (i.e., denomination) determined by a power of 10 (e.g., 10^0, or ones; 10^1, or tens; 10^2, or hundreds), and (b) the digit itself indicates number of the counts of a specific denomination (Lafay et al., 2023). Knowledge of the latter component implies that the value of a digit corresponds to its position (e.g., in the numeral 325, the digit 3 represents 300) and relatedly, that the digit 3 corresponds to three counts of 100.

Base-ten knowledge is foundational for children's future mathematics achievement (Bower et al., 2022; Götze and Baiker, 2021; Ho and Cheng, 1997; Mix et al., 2023). More specifically, the principle of position supports children's interpretations of symbolic representations of number (Mix et al., 2022) and knowledge of the relation between base-ten units can foster flexible ways to interpret quantities, both of which can support written and mental computation (Carpenter et al., 1997; Ladel and Kortenkamp, 2016; Lambert and Moeller, 2019). Despite the importance of base-ten numeration, many children struggle to learn its conceptual underpinnings (Cheung and Ansari, 2021; Jensen et al., 2024; Mix et al., 2022), particularly those who are at risk for mathematics learning difficulties (Lafay et al., 2023; Rojo et al., 2021).

Research in mathematical cognition has revealed that children with mathematics difficulties lag behind their typically developing (TD) peers in place-value knowledge (Lambert and Moeller, 2019). For one, they have greater difficulty identifying which digit in a numeral corresponds to denominations of specific sizes (i.e., the ones, tens, hundreds): Researchers have observed that relative to their TD peers, children who struggle in mathematics tend to produce higher proportions of so-called transcoding errors, characterized by writing the sequence of digits in a way that matches the way a number is heard rather than respecting place-value syntax (e.g., such as writing "204" when hearing "twenty-four"; Moura et al., 2013; Rousselle and Noël, 2007). Further, Landerl and Kölle (2009) found that children with mathematics difficulties were slower than their TD counterparts at judging which of two numerals was numerically larger when the tens digit was larger in one numeral and the ones digit was larger in the other (e.g., 37 vs. 81). This finding suggests that children with mathematics difficulties have relatively more trouble understanding that the digits' locations in a numeral represent units of specific sizes. More specifically, because the digits in the left-most place represent counts of a larger unit than the digits in the right-most place (i.e., tens vs. ones), the only digits that matter when comparing 37 and 81 are the 3 and the 8. In sum, researchers have inferred several gaps in the place-value knowledge of children with mathematics difficulties when they use and interpret symbolic representations of multidigit numerals (e.g., Cheung and Ansari, 2021), but substantially fewer studies have examined the perceptual characteristics of manipulatives that influence place-value performance and learning in children with mathematics difficulties.

1.2. Teaching place-value concepts with manipulatives

The impact of manipulatives on students' performance in mathematics has been shown to be in part contingent on their physical features, such as how perceptually rich they are or whether they contain extraneous details that could detract from the instructional objective (Marley and Carbonneau, 2015; McNeil et al., 2009; Petersen and McNeil, 2013; Uttal et al., 2013). Another way that the physical features of manipulatives can vary is the degree to which they make certain mathematical concepts visible. For example, base-ten blocks are specially designed to show the multilevel units embedded in base-ten quantities. A tens block with demarcations showing that 10 ones and one unit of ten are equivalent makes this equivalence visible.

Manipulatives have been viewed as "pedagogical analogies" (English, 2004), in which the physical objects serve as the source analog and the concepts they are intended to represent are abstracted by comparing the source to a target analog. Research in analogical reasoning has shown that it is easier for children to acquire the underlying conceptual structure between two analogs when they share perceptual similarities than when they do not (Kotovsky and Gentner, 1996; Siegler and Ramani, 2009). The same principle may extend to concrete objects: Preliminary evidence shows that children can learn place-value concepts when they are made visible through the perceptual characteristics of manipulatives. In one of three experiments with children aged 4 to 6 years, Yuan et al. (2021) investigated children's mappings between two representations of base-ten quantities: spoken number words and manipulatives. The manipulatives varied in the extent to which they directed attention to the target relations between denominations (i.e., that hundreds are larger than tens, and tens are larger than ones). A "deconstructed abacus," consisting only of discs of different sizes, yielded better performance than standard or modified abaci that included potentially redundant or distracting perceptual features (e.g., spatially separated denominations positioned at the left, middle, and right; the metal poles holding the discs). The deconstructed abacus also led to improved performance on a transfer task assessing mappings between number words and symbolic representations. Overall, the findings suggest that reducing extraneous perceptual information can facilitate learners' focus on the intended quantitative relations.

In previous work, we used the term "conceptual transparency" (Lafay et al., 2023) to refer to the degree to which the manipulatives render target mathematical concepts and relations visually salient (Chase and Abrahamson, 2013; Chen, 1996). We investigated the physical affordances of manipulatives on place-value understanding in TD children and those at risk for mathematics learning disabilities. Second graders were asked to represent numerals with manipulatives and to verbally interpret their physical representations. Results reported by Lafay et al. (2023) indicated that manipulatives that made the denominations visible (i.e., manipulatives that were already grouped in their respective denominations), but not the ones in the denominations, were responsible for more accurate representations of numbers and more frequent use of place-value concepts when interpreting their displays. A descriptive response analysis further revealed that the children at risk for mathematics learning disabilities were especially disadvantaged when using manipulatives that did not make the denominations visible. Indeed, they failed to use place-value concepts to interpret their displays but had less difficulty when using manipulatives that had the ones, tens, and hundreds already grouped.

Instructional factors have also been found to influence children's learning with manipulatives. The notion of highlighting the links between concrete objects and other representations, such as written symbols, has emerged from several theoretical accounts of mathematics learning, from Bruner (1966) to more recent perspectives, such as concreteness fading. Concreteness fading entails the gradual removal of perceptual details, which may support children's ability to make connections between concrete and symbolic representations of quantity (Fyfe and Nathan, 2019). Further, empirical evidence argues for the instructional practice of making explicit connections between the manipulatives and written symbols. Fuson and Briars' (1990) "mapping instruction," for example, emphasized the meanings of number words, written symbols, and base-ten blocks by making clear links among the three representational systems. In a study that involved children's addition and subtraction of multidigit quantities, the authors observed a relation between instruction that highlighted the connections between concrete objects and numerals and growth in the children's computation and knowledge of place-value concepts.

In another study on the impacts of explicit links in mathematics instruction, Donovan and Fyfe (2022) provided kindergarten, first-, and second-grade children with a place-value lesson in four instructional conditions, three of which highlighted the connections between base-ten blocks and written representations of multidigit quantities. The authors observed that regardless of instructional condition, children with a stronger grasp of the conceptual connections between base-ten blocks and multidigit numerals exhibited stronger performance on place-value assessments at posttest than did children with weaker knowledge of the connections. Finally, Osana et al. (2017) found that when base-ten blocks were presented after written symbols, second graders were only able to make the connections between the two representational systems when explicit conceptual links between them were provided.

In sum, the research on children's learning with mathematics manipulatives suggests that instruction is effective when clear and explicit connections are made between the objects and written representations of number. Little is known, however, about how the physical affordances (Glenberg et al., 2013) of manipulatives impact learning in the context of empirically validated instruction. For the present study, we hypothesized that instruction with manipulatives that are conceptually transparent, such as those that made the denominations visible in Lafay et al.'s (2023) study, would augment the instruction's effectiveness. The objective of the current study is to test this hypothesis directly.

1.3. Current study

The objective of the current study, which included second-grade TD children and children at-risk for mathematical disabilities, was to evaluate the affordances of the manipulatives used by Lafay et al. (2023) for learning about place value in an instructional context. Prior research has shown that place-value concepts are challenging for young children (e.g., Jensen et al., 2024). Although understanding the cognitive mechanisms underlying their challenges is an important area of inquiry, our focus in the current study was on supporting conceptual growth through the use of manipulatives, commonly used in the classroom by teachers to illustrate mathematical concepts.

The manipulatives we tested in the current study differed by their conceptual transparency—that is, by the extent to which place-value concepts were visible through their perceptual features. Incorporating the same two samples of children and the same experimental conditions, we administered an instructional intervention based on empirically validated approaches to teaching place-value concepts that highlighted the connections between concrete and written representations of quantities (Donovan and Fyfe, 2022; Fuson and Briars, 1990). The manipulatives provided in the three experimental conditions were the same as in the Lafay et al. (2023) study: In two conditions, the manipulatives made the denominations visible by having the tens and hundreds denominations already grouped. In one of the denominations-visible conditions, the ones in the larger denominations were visible and in the other, they were not. In the third condition, the participants were given individual beads that were not pre-grouped in their denominations but could be attached to form groups of tens and hundreds. We report the post-instruction performance of the children as a function of the physical features of the manipulatives, taking into account their initial performance, which was reported in detail by Lafay et al. (2023). We also investigated whether the features of the manipulatives had different effects on the at-risk children than on the TD children.

