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Article

Joint estimation of leverage thresholds in state-dependent capital structure dynamics: An (S,s) target zone framework for African firms

Oluseun Paseda[1,2,*], Peter Ashade[3], Charles Manasseh[4], Felicia Olokoyo[5], J. Abiodun Oladimeji[6], Fatimah Abdulazeez[1], Idowu Bosede Fashola[1]

[1] Department of Banking and Finance, University of Ibadan, Ibadan 200005, Nigeria; fatimoabdulazeez@gmail.com (FA); idowuorogun@gmail.com (IBF)

[2] Babcock Business School, Babcock University, Ilishan-Remo 121103, Nigeria

[3] Department of Finance, Babcock Business School, Babcock University, Ilishan-Remo 121103, Nigeria; peterashade@gmail.com

[4] Department of Banking & Finance, University of Nigeria, Nsukka 410001, Nigeria; charssille@gmail.com

[5] Department of Finance, Covenant University, Ota 112233, Nigeria; felicitymy79@gmail.com

[6] Department of Finance, University of Lagos, Lagos 101017, Nigeria; joladimeji@unilag.edu.ng

*** Corresponding authors:** Oluseun Paseda, oa.paseda@ui.edu.ng or seunpash@gmail.com

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Abstract: This study develops a joint-estimation framework for analysing state-dependent capital structure dynamics in African listed firms within an (S,s) target-zone setting. Unlike conventional speed-of-adjustment models that assume continuous convergence towards an optimal leverage ratio, the proposed framework allows firms to tolerate leverage drift within an inaction band and undertake refinancing only when upper or lower leverage thresholds are breached. The model jointly estimates target leverage, refinancing thresholds, and inaction-band width using dynamic panel techniques, constrained nonlinear estimation, and a state-dependent refinancing hazard with bootstrap-robust inference. The African setting provides a particularly suitable environment for investigating such dynamics because firms face episodic access to capital markets, exchange-rate and inflation shocks, shallow debt markets, and heterogeneous institutional environments that elevate refinancing frictions and encourage discontinuous adjustment behaviour. Empirical analysis is conducted using an unbalanced panel of listed non-financial firms from fifteen African stock exchanges over 1999–2024. The results reveal pronounced (S,s) behaviour characterised by wide periods of leverage inertia punctuated by discrete refinancing interventions. Inaction bands widen with firm-level volatility, financing deficits, and market-timing opportunities, but narrow significantly with liquidity, creditor-rights protection, governance quality, and market depth. Additional evidence indicates that upper-brink adjustments occur substantially more frequently than lower-brink adjustments, suggesting that deleveraging pressures dominate leverage-increasing episodes in African markets. Extensive Monte Carlo experiments, including misspecification tests against linear adjustment processes, demonstrate superior bias, RMSE (Root Mean Square Error), coverage, and classification performance relative to conventional alternatives while avoiding spurious threshold detection. The findings highlight the importance of institutional quality and financial-market development in shaping corporate refinancing behaviour and provide new evidence that leverage adjustment in African firms is inherently nonlinear, state dependent, and threshold driven.

Keywords: dynamic corporate financing; refinancing hazard; dynamic panel econometrics; leverage adjustment asymmetry; partial-adjustment benchmarks; African listed firms; state-dependent decisions; inaction bands

1. Introduction

Corporate capital-structure dynamics are traditionally modeled through smooth partial-adjustment (SOA) mechanisms, where leverage converges gradually toward a latent target. While analytically convenient, this framework conflicts with empirical evidence from both developed and emerging economies, where leverage behavior is episodic, clustered, and state dependent. Firms often tolerate prolonged periods of leverage drift before undertaking discrete refinancing moves when conditions tighten or opportunities arise—behavior consistent with (S,s) policies rather than continuous adjustment. These dynamics are especially salient in African capital markets, where intermittent market access, macro-financial volatility, information frictions, and heterogeneous institutional environments jointly create conditions under which firms face non-convex adjustment costs and respond through brink-triggered interventions rather than smooth corrections.

The central problem this study addresses is the absence of an integrated empirical framework capable of jointly identifying (i) the latent target leverage, (ii) the upper and lower brink thresholds that trigger refinancing, and (iii) the inaction-band width that governs drift dynamics. Existing approaches either impose linear SOA structure or estimate thresholds in isolation, thereby missing the interaction between fundamentals, frictions, and institutional forces that drive state dependence [1].

Recent advances in computational finance and intelligent scientific computing have expanded the range of tools available for modeling complex nonlinear systems. These developments include AI-enhanced symbolic derivation frameworks, neural-symbolic reasoning systems, bilinear neural network architectures, residual-learning approaches, and fractional sub-equation neural networks that integrate machine learning with the discovery of governing mathematical structures. Such methods have demonstrated strong predictive capabilities in highly nonlinear environments and have increasingly been applied to differential equations, dynamic systems, and data-rich forecasting problems. However, despite their flexibility, many of these approaches function primarily as black-box prediction tools and often provide limited economic interpretability regarding the structural mechanisms underpinning observed behavior. In the context of capital structure dynamics, policymakers and corporate finance researchers require interpretable estimates of target leverage, refinancing thresholds, adjustment hazards, and institutional frictions rather than prediction alone. Consequently, this study adopts a model-assisted econometric framework that preserves theoretical interpretability while accommodating nonlinear and state-dependent adjustment dynamics. The proposed framework should therefore be viewed as complementary to emerging intelligent measurement approaches rather than a substitute for them.

The African corporate landscape provides a particularly compelling setting for examining state-dependent capital structure behavior. Unlike firms in developed markets with relatively continuous access to deep capital markets, African firms frequently experience episodic and uneven access to external financing due to market segmentation, liquidity shortages, and recurrent macroeconomic disruptions.

Exchange-rate depreciations, inflationary pressures, and periodic financial shocks often cause leverage to drift away from desired levels, while limited refinancing opportunities prevent immediate adjustment. Recent evidence further shows that inflation shocks, financial-cycle instability, monetary-policy transmission asymmetries, and persistent crisis conditions can generate nonlinear balance-sheet adjustments and leverage instability, particularly in emerging and frontier markets [2–6].

At the same time, weak creditor-rights enforcement, lengthy judicial processes, and heterogeneous institutional quality across countries increase the costs and uncertainty associated with debt restructuring and refinancing decisions. These frictions effectively raise the threshold at which firms are willing or able to intervene, producing prolonged periods of inaction punctuated by discrete leverage adjustments. Furthermore, the relative underdevelopment of corporate bond markets and the limited depth of domestic debt markets amplify adjustment costs, making frequent rebalancing economically unattractive [7, 8]. As a result, African firms are more likely to operate within tolerance bands and undertake refinancing only when leverage deviations become sufficiently large, thereby providing a natural environment in which (S,s) target-zone dynamics are expected to emerge.

Consequently, Africa offers a unique laboratory for investigating whether capital structure adjustments are governed by continuous convergence mechanisms or by threshold-triggered interventions. The combination of macroeconomic volatility, intermittent market access, institutional heterogeneity, and elevated refinancing frictions creates conditions under which leverage dynamics are likely to be inherently nonlinear and state dependent. This perspective is consistent with emerging evidence that financial frictions and leverage dynamics are strongly state dependent and vary across macroeconomic regimes [1,9–11]. This study exploits these features to develop and test a joint estimation framework for target leverage, brink thresholds, and inaction-band width within an African setting where (S,s) behavior should be most observable.

This paper bridges corporate-finance theory with dynamic-panel and nonlinear time-series econometrics by developing a model-assisted estimation system that incorporates correlated auxiliary information—firm fundamentals, market conditions, and institutional covariates—to sharpen inference on the three latent components of the (S,s) mechanism. The proposed framework integrates: a hierarchical dynamic-panel model for targets; nonlinear, inequality-constrained estimation of brink distances; an explicit width equation capturing frictional tolerance; and a state-dependent hazard describing refinancing probability as a function of brink proximity. Heavy-tailed noise and cross-sectional dependence are handled through bootstrap-valid inference.

The study proceeds as follows. Section 2 presents the structural framework; Section 3 details the empirical design and simulation architecture; Section 4 reports empirical estimates for African firms; Section 5 validates the estimator via Monte Carlo experiments; Section 6 concludes with implications for research, policy, and corporate practice.

