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Article

A multi-stage decision-making model for urban fire emergency with multi-granularity uncertain linguistic information and prospect theory

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Abstract: Existing fire emergency decision-making models often struggle with accurately handling multi-granularity uncertain linguistic information, loss aversion, and a lack of adaptability to dynamic fire evolution. To address these gaps, this study adopts two-tuple linguistic representation (2TLR) for quantifying multi-granularity linguistic information and combines the Analytic Hierarchy Process (AHP) with the entropy weight method (EWM) to determine the ability weights of the experts. Furthermore, a six-dimensional dynamic reference point is generated via the random forest algorithm, and the integration of prospect theory (PT) with a sequential decision-making framework (SDF) is implemented for the dynamic optimization of response plans. Validation through real-world cases demonstrates that the proposed Multi-stage Prospect Selection (M-PS) model outperforms both the TOPSIS method and the single PT model, compared with these two methods, the proposed M-PS model can effectively prioritize the avoidance of high-risk scenarios, accurately reflect decision-makers' loss aversion tendency, and realize dynamic decision-making through updating the decision plan sequentially, thereby providing reliable support for urban fire emergency management. At the same time, in this study we conduct a comparative analysis of core metrics between existing methods and the proposed M-PS model. The evaluation across five dimensions demonstrates that the proposed M-PS model delivers superior performance.

Keywords: multi-granularity uncertain language; prospect theory; multi-stage decision-making; two-tuple linguistic model; random forest; urban fire emergency; dynamic reference point

1. Introduction

In the context of accelerating urbanization, fire is a potential risk that seriously threatens people's life and property safety and urban sustainable development. Furthermore, the uncertainty and complexity of information in fire rescue operations are also constantly evolving. With the continuous advancement of smart firefighting, many advanced technologies have been introduced in the field.

Multi-granularity uncertain linguistic information involves expressing uncertain information using linguistic terms at various levels of granularity, thereby enabling more flexible quantification and processing that aligns closely with human cognition. Wang [1] resolved cross-domain linguistic heterogeneity; Jin et al. [2] introduced "weakening modifiers" to adapt linguistic intensity, and Zhao et al. [3] proposed an entropy method for unbalanced granularity, breaking the uniform granularity assumption. Wei and Liao [4] proposed a multigranularity linguistic group decision-making method based on hesitant 2-tuple sets, enhancing the flexibility of linguistic information processing. Zhang and Guo [5] proposed a method for such group decision-making with incomplete weight information. Herrera et al. [6] presented a fusion approach for managing multi-granularity linguistic term sets in decision making. Cai et al. [7] studied the consistency measures of such group decision-making. Xu and Wang [8] gave an overview of managing multi-granularity linguistic information in qualitative group decision-making. Chen and Ben-Arieh [9] explored the fusion of multi-granularity linguistic label sets in group decision-making.

Prospect Theory [10] and Cumulative Prospect Theory (CPT) explain bounded rationality in risk decision-making, emphasizing a focus on changes in gains and losses rather than absolute levels. Werner and Zank [11] proposed an endogenous reference point identification method via probability midpoint consistency, addressing traditional PT's limitation and laying a rigorous foundation for its testing. Häckel et al. [12] applied CPT to quantify the energy efficiency gap, revealing that behavioral biases like loss aversion drive underinvestment, with CPT outperforming Expected Utility Theory. Ruggeri et al. [13] validated PT's core patterns through a multinational replication study, confirming cross-cultural robustness of loss aversion and framing effects. Kavya and Christopher [14] extended PT to interpretable decision systems, enhancing its practical applicability. These studies reinforce PT/CPT's enduring relevance across theoretical and applied domains. Levy [15] provided a foundational introduction to prospect theory within political psychology, contextualizing its implications for understanding political decision-making under risk. Barberis [16] offered a comprehensive review and assessment of prospect theory's 30-year impact on economics, synthesizing its theoretical evolution and empirical validation. Edwards [17] conducted a targeted literature review of prospect theory in finance, highlighting its applications to asset pricing and investor behavior. Bromiley [18] critically examined prospect theory's role in strategic management, evaluating its explanatory power for firm risk-taking and organizational outcomes. Collectively, these reviews underscore the breadth of PT/CPT's influence across diverse social science disciplines.