We assessed children's learning of place-value concepts using three measures. With the Representation Task, we measured gains in the accuracy of children's displays of multidigit numerals with the condition-specific manipulatives they used during instruction. We also examined gains on two symbolic assessments of place-value concepts. The Picture Place Value Test assessed children's knowledge of the size of the denominations (i.e., one, 10, and 100) corresponding to the position of the digits in a numeral. The Denominations Task assessed children's syntactic knowledge of base-ten numerals, namely that the value of each digit is equivalent to counts of units determined by the digit's position. These latter two place-value assessments measured children's transfer of the place-value concepts they acquired during the instruction to their symbolic representations of numbers. Finally, we conducted a descriptive response analysis to explore changes in the types of displays the children provided after instruction. In particular, we examined the errors the children in each mathematics ability group (i.e., TD and at-risk) produced to explain any potential condition differences. The research questions that guided the study were the following.

Research Question 1 (RQ1): Does conceptual transparency in manipulatives used during instruction support gains in (a) representation accuracy, (b) knowledge of denomination size, and (c) syntactic knowledge of place value? Based on the results of Lafay et al. (2023), we predicted that the manipulatives that were already grouped in their denominations (i.e., tens and hundreds, either with ones visible or not) would afford greater gains on all measures than manipulatives that were not grouped in base-ten denominations.

Research Question 2 (RQ2): Does conceptual transparency in manipulatives offer specific advantages in place-value learning to children at risk for mathematics learning disabilities? We predicted the same pattern of effects as for the TD children, but because of the difficulties observed by Lafay et al. (2023) in the at-risk children, we expected greater conceptual transparency advantages for the latter children.

Research Question 3 (RQ3): What is the nature of the change in children's displays, condition by condition, in the TD and at-risk children groups? More specifically, are there condition differences in the types of errors produced and do the patterns of errors observed differ by mathematics ability group?

2. Method

2.1. Participants

As was mentioned earlier, our sample included second-grade TD children and children at risk for mathematics learning disabilities (the latter hereafter referred to as at-risk). We were unable to use already-existing documentation to classify children into mathematics ability groups because diagnostic testing for mathematics learning disabilities is not systematically conducted in the province of Quebec in Canada. Instead, we used survey data from teachers and parents, and we confirmed their reports with measures that we administered to all children in the sample. Teachers and parents were given a paper-and-pencil survey asking about whether each child (a) had no mathematical difficulties, (b) struggled to learn mathematics compared to other children, or (c) had a documented diagnosis of mathematics learning disabilities. Teachers and parents gave similar responses for each child, and no child had a documented diagnosis of mathematics learning disabilities.

We administered the Tempo Test Rekenen (TTR; De Vos, 1992) and the Raven's Colored Progressive Matrices (Raven, 1977) to all children. The TTR is a paper-and-pencil test of arithmetic fluency, a measure used in previous research to determine the presence of mathematics learning disabilities (e.g., Träff et al., 2017). According to Dennis et al. (2016), the 25th percentile was the most frequently used cut-off point in the literature for mathematics difficulties identification. The Raven's was administered to assess the participants' nonverbal intelligence. Participants scoring below the 10th percentile would have been excluded from the analyses, but no participant fell into this category.

We used the TTR, together with the parent and teacher data, to separate the children into two mathematics ability groups. In our sample, teachers and parents reported mathematics difficulties for 32 children. Of these, 29 scored below the 25th percentile on the TTR, and were thus classified as at risk. The remaining three participants who scored above the 25th percentile on the TTR were excluded from the study. Finally, both the parents and teachers reported no mathematics difficulties in school for 97 children. Ninety-four of these children scored above the 25th percentile on the TTR and were classified as TD (the remaining three children were excluded from the study).

The sample (*N* = 123) consisted of 94 TD children and 29 at-risk children. Because of attrition and absence during the instruction and posttest phases of the study, six children were removed from the data set. The final sample (*N* = 117) thus consisted of 88 TD children (M_{age} = 92.4 months, *SD* = 4.8) and 29 at-risk children (M_{age} = 90.7 months, *SD* = 6.3). As reported in Lafay et al. (2023), the two groups did not differ statistically in mean age and were also equivalent in terms of gender distribution (47% girls in both mathematics ability groups).

2.2. Design

All children were recruited from six public schools. Within each mathematics ability group in each school, children were randomly assigned without restriction to three instructional conditions that differed in the manipulatives used (Lafay et al., 2023): (a) attachable beads that did not make the base-ten denominations or ones in the denominations transparent (*n* = 34; 25 TD and 9 at-risk), (b) pipe cleaners that made only the denominations transparent (*n* = 41; 33 TD and 8 at-risk), and (c) string beads that made both the denominations and the ones in the denominations transparent (*n* = 42; 30 TD and 12 at-risk). **Figure 1** presents the manipulatives used in the three instructional conditions.

(a)

(b)

(c)

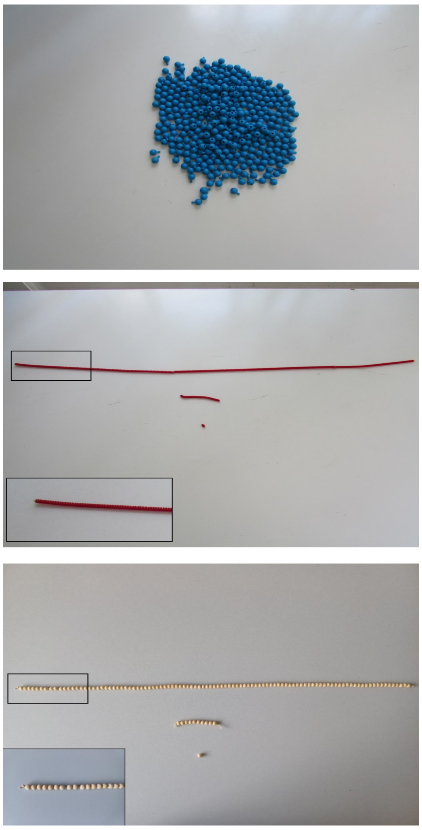


Figure 1. Manipulatives used in the instructional conditions: **(a)** attachable beads; **(b)** pipe cleaners; **(c)** string beads.

Children were given mapping instruction on place-value concepts with their condition-specific manipulatives. The instruction focused on demonstrating to the children how to represent two- and three-digit numerals with manipulatives and how to break down the numerals into parts that reflected base-ten denominations concretely and using written symbols (e.g., 47 = 40 + 7).

Before and after the instruction, we assessed the accuracy of the children's physical displays using condition-specific manipulatives on the Representation Task and documented the types of displays they produced. We also measured participants' knowledge of two specific place-value concepts using symbolic measures at both time points: (a) the idea that the position of each digit in a numeral corresponds to a specific denomination, using the Picture Place Value Test (PicPVT), and (b) the notion that the digit specifies the number of counts of a specific denomination with the Decomposition Task.

A researcher met with each child over two individual sessions. The pretest data were collected in the first session and the intervention was delivered at the start of the second session, followed immediately by the administration of the posttest measures. The administration of the Representation Task and the delivery of the intervention were video recorded. Eleven parents did not give consent for video recording, thus leaving 106 videos for further analysis. In these cases, the researcher met with the children with a second researcher who observed and took as many notes on the children's responses as possible.

The time between sessions was between 1 and 10 days for 89 children (79% of the sample; $M = 4.20$; $SD = 3.13$). For the entire sample (with five missing data points, $N = 112$), time between sessions ranged from 1 to 94 days, and did not differ by condition, $F(2, 106) < 1$, $p = 0.73$, or by mathematics ability group, $F(1, 106) = 3.05$, $p = 0.08$.

2.3. Instruction

The instruction in all conditions focused on representing numerals with concrete objects and using the objects to decompose the numeral using written notation to represent the hundreds, tens, and ones. It adhered to validated instructional principles for the development of conceptual knowledge in mathematics (Crooks and Alibali, 2014). The intervention incorporated explicit links between the numerals and the manipulatives by pointing and gesturing to the objects that represented the digits and explaining how the digits mapped to the physical displays (Donovan and Fyfe, 2022; Fyfe et al., 2017). What is more, corrective feedback was immediate, and whenever the child was at an impasse, the researcher provided conceptual explanations and modeled the solution again if necessary. We did this because feedback has been shown to be beneficial, particularly for children who struggle in mathematics (Fyfe et al., 2023; Kiru et al., 2018).

2.3.1. Demonstration and guided practice

The instruction consisted of a demonstration phase and a guided practice phase. The numerals presented in both phases were chosen according to the mathematics ability group (i.e., TD or at-risk) and performance on the PicPVT and Decomposition Task at pretest. Specifically, all children in the TD group, as well as those in the at-risk group who scored 70% or higher on the two-digit items on the PicPVT and Representation Task, worked primarily with three-digit numerals during the intervention (47 and 214 in the demonstration phase and 64, 156, 317, and 401 in the guided practice phase). All other children worked only with two-digit numerals (47 and 23 in the demonstration phase and 64, 18, 31, and 52 in the practice phase). In sum, during the instruction 18 at-risk children worked with only 2-digit numerals and 11 at-risk children worked with 3-digit numerals.

