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Article

Mapping and superposition of multi-modal data flows for system fault evolution

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Abstract: To study system fault evolution using multi-modal data, multi-modal data are associated with multiple factors, and a mapping and superposition method for multi-modal data flows and factors is established. Multi-modal data and their characteristics are discussed. The mapping between multi-modal data flows and factors and the superposition of mapping results are investigated. The robustness of superposition operators, dynamic system modeling of factor evolution, and denoising performance of the mapping-superposition strategy are theoretically analyzed. The function of mapping in system fault evolution is explained, and a case study is provided. Results show that disaster data exhibit multi-modal characteristics. A multi-modal data flow consists of multiple single-modal data flows, which can be further subdivided into multifactor value data flows. These establish a mapping relationship between factors and time calibration and form a factor mapping model for single-/multi-modal data flows. It is necessary to consider the superposition forms of multi-factor value data flows mapped to the same factor, including scalar, vector, and max-min superposition forms, with corresponding mathematical models and superposition processes provided. The framework is embedded into a continuous-time dynamic model based on differential equations, supporting state estimation and optimal control. The proposed method is applied to analyze the fault process of an unmanned monitoring aircraft. The case is modeled with linear differential equations to simulate factor state trajectories and fault events. The results can provide a multi-dimensional data interface for the study of system fault processes, facilitating the analysis, prediction, early warning, and intervention of system faults.

Keywords: safety system engineering; multi-modal data flow; factor mapping; superposition mode; system fault evolution

1. Introduction

In disaster and accident analysis, multi-modal data are important because they provide comprehensive and accurate information and enhance the perception and prediction capabilities of models. The fusion of different types of data, such as images, texts, and temperatures, enables a more precise description of the background, process, and impacts of addressing disasters and accidents, thereby improving the efficiency and accuracy of emergency decision-making. Any disaster or accident is the outcome of the fault evolution process of a certain system. It is more important to describe the system fault evolution process and its results using multi-modal data.

By integrating multi-modal data obtained from various equipment monitoring systems, a multi-dimensional, high-resolution system condition monitoring framework can be constructed. This helps capture the early signs of disasters and accidents and depicts their dynamic evolution from initiation to development. It provides strong data support for disaster and accident early warning, cause analysis, and intervention, thus improving system reliability and safety. However, some problems exist in describing system fault evolution using multi-modal data. Data quality affects accuracy and reliability. Different modal data may have different formats, structures, and dimensions, leading to difficulties in integration and processing. Algorithms for handling complex multi-modal data are insufficient. Hardware resource requirements are high.

Current research on disaster, accident, and fault processes using multi-modal data is growing rapidly.

Multi-modal data fusion serves as the foundation for factor mapping and superposition. Xu et al. [1] developed a multi-sensor signal-augmented multi-modal MAML-1DCNN-RBEAM deep transfer learning algorithm. Superposition mapping of features from different sensors enabled accurate fault diagnosis of wind turbine bearings under small-sample conditions. Wang et al. [2] proposed a comparative detection and multi-task learning framework for multi-modal defect data in drainage pipelines. Superposition mapping of visual and structural data improved defect recognition performance. Wang et al. [3] combined near-infrared spectroscopy with multi-modal data fusion and deep learning to achieve non-destructive detection of hollow defects in pecans, demonstrating the advantages of multi-source physical quantity superposition mapping. Zhu et al. [4] conducted research based on semantic-aware and enhanced cross-attention mechanisms. Interactive mapping between modalities improved the accuracy of multi-modal bearing fault diagnosis. Zhang et al. [5] developed the FCTransformer model. Fourier convolution and Transformer architecture were used for multi-sensor information fusion, realizing effective superposition of features in the frequency and spatial domains. Duan et al. [6] investigated smart bearings based on mosaic-patterned electrets. AI-driven fault diagnosis was achieved via multi-modal motion sensing, realizing superposition mapping of mechanical motion and electrical signals. Zheng et al. [7] proposed an enhanced Dempster-Shafer evidence theory for the condition monitoring of roller bearings, providing an effective evidence fusion framework for multi-source monitoring information. Na et al. [8] developed a deep mutual learning-based framework for multi-modal phased array ultrasonic data inspection. For wind turbine blade defects, feature superposition was enhanced through mutual learning between modalities. Lin et al. [9] conducted transfer learning based on multi-modal feature fusion. Intelligent detection of robotic MAG welding defects was realized under cross-condition and small-sample scenarios, verifying the effectiveness of multi-modal superposition in small-sample conditions. Chen et al. [10] constructed a multi-modal fault diagnosis dataset for three-phase asynchronous motors under variable working conditions, providing a standardized data basis for mapping and superposition of multi-modal data flows.

Mapping data flows to interpretable factors and forming semantic judgments through superposition is critical to improving the interpretability of diagnosis. Guo et al. [11] developed a knowledge-guided multi-modal large language model with a mixture-of-experts mechanism. Multi-modal data flows were mapped to a knowledge factor space to achieve interpretable defect detection and fault diagnosis. Yang et al. [12] proposed a vibration intelligent language model. An interpretable intelligent operation and maintenance system was built based on multi-modal data fusion, mapping vibration data flows to linguistic factor descriptions. Kiss and Nehéz [13] developed a Graph Transformer-based multi-modal fault diagnosis framework for microservice systems. A graph structure was used to map multi-modal data flows to causal correlations between factors. Hyla et al. [14] applied multi-modal infrared thermography. Superposition of information from different physical fields improved the precision of fatigue crack detection and validated inspection results. Yu et al. [15] performed hierarchical fault root cause diagnosis for multi-modal processes using direct causality and causal polarity analysis, mapping data flows to a causal relationship network. Zhang et al. [16] proposed a dynamic transfer entropy graph for fault root cause diagnosis in multi-modal industrial processes, mapping data flows to causal chains and enhancing the logical interpretability of diagnosis.

Some literature provides solid theoretical tools for factor mapping and superposition. Ma et al. [17] established a spatial factor mapping framework for indoor gamma-ray dose assessment after nuclear accidents by combining Monte Carlo simulation, GIS and remote sensing. Xu et al. [18] proposed a MaxEnt-TRIGRS hybrid model to realize dynamic safety factor mapping for rainfall-induced debris flow susceptibility assessment. Phansalkar and Han [19] developed a novel test method to measure time-cure superposition shift factors of filled-thermoset materials. Collectively, these studies enrich the methodologies of spatial mapping, hybrid modeling and time-superposition characterization for environmental and material research.

A profound understanding of the nature of system faults is a theoretical prerequisite for mapping multi-modal data to factors. Zhang et al. [20] developed an exergy-based unsupervised domain adaptive broad learning system framework for fault diagnosis in multi-modal batch manufacturing processes. Physical constraints improved the robustness of mapping. Hu et al. [21] proposed LWCNet, a physics-guided multi-modal few-shot learning framework for intelligent fault diagnosis. Physical laws were superimposed onto data-driven models. Liu J and Liu C [22] reviewed deep learning-based intelligent inspection of transmission lines, emphasizing application challenges and mapping requirements of multi-modal data fusion in complex scenarios. And some literature provides fundamental theoretical support [23].

These studies utilize multi-modal data and apply various methods to investigate disaster, accident, and fault processes in various systems. Significant progress has been achieved, particularly in the study of system fault processes within specific domains. However, as mentioned above, this field still has great development potential. Problems such as data quality, format and structure, insufficient algorithms, and hardware resources are essentially caused by analyzing multi-modal data as a whole. The granularity of multi-modal data is unsuitable for large-scale multi-dimensional analysis. Each modal data in multi-modal data should be further subdivided into different factors and their values. Factor value data flows should replace multi-modal data flows for the analysis of system fault evolution processes and results. This reduces data interference and improves data quality. No format issues exist based on the factors. Computational parameters are independent factors with simple algorithms, reducing time and space complexity. This is one approach to avoid the above problems when studying system fault evolution using multi-modal data flows.