2. Model-assisted statistical framework for state-dependent capital structure

2.1. Motivation: From smooth adjustment to state-dependent (S,s) policies

Canonical theories of capital structure—trade-off, pecking-order, agency and market-timing—predict systematic links between leverage and firm fundamentals, but the dynamics of adjustment often appear lumpy, episodic, and state-dependent rather than smooth [12–15]. Empirically, leverage exhibits long stretches of inertia interrupted by discrete jumps [16–20], especially around refinancing events, crises, or valuation windows—patterns that undermine linear speed-of-adjustment (SOA) models and support (S,s) behavior under fixed and non-convex adjustment costs [21–24]. Recent studies increasingly document nonlinear and threshold-driven adjustment mechanisms in financial and macroeconomic systems, reinforcing the notion that firm behaviour depends critically on prevailing states and constraints rather than constant adjustment speeds [4, 25, 26]. The African setting—with episodic market access, shallow corporate bond markets, currency and inflation shocks, and heterogeneous creditor protections—provides a natural environment for state-dependent corporate policies and brink-triggered refinancing [27].

We therefore embed leverage decisions in a target-zone (S,s) model in which firms allow leverage to drift within an inaction band and intervene only when upper or lower brink thresholds are hit. This paradigm is consistent with frictional dynamic optimization and the classic (S,s) results in inventory, cash-management, and pricing [1,28–31].

2.2. Economic structure and objects of inference

Let L_{it} denote leverage (book or market) for firm $i = 1, \dots, N$ and time $t = 1, \dots, T$. We posit a latent target $L^{\star}_{it}$, upper brink U_{it}, lower brink B_{it}, and width $W_{it} = U_{it} - B_{it} > 0$. Refinancing occurs if $L_{it} \geq U_{it}$ or $L_{it} \leq B_{it}$; otherwise, leverage drifts with cash flows, valuation, and financing availability. This specification follows the study's structural setup.

2.2.1. Model-assisted target equation

We use auxiliary (correlated) information to improve the projection of the latent target:

$$L^{\star}_{it} = x^{'}_{it}\beta + \mu_i + \tau_t + \varepsilon_{it}, \mathbb{E}[\varepsilon_{it} \mid x_{it}, \mu_i, \tau_t] = 0, \tag{1}$$

where x_{it} collects firm-level primitives (profitability, tangibility, liquidity, growth, payout, size/age), risk (earnings volatility), and macro-institutional auxiliaries (inflation, term spread, market depth, governance/creditor rights), while μ_i and τ_t capture unobserved heterogeneity and common shocks. The model-assisted gain arises because these auxiliaries shift targets in systematic ways predicted by corporate finance (trade-off, pecking-order, market-timing), thereby reducing projection error variance and sharpening downstream identification.

2.2.2. Brinks and inaction-band width

To respect positivity of brink distances and enable semi-elastic interpretation, we adopt exponential links:

$$U_{it} = L_{it}^{\star} + \exp(z_{it}^{'}\theta_U), B_{it} = L_{it}^{\star} - \exp(z_{it}^{'}\theta_B),$$

$$W_{it} = U_{it} - B_{it} = \exp(z_{it}^{'}\theta_U) + \exp(z_{it}^{'}\theta_B) > 0,$$

where z_{it} includes risk/friction proxies (EBITDA volatility, internal liquidity, financing deficit, payout), timing variables (valuation run-ups), and institutional depth (credit/GDP, disclosure/creditor-rights). This nests the economic content: volatility and financing deficits widen tolerance; liquidity and stronger institutions compress it. Theoretically, wider tolerance bands may emerge when costly external finance, liquidity constraints, or uncertain issuance conditions increase the value of waiting before adjustment [32–34].

2.2.3. State-dependent hazard of adjustment

Let distance to the nearest brink be

$$D_{it} = \min\{U_{it} - L_{it},\ L_{it} - B_{it}\}.$$

We specify a smooth threshold hazard of refinancing:

$$\Pr(\Delta L_{it} \neq 0 \mid \mathcal{F}_{t-1}) = \Lambda(\alpha_0 + \alpha_1 1\{D_{it} \leq \delta\} + \alpha_2 D_{it}^{-1}),$$

where $\Lambda(\cdot)$ is logit or probit, $\delta > 0$ defines a brink neighborhood, and D_{it}^{-1} captures sharply rising intervention propensity near thresholds. This provides a testable state-dependence distinct from linear SOA and aligns with event-clustered refinancing in empirical evidence (i.e., accords with the event-clustered refinancing observed empirically across African listed firms).

Corporate-finance mapping:

The structural elements of the hazard model align with multiple capital-structure theories:

Trade-off theory:

Tax shields and expected distress costs determine $L_{it}^{\star}$ and influence the size of the inaction band. Firms with large marginal tax benefits face tighter width W_{it} because deviating from target is costlier [14,35].

Pecking-order theory:

Internal financing shortfalls, cash-flow volatility and external-finance premia push firms toward upper brinks, widening W_{it}. Firms exhaust internal funds before issuing debt or equity [15,16].

Market-timing theory:

Favorable equity valuations increase equity issuance propensity, which shifts upper brinks outward, giving firms more room to deleverage without incurring adjustment costs [17]. This behavior is pronounced in African markets where valuation cycles and liquidity constraints amplify state-dependence.

Recent advances in leverage dynamics and financing under commitment constraints similarly predict nonlinear refinancing behaviour and adjustment asymmetries that are consistent with threshold-based intervention rules [1,10,32,36,37].

Agency theory:

Agency conflicts between managers and shareholders—or between shareholders and creditors—provide a fourth mechanism shaping brink behavior:

1. Managerial risk aversion and empire-building may cause managers to delay leverage reductions until L_{it} approaches the upper brink. They may prefer retaining financial slack [38,39].
2. Debt discipline can narrow the inaction band where leverage is used to constrain free-cash-flow overinvestment [39]. Firms facing severe managerial agency problems may choose a tighter band because the cost of drifting toward low leverage (reduced discipline) is high.
3. Creditor–shareholder agency conflicts (risk-shifting, under-investment) can shift either brink depending on covenant strength and creditor rights. In markets with weak creditor protection—characteristic of many African exchanges—the upper brink tends to be wider because firms can exploit debt without immediate refinancing pressure.
4. Information asymmetry—a core agency friction—interacts with the refinancing hazard. High opacity increases D_{it}^{-1}'s effect: firms wait longer to adjust due to adverse selection costs, then rebalance abruptly when brinks are breached.

Thus, agency theory explains why actual adjustments are lumpy, delayed, and brink-triggered, and why the hazard is steep: agency frictions encourage inaction until the expected agency cost exceeds the refinancing cost.

Integration:

Together, trade-off, pecking-order, market-timing, and agency mechanisms jointly determine:

- The position of brinks U_{it} and B_{it},
- The width W_{it} of tolerance for leverage drift,
- The shape of the refinancing hazard as firms approach brinks.

The Africa-wide evidence shows that these mechanisms are magnified where institutional quality is weak, market depth shallow, and macro-volatility high, making (S,s) behavior particularly salient in emerging and frontier markets.

2.3. Identification strategy

Identification exploits four complementary sources of variation:

1. Within-firm variation in $\mathrm{x}_{it}, \mathrm{z}_{it}$ and the two-way fixed effects (μ_i, τ_t) identify β in the target projection, with the auxiliaries providing model-assisted precision by shifting targets in predictable directions.
2. Inequality mapping from (θ_U, θ_B) to (U_{it}, B_{it}) under positivity constraints delivers sign-restricted brink distances that prevent pathological solutions and stabilize non-linear estimation.

3. Refinancing episodes are identified primarily using observed financing actions, including debt issuance, bond issuance, debt retirement, equity repurchases, and rights issues, where such information is available from firm disclosures and market records. These events provide an economically meaningful basis for identifying leverage-adjustment interventions independent of realized leverage changes. Deleveraging events provide information regarding proximity to the upper brink ($U_{it}-L_{it} \approx 0+$), while leverage-increasing transactions provide information regarding proximity to the lower brink ($L_{it}-B_{it} \approx 0+$). As a robustness exercise, changes in leverage ($\Delta L_{it} \neq 0$) are employed as a secondary event-identification criterion to assess the sensitivity of the results to alternative refinancing definitions.
4. Macro-institutional auxiliaries (market depth, creditor rights, disclosure quality) act as exogenous shifters of band width W_{it}, aiding separation between drift and trigger mechanisms across countries and time. This is central in African exchanges with evolving rules and liquidity.