Multistage decision-making refers to a decision process that requires a sequential series of interdependent steps or choices, where each stage's decision may influence the available options and overall outcomes in subsequent stages. Liang et al. [19] improved dynamic programming for multi-objective multi-stage decision-making; Azad and Moshkov [20] optimized decision and inhibitory trees for multi-stage problems; Polat et al. [21] designed a multi-stage learning framework combining CNN and a decision tree for attack detection. Additionally, Wang et al. [22] proposed the neighborhood rough decision tree, providing a new tool for multi-stage decision rule extraction; Chen et al. [23] evaluated the emergency rescue capability of urban fire stations; Ma et al. [24] developed a hybrid deep learning model for mountain slope displacement prediction, which provides reference for fire risk pre-warning. Delage and Iancu [25] explored robust multistage decision making. Kacprzyk [26] studied multistage decision making under fuzziness. Gerking [27] modeled multi-stage decision-making processes in multi-period energy models. Hotaling [28] proposed a cognitive model for on-the-fly planning in multistage decision making. Vlek [29] provided a multi-level perspective on risk assessment and decision-making in multi-stage processes.

A review of the literature reveals that existing decision-making models for fire rescue operations suffer from three main limitations.

- **Quantification dilemma of uncertain linguistic expressions:** The fire emergency decision-making process often involves decision-makers' subjective and uncertain linguistic expressions, whose granularity and connotative ambiguity are difficult to quantify precisely.
- **Handling irrational decision-making phenomena in high-risk contexts:** A notable behavioral phenomenon emerges in fire emergency scenarios with high risk and uncertainty—decision-makers often diverge from the assumptions of pure rationality in their choices, with loss aversion being a typical manifestation of such irrational tendencies.
- **Challenges in adaptive decision-making for dynamic and complex fire scenarios:** Fires themselves are highly dynamic and complex, and their development and evolution are affected by multiple factors, making it difficult for static information-based decision models to provide adequate responses.

In view of these limitations, in this study, we aim to construct a refined multi-stage decision-making model for urban fire emergencies with multi-granularity uncertain linguistic information and prospect theory. The model combines three key elements:

- **Multi-granularity uncertain linguistic information processing:** Quantifying uncertainty in subjective judgments using multi-granular uncertain linguistic information, applied to emergency decision-making in fire scenarios.
- **Decision plan selection with prospect theory:** Prospect theory's insights into irrational decision-making during fire emergencies to assist in optimal alternative selection through reference points.
- **Multi-stage sequential decision making:** A sequential decision-making approach to ensure optimal decisions at each stage, adapting to the dynamic nature of fire incidents.

2. Preliminaries

2.1. Consistent conversion of multi-granular linguistic information representation

The scientific formulation of fire plans needs to comprehensively consider factors from multiple dimensions. The multi-attribute decision matrix constructed based on decision-makers, alternatives, and evaluation factors can not only analyze alternative plans from multiple dimensions, enabling decision-makers to examine fire plans from different perspectives and provide a more comprehensive perspective, but also provide a clear structure for the complex decision-making process. Decision-makers can clearly see the performance of each alternative plan under different evaluation factors, facilitating comparison.

2.1.1. Calculation of expert's ability weights with subjective–objective fusion

In the construction of the multi-granular uncertain linguistic matrix, let $E = \{E_1, E_2, \dots E_i\}$ be a decision-making group composed of i interdisciplinary experts including fire engineering, emergency management, and disaster economics, and $\lambda = \{\lambda_1, \lambda_2, \dots, \lambda_s\}$ be the weight of each dimension.