To realize the above approach, within the framework of the proposed system fault evolution theory [23], this paper presents a method for mapping multi-modal data flows to factors and superposing mapping results. The method mainly addresses two issues: mapping between multi-modal data flows and factors, and superposition of factor mapping results. The research results extend the conceptual model, topological structure model, and mathematical model on the original theoretical basis. Contents such as the multi-modal data flow concept, complete many-to-many mapping between multi-modal data and multi-factors, and superposition of factor mapping results are added. This improves the accuracy and efficiency of system fault process analysis.

The primary contributions of this study (centered on mapping and superposition of multi-modal data flows and factors in system faults) are as follows.

- It reveals the composition mechanism of multi-modal data flows in system fault evolution, clarifying that multi-modal data can be decomposed into single-modal data flows and further refined into factor value data flows. They establish the mapping relationship between factors and time calibration, and form the factor mapping model for single-modal and multi-modal data flows.
- It constructs a factor mapping model and a mapping result superposition framework for multi-modal data flows: forms a many-to-many mapping relationship from multi-modal data to influencing factors, proposes scalar, vector and max-min superposition modes for factor mapping results, and matches them with corresponding physical significance and mathematical expressions.
- It expands the theoretical system of system fault evolution by introducing multimodal data layer, improves the concept model, topological structure model and mathematical model, establishes robustness criteria via Lipschitz continuity, proposes a dynamic system model based on differential equations, and verifies the denoising advantage of the mapping-superposition strategy, and verifies the practicability of the proposed method through the fault process of unmanned monitoring aircraft, providing a multi-dimensional data interface for fault analysis, prediction and early warning.

This study is organized as follows: Section 2 introduces multi-modal data and its characteristics, Section 3 elaborates the mapping method from multi-modal data flows to factors, Section 4 illustrates the superposition modes of mapping results, Section 5 presents the case analysis and discussions, and Section 6 provides the conclusions.

2. Multi-modal data and characteristics

Clear differences exist between single-modal data and multi-modal data. Single-modal data contains only one data recording mode, such as text, image, audio, or video. Multi-modal data includes two or more different data recording modes that jointly describe the same object or event. Multi-modal data are more comprehensive and abundant, with different data types complementing each other. Linear or non-linear mapping of single-modal data yields multiple features of the described object. Modal data and these features are integrated to achieve collaborative representation between modalities for data fusion and understanding. Meanwhile, data consistency between different modalities must be ensured. That is, data features are consistent when different modalities describe the same object. This is a basic condition for multi-modal data fusion.

System fault evolution often starts from abnormal changes in the natural environment, faults in production systems, human errors, or their combined effects, manifesting as disasters, accidents, and other forms. These initial states gradually accumulate and interact. Disasters and accidents erupt suddenly when a critical threshold is reached. Their impacts then spread rapidly, triggering secondary disasters and further worsening the severity and complexity of consequences. During this process, the nature and manifestations of disasters and accidents may change continuously, posing great challenges to rescue and response. Effective monitoring is critical to prevent disasters and accidents. Real-time monitoring of disaster and accident development enables timely detection and early warning of potential risks, gaining time and space for rescue and response. The collection and analysis of disaster and accident data reveal their evolution laws and characteristics, supporting disaster and accident analysis, prediction, intervention, and governance. Therefore, strengthening the monitoring of disasters and accidents is of great significance.

Due to the wide variety of disasters and large differences between accidents, comprehensive monitoring is required for disasters and accidents, with high demands on data. Disaster and accident process data have the following characteristics. 1) Large data volume: Disasters and accidents involve massive data, including geological, meteorological, and traffic data. Huge storage space and computing power are required for data collection, storage, and processing. 2) Diverse data types: Data come from a wide range of sources, including structured and unstructured data. Structured data includes seismic and meteorological data. Unstructured data includes video, audio, and sensor data with diverse types and formats. 3) Low value density: Valid information is hidden in massive irrelevant data. However, the relevance and consistency of these data are valuable for disaster early warning and emergency response. 4) Fast processing required: Data are real-time and must be processed and analyzed within a short time. The system must process data rapidly to respond quickly to sudden disaster events. 5) High data accuracy required: Accuracy is critical to disaster and accident prediction, analysis, and intervention. Therefore, strict specifications must be followed in data collection, storage, and processing to ensure data accuracy and reliability.

Thus, different types of data obtained in real time by different monitoring devices play a decisive role in disaster and accident research. However, these data show multi-modal characteristics owing to different monitoring devices and methods. Temporal and spatial data consistency must also be guaranteed. These multi-modal data must be consistent in time to form stable data flows. Multi-modal data includes different types and forms of data obtained by different devices, such as remote sensing images, meteorological data, hydrological data, geographic information, video surveillance data, and sensor monitoring data. These diverse data types together constitute disaster and accident data, describing the complete system fault evolution process. Remote sensing images are image data that monitor disasters and accidents in real time and intuitively display important information such as the scope, degree, and impacts of disasters and accidents. Meteorological data include temperature, humidity, wind speed, and other information. Hydrological data include water level, flow velocity, rainfall, and other information, which are useful for assessing the severity of disasters and accidents. Geographic information data includes topography, soil type, vegetation cover, and other information, which are important for analyzing disaster causes and conducting rescue operations. Video surveillance monitors disaster sites in real time and provides real-time image information. Sensors monitor various physical parameters at disaster sites in real time, such as changes in temperature, humidity, and pressure. In summary, data recorded by various devices are distributed in time series. Therefore, disaster and accident data are time-calibrated multi-modal data flows. These multi-modal data flows together form a complete dataset of disaster and accident data. Full use of multi-modal data flows enables more effective disaster early warning, emergency response, and post-disaster reconstruction, reducing the impacts of disasters and accidents on society and the environment.

Some research achievements on multi-modal data are presented in Section 1. Despite progress, challenges remain, especially in studying system fault evolution processes and resulting disasters and accidents using multi-modal data flows. These challenges include data flow consistency, mapping between data flows and factors, and superposition of factor mapping results. Data flow consistency exists in multi-modal data. Data must describe the same object or event in both time and space, namely data alignment. Spatial consistency of multi-modal data is easy to achieve in practice. Temporal consistency can be solved by calibrating any moment data in data flows using time. All multi-modal data flows are first decomposed into single-modal data flows. Various factor value data recording processes in single-modal data flows are calibrated by time. The calibrated time is used as the data alignment standard in subsequent analysis. Mapping between data flows and factors and the superposition of factor mapping results are discussed in Sections 2 and 3, respectively.

3. Mapping between multi-modal data flows and factors

After summarizing extensive literature, an in-depth study is conducted on the data characteristics contained in multi-modal data flows. These multi-modal data flows consist of multiple single-modal data flows. Each single-modal data flow contains multiple factor value data flows that describe the object state from different aspects.