Econometrically, the target projection can be estimated by FE or dynamic panel methods [40, 41] to accommodate persistence in leverage; the hazard by MLE for discrete choice; and brinks/width by constrained non-linear least squares (CNLS) or quasi-MLE under log-links [42, 43]. Cluster-robust covariance and wild bootstrap handle heavy tails typical in corporate data [44–46]. The use of dynamic panel estimators follows a long tradition in corporate-finance applications and remains particularly relevant where leverage persistence, endogenous financing choices, and unobserved heterogeneity coexist [47].

2.4. Joint estimation: "Two-nested-plus-one"

We propose a three-component system whose pieces exchange information to improve efficiency.

Stage A—Model-assisted target ($\widehat{L}^{\star}_{it}$):

Estimate

$$L^{\star}_{it} = x^{'}_{it}\beta + \mu_i + \tau_t + \varepsilon_{it} \tag{2}$$

by FE; if leverage is persistent, augment with lagged $L_{i,t-1}$ and estimate via System-GMM with internal instruments; report Hansen and AR(2) diagnostics to validate the moment set [40,41]. Save $\widehat{L}^{\star}_{it}$.

Stage B—Brinks and width via constrained nonlinear mapping:

Construct episode-wise proxies for brink distances using observed leverage levels (L_{it}) together with identified financing events, including debt issuance, debt retirement, equity repurchases, and rights offerings. Where event-level financing data are unavailable, leverage changes ($\Delta L_{it} \neq 0$) are used solely as a secondary robustness measure. This approach reduces concerns regarding circular identification between leverage movements and threshold estimation.

Estimate

$$U_{it} = \widehat{L}^{\star}_{it} + \exp\left(z^{'}_{it}\theta_U\right), B_{it} = \widehat{L}^{\star}_{it} - \exp\left(z^{'}_{it}\theta_B\right) \tag{3}$$

by CNLS with positivity constraints. To reduce sensitivity to local optima, estimation is initialized using a coarse grid-search procedure prior to application of the Levenberg-Marquardt optimization algorithm. This two-stage strategy improves convergence reliability and reduces dependence on arbitrary starting values.

The exponential links regularize the problem and deliver interpretable semi-elasticities for risk and institutional shifters. Compute $\widehat{W}_{it}$.

Stage C—State-dependent hazard:

Using $\widehat{D}_{it} = \min\{\widehat{U}_{it} - L_{it},\, L_{it} - \widehat{B}_{it}\}$,

Estimate the probability of a refinancing intervention conditional on observed financing actions. The primary hazard specification models the occurrence of debt issuance, debt retirement, equity repurchases, and rights issues. A supplementary specification based on $\Delta L_{it} \neq 0$ is reported as a robustness check.

$$\Pr\left(\Delta L_{it} \neq 0\right) = \Lambda\left(\alpha_0 + \alpha_1\, 1\left\{\widehat{D}_{it} \leq \delta\right\} + \alpha_2 \widehat{D}_{it}^{-1}\right) \tag{4}$$

by logit/probit MLE with firm-clustered standard errors. The slope around the brink neighborhood (δ) quantifies state-dependence and distinguishes (S,s) from linear SOA.

Joint covariance and inference. Obtain standard errors by (i) a block sandwich that stacks score contributions from A–C or (ii) a cluster-wild perturbation bootstrap (Rademacher weights at the firm level) that re-estimates A–C jointly, preserving cross-equation dependence [45,46].

2.5. Statistical properties and inference

2.5.1. Consistency and rates

Under standard conditions for FE and single-index GLM estimators (strict exogeneity of x_{it}, weak dependence in t, bounded moments) and regularity for smooth nonlinear maps $g(\cdot) = \exp(\cdot)$, the stacked estimator

$$(\widehat{\beta}, \widehat{\theta}_U, \widehat{\theta}_B, \widehat{\alpha})$$

is consistent as $N, T \to \infty$ with T moderate, and asymptotically normal with root-NT mixed rates. Width W_{it} inherits asymptotic normality via the delta method. The CNLS stage achieves standard parametric rates since the log-link is smooth and the positivity constraint binds only through the parameterization [42,48].

Recent machine-learning and high-dimensional approaches likewise emphasize the importance of robust inference under uncertainty, nonlinearity, and heterogeneous risk environments [49,50].

2.5.2. Robust covariance and heavy tails

Corporate outcomes exhibit heteroskedasticity and heavy tails. We therefore recommend firm-clustered (Arellano) covariance for A and C, and outer-product or Godambe matrices for B, along with a firm-clustered wild bootstrap to form percentile-t confidence intervals for non-linear functionals (e.g., width semi-elasticities). This bootstrap is first-order valid under heteroskedasticity and within-cluster dependence [45,46,51].

2.5.3. Specification checks and sensitivity

- Target step: test for serial correlation in residuals and weak instruments in Sys-GMM via Hansen–Sargan and AR(2).
- Brink step: compare log-link CNLS to piecewise-linear and log-linear alternatives; likelihood/AIC/BIC comparisons favor exponential mapping.
- Hazard step: ROC/marginal effects at brink deciles quantify state-dependence; sharp rises near δ corroborate (S,s) dynamics.

An essential component of the empirical strategy involves a series of specification checks and sensitivity diagnostics conducted at each stage of the model-assisted estimation framework. For the target-leverage step, diagnostic attention focuses on the statistical soundness of the dynamic panel estimators. Here, residual serial correlation and the validity of internal instruments are examined using the Hansen–Sargan over-identification test and the Arellano–Bond AR(2) statistic, ensuring that the System-GMM procedure is not compromised by weak instruments or misspecified error structures.

In the brink-estimation step, the stability and suitability of the exponential mapping are assessed by comparing the log-link CNLS specification against piecewise-linear and log-linear alternatives. Information-criterion comparisons—likelihood, AIC, and BIC—systematically favor the exponential formulation, reaffirming its empirical superiority within the target-zone structure of African firms.

The hazard-model step undergoes its own layer of validation. Here, ROC curves and marginal-effect analyses evaluated across brink-distance deciles provide direct visual and statistical confirmation of state dependence. The sharp rise in the probability of refinancing interventions as firms enter the δ-neighborhood of the brink offers compelling evidence consistent with (S,s) dynamics and strongly at odds with the smooth, continuous adjustment implied by classical SOA models.

2.6. Economic interpretation of parameters

The economic meaning of the estimated parameters follows naturally from the structure of the model. The target-leverage semi-elasticities (β) translate firm-level fundamentals—tangibility, profitability, liquidity, earnings volatility, payout intensity—together with macro-institutional factors such as inflation, the term structure, market depth, and governance quality, into long-run preferred leverage levels. Their signs and magnitudes echo core predictions of trade-off and pecking-order theories, reinforcing the idea that firms align target leverage with both cost-benefit considerations and financing-hierarchy frictions.

The brink semi-elasticities (θ_U, θ_B) operate on a different margin, capturing how risk exposures, timing incentives, and financing frictions shift the upper and lower trigger points for refinancing. The estimates show that heightened volatility and financing deficits tend to push the upper brink outward, expanding firms' tolerance for drift, while greater liquidity compresses both brinks, making firms more likely to intervene sooner. These results mirror theoretical expectations that firms buffer against risk by widening inaction regions but react quickly when internal liquidity renders adjustment less costly.

The elasticities of the inaction-band width summarize the structural determinants of firms' tolerance for leverage misalignment. These width parameters are of particular policy relevance, as they provide a quantitative measure of the frictions that inhibit timely rebalancing. In the African application, the pattern is especially revealing: narrower bands are systematically associated with stronger institutions and deeper financial markets, reinforcing the notion that well-functioning market infrastructure lowers refinancing frictions and encourages earlier intervention.

Finally, the slope of the hazard function offers a compact summarizing statistic of state-dependence in adjustment behavior. A steep hazard gradient near the brink demarcates an empirical boundary between (S,s) policies—where adjustments occur only when pressures accumulate sufficiently—and the SOA paradigm, where adjustments unfold smoothly and continuously.

2.7. Relation to time-series/panel econometrics and corporate finance

This model-assisted system unifies three strands:

Corporate finance: It nests trade-off (tax vs. distress), pecking-order (internal funds hierarchy), and market-timing (valuation windows) within a frictional control model that delivers endogenous inaction bands and brink triggers [14, 15, 18, 21, 35]. This unification is consistent with the influence of taxes and distress costs (trade-off), information asymmetry and financing hierarchies (pecking-order), and valuation cycles (market-timing), as well as the observed episodic refinancing documented in the literature.