The researcher began the demonstration phase by explaining to the child that together, they would use manipulatives to represent numerals. In each condition, the researcher put the object representing 1 on a sheet of paper on which "1—one" was printed. The same procedure was used for the objects representing 10 and 100 (with the sheets indicating "10—ten" and "100—one hundred," respectively). The researcher then explained that there were 10 ones in one ten by pointing to each one and counting each out loud. The researcher also gestured to each of 10 tens in one hundred with her thumb and index finger and counted the tens out loud (e.g., "1, 2, 3, 4…10").

Next, the researcher put a card with a written numeral on the table and read the target numeral (e.g., 47) out loud. She started by displaying 7 ones on the table with the manipulatives and counted them out loud (i.e., for seven: "1, 2, 3, 4, 5, 6, 7"). In the pipe cleaners and string beads condition, she then displayed 4 tens on the table and counted them out loud by tens (i.e., "10, 20, 30, 40"). In the attachable beads condition, she first constructed the groups of ten with the beads and then counted the groups. In all conditions, the researcher reminded the child of place-value principles by pointing to each digit in the numeral and the objects that represented the value of the digits. In this example, she pointed to the "7" in 47 and said, "Remember that the digit on the right represents the number of ones," and then pointed to the seven groups of ones represented with the objects. A similar explanation was provided for the tens. She then counted all the manipulatives representing 47 out loud, starting with the tens (e.g., 10, 20, 30, 40, 41, 42, 43, 45, 46, 47).

The second part of the demonstration phase entailed showing the child two different ways to represent the target numeral in writing. After each numeral was represented with the manipulatives and counted, the researcher wrote an expression using expanded notation that highlighted the values of the digits (e.g., 47 = 40 + 7). The researcher then wrote a second expression that further expanded the first to show the number of tens (e.g., 47 = 10 + 10 + 10 + 10 + 7). In this example, she explained, "So, 47 is 40 plus 7, but it is also 10 plus 10 plus 10 plus 10 plus seven." For each numeral, the concrete display stayed visible, and the researcher pointed and gestured back and forth between the symbolic and concrete representations to emphasize the conceptual link between the two.

In the practice phase, the children were given four numerals to represent first with the manipulatives and then symbolically in expanded form. For each numeral, the researcher asked the child to explain his or her thinking out loud. Corrective feedback was provided by the researcher whenever an error was made, either in the representation or in the concepts articulated by the child.

2.3.2. Implementation fidelity

The instructional intervention was delivered by the third author and six trained research assistants. All researchers followed a condition-specific script closely when delivering the instruction. To compute an estimate of implementation fidelity, 20 of the 106 (19%) videotaped instructional sessions were randomly sampled from each instructional condition. The third author and one of the research assistants independently coded each recording for specific instructional elements, including those related to verbal instructions, gestures, spatial arrangement of the manipulatives, and corrective feedback. **Table 1** presents an excerpt of the elements in the string beads condition that were used to calculate implementation fidelity. Agreement between the two coders averaged 87.7%; the discrepancies were resolved through discussion with the third author. Implementation fidelity was represented by the mean proportion of instructional elements present across all 20 sessions and was computed at 76.0%. The estimated fidelity rate of 76% falls within the moderate-to-high range commonly reported in educational intervention research (O'Donnell, 2008).

Table 1. Excerpt from the implementation fidelity coding rubric for the string beads condition.

Instructional element	Check "✓" if present
• Shows the number card	
• Says the number word out loud	
• Uses the word "number"	
• Points to the ones digit in the number	
• Says the number word for the ones digit	
• Uses the word "digit"	
• Counts the number of ones that correspond to the ones digit (e.g., 1, 2, 3 ones)	
• Points to each one in the physical display	
• Points to the tens digit in the number	
• Says the number word for the tens digit out loud	
• Uses the word "digit"	
• Counts the number of tens that correspond to the tens digit (e.g., 1, 2, 3 tens)	
• Gestures over each ten in the physical display	
• Places the manipulatives vertically by denomination: Tens on the top and the ones on the bottom	
• Reminds the child that the digit on left in the numeral represents the tens and the digit on the right represents the ones	
• While reminding the child [see above], points to the ones digit and the tens digit in the number on the card	
• Counts the number of tens in the physical display (e.g., 1, 2, 3 tens)	
• Counts the number of ones in the physical display (e.g., 1, 2, 3 ones)	
• Counts by ten while pointing to each ten in the physical display (e.g., 10, 20, 30…)	
• Counts by one while pointing to each one in the physical display (e.g., 1, 2, 3…)	
• Counts the entire quantity first by ten and then by one: (e.g., 10, 20, 30, 31, 32, 33)	

Table 1. *Cont.*

Instructional element	Check "✓" if present
• Says, "33 equals 30 plus 3"	
• Writes "33 = 30 + 3"	
• Makes a circular gesture around the tens digit in the numeral and around the ones digit in the numeral	
• Repeats the equality "33 equals 30 plus 3"	

2.3.3. Length of instruction

The length of the instruction was extracted from the videos and from written notes for the 11 participants who were not recorded. Instruction lasted from 7.7 to 59.2 min (*M* = 23.0; *SD* = 10.7), with comparable mean instructional session times in the two mathematics ability groups: 22.8 and 23.7 min for TD and at-risk, respectively; and for the 2-digit and 3-digit interventions, 19.4 and 23.7 min, respectively. A one-way analysis of variance (ANOVA), based on a Type I error (α) probability of 0.05, was conducted to test for differences in instructional time length among conditions. Unsurprisingly, a statistically significant condition effect was revealed, $F(2, 89) = 28.35$, $p < 0.001$, eta-squared = 0.39. Familywise Type I error-controlled Fisher Least Significant Difference (LSD) comparisons (Levin et al., 1994) revealed that intervention times were longer with the attachable beads (*M* = 33.9; *SD* = 11.7) than with string beads, $p < 0.001$ (*M* = 19.0; *SD* = 6.1) or with pipe cleaners, $p < 0.001$ (*M* = 18.9; *SD* = 7.7), with no difference between the latter two conditions, $p = 0.96$. Conditions-related ANOVAs with $\alpha = 0.05$ and, when statistically significant, Fisher LSD pairwise comparisons, were conducted and are reported in the Results section analyses that follow.

2.4. Measures

2.4.1. Representation task

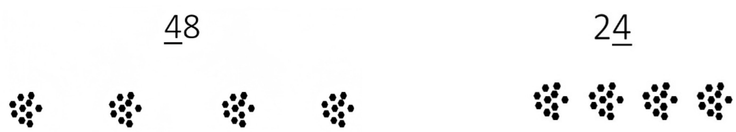
The researcher presented children with a numeral printed on an index card and asked them to use the manipulatives provided to represent it. There were four items: two 2-digit numerals and two 3-digit numerals. After constructing each representation, the children were asked to interpret how each digit in the numeral was represented in their displays. The items on the Representation Task were 19, 32, 127, and 208 at pretest and 18 and 138 at posttest. The task was videorecorded for subsequent data coding. Two pairs of trained researchers coded the complete data set and the percent agreement between the pairs was 94.4%. Additional details on task procedures can be found in Lafay et al. (2023).

The students' displays were classified in five categories (see Lafay et al., 2023), which were all used for the response analysis: (a) quantitatively accurate and place value-aligned responses (Optimal; e.g., representing 32 with 3 tens and 2 ones); (b) quantitatively accurate with incorrect place-value alignment (All-Ones; e.g., representing 32 with 32 ones); (c) quantitatively inaccurate with the appropriate object selected for each denomination but the incorrect number of groups (Grouping; e.g., representing 32 with 3 tens and 1 one or with 2 tens and 2 ones); (d) quantitatively inaccurate with no place-value alignment (Face-Value; e.g., representing 32 with 3 ones and 2 ones or with 3 tens and 2 tens); and (e) drawing of the numeral with the objects (Figural; e.g., representing 32 by "drawing" the digits 3 and 2 with the objects).

For the statistical analysis on the accuracy of the displays, one point was assigned to each Optimal response and 0 points for all responses in the other four categories. The Representation score was the proportion of correct responses across all items.

2.4.2. Picture place value test

The Picture Place Value Test (PicPVT; Osana and Lafay, 2019) was used to assess children's knowledge of the size of the denomination corresponding to each place in a numeral. The PicPVT consists of 20 randomly-ordered items, two of which are presented in **Figure 2**. On each item, children were shown a multidigit numeral with one of the digits underlined. An image consisting of sets of black hexagons was presented below the numeral. The number of sets in the image always matched the underlined digit, but the number of hexagons in each set was either 1, 10, or 100. Participants judged whether the image correctly matched the value of the underlined digit by verbally answering "yes" (**Figure 2a**) or "no" (**Figure 2b**).



(a) **(b)**

Figure 2. Sample PicPVT Items: **(a)** Sample "yes" item; **(b)** Sample "no" item.

Ten of the items on the PicPVT consisted of 2-digit numerals and ten items consisted of 3-digit numerals. All children completed the items in the same order. Half of the 2-digit items and half of the 3-digit items matched the corresponding image (e.g., 237 matched the image of two 2 sets of 100 hexagons). An isomorphic version of the pretest was administered at posttest. The PicPVT was administered using PowerPoint slides on a laptop computer that was placed in front of the participant. Each item was presented on a single slide. The PicPVT score was the proportion of correct responses.