For example, vehicle inertial navigation, eye trackers, and psychological data recorders were used to collect vehicle operation, eye movement, electrocardiogram, and other multi-modal driving data to identify dangerous driving behaviors [24]. Vehicle operation, eye movement, and electrocardiogram are three single-modal data flows. Vehicle operation includes three data flows: speed, longitudinal acceleration, and steering angle. Eye movement includes three data flows: left pupil diameter, right pupil diameter, and blink frequency. An electrocardiogram includes two data flows: heart rate and heartbeat interval. Obviously, these data flows represent the changes in basic factor values that affect dangerous driving behaviors. As another example, multi-modal data flows composed of images, audio, video, and text were used to analyze network public opinion [25]. Video data includes key frame and image frame data flows. Audio data includes speech recognition and auditory word data flows. Natural disasters are more complex. For example, remote sensing images, social media text, geographic information data, and other multi-modal data were used to study flood disaster processes. Data can be subdivided into 5 major categories and 15 subcategories: social data, surface data, historical disaster data, monitoring data, and geographic data. Current multi-modal data research is mostly based on multiple single-modal data, which are further subdivided into different data flows. A single data flow represents one factor affecting the analysis result. Thus, a single-modal data flow contains multiple factors. Meanwhile, different single-modal data flows may contain the same factors. That is, a single-modal data can contain multiple factors, and a single factor can exist in multiple modalities. Different modalities are divided by data collection methods to form multi-modal data flows. Factor impacts are divided by different roles to form multi-factors. This formally forms the correspondence between multi-modal data and multi-factors, and between multi-modal data flows and factor value changes, finally realizing the mapping between multi-modal data flows and factors.

Establishing mapping between multi-modal data flows and factors is of great significance, mainly reflected in the following aspects. The mapping relationship clearly identifies the correspondence between different modal data flows, enabling efficient fusion of multi-modal data. Fast data extraction, transformation, and loading are realized, reducing manual intervention and error rates. The mapping relationship helps convert data from original formats to formats easier to analyze and query, accelerating analysis speed and reducing system load. Meanwhile, integrating multi-modal data into a unified data model facilitates cross-modal data analysis and improves accuracy and comprehensiveness. That is, mapping improves data processing efficiency and accuracy. The mapping relationship is essentially a semantic mapping, enabling learning of semantic relationships between different modalities and enhancing cross-modal understanding and information retrieval capabilities. Multi-modal data fusion and generation technologies based on mapping relationships can generate high-quality cross-modal content. That is, mapping promotes cross-modal understanding and generation. The mapping relationship provides rich training data for models and optimizes model performance. It reduces training costs and improves development efficiency. It also promotes collaborative learning between different modalities, helps balance information volume between modalities, and improves generalization ability. That is, mapping optimizes model performance and reduces training costs. Under complex conditions, multi-modal data must be integrated for decision-making. The mapping relationship integrates different modal data for multi-dimensional analysis. In real-time monitoring and early warning systems, rapid identification and response to changes in different modal data are required. The mapping relationship enables real-time monitoring and analysis of multi-modal data, timely detection of potential risks, and early warning. That is, mapping supports decision-making under complex conditions. In summary, establishing a mapping relationship between multi-modal data flows and factors is of great significance in improving data processing efficiency and accuracy, promoting cross-modal understanding and generation, optimizing model performance and reducing training costs, and supporting decision-making under complex conditions.

To illustrate the factor mapping model of single-modal data flows and multi-modal data flows, an analysis parameter system is established as follows. Let the multi-modal set be $M = \{m_1, m_2, \ldots, m_J\}$, $m_j \in M$, $j = 1, \ldots, J$, where M is the number of single modalities. The factor set is $F = \{f_1, f_2, \ldots, f_I\}$, $f_i \in F$, $i = 1, \ldots, I$, where I is the number of factors. The event set is $E = \{e_1, e_2, \ldots, e_N\}$, $e_n \in E$, $n = 1, \ldots, N$, where N is the number of events. The event occurrence probability set is $Q = \{q_{n,i} \mid n = 1, \ldots, N, i = 1, \ldots, I\}$, where $q_{n,i}$ denotes the occurrence probability of the event e_n caused by factor f_i. The event occurrence probability distribution [26] set is $P = \{q_1, q_2, \ldots, q_N\}$. q_n depends on the combined effects of all $q_{n,i}$ of e_n. q_T is the final system function state of the system fault evolution process [23], namely the final event occurrence probability distribution.

Formally, each unimodal data flow can be represented as a function from the time domain T to a feature space $\mathbb{R}^{d_j}$, i.e., $m_j : T \to \mathbb{R}^{d_j}$, where d_j denotes the data dimension of the j-th modality. The factor extraction process can be viewed as a mapping operator $\mathit{\Pi}_i^{(j)} : \mathbb{R}^{d_j} \to \mathbb{R}$ (or a more general space), which extracts the part related to a specific factor f_i from the unimodal data flow $m_j(t)$, forming the factor value data flow $m_j(f_i, t) = \mathit{\Pi}_i^{(j)}(m_j(t))$. The mapping operator $\mathit{\Pi}_i$ is data-driven, can be linear or nonlinear, and operates between feature space and factor space to extract the component corresponding to the factor from the modal data flow. For the overall mapping from multimodal data flows to a factor f_i, a fusion operator M_i can be defined, acting on all relevant unimodal mapping results: $\mathit{\Gamma}_i(f_i(t)) = M_i(\{m_j(f_i, t)\}_{j=1}^{J})$, where $\mathit{\Gamma}_i$ represents the final state or influence quantity of the factor f_i. Equations (1) and (2) demonstrate the concrete implementation of this mapping relationship and its possible values (related or unrelated).

The relationships between the above variables are shown in **Figure 1**.

Figure 1 shows a complete graphical representation of the system fault evolution process based on multi-modal data. It is divided into six layers: modal layer, factor layer, event layer, occurrence probability layer, probability distribution layer, and system function state layer. The modal layer contains all modalities and collectively represents multi-modal data. The factor layer contains all factors related to system fault evolution. The event layer contains all events occurring in the system's fault evolution. The occurrence probability layer stores the occurrence probabilities of all events affected by factors, with a value range of [0,1]. The probability distribution layer stores the occurrence probability distributions of all events. All events are processed according to the evolution structure to obtain the system function state, namely the final event occurrence probability distribution. For specific concepts, refer to Section 4.

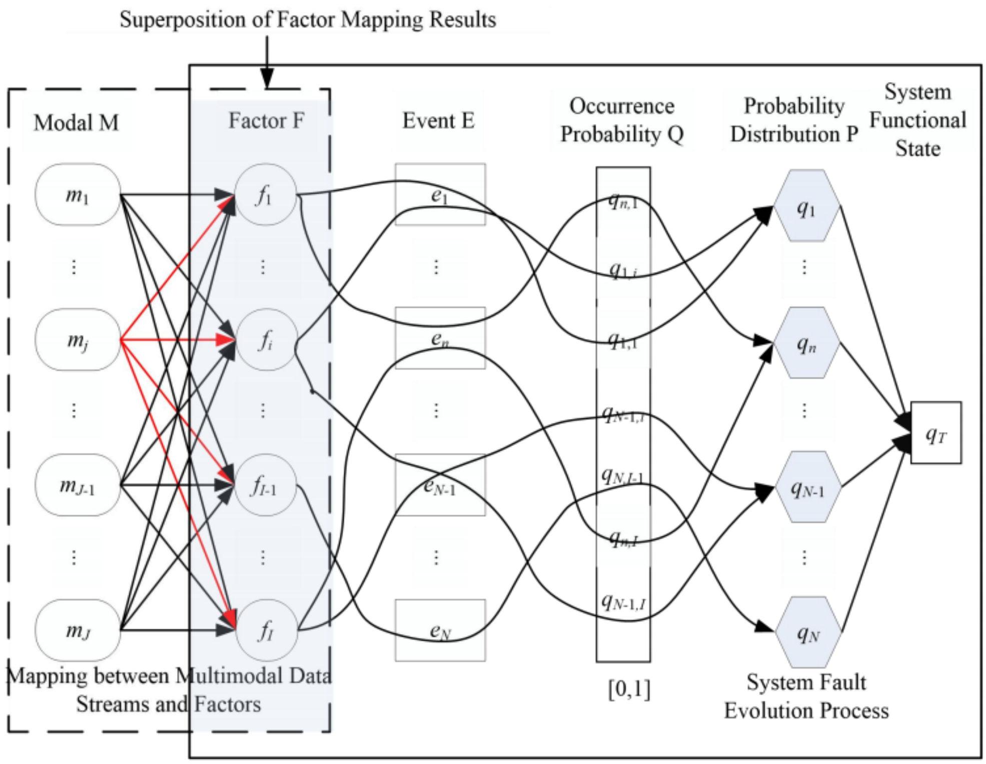

Figure 1. System fault evolution process based on multi-modal data.