Dynamic panel econometrics: It leverages FE/Sys-GMM targets with internal instrumentation and robust diagnostics, conforms to nonlinear M-estimation for brinks/width, and uses discrete-choice hazard modeling for state-dependent adjustment [40–42, 48].

Statistics of complex systems: The use of auxiliary (correlated) information and structural positivity constraints is quintessential model-assisted statistics, yielding variance reduction and stability in non-linear estimation under heteroskedastic heavy-tail noise; joint inference relies on sandwich/Godambe and bootstrap machinery suited to clustered panels [43–46].

2.8. Practical guidance for implementation (replicable recipe)

A transparent implementation pathway facilitates replication. The pre-processing stage includes winsorizing extreme values, defining leverage measures (book, market, net), calculating rolling volatility and financing deficits, constructing timing proxies, and merging macro-institutional datasets.

Stage A estimates the model-assisted target using FE or System-GMM (FE or Sys-GMM for $\widehat{L}^{\star}_{it}$) while reporting Hansen and AR(2) diagnostics.

Stage B applies CNLS with exponential links and positivity constraints—initialized via log-linear OLS—to estimate brink distances and compute the inaction-band width (initialize by log-linear OLS; compute $\widehat{W}_{it}$).

Stage C fits the logit or probit hazard model using estimated brink distances and reports marginal effects evaluated close to the brink (Logit/probit hazard with $\widehat{D}_{it}$; report marginal effects near brinks).

Inference relies on cluster-robust variance estimators and a firm-clustered wild bootstrap to correctly approximate uncertainty in nonlinear functionals and across equations. Sensitivity checks benchmark alternative link functions and leverage measures.

2.9. Why model-assisted? Gains over SOA and unconstrained thresholds

Relative to stand-alone SOA or unconstrained threshold fits, the macro-institutional auxiliaries and structural positivity (log-links) yield: (i) variance reduction in $\widehat{L}^{\star}_{it}$; (ii) stable identification of brink distances and width; and (iii) sharper hazard discrimination near thresholds. The simulation and empirical sections show material improvements in MSE, coverage, and interpretability—particularly for African markets where institutions and liquidity are decisive shifters of band width and brink locations.

2.10. Intelligent and data-driven approaches to nonlinear financial modeling

Artificial intelligence and machine learning in finance:

Recent advances in artificial intelligence (AI) and machine learning have transformed financial research by enabling the analysis of highly complex, nonlinear, and high-dimensional datasets that often challenge conventional econometric techniques. Machine-learning algorithms are increasingly employed for credit-risk assessment, asset-pricing prediction, portfolio optimization, corporate forecasting, and financial distress modelling because of their ability to uncover latent patterns and nonlinear interactions within large datasets. As financial systems become more interconnected and data-rich, AI-driven methods have emerged as valuable tools for enhancing predictive performance, identifying hidden relationships, and supporting data-informed decision-making under uncertainty.

Neural-network approaches to nonlinear dynamic systems:

Within the broader AI ecosystem, neural-network architectures have attracted substantial attention for modelling nonlinear dynamic systems characterized by complex interactions and state-dependent behaviour. Recent developments such as bilinear neural networks, bilinear residual networks, and fractional sub-equation neural networks (fSENNs) have demonstrated strong capabilities in approximating intricate functional relationships and solving nonlinear mathematical systems. These approaches offer considerable flexibility in capturing nonlinear adjustment processes, regime shifts, and dynamic transitions that may be difficult to represent within traditional linear modelling frameworks, thereby expanding the methodological frontier for empirical analyses involving complex economic and financial phenomena.

Symbolic artificial intelligence and neuro-symbolic reasoning:

A parallel stream of research has focused on symbolic artificial intelligence and neuro-symbolic reasoning frameworks that combine machine-learning capabilities with formal mathematical and logical representations. Advances in AI-enhanced mathematical derivation, multi-modal neuro-symbolic reasoning algorithms, and neural-network-based symbolic computation have sought to improve model interpretability while preserving the flexibility of modern learning systems. Unlike purely data-driven approaches, neuro-symbolic methods attempt to discover underlying structural relationships, generate interpretable mathematical expressions, and incorporate domain knowledge into the modelling process, thereby addressing concerns regarding the opacity and explainability of conventional black-box algorithms.

Positioning of the present study:

While intelligent measurement approaches have demonstrated considerable success in solving high-dimensional nonlinear systems, the objective of the present study differs fundamentally from a purely predictive exercise. Rather than maximizing forecasting accuracy, this research seeks to recover economically interpretable estimates of target leverage, refinancing thresholds, inaction-band width, and state-dependent refinancing hazards within an integrated corporate-finance framework. Consequently, the proposed model-assisted estimation system occupies an intermediate position between traditional structural econometrics and emerging AI-driven modelling approaches by retaining theoretical transparency, policy relevance, and economic interpretability while simultaneously accommodating nonlinear, threshold-driven, and state-dependent capital-structure dynamics.

3. Data, design, and simulation architecture

This section describes the empirical and computational foundations supporting the model-assisted estimation framework. We integrate the data architecture and simulation study into a single methodological core, thereby linking the structural properties of the African firm-level panel to the Monte Carlo experiments that validate the estimator's performance.

3.1. Data architecture

We construct an unbalanced panel of African listed non-financial firms over the period

$$t = 1999, \ldots, 2024,$$

covering approximately fifteen exchanges—including South Africa, Nigeria, Kenya, Egypt, Morocco, Ghana, Botswana, Namibia, Rwanda, Côte d'Ivoire, Mauritius, Tanzania, Uganda, Zambia, and Tunisia. Firm-level accounting and market variables are sourced from Refinitiv, Bloomberg, and exchange filings. After harmonization, variables are winsorized at standard cutoffs, and leverage is computed in both book and market forms.

To capture the frictions that govern state-dependent dynamics, we merge the panel with macro-institutional auxiliaries comprising:

- Inflation and term-spread dynamics,
- Foreign-exchange volatility,
- Equity-market depth and turnover,
- Governance and creditor-rights indices.

These auxiliaries play a structural role in the estimation of inaction-band width and hazard steepness.

Firm-specific volatility is proxied using rolling EBITDA variability; timing variables are constructed from valuation run-ups; and financing deficits follow standard investment-cashflow-gap measures.

Macroeconomic variables are particularly important because inflation, monetary-policy conditions, exchange-rate instability, and financial-cycle fluctuations have been shown to exert significant effects on leverage decisions and broader balance-sheet dynamics [3,52–54].

Although the theoretical contribution is general, the African context is uniquely suited to evaluating state-dependence in leverage behavior because of its combination of macroeconomic volatility, institutional heterogeneity, liquidity constraints, and episodic market access. These features generate rich variation in brink distances, hazard slopes, and refinancing incidence.

3.2. Simulation study

To assess the small-sample reliability and structural identification of the joint estimator, we conduct Monte Carlo experiments under three canonical data-generating processes (DGPs). Each DGP is designed to isolate a different dynamic mechanism implied by the theoretical model.

3.2.1. DGP S (true (S, s) model)

The data follow a genuine target-zone structure**:**

$$U_{it} = L^{\star}_{it} + \exp(z^{'}_{it}\theta_U), B_{it} = L^{\star}_{it} - \exp(z^{'}_{it}\theta_B),$$

with refinancing occurring when

$$L_{it} \notin [B_{it}, U_{it}],$$

and the hazard of adjustment rising sharply as distance to the nearest brink,

$$D_{it} = \min\{U_{it} - L_{it},\ L_{it} - B_{it}\},$$

approaches zero. Errors ε_{it} follow heavy-tailed t_5 distributions to emulate empirical kurtosis.

The simulation architecture is also motivated by recent stochastic and scenario-based modelling approaches used to analyse financial stability, credit risk, and macro-financial shocks [55,56].

3.2.2. DGP M (mixed regime model)

This process alternates between periods of smooth SOA dynamics and distinct brink-triggered episodes. It mimics empirical leverage patterns in calm versus stressed macro-financial regimes, capturing switching behavior not detectable under purely linear specifications.

3.2.3. DGP L (linear SOA model)

A conventional partial-adjustment model provides a falsification environment:

$$L_{it} - L_{i,t-1} = \lambda(L^{\star}_{it} - L_{i,t-1}) + u_{it}.$$

This DGP evaluates whether the model-assisted estimator collapses appropriately to the SOA benchmark when no threshold behavior is present.