Definition 1. *The expert's ability weights are calculated by the Analytic Hierarchy Process (1), entropy weight methods (2), and (3).* w_{is} *is the score of the* i*-th expert in the* s*-th dimension [30].*

$$w_{i\lambda} = \sum_{s=1}^{z} \lambda_s w_{is} \tag{1}$$

$$H_i = -\frac{1}{\ln k} \sum_{j=1}^{k} p_{ij} \ln p_{ij} \tag{2}$$

$$w_i^{obj} = \frac{1 - H_i}{\sum_{i=1}^{5} (1 - H_i)} \tag{3}$$

2.1.2. Normalization of the ability weights of the experts

Definition 2. *The fusion weight calculation adopts a linear weighted fusion approach, as shown in Equation (4). The subjective weight is assigned a weight of 0.6, and the objective weight is 0.4. Subsequently, a normalization Equation (5) is applied to map the fusion weights to the [0,1] interval, ensuring the sum of the final expert's ability weights is 1 [31].*

$$w_i^{\text{fusion}} = 0.6 \times w_i^{\text{subj}} + 0.4 \times w_i^{\text{obj}} \tag{4}$$

$$\bar{w}_i^{\text{fusion}} = \frac{w_i^{\text{fusion}}}{\sum_{i=1}^{m} w_i^{fusion}} \tag{5}$$

2.1.3. Multi-granular uncertain linguistic terms

Definition 3. *A set of linguistic terms used to express the subjective judgment of decision makers [32]. The multi-granularity uncertainty language given in this paper is shown in* ***Table 1****, and its core lies in:*

Granularity variability: The term set contains multiple levels, allowing the number of terms and the fineness of expression to vary.

Uncertainty: Terms are inherently ambiguous, subjective, and their precise semantics depend on human understanding.

Flexibility: Experts can autonomously choose their own set of terms based on their familiarity with the evaluation dimensions.

Table 1. Multi-granularity nondeterministic languages.

Granularity type	Number of terms	Term set example
Coarse-grained	3	Mild, Moderate, Severe
Medium-grained	4	Very Mild, Mild, Moderate, Severe
Fine-grained	5	Relatively Mild, Mild, Moderate, Severe, Very Severe

2.1.4. Two-tuple semantic model

Definition 4. *The 2-tuple linguistic model quantifies linguistic terms via a 2-tuple* (i, α) *(term index* i*, deviation* α*) with the following core steps [33].*

- ***Cross-granularity mapping****: Map experts'* m *terms to the reference term set's numerical interval, obtain the numerical position* y *under the original granularity, and convert it to* x *in the reference set via linear transformation.*
- ***Solve for*** i*: Determine the benchmark term index* i *as in Equation (6).*

$$i = \lfloor p(s_{b_n}^{a}) \cdot (n-1) + 0.5 \rfloor + 1 \tag{6}$$

  $p(s_{b_n}^{a})$ *is the numerical result of evaluation terms given by experts, and* $s_{b_n}^{a}$ *is the evaluation term of experts.*
- ***Calculate*** α*: The offset is calculated as in Equation (7).*

$$\alpha = \lfloor p(s_{b_n}^{a}) \cdot (n-1) - i \rfloor \tag{7}$$

- ***Result****: Terms are represented as* (i, α)*, where* $i \in \{0, 1, \ldots, n-1\}$ *(or equivalent range) and* $\alpha \in [-1, 1]$.

2.1.5. Multi-attribute same-granularity decision matrix

Definition 5. *To better evaluate decision-making alternatives, in this paper, we propose a dynamic decision matrix under multi-granularity uncertain linguistic environments. An* $n \times m$ *matrix (denoted as* Z^k*, as shown in Equation (8)), where* n *is the number of alternative plans,* m *is the number of evaluation dimensions, and* $k \in K$ *represents the* k*-th decision-maker (*K *is the set of decision-makers). Each element* z_{ij}^{k} *in the matrix denotes the same-granularity numerical evaluation value of plan* i *under dimension* j*, given by the* k*-th decision-maker. These values are processed via multi-granularity linguistic conversion, digitization, and unification, ensuring consistent dimension and granularity.*

$$Z^k = \left(z_{ij}^{k}\right)_{n \times m} = \begin{bmatrix} z_{11}^{k} & z_{12}^{k} & \cdots & z_{1m}^{k} \\ z_{21}^{k} & z_{22}^{k} & \cdots & z_{2m}^{k} \\ \vdots & \vdots & \ddots & \vdots \\ z_{n1}^{k} & z_{n2}^{k} & \cdots & z_{nm}^{k} \end{bmatrix}, \, k \in K \tag{8}$$