A factor mapping model of single-modal data flows is established. Taking m_j in **Figure 1** as an example, a mapping model is constructed. m_j is mapped to all factors $f_1, \ldots, f_I$. Meanwhile, for data alignment, the data flow of m_j is recorded with time *t* as calibration. The factor mapping model of single-modal data flows is shown in Equation (1).

$$m_j(t) = \begin{cases} m_j(f_1, t) \\ \vdots \\ m_j(f_I, t) \end{cases} \tag{1}$$

Equation (1) is the factor mapping model for a unimodal data flow. $m_j(t)$ is the *j*-th time-calibrated unimodal data flow; $m_j(f_i, t)$ is its component on factor f_i; non-zero means relevant, zero means irrelevant. $m_j(t)$ represents the data flow of the *j*-th modality calibrated by time *t*. The object is monitored and measured continuously as time *t* changes, forming an ordered data set, namely the $m_j(t)$ data flow. The $m_j(t)$ data flow may be related to all factors $f_1, \ldots, f_I$ or some of them. The data part of $m_j(t)$ related to factor f_i forms the component of $m_j(t)$ on f_i, denoted as $m_j\left(f_i, t\right)$, which becomes the f_i factor value data flow. Furthermore, the components of $m_j(t)$ on all factors $f_1, \ldots, f_I$ are denoted as $m_j\left(f_1, t\right), \ldots, m_j\left(f_I, t\right)$ in sequence. If $m_j(t)$ is unrelated to f_i, the component of $m_j(t)$ on f_i is $m_j\left(f_i, t\right) = 0$. Otherwise, $m_j(t)$ is related to f_i, and $m_j\left(f_i, t\right) \neq 0$. Finally, the factor mapping model of single-modal data flows in Equation (1) is formed. This shows the correlation between $m_j(t)$ and all factors $f_1, \ldots, f_I$, as indicated by the red directed line segments in **Figure 1**. Similarly, a factor mapping model of multi-modal data flows can be derived, as shown in Equation (2).

$$\begin{cases} m_1(t) = \begin{cases} m_1(f_1, t) \\ \vdots \\ m_1(f_I, t) \end{cases} \\ \vdots \\ m_J(t) = \begin{cases} m_J(f_1, t) \\ \vdots \\ m_J(f_I, t) \end{cases} \end{cases} \Rightarrow \begin{cases} \mathit{\Gamma}_1(f_1(t)) = \mathit{\Theta}_{j=1}^{J} \mathit{\Gamma}_1(m_j(f_1, t)) \\ \vdots \\ \mathit{\Gamma}_I(f_I(t)) = \mathit{\Theta}_{j=1}^{J} \mathit{\Gamma}_I(m_j(f_I, t)) \end{cases} \tag{2}$$

Equation (2) is the factor mapping model for multimodal data flows. $\mathit{\Gamma}_i(f_i(t))$ is the fused state of factor f_i; $\mathit{\Theta}$ denotes the superposition operator for fusing multimodal mapping components. It should be noted that the binary representation (0 or $\neq 0$) in Equations (1) and (2) is a simplification for clearly defining the fundamental 'related' or 'unrelated' relationship between modalities and factors within the conceptual and topological models. In practical applications, this mapping relationship is far from binary. Specifically, when modality m_j is unrelated to factor f_i, the mapped value $m_j(f_i, t) = 0$; when they are related, $m_j(f_i, t)$ is not merely a non-zero flag but a quantity that varies continuously within a specific numerical range. It represents the quantified information attributable to a factor f_i, extracted from the raw data flow $m_j(t)$. This value can be a direct physical reading (e.g., temperature, pressure) or a numerical feature after extraction (e.g., spectral features, texture features), and its variation reflects the dynamic evolution of the factor's state. Therefore, the essence of mapping is to parse multimodal data flows into a series of continuously varying factor data flows, providing rich and interpretable inputs for subsequent superposition analysis.

According to the factor mapping model of single-modal data flows, mapping relationships are established between $m_j(t)$, ($j = 1, \ldots, J$) and all factors $f_1, \ldots, f_I$, respectively. Modalities are divided according to data collection methods, forming a multi-modal data flow; factors are divided according to their roles, forming a multi-factor system. This forms a many-to-many mapping relationship between the modal layer M and the factor layer F in **Figure 1**. If all $m_j\left(f_i, t\right)$ are related to all $f_1, \ldots, f_I$, there will be $J \times I$ mappings. At this time, the correlation between modalities and factors is considered to determine the specific value of $m_j\left(f_i, t\right)$. All $m_j\left(f_i, t\right)$ ($j = 1, \ldots, J,\ i = 1, \ldots, I$) are data flows calibrated by time *t*, thus achieving alignment of all data flows $m_j\left(f_i, t\right)$.

Decomposing multi-modal data flows into single-modal data flows is straightforward. However, further splitting single-modal data flows into factor value data flows is uncertain. Clear factors are provided in the three examples at the beginning of this section, so single-modal data flows can be split directly. If factors are unclear, factor value data flows can be established by analyzing the correlation between events and factors focused on in the research. The mapping between multi-modal data flows and factors can also be realized using algorithms such as neural networks and deep learning. Multi-modal data flows are used as input data, and factor value data flows are used as output data for learning to establish a mapping network structure.

The identifiability of the factor mapping process $m_j \mapsto \{m_{j,i}\}$, i.e., the conditions under which this decomposition is unique or well-defined, is an important theoretical issue. It relates to how to uniquely invert the independent contributions of each factor from the observed data. In practical applications, ensuring identifiability typically requires satisfying one or a combination of the following conditions:

Factor Orthogonality Assumption: The influences of different factors f_i and f_k ($i \neq k$) on the same modality m_j are linearly independent. This implies that the components $m_j(f_i, t)$ and $m_j(f_k, t)$ extracted from the data flow $m_j(t)$, representing different factors, are orthogonal or approximately orthogonal in the function space.

Modality Specificity Assumption: Certain factors are sensitively reflected only by specific modalities. For example, a factor f_1 may be primarily reflected in modality m_1 and have a weak or zero response in others. This introduces a sparsity structure into the mapping matrix.

Prior Knowledge or Constraints: Utilizing domain knowledge about the system's physical mechanism to impose constraints on the form or interrelationships of the factor components can narrow the solution space, thereby facilitating the attainment of a unique solution.

If the above conditions are not met, the decomposition may not be unique, with multiple factor assignment schemes capable of explaining the observed data. In such cases, additional regularization terms or inference within a probabilistic framework may be required. When applying this framework, one should combine it with the specific physical context of the problem to assess and, as far as possible, satisfy the aforementioned identifiability conditions to ensure the reliability and interpretability of the factor analysis results.