2.4.3. Decomposition task

The Decomposition Task was designed by the authors to assess the children's syntactic knowledge of multidigit base-ten numerals. The task consisted of 20 items; the first 10 items presented 2-digit numerals, followed by 10 3-digit numerals. An isomorphic version of the pretest was administered at posttest. On each trial, children were shown a target numeral followed by four mathematical expressions from which they chose one that was equivalent to the value of the numeral. Each item contained the correct response, such as 20 + 4 for the numeral 24 or 9 + 10 + 500 for 519. The distractors were constructed using the same error categories across the items. For 2-digit items, two of the distractors were "same denomination" representations, which were incorrect representations in which the same denominational value, either one or ten, was used for each digit (e.g., 2 + 4 and 20 + 40 for 24). The third distractor was a "different denomination" representation, which was an incorrect representation that accorded a different denominational value to each digit (e.g., 2 + 40).

For 3-digit numerals (e.g., 519), the choices on each item were the correct response (e.g., 9 + 10 + 500) and three distractors. One of the distractors was a same denomination representation, using either ones, tens, or hundreds (e.g., 500 + 100 + 900), and the remaining two distractors were different denomination representations, which were incorrect representations using two or more distinct denominational values (e.g., 50 + 10 + 9; 500 + 1 + 90; 5 + 100 + 90; 1 + 900 + 50). On 16 of the 20 items, at least one of the options, whether the correct response or a distractor, was a representation that included further decomposition of one of the denominations (e.g., for 37, 10 + 10 + 10 + 7; for 324, 30 + 100 + 100 + 4). Choosing the same or different denomination representations would indicate difficulty associating the number of counts (i.e., the face values of one or more digits) with the correct unit being counted. For 519, for instance, selecting 50 + 10 + 9 would show difficulty understanding the value of the 5 digit. The Decomposition score was the proportion of correct answers.

2.4.4. Social validity

A questionnaire was administered to children to assess the extent to which they valued using the manipulatives during the intervention. Children's perceptions of the objects used during instruction were used to address possible alternative explanations for any condition effects. The questionnaire contained four items on a five-point Likert scale with a statement about the manipulatives they used while working with the researcher. The statements aimed to assess the degree to which the children: (a) enjoyed using the objects, (b) perceived them to be helpful for composing and decomposing numerals, (c) believed that the objects would be helpful and enjoyable when composing and decomposing numerals in class, and (d) wished that they had the objects in their class. The dependent variable was the mean rating across the four items, with a minimum score of 1 and a maximum of 5.

2.5. Data analysis

To answer the first research question, we conducted separate 2 (mathematics ability group: TD, at-risk) by 3 (instructional condition: attachable beads, string beads, pipe cleaners) analyses of variance (ANOVA) tests based on $\alpha = 0.05$, assessing gains from pretest to posttest on three dependent measures, namely Representation score, PicPVT score, and Decomposition score. As was mentioned earlier, when statistically significant, each ANOVA was followed up with Type I error-controlled Fisher LSD pairwise comparisons to test whether the amount of positive change was equivalent in the three instructional conditions. Tests of interactions in each of the three ANOVAs served to address the second research question about whether manipulatives that made place-value concepts visible would support positive change to a greater extent for at-risk children than for TD children. Finally, to address the third research question, we coded the types of displays provided by the children before and after instruction using the same coding rubric as in Lafay et al. (2023). The analysis focused on positive change in the children's application of place-value concepts in the displays as a function of manipulative type, particularly that of the children in the at-risk group, who were likely to make greater gains from pretest to posttest than those in the TD group.

3. Results

The means and standard deviations of the scores on the Representation Task, PicPVT, and Decomposition Task at pretest and posttest for the TD and at-risk mathematics ability groups by instructional condition are presented in **Table 2**.

Table 2. Means and standard deviations of pretest and posttest scores on the Representation Task, PicPVT, and Decomposition Task by mathematics ability group and instructional condition.

Mathematics ability group	Instructional condition	Pretest	Posttest
	Representation Task		
TD	Attachable beads	0.69 (0.32)	0.98 (0.10)
	String beads	0.93 (0.18)	0.92 (0.19)
	Pipe cleaners	0.89 (0.17)	0.95 (0.15)
At-risk	Attachable beads	0.37 (0.33)	0.83 (0.25)
	String beads	0.63 (0.27)	0.79 (0.40)
	Pipe cleaners	0.69 (0.35)	0.69 (0.37)
	PicPVT		
TD	Attachable beads	0.81 (0.18)	0.96 (0.10)
	String beads	0.79 (0.22)	0.92 (0.15)
	Pipe cleaners	0.86 (0.17)	0.89 (0.18)
At-risk	Attachable beads	0.64 (0.14)	0.71 (0.23)
	String beads	0.52 (0.13)	0.68 (0.20)
	Pipe cleaners	0.63 (0.23)	0.76 (0.26)
	Decomposition Task		
TD	Attachable beads	0.79 (0.25)	0.86 (0.20)
	String beads	0.76 (0.25)	0.83 (0.22)
	Pipe cleaners	0.81 (0.22)	0.89 (0.18)
At-risk	Attachable beads	0.49 (0.29)	0.48 (0.32)
	String beads	0.42 (0.27)	0.53 (0.27)
	Pipe cleaners	0.58 (0.31)	0.61 (0.34)

Note: In the TD group, one participant in the attachable beads condition and two participants in the pipe cleaners condition did not complete the Representation Task at posttest. Also in the TD group, one additional participant in the pipe cleaners condition did not complete the Decomposition Task at posttest.

3.1. Effects of conceptual transparency on performance

According to the data presented in **Table 2**, ceiling effects were observed in the TD group at pretest in the pipe cleaners and string beads conditions on the Representation Task. In contrast, the participants in the at-risk group showed room for improvement from pretest to posttest. To maximize the interpretability of the analysis, we chose to conduct a one-way ANOVA on the change scores of the Representation Task (posttest–pretest) for the at-risk group only. A statistically significant effect of condition was found, $F(2, 26) = 6.85$, $p = 0.004$, eta-squared = 0.35. Follow-up Fisher LSD comparisons revealed that, contrary to our expectations, children in the attachable beads condition improved their performance from pretest to posttest statistically more than those in the string beads condition, M changes = 0.46 and 0.16, respectively, $t(26) = 2.55$, $p = 0.02$, $d = 1.12$, and also more than in the pipe cleaners condition, M change = 0.00, $t(26) = 3.62$, $p = 0.001$, $d = 1.75$. The latter two conditions did not differ statistically, $t(26) = 1.38$, $p = 0.18$.

No ceiling effects were observed at pretest on either the PicPVT or Decomposition Task in either mathematics ability group or instructional condition. As such, we ran two separate 2 (mathematics ability group: TD, at-risk) × 3 (instructional condition: attachable, string, pipe cleaners) ANOVAs, one on PicPVT gain scores and the second on Decomposition Task gain scores. On the PicPVT, no statistically significant effects were found. The results showed that for the main effect of mathematics ability group, $F < 1$; for the main effect of instructional condition, $F < 1$; and for the interaction, $F(2, 111) = 1.06$, $p = 0.35$. Similarly, no effects were found on the Decomposition scores: for the main effect of mathematics ability group, $F < 1$; for the main effect of instructional condition, $F(2, 109) = 1.28$, $p = 0.28$; and for the interaction, $F < 1$.

A reviewer of an earlier version of this article was concerned that the preceding analyses were based on simple gain scores that did not take into account the children's initial levels of proficiency and that might be susceptible to regression-to-the-mean effects. Consequently, we conducted analyses of covariance on the three outcome measures, separately for the two mathematical ability groups. In those analyses, the pretest was the covariate to control for the children's differing initial levels and the posttest was the outcome measure. On all three outcome measures, the results were completely consistent with the just-reported results based on ANOVAs of the simple gain scores.

3.2. Social validity

Finally, the participants reported a high level of satisfaction using the manipulatives during the intervention across the whole sample ($M = 4.47$ on the 5-point scale; $SD = 0.65$). A 2 (mathematics ability group: TD, at-risk) × 3 (instructional condition: attachable, string, pipe cleaners) ANOVA revealed no main effects of instructional condition, $F(2, 110) = 1.19$, $p = 0.31$, or mathematics ability group, $F(1, 110) = 1.19$, $p = 0.28$, nor was the mathematics ability group by instructional condition interaction statistically significant, $F < 1$. These results suggest that the children in the attachable beads condition, who worked to manually construct groups of tens and hundreds, enjoyed using the beads and thought they were helpful when learning mathematics.

3.3. Response analysis

To explore the nature of the gains in the types of displays produced, a descriptive analysis was conducted using students' aggregated responses as the outcome measures. In each mathematics ability group at pretest, there were no differences in the proportions of any type of display between the pipe cleaners and string beads conditions (i.e., in the TD group, the maximum difference in percentage points between the pipe cleaners and string beads conditions was 4% in optimal displays; in the at-risk group, the maximum different in percentage points between the two conditions was 7% in optimal displays). Because there were also no differences between the same two conditions at posttest (i.e., in the TD group, the maximum difference was 3 percentage points in optimal and face-value responses; in the at-risk group, the maximum difference was 9 percentage points in face-value displays), we collapsed the data to form one condition called denominations.