2.2. Prospect theory

2.2.1. Value function

Definition 6. *The mathematical expression of the value function of prospect theory is given in Equation (9) [10],*

$$v_{ij}(\Delta x) = \begin{cases} \Delta x^{\alpha}, \, \Delta x \geq 0 \\ -\lambda(-\Delta x)^{\beta}, \, \Delta x < 0 \end{cases} \tag{9}$$

where Δx denotes the deviation from the reference point x_0 (positive values represent gains and negative values represent losses); α and β are parameters indicating the decision-maker's sensitivity to gains and losses. The relation $\alpha < \beta$ implies that decision-makers attach greater importance to the subjective experience caused by losses than to that from gains. λ reflects the degree of loss aversion of the decision-maker, where $\lambda > 1$ indicates the presence of loss aversion. Based on empirical evidence and experimental data, typical values are $\alpha = 0.89$, $\beta = 0.92$, and $\lambda = 2.25$.

1. *Gains and losses of outcomes are determined by their deviations from the reference point. Decision-makers tend to be risk-averse when facing gains, preferring certain small gains; conversely, they tend to be risk-seeking when confronting losses, hoping to reverse the situation and secure gains.*
2. *The geometric shape of the value function is S-shaped: it is concave in the gain domain and convex in the loss domain. This leads decision-makers to display loss aversion when anticipating gains and risk preference when perceiving potential losses.*
3. *Decision-makers are more sensitive to losses than to equivalent gains.*

2.2.2. Probability weighting function

Definition 7. *The probability weight functions of gain and loss in prospect theory are given by Equations (10) and (11), respectively [10].*

$$\pi^{+}(p) = \frac{p^{\tau}}{\left(p^{\tau} + (1-p)^{\tau}\right)^{\frac{1}{\tau}}} \tag{10}$$

$$\pi^{-}(p) = \frac{p^{\delta}}{\left(p^{\delta} + (1-p)^{\delta}\right)^{\frac{1}{\delta}}} \tag{11}$$

where τ and δ denote the attitude coefficients toward risky gains and losses, respectively. Based on empirical evidence and experimental results, $\tau = 0.61$ and $\delta = 0.69$.

The probability weight function exhibits three key characteristics:

It is a monotonically increasing and non-linear function of probability, which does not represent the objective likelihood of an event occurring.

Under risky conditions, decision-makers typically pay more attention to low-probability events—they tend to prefer low-probability gain events while avoiding low-probability risk events.

For all mutually exclusive events, the sum of their decision weights is less than the weight of a certain event, a phenomenon known as subcertainty.

3. Construction of a multi-stage decision-making model for urban fire safety

3.1. Assumptions of the model and overall framework

3.1.1. Assumptions of model

To ensure the scientific validity and operability of the proposed model, and in consideration of the practical scenarios of urban fire emergency decision-making, the following assumptions are made:

- **Expert team composition**: The decision-making group comprises interdisciplinary experts from various fields, including Fire Engineering, Emergency Management, Disaster Economics, and Law. It is posited that the evaluation information provided by these experts is independent, representative, highly professional, and comprehensively covers the dimensions of core fire emergency decision-making.
- **Staged evolution of fire incidents**: The development and evolution of urban fires can be decomposed into a series of continuous stages with distinct boundaries.
- **Decision-maker's risk attitude and parameter calibration**: It is assumed that decision-makers' risk attitudes conform to the characteristics of the "Quadrant Model" within Prospect Theory. Core parameters of the model, such as the loss aversion coefficient and sensitivity to gains, are calibrated based on relevant experimental data and expert consensus within the field of fire emergency decision-making.
- **Fire information timeliness**: It is assumed that the Fire information obtained at each decision-making stage is timely and effective.
- **Semantic consensus**: It is assumed that consensus has been reached within the decision-making group regarding the semantic interpretation of multi-granularity linguistic term sets.

3.1.2. Overall framework

The urban fire multi-stage sequential decision-making model proposed in this study is underpinned by a core logic integrating three key components (see **Figure 1**).