4. Superposition of factor mapping results

The above process solves two major problems: consistency of multi-modal data flows and mapping between data flows and factors. Based on the obtained mapping relationship between multi-modal data flows and factors, multiple different $m_j(f_i, t)$ values will appear for the factor f_i. This is because multiple single-modal $m_j(t)$ are correlated with f_i. Therefore, the relationship between f_i and multiple single-modal $m_j(t)$ needs to be further determined. In other words, in the data flow, multiple $m_j(f_i, t)$ explain f_i under the same time calibration, and these explanations obviously have coordination and contradiction issues. That is, the superposition mode of mapping results $m_j(f_i, t)$ of multiple $m_j(t)$ to factor f_i.

Superposition requires analysis based on the characteristics of $m_j(f_i, t)$ data flows to determine a suitable superposition mode, finally forming the derived formula on the right side of Equation (2). $\mathit{\Gamma}_i(f_i(t)) = \mathit{\Theta}_{j=1}^{J} \mathit{\Gamma}_i(m_j(f_i, t))$ represents the data flow superposition form of all $m_j(f_i, t)$ $(j = 1, \ldots, J)$ related to the factor f_i. In the formula, $\mathit{\Gamma}_i()$ denotes the equivalent formula between $f_i(t)$ and all $m_j(f_i, t)$. $\mathit{\Theta}$ denotes the superposition mode of factor mapping results, which traverses all $m_j(f_i, t)$ $(j = 1, \ldots, J)$ in this superposition mode.

Based on the mapping relationship established in Section 3, for multiple unimodal data flow components $m_j(f_i, t)$ (j = 1,..., J) mapped to the same factor f_i, their superposition process can be uniformly expressed as the action of a superposition operator Θ_i, $\Gamma_i(f_i(t)) = \Theta_i(m_1(f_i,t), m_2(f_i,t), ..., m_J(f_i,t))$. The specific form of the superposition operator Θ_i depends on the physical nature of the factor f_i, the relationship among the data flow components $m_j(f_i,t)$, and the analysis objectives. Common Θ_i operators need to satisfy certain mathematical properties, for example:

The scalar, vector, and max–min superposition frameworks are unified as specific implementations of the general superposition operator Θ_i. In practice, the appropriate form is selected according to the physical attribute of the factor: scalar superposition for scalar factors, vector superposition for vector factors, and max–min superposition for consistent measurements.

Additivity/Linearity: When the contributions of each component to the factor are independent and can be linearly added, Θ_i manifests as a summation operation, i.e., scalar superposition.

Vector Composition Rules: When the factor itself is a vector quantity, and each component represents an action in a different direction, Θ_i must follow vector composition rules (e.g., the parallelogram law).

Extreme Value Selection: When it is necessary to determine a representative value from multiple potentially inconsistent measurements, Θ_i can manifest as taking the maximum or minimum operation.

Continuity: To ensure that the superposition result is insensitive to small perturbations in the input data, Θ_i should generally be continuous.

Monotonicity: In some application scenarios, Θ_i may need to satisfy monotonicity, meaning that an increase in any input component does not lead to a decrease in the output result.

In practical applications, corresponding mathematical definitions and constraints need to be assigned to Θ_i based on the specific physical background and analysis requirements. The following text will elaborate on three typical cases, specifically explaining the scalar, vector, and max-min superposition modes and their corresponding implementations of Θ_i.

Current superposition modes mainly include scalar superposition, vector superposition, and max-min superposition.

Scalar superposition is mainly used when multiple single-modal data flows in a multi-modal data flow involve the same factor, and these single-modal data flows are additive for this factor. For example, the light energy is released by the flame of a burning substance. The wavelengths of light released by different burning substances are different, so the energy calculated at different wavelengths is additive.

As shown in **Figure 2**, four video recording devices are set in the same direction as the burning substance. Each device only records images within its respective wavelength range. Devices 1–4 provide video image data flows $m_1(t)$, $m_2(t)$, $m_3(t)$ and $m_4(t)$, respectively, form a multi-modal data flow, with data aligned by shooting time *t*. For the correlation between the multi-modal data flows $m_1(t)$, $m_2(t)$, $m_3(t)$, $m_4(t)$ and the wavelength λ factor, factor λ value data flows $m_1(\lambda_1,t)$, $m_2(\lambda_2,t)$, $m_3(\lambda_3,t)$, and $m_4(\lambda_4,t)$ are obtained, respectively. Obviously, the energies of different wavelengths are independent and can be directly summed. The equivalent formula $\varGamma_i()$ is the energy calculation formula $E = \frac{hc}{\lambda}$ kJ. Here, h is the Planck constant, $h = 6.62 \times 10^{-34}$/J · s; c is the speed of light, $c = 3 \times 10^8$ m/s; λ is the wavelength, in um. Then $m_1(\lambda_1, t)$, $m_2(\lambda_2, t)$, $m_3(\lambda_3, t)$, and $m_4(\lambda_4, t)$ are data flows of different wavelengths in flame images obtained by different devices. The total light energy of the burning substance in the observation direction is $E_{\text{total}} = \sum_{j=1}^{4} \frac{hc}{m_j(\lambda,t)}$, namely $\varGamma(f(t)) = E_{\text{total}}(\lambda(t)) = \sum_{j=1}^{4} \frac{hc}{m_j(\lambda,t)} = \varTheta_{j=1}^{4} \varGamma(m_j(\lambda, t))$. $\varTheta$ represents scalar superposition, namely the summation operation. Therefore, scalar superposition is suitable for cases of the same factor with different ranges.

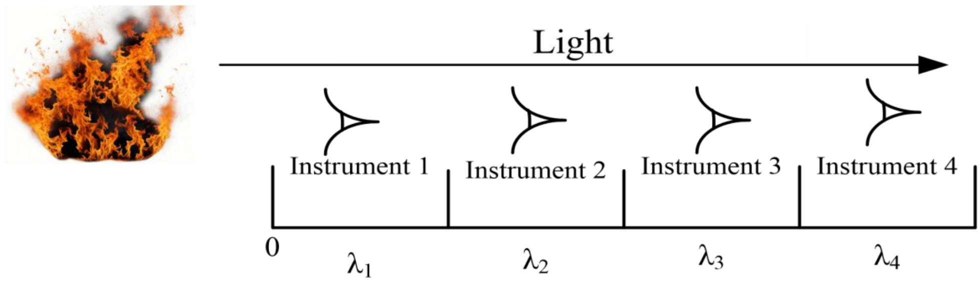

Figure 2. Scalar superposition.

Vector superposition is also common, such as the superposition of force actions, as shown in **Figure 3**. In **Figure 3**, an object on a plane is acted upon by two forces F_1 and F_2. The magnitudes of these two forces are displayed by spring dynamometers. Two video recording devices are set to record the changes in these two forces at different times. They provide video image data flows $m_1(t)$ and $\boldsymbol{m}_2(\boldsymbol{t})$, respectively, form a multi-modal data flow, with data aligned by shooting time *t*. For the correlation between the multi-modal data flows $m_1(t)$ and $\boldsymbol{m}_2(\boldsymbol{t})$ and the spring reading *x* factor, factor value data flows $m_1(x_1, t)$ and $\boldsymbol{m}_2(\boldsymbol{x}_2, t)$ are obtained, respectively. Obviously, forces in different directions on the plane are superimposed as vectors. The equivalent formula $\varGamma_i()$ is $F = kx$, where *x* is the spring elongation and *k* is the spring stiffness coefficient. Then the forces in the two observation directions are $F_1 = k_1 m_1(x_1, t)$ and $F_2 = k_2 m_2(x_2, t)$, respectively. The resultant force on the object is $F_{\text{total}} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha}$, namely $\varGamma(f(t)) = F_{\text{total}}(f(t)) = \varTheta_{j=1}^{2} \varGamma(m_j(x, t))$. $\varTheta$ represents vector superposition. Therefore, vector superposition is suitable for characterizing the combined effect of vector factors.