3.2.4. DGP X (model misspecification test)

To assess whether the proposed framework spuriously detects threshold behavior when none exists, an additional misspecification experiment is conducted. Specifically, leverage dynamics are generated from a conventional linear partial-adjustment process:

$$L_{it} - L_{it-1} = \lambda(L^{*}_{it} - L_{it-1}) + u_{it}$$

where no underlying threshold structure exists. The proposed (S,s) estimation framework is then applied to the simulated data.

Three diagnostic measures are evaluated:

- False threshold detection rate.
- Width estimation bias.
- Hazard-significance frequency.

The objective is to determine whether the estimator incorrectly imposes nonlinear threshold dynamics on fundamentally linear processes.

3.3. Experimental design

Simulations consider:

$$N \in \{200, 500\}, T \in \{10, 20\},$$

with covariates drawn to mimic the empirical correlation matrices of African firms. Macro-institutional variables act as exogenous shifters of band width and hazard curvature, mirroring their empirical role. Heavy-tailed shocks (scaled t_5) ensure robustness under non-Gaussian noise, and refinancing episodes emerge endogenously from the latent structural model.

3.4. Performance metrics

For each DGP, we compute the bias, RMSE, and empirical size/power for:

- The target-semi-elasticity vector β,
- Brink parameters θ_U and θ_B,
- Derived band-width elasticities,
- Hazard parameters $\alpha_0, \alpha_1, \alpha_2$.

We benchmark the estimator against:

1. A standard SOA estimator,
2. An unconstrained threshold model replacing exp$(\cdot)$ with identity,

3. A no-auxiliaries model, excluding macro-institutional shifters.

This allows us to separate the contributions of (i) positivity constraints, (ii) auxiliary-information channels, and (iii) the joint estimator's treatment of nonlinear structure.

3.5. Simulation results

Two robust findings emerge.

(i) Performance under nonlinear DGPs (S and M)
Under true and mixed (S, s) environments, the joint model-assisted estimator uniformly dominates alternatives in RMSE, bias, and confidence-interval coverage. The SOA estimator systematically exhibits pseudo-convergence, delivering near-unity adjustment speeds that mistakenly interpret within-band drift as continuous convergence to target.
The exponential positivity constraint significantly reduces the variance of estimated brink distances and produces well-behaved width estimates.
Including macro-institutional auxiliaries improves the identification of state-dependence: the hazard slope steepens appropriately near brinks, and the classifier correctly differentiates adjustment episodes from passive drift.

(ii) Performance under the linear DGP (L)
When the true data-generating process is linear, the model-assisted system nests the SOA case correctly. Inferred width collapses toward zero, brink parameters degenerate appropriately, and the hazard function becomes flat. No spurious threshold behavior is detected, demonstrating robustness and correct specification under linearity.

3.6. Synthesis

The integration of a rich African dataset with controlled simulations provides strong support for the joint estimator's identification, stability, and interpretability. The data environment generates genuine state-dependent leverage dynamics, while the simulations confirm that the estimation strategy recovers structural features across nonlinear, mixed, and linear regimes. Together, they establish a coherent empirical foundation for the model-assisted framework developed in the preceding sections.

4. Empirical results: African listed firms, 1999–2024

This section reports the structural empirical estimates obtained from the model-assisted target-zone system. The results confirm the presence of pronounced state dependence in corporate leverage policies across African markets and demonstrate the empirical value of macro-institutional auxiliaries for identifying brink thresholds, width parameters, and hazard curvature.

4.1. Target-leverage estimates

We first estimate the model-assisted target

$$\widehat{L}^{\star}_{it} = x'_{it}\widehat{\beta} + \widehat{\mu}_i + \widehat{\tau}_t,$$

using FE and System-GMM specifications. **Table 1** summarizes the coefficient estimates.

Table 1. Target leverage equation (FE/system-GMM).

Variable	Coefficient (β)	Std. error	z-stat	*p*-value	Interpretation
Tangibility	0.118	0.028	4.21	0.000	Collateral ↑ → Target ↑
Profitability	−0.062	0.015	−4.13	0.000	Internal funds ↓ debt need
Liquidity	−0.087	0.021	−4.14	0.000	Liquidity substitutes for leverage
Volatility	−0.044	0.017	−2.59	0.009	Risk reduces target leverage
Size (log assets)	0.071	0.013	5.46	0.000	Scale ↑ debt capacity
Payout ratio	−0.031	0.014	−2.17	0.030	Dividends reduce debt demand
Inflation	0.009	0.004	2.25	0.024	Nominal leverage tolerance rises. The positive role of inflation is broadly consistent with recent evidence showing that inflation shocks can materially influence asset valuations, financing conditions, and leverage behaviour across economies [3]
Term spread	0.014	0.006	2.33	0.020	Cheaper long-term funding
Governance index	0.021	0.009	2.33	0.020	Institutions strengthen target stability
Creditor rights	0.036	0.012	3.00	0.003	Higher debt capacity

Note: Dependent variable: $\widehat{L}^{\star}_{it}$. AR(2) *p*-value: 0.221. Hansen *p*-value: 0.412. These diagnostics indicate valid instruments and no second-order serial correlation, aligning with the requirements for reliable System-GMM estimation.

4.2. Brink parameters and inaction-band width

The structural brinks

$$\widehat{U}_{it} = \widehat{L}^{\star}_{it} + \exp(z'_{it}\widehat{\theta}_U), \widehat{B}_{it} = \widehat{L}^{\star}_{it} - \exp(z'_{it}\widehat{\theta}_B),$$

produce the semi-elasticities reported in **Table 2**.

Table 2. Brink semi-elasticities (θ_U and θ_B).

Variable	$\widehat{\theta_U}$	SE	Sig.	$\widehat{\theta_B}$	SE	Sig.
Volatility	0.412	0.091	***	0.131	0.044	**
Financing deficit	0.355	0.073	***	0.097	0.038	**
Liquidity	−0.228	0.062	***	−0.301	0.071	***
Tangibility	0.042	0.018	*	−0.096	0.024	**
Timing variable	0.276	0.057	***	0.033	0.012	**
Creditor rights	−0.144	0.049	***	−0.162	0.052	***
Market depth	−0.201	0.065	***	−0.187	0.067	***

Note: *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

The upper brink expands with volatility, financing deficits, and timing opportunities—consistent with precautionary and market-timing motives.

The lower brink contracts substantially in the presence of liquidity and tangibility, reflecting lower cost of levering up.

The inaction-band width

$$W_{it} = \exp(z'_{it}\widehat{\theta}_U) + \exp(z'_{it}\widehat{\theta}_B)$$

exhibits the elasticities shown in **Table 3**.

Table 3. Width elasticities (|∂W/W|).

Driver	Elasticity	Interpretation
Volatility	+0.218	Precautionary widening
Timing	+0.144	Option-value widening
Liquidity	−0.167	Lower adjustment costs
Marginal tax rate	−0.091	Incentive to align with target
Governance	−0.132	Institutional compression
Market depth	−0.188	Liquidity improves rebalancing

Note: Reported estimates represent marginal effects (∂W/∂z) rather than log-log elasticities. They measure the absolute change in inaction-band width associated with a one-unit change in the explanatory variable while holding other determinants constant.

The observed narrowing of inaction-band width reflects two distinct mechanisms. First, active compression arises from economic forces that directly reduce refinancing frictions, including improved liquidity, deeper capital markets, and stronger creditor-rights protection. Under these conditions firms can undertake leverage adjustments more readily, reducing the tolerance for leverage drift.

Second, passive compression emerges from regulatory and institutional constraints such as prudential leverage requirements, stock-exchange listing standards, and supervisory oversight mechanisms that effectively force firms to maintain leverage within narrower bounds. Distinguishing these mechanisms is important because narrower bands do not necessarily imply lower transaction costs.

To explore this distinction, subsample analyses are conducted based on market-depth indicators and regulatory-strength proxies. Results indicate that active compression effects dominate in financially developed markets, while passive compression plays a larger role in more heavily regulated environments (**Table 4**).

Table 4. Active versus passive width compression.

Compression channel	Primary drivers	Width effect
Active Compression	Liquidity, market depth, creditor rights	Significant narrowing
Passive Compression	Regulatory limits, prudential oversight, exchange regulation	Moderate narrowing

4.3. State-dependent adjustment hazard

The estimated hazard function

$$\Pr(\Delta L_{it} \neq 0) = \Lambda(\widehat{\alpha}_0 + \widehat{\alpha}_1 1\{D_{it} \leq \delta\} + \widehat{\alpha}_2 D_{it}^{-1})$$

is reported in **Table 5**.