The proportions of response types on the Representation Task within instructional condition (i.e., attachable beads, denominations) by mathematics ability group at pretest and posttest are presented in **Table 3**. **Figure 3** presents the change from pretest to posttest in display type proportions for the TD and at-risk groups by instructional condition (i.e., attachable beads vs. denominations).

Table 3. Proportions of response types on the Representation Task within mathematics ability group and instructional condition at pretest and posttest.

Mathematics ability group	**Instructional condition (*n*)**	**Optimal**	**All ones**	**Grouping**	**Face value**
		Pretest			
TD	Attachable beads (98)	61.20	26.50	7.10	5.10
	Denominations (250)	90.87	0.00	3.56	5.58
At-risk	Attachable beads (31)	38.70	25.80	9.70	25.80
	Denominations (77)	65.49	0.00	6.01	28.51
		Posttest			
TD	Attachable beads (47)	97.87	0.00	2.13	0.00
	Denominations (119)	93.26	0.00	5.04	1.69
At-risk	Attachable beads (17)[a]	82.35	5.88	0.00	11.76
	Denominations (34)	73.21	0.00	2.50	24.29

Note: Proportions were calculated by dividing the total number of instances of each response type by the total number of responses in each condition within mathematics ability group. [a] Row sum does not add up to 100.00 because of rounding. Missing data at pretest were 20 responses in the TD group using attachable beads and 6 responses in the denominations condition. In the at-risk group at pretest, missing data were 5 responses in the attachable beads condition and 3 responses in the denominations condition. At posttest, 3 responses were missing in the TD group when using attachable beads and 7 responses in the denominations condition. One datum was missing in the at-risk group using attachable beads and 6 responses were missing in the denominations condition.

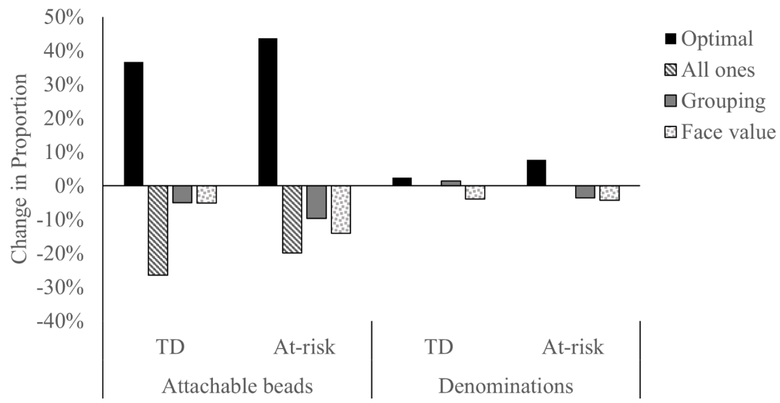


Figure 3. Change in proportions of response types on the Representation Task within mathematics ability group by instructional condition.

We conducted two analyses of display types. In the first, we examined group differences in the types of displays produced. Because of ceiling effects in the denominations condition in the TD group at pretest (see **Table 2**), we removed the condition in the first analysis. We thus compared the display types produced by the two mathematics ability groups in the attachable beads condition only. In the second analysis, we examined condition differences in the types of displays produced in the at-risk group only.

3.3.1. Mathematics ability group differences

Exploring the change in the types of displays between the TD and at-risk groups in the attachable beads condition, we first observed comparable gains in optimal responses in both mathematics ability groups (gains of 36.7 percentage points in the TD group and 43.7 points in the at-risk group). Further, the proportion of all error types decreased in both mathematics ability groups, with the largest reduction in the proportion of all-ones responses, which decreased by 26.5 percentage points in the TD group and 19.9 points in the at-risk group; in fact, all-ones responses disappeared entirely after the intervention in the TD group and went down to only 5.9% of all responses in the at-risk group.

3.3.2. Condition differences

The second analysis focused on condition differences (i.e., attachable beads vs. denominations) in the display types produced in the at-risk group. The gains in proportion of optimal responses were substantially larger in the attachable beads condition than in the denominations condition: 43.8%, from 38.7% to 82.5% of all responses in the attachable beads condition vs. 7.7%, 65.5% to 73.2% in the denominations condition. Further, there was an appreciable reduction in all-ones responses in the attachable beads condition (from 25.8% at pretest to 5.9% at posttest), but no change in the denominations condition. The latter observation is not meaningful, however, because there were no all-ones errors produced in the denominations condition at either pretest or posttest. Finally, a notable reduction was observed in the proportion of face-value errors (from 26% at pretest to 12% at posttest) in the attachable beads condition, amounting to slightly more than a 50% decrease, whereas the proportion was stable in the denominations condition.

In sum, the response analyses suggest that the improvement observed in the attachable beads condition was because children in both groups were better able to display quantities using base-ten units (i.e., tens and hundreds) at posttest than at pretest, as evidenced by a reduction in the proportion of all-ones responses after instruction. A similar conclusion can be drawn from the analysis of condition differences in the at-risk group: The reduction in all-ones and face-value responses demonstrated that children in the at-risk group gained knowledge of the unit structure of the base-ten system when they received instruction with attachable beads.

4. Discussion

In the present study, we investigated the impact of instruction with manipulatives that made the concepts of place value conceptually transparent on children's representations of quantities and place-value knowledge. In a previous study (Lafay et al., 2023), we found that when second graders represented multidigit quantities with manipulatives, they were more successful when they used manipulatives that were conceptually transparent, namely those that had the tens and hundreds denominations already attached than when they were required to construct the denominations themselves. Our objective in the present study was to test whether instruction with the same conceptually transparent manipulatives had a similar effect on children's learning about place value as their performance before instruction.

We delivered an instructional intervention to the same sample of children in Lafay et al. (2023) using the same manipulatives that were provided at pretest (i.e., attachable beads, string beads, and pipe cleaners). We assessed improvement in children's representations of quantities and knowledge of place-value concepts after instruction and compared the learning effects of TD children and those at risk for mathematics learning disabilities.

We initially had predicted that the children's improvement in representations and place-value knowledge across the sample would be greater when the children used conceptually transparent manipulatives during instruction, namely those that were already grouped in their denominations, than when they used manipulatives that were less conceptually transparent. Because of ceiling effects at pretest in the TD group, the test of conceptual transparency was conducted in the at-risk group only. We found the reverse of our expectations: The children at risk for mathematics learning disabilities who used attachable beads during instruction had statistically higher gain scores on the Representation Task than those who used manipulatives that were already arranged in their groupings of tens and hundreds. This finding suggests that the act of constructing tens and hundreds by attaching the beads into groups resulted in greater improvement than when the children worked with ready-made denominations during instruction, consistent with research stemming from embodiment theory (e.g., Abrahamson et al., 2020; Glenberg et al., 2013; Pouw et al., 2014). The finding is also in alignment with recent research showing that some actions are more helpful than others for fostering understanding of place-value concepts (e.g., Donovan and Alibali, 2021), perhaps because constructing the denominations "sparked" new ideas (Martin and Schwartz, 2005) and supported students' attempts to make sense of the target content through generative learning processes (Fiorella, 2023).

One explanation for the unanticipated findings is that the children who constructed the denominations with the attachable beads spent more time on base-ten tasks during instruction than did those in the other instructional conditions. Indeed, the children may have been encouraged to spend more time with the attachable beads when the beads were presented during the instruction. An alternative, and perhaps related, explanation is that those who received instruction with the attachable beads might have engaged in more meaningful encoding of the tens and hundreds units by constructing them than the children who simply selected the appropriate number of ready-made units to represent quantities (Castro-Alonso et al., 2024; Fiorella, 2021). Deep processing of stimuli can result in meaningful encoding, resulting in durable memory traces that result in enhanced performance (Craik and Tulving, 1975; Dinsmore and Alexander, 2012). Aligned with prior research on the Time-on-Task hypothesis (Godwin et al., 2021), these findings lead us to speculate that when children spend more time interacting with manipulatives during instruction, their attention to what the objects represent increases, resulting in more effective learning.

At the same time, depth of processing alone cannot be used to explain why the actions that took place with the attachable beads during instruction did not have a similar effect as the actions performed with the same manipulatives at pretest (Lafay et al., 2023). According to the theory of cognitive alignment (Laski and Siegler, 2014), the greater the alignment of the perceptual features of instructional materials with the desired mental representations, the greater the chances of effective learning. Here, before the mapping instruction was delivered, it is possible that the manipulatives that were conceptually transparent supported performance because the ready-made denominations mapped well, at least visually, to the place-value concepts that were required for constructing accurate displays of base-ten quantities.