In the third case, max-min superposition can be used when the same property of the same object is measured through different single-modal data flows. For example, flame temperature can be measured by contact thermometry (using metal wires, thermocouples, etc.) and optical measurement (non-contact thermometry, such as the sodium line self-absorption method, relative emission intensity method of spectral lines, relative atomic absorption intensity method). These methods obtain single-modal data flows through different devices, which together form a multi-modal data flow. In practice, the flame temperatures measured by these methods and devices may be different, and the maximum or minimum value can be selected according to actual needs. In this process, the equivalent formula $\varGamma_i()$ is the function between instrument readings and flame temperature, and $\varTheta$ is a max or min superposition. Max-min superposition is suitable for cases where different single-modal data flows represent the same factor of the same object.

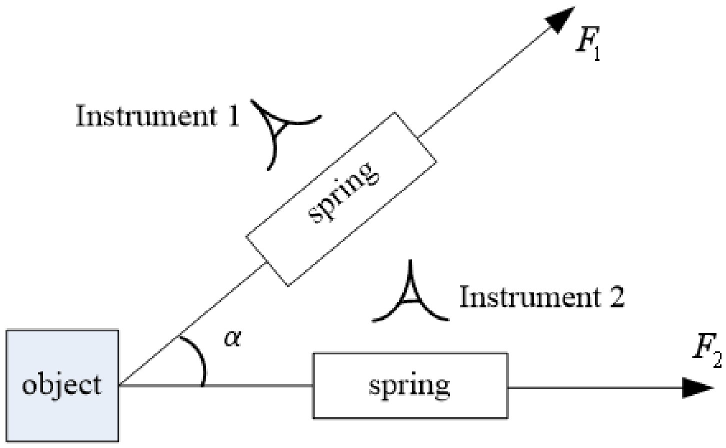

Figure 3. Vector superposition.

In practice, many forms of mapping results from multi-modal data to factors exist, not limited to the above three superposition modes. $\Gamma_i()$ and Θ can be determined according to actual physical meanings. To ensure system robustness, the superposition operator Θ should satisfy Lipschitz continuity to control the output error caused by input perturbation. The above research realizes the content in the dashed box in **Figure 1**. Its important significance is to advance the research of system fault evolution theory from the front-end factor layer to the modal layer of multi-modal data flows. Furthermore, the factor mapping and superposition processes can be embedded into a continuous-time dynamic system model, which provides a theoretical basis for state estimation, prediction and optimal control based on differential equations. This is an important extension of system fault evolution theory, establishing the correlation between data and factors. On the other hand, multi-modal data flows are the data form of current practical disaster and accident monitoring systems, which are the direct source of data. Multi-modal analysis methods can also be applied. These are conducive to the analysis, prediction, early warning, and intervention of system fault evolution.

To theoretically analyze the robustness of the proposed framework, we consider the mathematical properties of the superposition operator Θ_i and its response to input perturbations. Assume the mapped factor data flow components $m_j(f_i, t)$ belong to a certain function space, e.g., $L^p(T)$. If the superposition operator Θ_i is Lipschitz continuous, i.e., there exists a constant $L_i > 0$ such that for any two sets of inputs $\{m_j^{(1)}(f_i,t)\}$ and $\{m_j^{(2)}(f_i,t)\}$, we have $\|\Theta_i(\{m_j^{(1)}(f_i,t)\}) - \Theta_i(\{m_j^{(2)}(f_i,t)\})\| \leq L_i \sum_{j=1}^{J} \|m_j^{(1)}(f_i,t) - m_j^{(2)}(f_i,t)\|$, where $\|\cdot\|$ denotes the norm in the $L^p(T)$ space. Then, when the input data is perturbed by $\epsilon_{j,i}(t)$, i.e., the observed data is $\widetilde{m}_j(f_i,t) = m_j(f_i,t) + \epsilon_{j,i}(t)$, the error between the estimated final factor state $\widetilde{\Gamma}_i(f_i(t)) = \Theta_i(\{\widetilde{m}_j(f_i,t)\})$ and the true value $\Gamma_i(f_i(t)) = \Theta_i(\{m_j(f_i,t)\})$ satisfies the stability estimate $\|\widetilde{\Gamma}_i(f_i(t)) - \Gamma_i(f_i(t))\| \leq L_i \sum_{j=1}^{J} \|\epsilon_{j,i}(t)\|$.

This result indicates that as long as the superposition operator satisfies Lipschitz continuity, the entire mapping and superposition process is stable with respect to small perturbations in the input data, and the output error can be controlled by a weighted sum of the input errors. This provides a theoretical foundation for the framework to handle measurement noise and data uncertainty in practical applications. In practice, when designing or selecting Θ_i, the Lipschitz property should be considered to ensure system robustness.

The aforementioned superposition models describe the synthesis of factor states under static or quasi-static conditions. From a dynamic system and control perspective, the evolution of factor states $\Gamma_i(f_i(t))$ itself can be modeled as a controlled dynamical system. Consider the factor state vector $x(t) = [\Gamma_1(f_1(t)), \ldots, \Gamma_I(f_I(t))]^T \in \mathbb{R}^I$. Its change over time can be described by a differential equation, $\frac{dx(t)}{dt} = F(x(t), u(t), t)$, where F is a function describing the internal dynamics and external disturbances of the system, and $u(t)$ represents the control input or driving term aggregated from the mapping results of multimodal data flows $\{m_j(f_i, t)\}$ via the superposition operators Θ_i. Specifically, the i-th component of $u(t)$, $u_i(t)$, can be defined as $u_i(t) = \Theta_i(m_1(f_i, t), \ldots, m_J(f_i, t))$, which is the instantaneous target or disturbance value for $\Gamma_i(f_i(t))$.

Furthermore, the system fault evolution process can be viewed as the state trajectory of this dynamical system driven by the control input $u(t)$. The system function state T_q can then correspond to the output of this dynamical system at a specific time t_f, $y(t_f) = H(x(t_f))$, where H is an observation function.

To more concretely demonstrate the application of differential equations, consider a linearized approximation model, $\frac{dx(t)}{dt} = Ax(t) + Bu(t) + w(t)$, where A is the system matrix describing the coupling between factors; B is the input matrix describing the influence of superposition results on the rate of change of factor states; and $w(t)$ is process noise. This model provides a direct foundation for state estimation and prediction using algorithms such as the Kalman filter. Moreover, from a control theory perspective, control objectives can be defined, such as making the system state $x(t)$ track a desired trajectory $x_{des}(t)$, or optimizing a performance index $J = \int_{t_0}^{t_f} L(x(t), u(t), t)dt$ related to the system function state T_q. This naturally leads to optimal control problems.

This framework naturally embeds the factor mapping and superposition processes into a continuous-time dynamic model, providing a theoretical foundation for differential equation-based system analysis, state estimation, and optimal control, and establishing a rigorous mathematical bridge from multimodal data flows to system dynamic behavior.

5. Role of factor mapping of multi-modal data flows in system fault evolution

The system fault evolution process theory is a theoretical system proposed by the authors to describe the change in the ability of a system to achieve predetermined functions during operation [23]. It includes three levels. The system fault evolution is a conceptual model describing the system fault process. The spatial fault network is an abstract network topological model based on the conceptual model. A mathematical calculation model is formed based on the topological structure to realize qualitative and quantitative analysis.