Table 5. State-dependent refinancing hazard.

Parameter	Estimate	SE	*p*-value	Interpretation
α_0	−3.612	0.502	0.000	Adjustments rare far from brink
α_1	1.447	0.211	0.000	Large jump in hazard near brink
α_2	0.823	0.147	0.000	Strong inverse-distance curvature

Note: Marginal effect ($D \leq \delta$): +0.27. ROC AUC = 0.82, indicating excellent classification.

The results reveal a steep, nonlinear rise in adjustment probability within a narrow 1–2 p.p. corridor around the brinks—a hallmark of (S,s) behavior. The pronounced nonlinearity is consistent with recent studies documenting state-dependent responses to financial and macroeconomic shocks in both firm-level and aggregate settings [4,9].

4.4. Cross-country institutional heterogeneity

Table 6's patterns track institutional depth and are highly consistent with the structural prediction that creditor-rights protection and market liquidity reduce the tolerance for misalignment, thereby narrowing the inaction zone.

Table 6. Institutional determinants of width.

Country group	Avg width (p.p.)	Width elasticity wrt governance	Interpretation
South Africa, Mauritius	2.8	−0.31	Strong institutions → tight bands
Egypt, Morocco, Kenya	4.9	−0.17	Moderate compression
Nigeria, Ghana, Tanzania	6.5	−0.11	Higher frictions
Zambia, Uganda, Côte d'Ivoire	7.4	−0.08	Weakest institutions

4.5. Robustness analysis

Table 7 shows the robustness summary.

Table 7. Robustness summary.

Specification	Key result
Leverage definitions	Coefficients stable; sign invariant
Alternative links	Exponential mapping preferred (AIC ↓ 18–27)
GMM vs. FE targets	$\hat{\beta}$ correlation ≈ 0.93
Wild bootstrap	CI widths widen ~9%; inference unchanged
Influence diagnostics	No exchange drives $\hat{\theta}$ by >5%

Across all specifications, the brink, width, and hazard-slope results remain materially intact.

Table 8 shows the regional groupings.

Table 8. Regional groupings.

Group	Countries
West Africa	Nigeria, Ghana, Côte d'Ivoire
East Africa	Kenya, Tanzania, Uganda, Rwanda
Southern Africa	South Africa, Namibia, Botswana, Zambia
North Africa	Egypt, Morocco, Tunisia

Table 9 shows the regional robustness analysis: Institutional determinants of inaction-band width.

Table 9. Regional robustness analysis: Institutional determinants of inaction-band width.

Regional group	Countries	Avg width (p.p.)	Width elasticity wrt governance	Interpretation
Southern Africa	South Africa, Namibia, Botswana, Zambia	3.2	−0.28	Strong institutions and deeper markets compress adjustment bands
North Africa	Egypt, Morocco, Tunisia	4.3	−0.19	Moderate institutional quality associated with narrower widths
West Africa	Nigeria, Ghana, Côte d'Ivoire	6.1	−0.12	Greater market frictions and financing constraints widen bands
East Africa	Kenya, Tanzania, Uganda, Rwanda	6.8	−0.09	Weaker market depth and institutional heterogeneity generate wider bands

Source: Authors' estimates based on regional aggregation of African exchanges.

Interpretation:

The regional decomposition reinforces the main finding that institutional quality plays a central role in shaping leverage-adjustment behavior. Southern African firms exhibit the narrowest inaction bands and the largest governance elasticity, indicating that stronger creditor-rights protection, more liquid capital markets, and deeper financial systems reduce firms' tolerance for leverage misalignment and encourage more timely refinancing interventions. North African firms display similar but less pronounced effects, reflecting intermediate levels of financial development and institutional effectiveness.

In contrast, West and East African firms operate with substantially wider inaction bands, implying greater reluctance or inability to undertake frequent leverage adjustments. The relatively small governance elasticities in these regions suggest that adjustment costs remain elevated due to market illiquidity, intermittent access to external finance, weaker enforcement mechanisms, and higher refinancing frictions. Overall, the regional evidence confirms that improvements in institutional quality and market depth systematically compress inaction-band width, consistent with the theoretical prediction that stronger financial and legal infrastructures reduce adjustment costs and facilitate more responsive capital-structure management (**Table 10**).

Table 10. Regional robustness of state-dependent capital structure estimates.

Specification	Southern Africa	North Africa	West Africa	East Africa
Target-leverage coefficients	Stable	Stable	Stable	Stable
Width determinants	Sign invariant	Sign invariant	Sign invariant	Sign invariant
Governance effect on width	Strongly negative	Negative	Negative	Negative
Hazard slope (α_2)	Significant	Significant	Significant	Significant
Width compression effect	Strongest	Moderate	Moderate	Weakest
Model classification accuracy	High	High	Moderate-High	Moderate-High

Regional Robustness Summary:

The regional estimations provide strong support for the generality of the proposed (S,s) framework across African capital markets. Although the magnitude of the coefficients varies according to institutional development and market maturity, the signs and economic interpretations remain unchanged. In every regional subsample, leverage adjustments exhibit pronounced state dependence, refinancing hazards increase sharply as firms approach estimated thresholds, and institutional quality contributes to narrower inaction bands.

Importantly, no single region drives the baseline findings. The persistence of negative governance effects on band width and positive brink-proximity effects on refinancing probability suggests that the documented nonlinear leverage dynamics represent a continent-wide phenomenon rather than the experience of a small subset of more developed exchanges. These results alleviate concerns regarding sparse country-level observations and demonstrate that the principal conclusions remain robust when the analysis is conducted at broader sub-regional levels.

4.6. Asymmetric threshold dynamics

New Hypothesis:

H1. *Upper-brink adjustments occur more frequently than lower-brink adjustments among African listed firms.*

A distinctive feature of African capital markets is the greater prevalence of financing frictions that constrain firms' ability to raise new debt relative to their ability to reduce existing leverage. Consequently, firms may be more likely to encounter upper-brink refinancing triggers than lower-brink triggers. The prevalence of upper-brink interventions is consistent with theories emphasizing borrowing constraints, debt overhang risks, covenant pressures, and monetary-financial interactions that make debt reduction more common than leverage expansion during periods of financial stress [34,52,53].

To evaluate this possibility, the refinancing hazard model is estimated separately for upper-threshold events (deleveraging interventions) and lower-threshold events (leverage-increasing interventions) (**Table 11**).

Table 11. Asymmetric threshold frequency.

Measure	Percentage
Upper-brink events	67.4%
Lower-brink events	32.6%
Upper/Lower ratio	2.07
Difference test (*p*-value)	0.003

Interpretation:

The results indicate a pronounced asymmetry in leverage-adjustment behavior. Approximately two-thirds of observed refinancing interventions occur near the upper brink, suggesting that deleveraging episodes substantially outnumber leverage-increasing episodes. This pattern is consistent with limited borrowing capacity, intermittent market access, and elevated financing frictions across African markets. The evidence therefore supports H1 and suggests that the upper threshold plays a more active role in shaping observed leverage dynamics.

5. Simulation evidence

Monte Carlo simulations evaluate estimator reliability under nonlinear (DGP S), mixed (DGP M), and linear SOA (DGP L) environments.

5.1. Performance under nonlinear and mixed DGPs

Table 12 presents the synthetic evaluation statistics.

Table 12. Simulation metrics (DGP S and DGP M).

Parameter	True	Mean	Bias	RMSE	Coverage
β_1	0.12	0.118	−0.002	0.019	0.94
β_2	−0.08	−0.079	+0.001	0.017	0.95
θ_U	0.40	0.412	+0.012	0.066	0.93
θ_B	−0.30	−0.301	−0.001	0.053	0.95
α_1	1.50	1.447	−0.053	0.142	0.91
α_2	0.80	0.823	+0.023	0.101	0.94

Note: 200 replications, N = 500, T = 20. The joint estimator performs strongly: bias is negligible, RMSE low, and coverage close to nominal levels.

5.2. Comparison with alternative estimators

Table 13 shows the estimator comparison.

Table 13. Estimator comparison.

Estimator	RMSE (θ_U)	RMSE (Width)	Misclassification	Notes
Model-assisted	0.066	0.044	0.12	Best overall
SOA	0.144	—	0.41	Pseudo-convergence
Unconstrained threshold	0.119	0.097	0.26	High variance
No auxiliaries	0.103	0.082	0.23	Precision loss

Note: The exponential positivity constraint and auxiliary information yield decisive improvements.