As Laski and Siegler (2014) found, however, perceptual features are only as effective as the extent to which the information in the features is encoded. In one of a series of studies, the authors taught kindergarteners to move tokens on a number board either using a count-on strategy (i.e., with each roll of the die, counting on from the number on which their token was located) or a counting-from-1 strategy, which required them to count from 1 each time it was their turn to move the token. The authors found that the children who used a count-on strategy learned more about number magnitude than when they used a count-from-1 strategy and concluded that counting on enabled better encoding of the spatial configurations of the numbers on the board, which resulted in better learning. Similarly, here, the actions the children used to construct tens and hundreds denominations during mapping instruction possibly encouraged the meaningful encoding that was required for the place-value concepts to be learned. Thus, we speculate that at pretest, conceptual transparency was important because it was the only cue to the mapping between the perceptual features and the place-value concepts they were meant to illustrate, but the manipulative actions, coupled with explicit mapping instruction that gave meaning to the actions (Nathan et al., 2014) resulted in the greatest learning gains.

As was noted earlier, our findings are in line with the literature on embodied cognition, indicating that children can acquire mathematical understanding from acting on physical objects (Glenberg et al., 2013; Martin and Schwartz, 2005; Nathan, 2021). Relevant to the present study, Donovan and Alibali (2022) further argued that the influences of perceptual features of manipulatives on learning must be examined in light of the actions that the features afford. Perceptual features afford different actions, and if the actions are well aligned with the concepts being taught, it is those actions that are ultimately responsible for the learning that takes place (Castro-Alonso et al., 2024). In our study, it is possible that the ready-made denominations may have provided information about place-value concepts "for free" (Martin and Schwartz, 2005; Pouw et al., 2014), whereas when engaging in actions to construct the denominations themselves, children were provided with more opportunities to adapt their actions to support new interpretations of place-value concepts.

We also had specific predictions about the role of conceptual transparency on tasks that required the children to transfer newly learned place-value concepts to assessments with more abstract representations that incorporated pictures and symbols. However, no effect of conceptual transparency or of mathematics ability group was found on either transfer measure. One possible reason for the lack of transfer effects is that more instructional support was needed. Although explicit instruction with manipulatives that required the construction of the denominations was an effective support for learning, more focused self-explanation prompts from the instructor might have resulted in greater gains on the transfer tasks than the comparatively "open" self-explanation prompts that were provided (Berthold et al., 2009; Rittle-Johnson et al., 2017). These interpretations are consistent with the findings, but based on the present study they are only speculative rather than experimentally supported and should be examined in future research on place-value instruction.

Another possible explanation is that the transition from physical materials to more abstract representations of quantities was too large for children to achieve independently (e.g., Alfieri et al., 2013). The findings are consistent with research suggesting that transfer may be enhanced when instruction guides students more gradually toward formal representations of quantities (Donovan and Fyfe, 2022; Fyfe and Nathan, 2019; Sweller et al., 2019), particularly for children with mathematics difficulties (Fiorella, 2023; Witzel et al., 2008). In addition, the embodied nature of the instructional activities—especially those involving the attachable beads—may have situated learning within specific action contexts, potentially limiting transfer to more abstract representations. Theories of embodied cognition propose that mathematical ideas are grounded in action and perception (Glenberg et al., 2013; Nathan, 2024) and that transfer is closely tied to the contexts in which learning occurs (Nathan and Alibali, 2021). From this perspective, additional instructional experiences explicitly linking manipulatives to symbolic representations may support transfer. Because these mechanisms were not directly examined in the present study, future research is needed to test these theoretically-informed explanations.

4.1. Contributions and implications for practice

A contribution of the present research to the literature on the affordances of perceptual features of manipulatives is that the impact of external representations varies depending on the way in which students' knowledge is assessed. More specifically, manipulatives that make target concepts visible might be effective for performance outside of instruction because the objects' perceptual features are aligned with desired place-value concepts. In the context of mapping instruction, however, the children's encoding of those perceptual features varied depending on how they interacted with the manipulatives. In line with the theory of cognitive alignment (Laski and Siegler, 2014), the actions of constructing the denominations might have resulted in meaningful encoding of the relevant dimensions of the base-ten structure. Further investigations into these findings are needed, specifically to examine the nature of the encoding that may have taken place during instruction. Nevertheless, we recommend that teachers attend to what children are actually processing during instruction – in tandem with the materials that promote that processing – given its apparent importance for learning (Fyfe et al., 2014).

Our response analysis contributes to the literature in mathematical cognition (e.g., Cheung and Ansari, 2021; Landerl and Kölle, 2009; Rousselle and Noël, 2007) on the place-value knowledge of children who struggle in mathematics by providing a more detailed view of the nature of their difficulties and by extending this literature by examining the impact of instruction. Using more direct assessments of place-value than those often used (e.g., transcoding and number magnitude tasks), the preponderance of all-ones responses in the at-risk group confirms existing findings that children with mathematics difficulties struggle to understand that the locations of the digits in a numeral represent units of specific sizes. Further, after mapping instruction with the attachable beads, all-ones responses almost disappeared, which, together with the increase in optimal responses, suggests a shift from a unitary conception of number (i.e., all ones; Fuson, 1998) to conceptions that take base-ten units (e.g., tens, hundreds) into account. Both before and after instruction, children at risk for mathematics learning disabilities also demonstrated a larger proportion of face-value errors than did TD children. Face-value errors were described by Fuson (1998) as the "concatenated digit" view of numerals, which is the misconception that all the digits in a numeral represent counts of one (e.g., that 356 represents 3 ones + 5 ones + 6 ones). Using attachable beads during instruction was more beneficial for correcting the misconception than were the manipulatives already grouped in tens and hundreds denominations. An educational implication of these findings is that it may be beneficial for teachers to attend to the diversity in children's responses during place-value instruction and to adjust their instructional materials accordingly: Less perceptually salient manipulatives may in fact be more effective during instruction than manipulatives that appear on the surface to illustrate place-value concepts.

Finally, although additional evidence is needed to support the speculation that the action of constructing the base-ten units was responsible for the children's learning, the results of the present study support the idea that students who have difficulties learning mathematics would benefit from spending extended time working with manipulatives during instruction that supports the meaningful processing of the target concepts (Laski et al., 2015). By itself, the time spent by the present students building denominations during instruction may explain their learning, but we argue that how they spent their time, particularly in the context of verbal explanations and prompts (Nathan et al., 2014), afforded the processing that was required for the observed gains in place-value understanding. As well, the students expressed high satisfaction using all three types of manipulatives, suggesting that constructing the denominations did not frustrate or otherwise discourage them from interacting with the objects.

4.2. Strengths and limitations

A strength of the present study is our investigation of children's place-value knowledge by adopting more direct assessments of their conceptual knowledge. Aside from the contributions of these data to theoretical accounts of children's knowledge, we propose that the descriptions reported here are particularly useful for teachers in terms of attending to their students' processing during instruction and in supporting the development of more sophisticated conceptions of number. Another present-study strength lies in the manipulatives that we constructed to test specific hypotheses about conceptual transparency in mathematics manipulatives. A consideration for future research is to test the findings in actual classroom settings, which would increase the ecological validity of the study and generate additional recommendations for practice.

Further, the expected transfer effect was not supported by the data. It is possible that the instruction we provided did not effectively promote the shift from concrete to symbolic representations of quantities. Another potential explanation is that the present transfer measures did not represent valid assessments of denomination size and syntactic knowledge of base-ten numerals. The limitations of the present study should be considered in future studies on children's place-value learning.

Other limitations are methodological in nature. First, one way to strengthen the study further would be to increase the number of participants assigned to each instructional condition, and in particular, to recruit larger numbers of children who struggle in mathematics. Another limitation is the ceiling effects we observed among the TD children in their performance with the conceptually transparent manipulatives at pretest, which made it problematic to test the impact of conceptual transparency on the accuracy of their displays. Moreover, the observed ceiling effects restricted our ability to compare the performance of children at risk with that of typically developing children. Without such a measurement limitation, the data could have revealed finer distinctions between the ways in which the two populations learn about place value with manipulatives. In future studies, therefore, measures of children's place-value knowledge with a higher ceiling are needed to provide greater variability in second graders' performance. It is also probable that replicating the study with younger participants would reduce such concerns.

Another limitation of the present study is related to the rate of implementation fidelity, which fell within the moderate-to-high range. Nevertheless, we contend that the observed rate is acceptable because the instructional elements were coded with a high degree of precision, making minor deviations from the scripted protocol a natural consequence of interacting with young children. An additional limitation is that the instructional time was longer when the children used the attachable beads than when they used the manipulatives assigned to the other two conditions. Although this is not a surprising finding, it limits the conclusions that can be drawn about manipulative design alone, independent of how these materials might be used in the classroom. Despite this limitation, the finding that interacting with the attachable beads was associated with improved learning outcomes is consistent with embodied cognition perspectives on learning with instructional manipulatives (Pouw et al., 2014). Donovan and Alibali (2022) argued that, when choosing manipulatives for classroom use, teachers should consider not only the design of the objects, but also the ways in which their perceptual features afford actions that lead to desired outcomes. Accordingly, these findings suggest that conclusions about the design of manipulatives, in conjunction with the ways they would likely be used in the classroom, may have important educational implications.