The system fault evolution conceptual model describes the system fault evolution process, including four concepts: experienced events, influencing factors, logical relationships, and evolution conditions. Experienced events exist in the event layer in **Figure 1** and are landmark states or behaviors of the evolution process. Events are divided into basic cause events, process events, and final result events. The final result event represents the outcome of system evolution, namely the system function state, as shown in **Figure 1**. The system function state is a snapshot of the system fault evolution process at a certain moment. The evolution process at different moments is characterized by the time-calibrated system function state. This is consistent with data alignment achievable by multi-modal data flows. Influencing factors are in the factor layer in **Figure 1**. Influencing factors act on events and can change the occurrence state of events, so they are also the driving force of system fault evolution. Logical relationships are the comprehensive modes of action between events, such as Boolean logic and flexible logic [27]. Evolution conditions are the possibility of cause events leading to result events.

The spatial fault network is a network topological structure formed by abstracting the evolution process. Events are represented by rectangular symbols, influencing factors by circular symbols, logical relationships are implied in the subscripts of rectangular symbols, and evolution conditions are represented by directed line segments. **Figure 1** is a complete schematic diagram of the system fault evolution process, not a spatial fault network diagram. In **Figure 1**, the modal layer is represented by elliptical symbols, and the factor layer by circular symbols, which are the main research objects of this paper. The occurrence probability layer has a value range of [0,1]. Event occurrence probability distribution is represented by hexagons, and system function state by squares.

The mathematical model mainly includes characteristic functions, event occurrence probability distribution, and system function state distribution. The characteristic function is a function of the change in event occurrence probability caused by the influence of a certain factor, namely the curved part between the factor layer and the occurrence probability layer in **Figure 1**. The event occurrence probability distribution is formed by superposing the characteristic functions of relevant influencing factors, obtaining a distribution surface in the space constructed by these factors. According to the four-element relationship of the system fault evolution process, the system function state expression is finally obtained, namely the occurrence probability distribution of the final result event. A mathematical model system is formed based on the above concepts.

The research results of this paper extend the system fault evolution theory in the above three aspects. The conceptual model adds modal concepts to describe single-modal and multi-modal data. The topological structure model adds a modal layer outside the factor layer, realizing a complete many-to-many mapping between multi-modal data and multi-factors. The mathematical model adds a mathematical model for the superposition of factor mapping results. Therefore, its important significance is to expand the original research process from factors to system function state to the research process from multi-modal data flows to system function state.

Figure 4 is divided into four layers. The data flow layer represents multi-modal data, which and the factor layer are the research focus of this paper. The basic cause event and final result event layers are used to study the specific system fault evolution process, which is not discussed in this paper and is only provided for process integrity. The modal layer represents the data recording forms and variation ranges of different devices. Some factor values are fixed and do not require time calibration; others are variable ranges and require time calibration. The main data recording devices include antenna equipment, radar equipment, and signal detection equipment. The obtained data flows are $m_1(t)$ (single-modal data flow of antenna equipment), $m_2(t)$ (single-modal data flow of radar equipment), and $m_3(t)$ (single-modal data flow of signal detection equipment), respectively. The formed multi-modal data flow is $M = \{m_1(t), m_2(t), m_3(t)\}$. $m_1(t)$ is subdivided into $m_1(\theta_1)$, $m_1(R_1, t)$, and $m_1(\theta_2, t)$ according to factor correlation. The factors represent the antenna azimuth coverage range θ_1/°, elevation coverage range θ_2/°, and detection range R_1/km, respectively. $m_2(t)$ is subdivided into $m_2(f)$, $m_2(\varDelta f)$, $m_2(f_r)$, and $m_2(\varDelta f_r)$ according to factor correlation. The factors represent the monitoring equipment frequency coverage range f/MHz, instantaneous operating bandwidth $\varDelta f$/MHz, radar frequency band f_r/MHz, and radar instantaneous operating bandwidth $\varDelta f_r$/MHz, respectively. $m_3(t)$ is subdivided into $m_3(P_m, t)$, $m_3(P_0, t)$, and $m_3(T_0, t)$ according to factor correlation. The factors represent the loss probability P_m caused by signal overlap, coincidence probability P_0 at any time, and coincidence period T_0 (min), respectively. Among the above factor value data flows, time variables are removed for fixed values. In addition, the maximum detection range of enemy radar R_0/km, UAV(Unmanned Aerial Vehicle) operating time T/min, and information processing failure probability q_I are measured by other devices and not included in the analysis.

To analyze the denoising potential of mapping and superposition from a mathematical structure, consider the model: $\widetilde{m}(t) = m(t) + n(t)$, where $m(t)$ is the true signal and $n(t)$ is zero-mean noise. Assume $m(t)$ can be decomposed into a sum of I uncorrelated factor components: $m(t) = \sum\limits_{i=1}^{I} m_i(t)$. Compare two estimation strategies: Global Estimation (GE): Design an optimal linear estimator W_g directly for $\widetilde{m}(t)$, yielding $\widehat{m}_g(t) = W_g \circ \widetilde{m}(t)$. Its minimum MSE is $\mathcal{E}_g^{min} = \int \frac{S_m(\omega) S_n(\omega)}{S_m(\omega) + S_n(\omega)} d\omega$. Mapping-Superposition Estimation (MSE): First, map the observation to each factor space via orthogonal operators $\{\varPi_i\}$: $\widetilde{m}_i(t) = \varPi_i \circ \widetilde{m}(t) = m_i(t) + n_i(t)$. Then, design optimal estimators W_i for each $\widetilde{m}_i(t)$, yielding $\widehat{m}_i(t) = W_i \circ \widetilde{m}_i(t)$, and finally superimpose: $\widehat{m}_s(t) = \sum\limits_{i=1}^{I} \widehat{m}_i(t)$. Its minimum MSE is $\mathcal{E}_s^{min} = \sum\limits_{i=1}^{I} \int \frac{S_{m_i}(\omega) S_{n_i}(\omega)}{S_{m_i}(\omega) + S_{n_i}(\omega)} d\omega$.

Comparing $\mathcal{E}_g^{min}$ and $\mathcal{E}_s^{min}$ reveals potential advantages of the MSE strategy: Directed Noise Suppression: If the noise $n(t)$ contributes weakly in some factor subspaces (i.e., $S_{n_i}(\omega)$ is small), the corresponding error term is significantly reduced. Refined Use of Signal Prior: Factor decomposition provides a more structured prior than $S_m(\omega)$, allowing each sub-estimator W_i to be more focused.