5.3. Linear DGP robustness (DGP L)

Table 14 shows the linear consistency check.

Table 14. Linear consistency check.

Criterion	Expected	Actual outcome
Width W → 0	Collapse	Est. mean 0.31 p.p.
Hazard slope → 0	Flat	α_2 = 0.07 (insignificant)
β̂ stability	Consistent	All β̂ within ±0.01

Note: The estimator nests the linear model cleanly and does not spuriously detect thresholds.

Summary:

With empirical findings validated through simulation, the evidence strongly supports a state-dependent (S,s) capital-structure model for African firms. The model-assisted system delivers superior identification, lower RMSE, tighter confidence intervals, and more accurate classification of refinancing events relative to SOA and unconstrained alternatives.

5.4. Misspecification diagnostics

Table 15 shows the misspecification diagnostics (DGP X).

Table 15. Misspecification diagnostics (DGP X).

Measure	Result
False threshold detection rate	4.1%
Mean width estimate	0.29 p.p.
Width bias	0.03 p.p.
Hazard significance rate (5% level)	4.6%
Classification accuracy	Consistent with random assignment

Interpretation:

The false threshold detection rate remains below 5%, indicating that the proposed framework does not spuriously generate threshold behavior when the underlying process is linear. Estimated widths collapse toward zero, hazard parameters become statistically insignificant, and adjustment behavior resembles the conventional SOA benchmark. These findings strengthen confidence that the documented African (S,s) dynamics reflect genuine nonlinear behavior rather than mechanical features of the estimation procedure.

6. Conclusion

This paper develops a model-assisted econometric framework for state-dependent capital-structure dynamics in which leverage adjusts through target alignment, brink-triggered intervention, and a structurally identified inaction-band width. By jointly estimating these three latent components—and by allowing institutional, macro-financial, and firm-specific auxiliaries to inform each stage—the framework delivers a unified representation of how firms tolerate leverage drift and when they intervene.

The empirical analysis using a large, multi-country African panel demonstrates that capital-structure adjustments are episodic, discontinuous, and sharply state-dependent. Firms drift within narrow corridors but rebalance abruptly once upper or lower brink thresholds are breached, producing the clear "drift → brink → jump" signature of an (S,s) process. The estimated band widths widen in the presence of volatility and timing opportunities but compress with liquidity, tax incentives, and—critically—with improvements in institutional quality and market depth. These findings confirm that macro-institutional frictions materially shape corporate adjustment behavior, and they highlight the structural channels through which financial-market development affects refinancing dynamics.

The simulation study reinforces these insights. Across nonlinear and mixed data-generating processes, the proposed estimator achieves superior RMSE, bias, coverage, and episode-classification performance relative to standard partial-adjustment and unconstrained threshold models. The estimator correctly nests the linear SOA benchmark when the true process is smooth, confirming the design's robustness and its ability to avoid spurious detection of thresholds. The simulations also show that exponential positivity constraints and macro-institutional auxiliaries play a decisive role in stabilizing brink and width estimation under heavy-tailed shocks.

The novelty of the paper lies in treating targets, thresholds, and widths as jointly estimable structural objects—each informed by correlated auxiliary information and each contributing to a coherent model of nonlinear adjustment. This contrasts sharply with traditional approaches that impose continuous adjustment or estimate thresholds in isolation. By integrating dynamic-panel methods, nonlinear constrained estimation, and state-dependent hazards within a single system, the paper provides a general blueprint for analyzing lumpy, threshold-driven corporate policies.

The results carry important implications. For empirical researchers, the model offers a tractable strategy for recovering latent nonlinear dynamics using observed firm- and macro-level covariates. For policymakers, the compression of width with institutional and market improvements highlights concrete channels through which reforms can reduce refinancing frictions. For practitioners, the findings underscore that leverage should be monitored as a tolerance range rather than a point, with refinancing decisions governed by proximity to brinks rather than by smooth convergence pressures. More broadly, the findings complement recent research emphasizing that leverage, monetary transmission, financial-cycle dynamics, and macroeconomic shocks interact through fundamentally nonlinear channels, particularly in emerging markets characterized by institutional and financing frictions [6,26,52].

Future research may extend the framework in several directions. First, future studies could integrate machine-learning and neural-symbolic estimation techniques into the target-zone architecture to examine whether AI-assisted approaches improve threshold detection and hazard classification while preserving economic interpretability. Second, the framework could be adapted to banking institutions, financial firms, and privately held firms where refinancing constraints differ substantially from those observed in listed non-financial corporations. Third, future work may incorporate climate-transition risk, sustainability-related financing constraints, and geopolitical uncertainty as additional determinants of leverage thresholds and inaction-band width. Finally, cross-regional comparisons involving African, Asian, Latin American, and Middle Eastern markets would provide valuable insights into how institutional development shapes nonlinear capital-structure adjustment behavior across emerging economies.

In sum, the proposed framework demonstrates how combining economic structure, auxiliary information, and joint estimation yields a powerful lens for understanding capital-structure behavior in environments where adjustments are inherently state-dependent, infrequent, and driven by meaningful frictions. Please see the **Appendices A** and **B** for more information.

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References

1. Korteweg A, Strebulaev IA. An empirical target zone model of dynamic capital structure. SSRN. 2012. Available online: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1787001
2. Gwatidzo T, Ojah K. Firms' debt choices in Africa: Are institutional infrastructure and non-traditional determinants important? International Review of Financial Analysis. 2014; 31: 152–166.
3. Ayadi E, Ben Jedidia L, Ben Mbarek N. Inflation Shocks and Equity Vulnerability: Regime, Sign, and Cross-Country Asymmetries in the G7. Economies. 2026; 14(2): 55. doi: 10.3390/economies14020055
4. Doojav GO, Juragat A. Nonlinearities and state-dependence in the monetary transmission mechanism: Evidence from a commodity-dependent economy. International Economics. 2025; 184: 100640. doi: 10.1016/j.inteco.2025.100640
5. Kuehn LA, Schreindorfer D, Schulz F. Persistent crises and levered asset prices. The Review of Financial Studies. 2023; 36(6): 2571–2616. doi: 10.1093/rfs/hhac081
6. Magubane K. The stability of the financial cycle: Insights from a Markov switching regression in South Africa. Journal of Risk and Financial Management. 2025; 18(2): 76. doi: 10.3390/jrfm18020076
7. Khemiri W, Noubbigh H. Determinants of capital structure: Evidence from Sub-Saharan African firms. Quarterly Review of Economics and Finance. 2018; 70: 150–159.
8. Kamepalli SK. Essays in Macroeconomics of Microeconomic Frictions [PhD Thesis]. Columbia University; 2026.
9. Harding M, Wouters R. Risk and State-Dependent Financial Frictions (No. 2022-37). Bank of Canada; 2022.
10. Li S, Yang J, Zhao S. Robust leverage dynamics without commitment. Economic Theory. 2022; 74(2): 643–679.
11. Frank MZ, Goyal VK. Empirical corporate capital structure. In: Handbook of Corporate Finance. Edward Elgar Publishing; 2024. pp. 27–125.
12. Modigliani F, Miller MH. The cost of capital, corporation finance, and the theory of investment. American Economic Review. 1958; 48(3): 261–297.
13. Modigliani F, Miller MH. Corporate income taxes and the cost of capital: A correction. American Economic Review. 1963; 53(3): 433–443.
14. Miller MH. Debt and taxes. Journal of Finance. 1977; 32(2): 261–275. doi: 10.1111/j.1540-6261.1977.tb03267.x
15. Myers S. The capital structure puzzle. Journal of Finance. 1984; 39(3): 575–592. doi: 10.2307/2327916
16. Myers S, Majluf N. Corporate financing and investment decisions when firms have information that investors do not have. Journal of Financial Economics. 1984; 13(2): 187–221. doi: 10.1016/0304-405X(84)90023-0
17. Baker M, Wurgler J. Market timing and capital structure. Journal of Finance. 2002; 57(1): 1–32. doi: 10.1111/1540-6261.00414.
18. DeAngelo H, Roll R. How stable are corporate capital structures? Journal of Finance. 2015; 70(1): 373–418. doi: 10.1111/jofi.12163
19. DeAngelo H. Corporate financial policy: What really matters? Journal of Corporate Finance. 2021; 68: 101925. doi: 10.1016/j.jcorpfin.2021.101925
20. DeAngelo H. The capital structure puzzle: What are we missing? Journal of Financial and Quantitative Analysis. 2022; 57(2): 413–454. doi: 10.1017/S002210902100079X
21. Lambrecht BM, Myers SC. The dynamics of investment, payout, and debt. Review of Financial Studies. 2017; 30(11): 3759–3800. doi: 10.1093/rfs/hhx045
22. Paseda OA, Ayadi OF. Capital structure instability: An empirical investigation among non-financial quoted firms in Nigeria. Nigerian Journal of Economic and Social Studies. 2023; 65(1): 72–128.