5. Conclusion

In this study, we investigated the impact of mapping instruction with second graders using manipulatives that transparently illustrated place-value concepts, specifically that the size of the denominations depends on the position of the digits in a numeral and that the digits represent the number of counts of their corresponding denominations. We compared the learning of TD children to that of children at risk for mathematics learning disabilities. Contrary to our expectations, greater gains were observed when the at-risk children used manipulatives that did not make place-value concepts visible than when the objects' perceptual features displayed the tens and hundreds denominations. A response analysis revealed gains in place-value knowledge in the TD group using the manipulatives that required the same constructions. Those response-analysis data also revealed shifts in both groups from a less sophisticated understanding of number, such as unitary conceptions, to interpreting numerals in terms of their base-ten units. A major contribution of the present study to the literature is that, in the context of mapping instruction, young children using manipulatives to construct base-ten denominations improved their learning of place-value concepts. We speculate that the actions involved in those constructions provided opportunities for the children to encode the materials' salient features in ways that led to the greatest learning benefits.

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Data availability statement: The participants of this study did not give written consent for their data to be shared publicly. Because of ethical obligations, supporting data are not available. A portion of the data reported in this manuscript was presented at the 2019 Annual Meeting of the Mathematical Cognition and Learning Society in Ottawa, Canada.

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References

Abrahamson, D., Nathan, M. J., Williams-Pierce, C., et al. (2020). The future of embodied design for mathematics teaching and learning. Frontiers in Education, 5, 147. https://doi.org/10.3389/feduc.2020.00147

Alfieri, L., Nokes-Malach, T. J., & Schunn, C. D. (2013). Learning through case comparisons: A meta-analytic review. Educational Psychologist, 48(2), 87–113. https://doi.org/10.1080/00461520.2013.775712

Berthold, K., Eysink, T. H., & Renkl, A. (2009). Assisting self-explanation prompts are more effective than open prompts when learning with multiple representations. Instructional Science, 37(4), 345–363. https://doi.org/10.1007/s11251-008-9051-z

Bower, C. A., Mix, K. S., Yuan, L., et al. (2022). A network analysis of children's emerging place-value concepts. Psychological Science, 33(7), 1112–1127. https://doi.org/10.1177/09567976211070242

Bruner, J. S. (1966). Towards a Theory of Instruction. Cambridge, MA: Harvard University Press.

Byrge, L., Smith, L. B., & Mix, K. S. (2014). Beginnings of place value: How preschoolers write three-digit numbers. Child Development, 85(2), 437–443. https://doi.org/10.1111/cdev.12162

Byrne, E. M., Jensen, H., Thomsen, B. S., et al. (2023). Educational interventions involving physical manipulatives for improving children's learning and development: A scoping review. Review of Education, 11(2), e3400. https://doi.org/10.1002/rev3.3400

Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. Journal of Educational Psychology, 105(2), 380–400. https://doi.org/10.1037/a0031084

Carpenter, T. P., Franke, M. L., Jacobs, V. R., et al. (1997). A longitudinal study of invention and understanding in children's multidigit addition and subtraction. Journal for Research in Mathematics Education, 29(1), 3–20. https://doi.org/10.5951/jresematheduc.29.1.0003

Castro-Alonso, J. C., Ayres, P., Zhang, S., et al. (2024). Research avenues supporting embodied cognition in learning and instruction. Educational Psychology Review, 36(1), 10. https://doi.org/10.1007/s10648-024-09847-4

Chase, K., & Abrahamson, D. (2013). Rethinking transparency: Constructing meaning in a physical and digital design for algebra. In: Proceedings of the 12th International Conference on Interaction Design and Children; 24–27 June 2013; New York, NY, USA. pp. 475–478. https://doi.org/10.1145/2485760.2485834

Chen, Z. (1996). Children's analogical problem solving: The effects of superficial, structural, and procedural similarity. Journal of Experimental Child Psychology, 62(3), 410–431. https://doi.org/10.1006/jecp.1996.0037

Cheung, P., & Ansari, D. (2021). Cracking the code of place value: The relationship between place and value takes years to master. Developmental Psychology, 57(2), 227–240. https://doi.org/10.1037/dev0001145

Craik, F. I. M., & Tulving, E. (1975). Depth of processing and the retention of words in episodic memory. Journal of Experimental Psychology: General, 104(3), 268–294. https://doi.org/10.1037/0096-3445.104.3.268

Crooks, N. M., & Alibali, M. W. (2014). Defining and measuring conceptual knowledge in mathematics. Developmental Review, 34(4), 344–377. https://doi.org/10.1016/j.dr.2014.10.001

De Vos, T. (1992). Tempo Test Rekenen (TTR). Arithmetic Number Fact Test. Nijmegen: Berkhout.

Dennis, M. S., Sharp, E., Chovanes, J., et al. (2016). A meta–analysis of empirical research on teaching students with mathematics learning difficulties. Learning Disabilities Research & Practice, 31(3), 156–168. https://doi.org/10.1111/ldrp.12107

Dinsmore, D. L., & Alexander, P. A. (2012). A critical discussion of deep and surface processing: What it means, how it is measured, the role of context, and model specification. Educational Psychology Review, 24(4), 499–567. https://doi.org/10.1007/s10648-012-9198-7

Donovan, A. M., & Alibali, M. W. (2021). Toys or math tools: Do children's views of manipulatives affect their learning? Journal of Cognition and Development, 22(2), 281–304. https://doi.org/10.1080/15248372.2021.1890602

Donovan, A. M., & Alibali, M. W. (2022). Manipulatives and mathematics learning: The roles of perceptual and interactive features. In: Macrine, S. L., Fugate, J. M. B. (editors). Movement Matters: How Embodied Cognition Informs Teaching and Learning. MIT Press. pp. 147–161. https://doi.org/10.7551/mitpress/13593.003.0017

Donovan, A. M., & Fyfe, E. R. (2022). Connecting concrete objects and abstract symbols promotes children's place value knowledge. Educational Psychology, 42(8), 1008–1026. https://doi.org/10.1080/01443410.2022.2077915

English, L. D. (2004). Mathematical and Analogical Reasoning in Early Childhood. In: English, L. (editor). Mathematical and Analogical Reasoning of Young Learners. Routledge. pp. 1–22.

Fiorella, L. (2021). The embodiment principle in multimedia learning. In: Mayer, R. E., Fiorella, L. (editors). The Cambridge Handbook of Multimedia Learning, 3rd ed. Cambridge University Press. pp. 286–295. https://doi.org/10.1017/9781108894333.030

Fiorella, L. (2023). Making sense of generative learning. Educational Psychology Review, 35(2), 50. https://doi.org/10.1007/s10648-023-09769-7

Fuson, K. C. (1998). Pedagogical, mathematical, and real-world conceptual-support nets: A model for building children's multidigit domain knowledge. Mathematical Cognition, 4(2), 147–186. https://doi.org/10.1080/135467998387370

Fuson, K. C., & Briars, D. J. (1990). Using a base-ten blocks learning/teaching approach for first-and second-grade place-value and multidigit addition and subtraction. Journal for Research in Mathematics Education, 21(3), 180–206.

https://doi.org/10.5951/jresematheduc.21.3.0180

Fyfe, E. R., Alibali, M. W., & Nathan, M. J. (2017). The promise and pitfalls of making connections in mathematics. In: Proceedings of the 39th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education; 5–8 October 2017; Indianapolis, IN USA. pp. 717–724.

Fyfe, E. R., Borriello, G. A., & Merrick, M. (2023). A developmental perspective on feedback: How corrective feedback influences children's literacy, mathematics, and problem solving. Educational Psychologist, 58(3), 130–145. https://doi.org/10.1080/00461520.2022.2108426

Fyfe, E. R., DeCaro, M. S., & Rittle-Johnson, B. (2014). An alternative time for telling: When conceptual instruction prior to problem solving improves mathematical knowledge. British Journal of Educational Psychology, 84(3), 502–519. https://doi.org/10.1111/bjep.12035

Fyfe, E. R., & Nathan, M. J. (2019). Making "concreteness fading" more concrete as a theory of instruction for promoting transfer. Educational Review, 71(4), 403–422. https://doi.org/10.1080/00131911.2018.1424116

Glenberg, A. M., Witt, J. K., & Metcalfe, J. (2013). From the revolution to embodiment: 25 years of cognitive psychology. Perspectives on Psychological Science, 8(5), 573–585. https://doi.org/10.1177/1745691613498098

Godwin, K. E., Seltman, H., Almeda, M., et al. (2021). The elusive relationship between time on-task and learning: Not simply an issue of measurement. Educational Psychology, 41(4), 502–519. https://doi.org/10.1080/01443410.2021.1894324

Götze, D., & Baiker, A. (2021). Language-responsive support for multiplicative thinking as unitizing: Results of an intervention study in the second grade. ZDM – Mathematics Education, 53(2), 263–275. https://doi.org/10.1007/s11858-020-01206-1

Ho, C. S.-H., & Cheng, F. S.-F. (1997). Training in place-value concepts improves children's addition skills. Contemporary Educational Psychology, 22(4), 495–506. https://doi.org/10.1006/ceps.1997.0947

Jensen, S., Gasteiger, H., & Bruns, J. (2024). Place value and regrouping as helpful constructs to diagnose difficulties in understanding the place value system. Journal für Mathematik-Didaktik, 45(2), 11. https://doi.org/10.1007/s13138-024-00234-8

Kiru, E. W., Doabler, C. T., Sorrells, A. M., et al. (2018). A synthesis of technology-mediated mathematics interventions for students with or at risk for mathematics learning disabilities. Journal of Special Education Technology, 33(2), 111–123. https://doi.org/10.1177/0162643417745835

Kotovsky, L., & Gentner, D. (1996). Comparison and categorization in the development of relational similarity. Child Development, 67(6), 2797–2822. https://doi.org/10.1111/j.1467-8624.1996.tb01889.x

Ladel, S., & Kortenkamp, U. (2016). Development of a flexible understanding of place value. In: Meaney, T., Helenius, O., Johansson, M. L., et al. (editors). Mathematics Education in the Early Years. Springer. pp. 289–307.