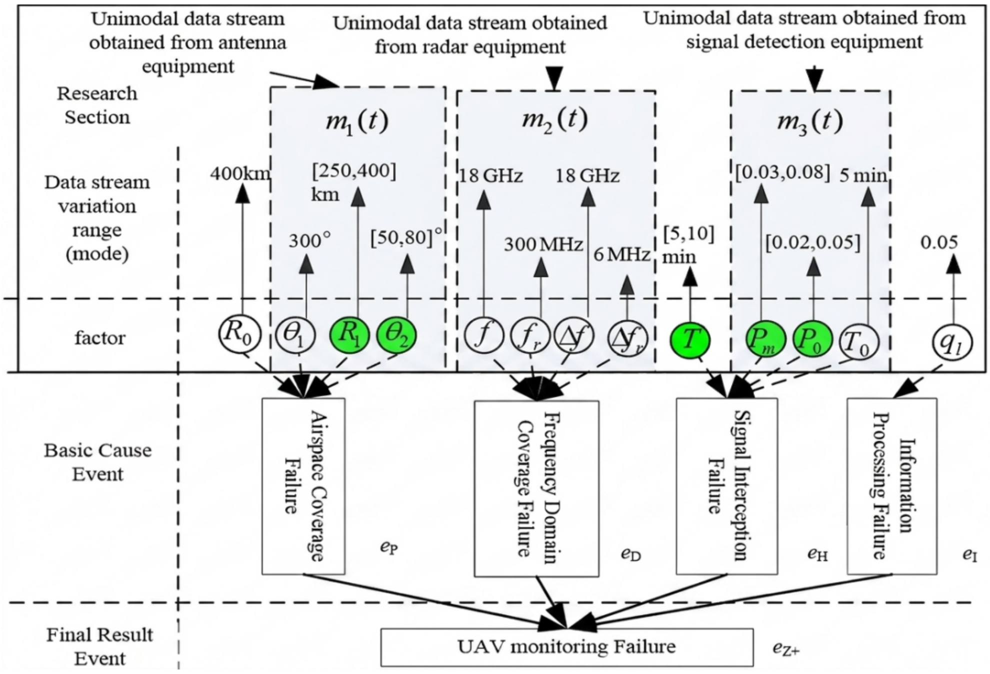

Figure 4. Fault process of unmanned monitoring aircraft.

Thus, when the signal is decomposable and noise is unevenly distributed across factor spaces, the mapping-superposition strategy can theoretically achieve superior denoising performance. This structurally reveals the potential of our framework.

Based on the dynamic system model proposed in Section 4, we can provide a deeper mathematical characterization of the aforementioned UAV fault evolution case. Key factors are identified, e.g., f_1: Engine Performance Factor, f_2: Communication Link Factor, f_3: Navigation System Factor. The corresponding factor state vector is $x(t) = [\varGamma_1, \varGamma_2, \varGamma_3]^T$.

Various modal data flows (vibration, temperature, electromagnetic signal, GPS data, etc.) are transformed into data flow inputs $m_j(f_i, t)$ for each factor via mapping operators $\varPhi_{j,i}$, and then aggregated into control inputs $u_i(t) = \varTheta_i(...)$ via case-specific superposition operators $\varTheta_i$ (e.g., weighted average, logical AND). For instance, the control input $u_1(t)$ for the Engine Performance Factor f_1 might be composed of the weighted fusion result of vibration and temperature data.

Assuming the system behavior can be approximated as linear during the initial fault evolution phase, its dynamics can be described by the following differential equation:

$$\frac{dx(t)}{dt} = \begin{bmatrix} -a_1 & 0 & 0 \\ 0 & -a_2 & 0 \\ 0 & 0 & -a_3 \end{bmatrix} x(t) + \begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \end{bmatrix} u(t) + w(t).$$

Here, the negative elements $(-a_i)$ of the diagonal matrix A indicate a natural decay trend for each factor state in the absence of external input (e.g., gradual performance degradation); the diagonal matrix B indicates that the control input for each factor directly affects its own state's rate of change; $w(t)$ represents unmodeled dynamics and noise.

The communication link interruption event can be modeled as a step decrease (e.g., dropping near zero) in the input $u_2(t)$ at time t_{event}, causing $\frac{dx_2}{dt}$ to become negative, and thus leading to a rapid decay of $x_2(t)$ (the Communication Link Factor state). By solving this system of differential equations, the trajectory of the factor states $x(t)$ over time can be simulated. Furthermore, through an observation function H (e.g., a setting that when $x_2(t)$ falls below a threshold θ_{comm}, the probability of the communication interruption event q_{event} increases significantly), it can be linked to changes in the system function state T_q.

This analysis demonstrates the possibility of transforming specific multimodal monitoring data, through the mapping and superposition framework, into the input $u(t)$ of a dynamic system model, and subsequently utilizing differential equation tools to model, simulate, and predict the system fault evolution process.

The above analysis mainly focuses on the mapping between multi-modal data flows and factors. Multi-modal data flows come from antenna equipment, radar equipment, and signal detection equipment. The single-modal data flows of these devices correspond to airspace coverage failure events, frequency domain coverage failure events, and interception information failure events, respectively. The monitoring failure event of the unmanned monitoring aircraft depends entirely on the occurrence of these events. Of course, it is also related to R_0, T, and q_1, which are data flows provided by other devices. In addition, since there are no repeated factors among these factors, the superposition process of factor mapping results is not involved. However, the three examples given in Section 3 can also illustrate the superposition process of mapping results of the same factor from different single-modal data flows. After all factor value data flows are formed, calculating event occurrence probability and system function state is the work of the existing mathematical model of the system fault evolution process. Based on the dynamic system model, the UAV fault process can be further modeled by linear differential equations to simulate the time trajectory of factor states and describe the occurrence mechanism of fault events, such as communication interruption. This part has been initially realized and can be found in literature [26,27], so it is not discussed here.

The main significance of this paper is to provide multi-faceted data interfaces for system fault process research. At present, diverse and powerful monitoring devices exist in fields such as natural disaster monitoring, industrial production processes, public safety, and personnel operations. These devices respectively obtain single-modal data, forming multi-modal data flows. Meanwhile, single-modal data flows contain multiple factors, and a single factor may also appear in different single-modal data. This forms a full mapping relationship between multi-modal data flows and multiple factors. In subsequent system fault process research, monitoring devices can be used to start analysis from multi-modal data flows, replacing the original process of analyzing system faults from factors. This helps integrate multi-modal analysis methods such as deep learning and improves the accuracy and efficiency of system fault process analysis.

6. Conclusion

1) Multi-modal data contain various types of information and are more comprehensive and abundant. Disaster and accident data exhibit multi-modal characteristics. Multi-modal data flows play a key role in disaster early warning and emergency response. However, challenges such as data flow consistency, factor mapping, and action superposition remain.
2) Mapping between multi-modal data flows and factors is investigated. A multi-modal data flow consists of multiple single-modal data flows, which further contain multiple factor value data flows describing the object state from different aspects. Factor mapping models for single-modal and multi-modal data flows are established. Data flows are mapped to all relevant factors through time calibration, realizing the conversion from multi-modal data flows to factor value data flows. This improves data processing efficiency and accuracy, cross-modal understanding and generation, optimizes performance and reduces training costs, and supports decision-making under complex conditions.
3) Superposition modes of mapping results between multi-modal data flows and factors are investigated. The relationship between the results of multiple single-modal data flows mapped to the same factor must be considered to solve coordination and contradiction issues. Superposition modes include scalar, vector, and max-min superposition. Scalar superposition is suitable for cases of the same factor with different ranges. Vector superposition is suitable for characterizing the combined effect of vector factors. Max-min superposition is suitable for cases where different single-modal data flows represent the same factor of the same object.
4) Factor mapping and result superposition of multi-modal data flows play an important role in system fault evolution research. This study extends the conceptual model, topological structure model, and mathematical model on the original basis. Contents such as the multi-modal data flow concept, complete many-to-many mapping between multi-modal data and multi-factors, superposition of factor mapping results, robustness analysis under Lipschitz continuity, dynamic system modeling with differential equations, and theoretical denoising performance comparison are added. It advances system fault evolution research and facilitates the analysis, prediction, early warning, and intervention of system faults. The case analysis demonstrates the application of mapping between multi-modal data flows and factors in UAV monitoring failure analysis, and realizes dynamic simulation of fault evolution through linear differential equation modeling. The results can provide multi-faceted data interfaces for system fault process research, integrate multi-modal analysis, and improve accuracy and efficiency.

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