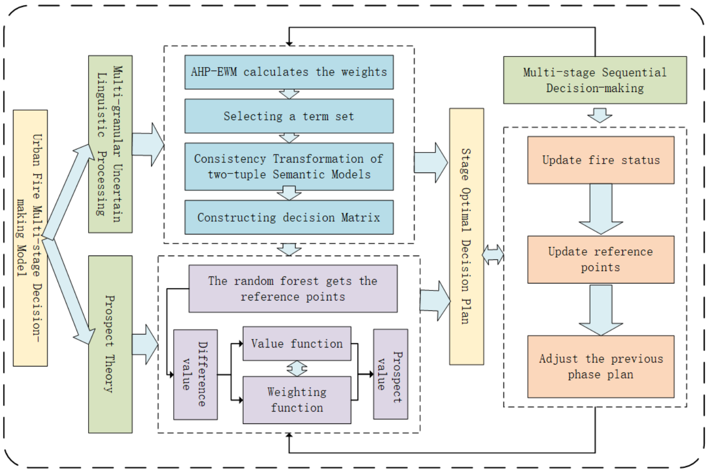

Figure 1. The overall framework of the proposed multi-stage decision model.

The figure above illustrates the framework of the proposed Multi-stage Prospect Selection (M-PS) model. The framework consists of three integrated components: the multi-granular uncertain linguistic processing module in the upper-left section, the prospect theory model in the lower-left section, and the multi-stage sequential decision-making module on the right. These three components work in concert to generate decision alternatives for each stage.

- **Quantifying subjective preferences with multi-granularity uncertain linguistic information:** This phase first determines the expert's ability weights through the Analytic Hierarchy Process. Subsequently, the Dual Hesitation Semantic Model is adopted to realize consistent transformation of multi-granular linguistic information. This process effectively converts subjective and uncertain evaluations into quantifiable numerical data.
- **Characterizing irrational decision-making behavior with prospect theory:** Leveraging Prospect Theory, this phase constructs a value function and a probability weighting function, which are specifically designed to quantify decision-makers' irrational risk attitudes under both gain and loss scenarios. Based on these functions and the transformed data, the prospect value of each alternative decision plan is then calculated.
- **Multi-stage sequential decision-making with the random forest algorithm:** Supported by a Random Forest model, this phase dynamically updates the reference points for each decision stage using real-time fire status data. Through multi-stage sequential iteration, the model achieves continuous optimization of decision plans until the fire is fully controlled. This iterative mechanism ensures the model's adaptability to the dynamically evolving fire situation.

3.2. Multi-granularity uncertain linguistic information processing

3.2.1. The ability weights of the experts are determined and normalized

Let the decision-making group be comprised of i interdisciplinary experts, denoted as $E = \{E_1, E_2, \dots, E_i\}$. An expert evaluation index system is constructed based on five dimensions: professional qualifications, practical experience, technical proficiency, coordination ability, and innovative thinking. The score assigned by expert E_i on the s-th dimension is denoted as a_{is} (where $i = 1, 2, \dots, 5$). Utilizing the Analytic Hierarchy Process, a judgment matrix is developed. Subsequently, the comprehensive weight for each expert, denoted as w_{is}, is calculated to reflect their relative importance in the decision-making process.

To address the issue of inconsistent dimensionality and values exceeding the $[0, 1]$ range in the initial weights calculated by the AHP method, a linear normalization technique is employed. This method maps the initial weights to a standard interval. Specifically, for assessments within the same dimension, this process ensures that the sum of the weights assigned to all experts equals 1.

To improve the efficiency of real-time decision-making, an offline pre-calculation strategy for experts' basic weights is adopted. For a fixed interdisciplinary expert team, the construction of the expert's ability judgment matrix and the calculation of initial weights are completed offline, and the normalized basic weight vector $W_{\text{base}} = [w_{1,\text{base}}, w_{2,\text{base}}, \ldots, w_{i,\text{base}}]$ is stored in the model database, where $w_{i,\text{base}}$ denotes the offline pre-calculated basic weight of expert E_i.