Lafay, A., Osana, H. P., & Levin, J. R. (2023). Does conceptual transparency in manipulatives afford place-value understanding in children at risk for mathematics learning disabilities? Learning Disability Quarterly, 46(2), 92–105. https://doi.org/10.1177/07319487221124088

Lambert, K., & Moeller, K. (2019). Place-value computation in children with mathematics difficulties. Journal of Experimental Child Psychology, 178, 214–225. https://doi.org/10.1016/j.jecp.2018.09.008

Landerl, K., & Kölle, C. (2009). Typical and atypical development of basic numerical skills in elementary school. Journal of Experimental Child Psychology, 103(4), 546–565. https://doi.org/10.1016/j.jecp.2008.12.006

Laski, E. V., Jor'dan, J. R., Daoust, C., et al. (2015). What makes mathematics manipulatives effective? Lessons from cognitive science and Montessori education. SAGE Open, 5(2), 1–8. https://doi.org/10.1177/2158244015589588

Laski, E. V., & Siegler, R. S. (2014). Learning from number board games: You learn what you encode. Developmental Psychology, 50(3), 853–864. https://doi.org/10.1037/a0034321

Laski, E. V., Vasilyeva, M., & Schiffman, J. (2016). Longitudinal comparison of place-value and arithmetic knowledge in Montessori and non-Montessori students. Journal of Montessori Research, 2(1), 1–15.

Levin, J. R., Serlin, R. C., & Seaman, M. A. (1994). A controlled, powerful multiple-comparison strategy for several situations. Psychological Bulletin, 115(1), 153–159. https://doi.org/10.1037/0033-2909.115.1.153

Marley, S. C., & Carbonneau, K. J. (2015). How psychological research with instructional manipulatives can inform classroom learning. Scholarship of Teaching and Learning in Psychology, 1(4), 412–424. https://doi.org/10.1037/stl0000047

Martin, T., & Schwartz, D. L. (2005). Physically distributed learning: Adapting and reinterpreting physical environments in the development of fraction concepts. Cognitive Science, 29(4), 587–625. https://doi.org/10.1207/s15516709cog0000_15

McNeil, N. M., Uttal, D. H., Jarvin, L., et al. (2009). Should you show me the money? Concrete objects both hurt and help performance on mathematics problems. Learning and Instruction, 19(2), 171–184. https://doi.org/10.1016/j.learninstruc.2008.03.005

Mix, K. S., Bower, C. A., Hancock, G. R., et al. (2022). The development of place value concepts: Approximation before principles. Child Development, 93(3), 778–793. https://doi.org/10.1111/cdev.13724

Mix, K. S., Bower, C. A., Yuan, L., et al. (2023). Predictive relations between early place value understanding and multidigit calculation: approximate versus syntactic measures. Educational Psychology, 43(7), 795–813. https://doi.org/10.1080/01443410.2023.2254528

Mix, K. S., Smith, L. B., & Crespo, S. (2019). Leveraging relational learning mechanisms to improve place value instruction. In: Norton, A., Alibali, M. W. (editors). Constructing Number: Merging Perspectives from Psychology and Mathematics Education. Springer. pp. 87–121. https://doi.org/10.1007/978-3-030-00491-0_5

Moura, R., Wood, G., Pinheiro-Chagas, P., et al. (2013). Transcoding abilities in typical and atypical mathematics achievers: The role of working memory and procedural and lexical competencies. Journal of Experimental Child Psychology, 116(3), 707–727. https://doi.org/10.1016/j.jecp.2013.07.008

Nathan, M. J. (2021). Foundations of Embodied Learning: A Paradigm for Education. New York, NY: Routledge. https://doi.org/10.4324/9780429329098

Nathan, M. J. (2024). Advances in grounded and embodied mathematical reasoning: Theory, technology, and research methods. In: Shapiro, L., Spaulding, S. (editors). The Routledge Handbook of Embodied Cognition, 2nd ed. Routledge. pp. 260–278. https://doi.org/10.4324/9781003322511-28

Nathan, M. J., & Alibali, M. W. (2021). An embodied theory of transfer of mathematical learning. In: Hohensee, C., Lobato, J. (editors). Transfer of learning: Progressive Perspectives for Mathematics Education and Related Fields. Springer. pp. 27–58. https://doi.org/10.1007/978-3-030-65632-4_2

Nathan, M. J., Walkington, C., Boncoddo, R., et al. (2014). Actions speak louder with words: The roles of action and pedagogical language for grounding mathematical proof. Learning and Instruction, 33, 182–193. https://doi.org/10.1016/j.learninstruc.2014.07.001

O'Donnell, C. L. (2008). Defining, conceptualizing, and measuring fidelity of implementation and its relationship to outcomes in K–12 curriculum intervention research. Review of Educational Research, 78(1), 33–84. https://doi.org/3102/0034654307313793

Osana, H. P., Adrien, E., & Duponsel, N. (2017). Effects of instructional guidance and sequencing of manipulatives and written symbols on second graders' numeration knowledge. Education Sciences, 7(2), 52–74. https://doi.org/10.3390/educsci7020052

Osana, H. P., & Lafay, A. (2019). The relationship between analogical reasoning and second-graders' structural knowledge of number. In: Proceedings of the Forty-First Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education; 14–17 November 2019; St Louis, MO, USA. pp. 201–205.

Petersen, L. A., & McNeil, N. M. (2013). Effects of perceptually rich manipulatives on preschoolers' counting performance: Established knowledge counts. Child Development, 84(3), 1020–1033. https://doi.org/10.1111/cdev.12028

Pouw, W. T. J. L., Van Gog, T., & Paas, F. (2014). An embedded and embodied cognition review of instructional manipulatives. Educational Psychology Review, 26(1), 51–72. https://doi.org/10.1007/s10648-014-9255-5

Raven, J. C. (1977). Standard Progressive Matrices. Paris: Editions Scientifiques et Psychologiques.

Rittle-Johnson, B., Loehr, A. M., & Durkin, K. (2017). Promoting self-explanation to improve mathematics learning: A meta-analysis and instructional design principles. ZDM Mathematics Education, 49(4), 599–611. https://doi.org/10.1007/s11858-017-0834-z

Rojo, M. M., Knight, B., & Bryant, D. P. (2021). Teaching place value to students with learning disabilities in mathematics. Intervention in School and Clinic, 57(1), 32–40. https://doi.org/10.1177/1053451221994827

Ross, S. H. (1989). Parts, wholes, and place value: A developmental view. Arithmetic Teacher, 36(6), 47–51. https://doi.org/10.5951/AT.36.6.0047

Rousselle, L., & Noël, M. P. (2007). Basic numerical skills in children with mathematics learning disabilities: A comparison of symbolic vs non-symbolic number magnitude processing. Cognition, 102(3), 361–395. https://doi.org/10.1016/j.cognition.2006.01.005

Siegler, R. S., & Ramani, G. B. (2009). Playing linear number board games—but not circular ones—improves low-income preschoolers' numerical understanding. Journal of Educational Psychology, 101(3), 545–560. https://doi.org/10.1

037/a0014239

Sweller, J., van Merrienboer, J. J., & Paas, F. (2019). Cognitive architecture and instructional design: 20 years later. Educational Psychology Review, 31(2), 261–292. https://doi.org/10.1007/s10648-019-09465-5

Träff, U., Olsson, L., Östergren, R., et al. (2017). Heterogeneity of developmental dyscalculia: Cases with different deficit profiles. Frontiers in Psychology, 7(2000), 1–16. https://doi.org/10.3389/fpsyg.2016.02000

Uttal, D. H., Amaya, M., del Rosario Maita, M., et al. (2013). It works both ways: Transfer difficulties between manipulatives and written subtraction solutions. Child Development Research, 2013(1), 216367. https://doi.org/10.1155/2013/216367

Witzel, B. S., Riccomini, P. J., & Schneider, E. (2008). Implementing CRA with secondary students with learning disabilities in mathematics. Intervention in School and Clinic, 43(5), 270–276. https://doi.org/10.1177/1053451208314734

Yuan, L., Prather, R., Mix, K., et al. (2021). The first step to learning place value: A role for physical models? Frontiers in Education, 6, 683424. https://doi.org/10.3389/feduc.2021.683424