23. Paseda O. The speed of adjustment of capital structure of Nigerian quoted firms. Journal of Developing Areas. 2025; 59(1): 207–231. doi: 10.1353/jda.2025.a952661
24. Paseda O, Ashiru O, Balogun G. Financial innovation and bank financial performance: Further evidence from Nigerian deposit money banks. Journal of Developing Areas. 2026; 60(2): 109–144. doi: 10.1353/jda.2026.a992557
25. Fontalvo HR, de la Puente M, Torres J, et al. Foreign Direct Investment Under Uncertainty: A Regime-switching, Threshold and Dynamic Correlation Analysis of Emerging Economies. Global Business Review. 2026. doi: 10.1177/09721509261418489
26. Nyati MC, Muzindutsi PF, Tipoy C. Interactions between Business Cycles, Financial Cycles and Monetary Policy in South Africa. Forecasting. 2026; 8(3): 51. doi: 10.3390/forecast8030051
27. Mudiangombe BM, Muteba Mwamba JW. Dynamic asymmetric effect of currency risk pricing of exchange rate on equity markets: a regime-switching based C-vine copulas method. International Journal of Financial Studies. 2022; 10(3): 72. doi: 10.3390/ijfs10030072
28. Arrow KJ, Harris T, Marschak J. Optimal inventory policy. Econometrica. 1951; 19(3): 250–272. doi: 10.2307/1906813
29. Miller MH, Orr D. A model of the demand for money by firms. Quarterly Journal of Economics. 1966; 80(3): 413–435. doi: 10.2307/1880728
30. Caplin A, Leahy J. Economic theory and the world of practice: A celebration of the (S, s) model. Journal of Economic Perspectives. 2010; 24(1): 183–202. doi: 10.1257/jep.24.1.183
31. Burdett K, Menzio G. The (Q,S,s) pricing rule. Review of Economic Studies. 2018; 85(2): 892–928. doi: 10.1093/restud/rdx048
32. Lee J, Ying C. Leverage Dynamics and Liquidity Management without Commitment. SSRN. 2026. Available online: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=6193739
33. Li Y, Yang J, Zhou W. Robust Leverage Dynamics under Costly Equity Issuance. SSRN. 2026. Available online: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=6305980
34. Greenwald D. Firm debt covenants and the macroeconomy: The interest coverage channel. SSRN. 2019. Available online: https://ssrn.com/abstract=3535221
35. Fama EF, French KR. Capital structure choices. Critical Finance Review. 2012; 1(1): 59–101. doi: 10.1561/104.00000002.
36. Paseda O. The impact of firm-specific characteristics on the capital structure of Nigerian quoted firms. SSRN. 2016. Available online: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2889419
37. Paseda O. Firm size, asset tangibility, growth, volatility, dividends and the capital Structure of Nigerian quoted firms. Journal of Finance and Accounting. 2021; 9(2): 36–52. doi: 10.11648/j.jfa.20210902.13
38. Jensen MC, Meckling WH. Theory of the firm: Managerial behavior, agency costs, and ownership structure. Journal of Financial Economics. 1976; 3(4): 305–360. doi: 10.1016/0304-405X(76)90026-X.
39. Jensen MC. Agency costs of free cash flow, corporate finance, and takeovers. American Economic Review. 1986; 76(2): 323–329.
40. Arellano M, Bond S. Some tests of specification for panel data: Monte Carlo evidence and an application to employment equations. Review of Economic Studies. 1991; 58(2): 277–297. doi: 10.2307/2297968
41. Blundell R, Bond S. Initial conditions and moment restrictions in dynamic panel data models. Journal of Econometrics. 1998; 87(1): 115–143. doi: 10.1016/S0304-4076(98)00009-8
42. Amemiya T. Advanced Econometrics. Harvard University Press; 1985.
43. Gallant AR. Nonlinear Statistical Models. Wiley; 1987.
44. White H. A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity. Econometrica. 1980; 48(4): 817–838. doi: 10.2307/1912934
45. Mammen E. Bootstrap and wild bootstrap for high dimensional linear models. Annals of Statistics. 1993; 21(1): 255–285. doi: 10.1214/aos/1176349025
46. Cameron AC, Gelbach JB, Miller DL. Bootstrap-based improvements for inference with clustered errors. Review of Economics and Statistics. 2008; 90(3): 414–427. doi: 10.1162/rest.90.3.414
47. Strebulaev IA, Whited TM. Dynamic models and structural estimation in corporate finance. Published in Foundations and Trends in Finance. 2012; 6: 1–163.
48. Wooldridge JM. Econometric Analysis of Cross Section and Panel Data, 2nd ed. MIT Press; 2010.
49. Ali H, Naz S. Interpretable deep learning for modeling policy uncertainty and firm-specific risk: Evidence from advanced and emerging markets. Data Science in Finance and Economics. 2026; 6(1): 147–182.

50. Moroke ND. Cognitive Big Data Architecture for Daily Operational Jamming Transition Detection with Low-Latency Inference in Infrastructure-Constrained Financial Markets: The MERI Framework. Big Data and Cognitive Computing. 2026; 10(7): 240. doi: 10.3390/bdcc10070240
51. Davidson R, MacKinnon JG. Bootstrap methods in econometrics. In: Mills TC, Patterson K (editors). Palgrave Handbook of Econometrics. Palgrave Macmillan; 2006. pp. 812–838.
52. Acharya VV, Plantin G, Wang O. Indebted Supply and Monetary Policy: A Theory of Financial Dominance (No. w34798). National Bureau of Economic Research; 2026.
53. Goodhart CA, Peiris MU, Tsomocos DP, et al. HKC05-Household Portfolios, Corporate Leverage, and the Supply Side of Monetary Policy. Oberlin College Kasper Economics and Business; 2026. Available online: https://digitalcommons.oberlin.edu/economics_wps/5
54. Nguyen Khac Quoc B, Pham Duy T. Heterogeneous Impacts of Macroprudential Policy on GDP Tails: The Role of Credit Cycle, Financial Cycle and Financial Leverage in Vietnam. Emerging Markets Finance and Trade. 2025; 62(11): 4073–4093. doi: 10.1080/1540496X.2025.2604596
55. Ahmed ZS, Ibrahim O. Credit Risk Dynamics and Macroeconomic Shocks in the European Union: A Stochastic Simulation Framework for Banking Stability and Policy Analysis (2000–2025). Research Square. 2026.
56. Fusari N, Lamichhane S. Structural Equity Option Pricing: Implications for Credit Risk. SSRN. 2025; 5393252.

Appendix A. Dynamic panel specification comparisons

Table A1. Difference-GMM versus system-GMM comparison.

Measure	Difference GMM	System GMM
Hansen *p*-value	0.218	0.412
AR(1) *p*-value	0.000	0.000
AR(2) *p*-value	0.134	0.221
Number of Instruments	48	63
Target Leverage Estimate (Lagged Leverage)	0.71	0.83
Std. Error	0.078	0.052

Source: Empirical Results.

A Hausman-type comparison between Difference-GMM and System-GMM suggests that System-GMM delivers more stable coefficient estimates, improved instrument relevance, and lower finite-sample variability. While both estimators generate qualitatively consistent results, System-GMM exhibits stronger persistence estimates and more precise inference. Consequently, the baseline specifications rely on System-GMM, with Difference-GMM results reported as additional robustness evidence.

Appendix B. Starting value sensitivity analysis

Table A2. CNLS initialization robustness.

Initialization method	Objective function value	Converged?
Grid-search initialization	1,241.3	Yes
OLS initialization	1,244.5	Yes
Random initialization	1,247.1	Yes

Interpretation:

The resulting parameter estimates exhibit minimal variation across alternative starting values, indicating that the CNLS optimization procedure converges to a stable solution rather than a local optimum. The grid-search initialized solution consistently produces the lowest objective-function value and is therefore retained as the baseline specification.