3.2.2. Design of linguistic terminologies with multi-granularity uncertainty

Determination of the Evaluation Indicator System: The evaluation indicator system for urban fire decision-making is constructed following the principles of "full life-cycle coverage and core dimension focus." It comprises six dimensions:

Personnel Casualties (F_1); Economic Losses (F_2); Emergency Effectiveness (F_3); Social Recovery (F_4); Responsibility Attribution (F_5); Ethical Implications (F_6).

The weights for each indicator are determined using the AHP. The resulting weight vector is $W_k = [0.3, 0.15, 0.1, 0.15, 0.2, 0.1]$. Notably, F_1 (Personnel Casualties) has the highest weight, reflecting the fire decision-making principle of "life safety first."

Three kinds of language terminology sets with typical granularity are provided for experts to choose independently to meet the differences in expression habits of different experts.

The two-tuple semantic model is used to realize the cross-granularity conversion of multi-granularity language information, and the core goal is to retain the original semantics completely in the quantization process and avoid information distortion. The transformation steps are shown in **Figure 2**.

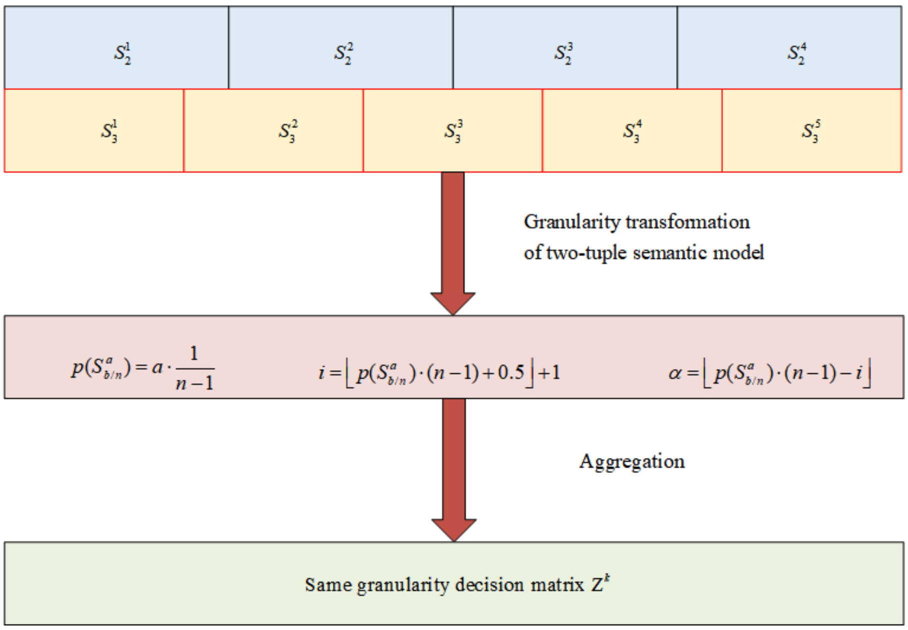

Figure 2. The same granularity conversion process.

To optimize the real-time performance of the model, this paper defines a term and a quantitative value mapping table based on the terminology set commonly used by experts. For commonly used linguistic terms with explicit semantics, their tuple-based quantitative results are calculated offline and stored in a dictionary. During emergency decision-making, the quantitative values of these terms can be directly retrieved from the table without the need to repeatedly perform the online calculations of Equations (3) and (4). A simplified example of the mapping table is shown in the **Table 2** below.

Table 2. Quantification results of linguistic terms via two-tuple semantic model.

Original term	Granularity	2-tuple	Numerical result
$s^1_{b/1}$	3	$(1, 0)$	0
$s^2_{b/1}$	3	$(3, 0)$	0.1
$s^3_{b/1}$	3	$(5, 0)$	1
$s^1_{b/2}$	4	$(1, 0)$	0
$s^2_{b/2}$	4	$(2, 0.332)$	0.333
$s^3_{b/2}$	4	$(4, -0.336)$	0.666
$s^4_{b/2}$	4	$(5, 0)$	1

3.3. Decision plan selection with prospect theory

3.3.1. Reference point dynamic update mechanism

Reference points are the core elements of prospect theory and directly affect decision-makers' judgments of gains and losses. Considering the dynamic evolution characteristics of urban fires, a random forest model is adopted to construct a multi-dimensional fire impact prediction model, so as to realize the dynamic update of reference points.

Model input and preprocessing: Taking fire area, population density, and asset density as core features, combined with auxiliary features such as meteorological conditions, building structures, and firefighting facility configurations, an input feature set $X = \{x_1, x_2, ..., x_i\}$ is constructed. The original data is preprocessed as follows: missing values are filled with medians, and Z-score standardization is used to eliminate dimension differences (12) :

$$X_{norm} = (X - \widetilde{X})/\sigma_X \tag{12}$$

where $\widetilde{X}$ denotes the median of feature X and σ_X denotes the standard deviation.

Construction of random forest model: An ensemble learning framework containing 150 decision trees is built, and a feature importance-weighted splitting strategy is adopted to optimize the tree structure. The maximum depth is set to 10 layers, and the minimum number of splitting samples per node is set to 5, so as to balance the model complexity and generalization ability. The model outputs the predicted values of 6 evaluation indicators, namely the reference point vector $\phi_q = (\phi_1, \phi_2, ..., \phi_s)$, which is the quantified result of the reference point for indicator F_s.

The ensemble learning framework is used to construct a multi-target prediction model as shown in Equation (13):

$$\hat{Y} = \frac{1}{K}\sum_{k=1}^{K} T_k(X_{\text{norm}}), K = 150, X_{\text{norm}} \in R^{n\times 3} \tag{13}$$

The dynamic update mechanism of the random forest model is highly dependent on the timeliness and accuracy of real-time data. To quantify the impact of data acquisition delays on model decision-making, this paper conducts an analysis with the actual characteristics of fire emergency scenarios, covering multi-dimensional experimental scenarios with different delay durations, key data types, and typical scenarios.

Define data delay scenarios:

Delay duration: Five gradient levels are set in accordance with relevant literature (0 s, 15 s, 30 s [34,35], 1 min [36], 3 min) to cover both short and medium delays.

Types of delayed data: Focus on two categories of critical data, namely fire area and number of trapped people; auxiliary data, which have a relatively minor impact, are not prioritized for analysis in this study. Scenarios of data delay and credibility weights are shown in **Table 3**.

Table 3. Delay variable setting table.

Data type	Delay scenario	Typical scenario	Credibility weight
Fire Area	0 s	Real-time data collection	1.0
Fire Area	15 s	Sensor transmission delay	0.95
Fire Area	30 s	Photo feedback delay	0.9
Fire Area	1 min	Data aggregation delay	0.8
Fire Area	30 s	Signal interference in complex buildings	0.6
People	15 s	Direct counting	0.95
People	30 s	Scattered trapped people	0.9
People	1 min	Partial trapped people lost contact	0.8
People	3 min	Complex building structure	0.6

3.3.2. Construction of value function

Editing stage: The decision-maker selects a reference point, codes the outcomes, and forms gains or losses.

If the decision outcome is above the reference point, it is a gain; if it is below the reference point, it is a loss.

Evaluation stage: Prospects are assessed and selected using a value function and a probability weighting function. The prospect value model is given by Equation (14):

$$V = \sum_{i=1}^{k} (\pi(p_i)v(\Delta x_i)) \tag{14}$$

where V represents the prospect value, v is the value function, Δx_i denotes the difference from the reference point, and $\pi(p_i)$ is the probability weighting function applied to the probability p_i.

3.4. Multi-stage sequential decision process

The evolution and development of urban fire incidents and urban fire emergency response decisions constitute a dynamic, multi-stage sequential relationship. In this process, fire incidents, as the subject of natural evolution, continuously change their state over time. Concurrently, the emergency response decision-making entity dynamically adjusts its strategies based on incrementally acquired information. This entire process can be viewed as an emergency response mechanism that is multi-stage, characterized by progressively complete information, and involves continuous strategy optimization.

As depicted in **Figure 3**, the entire sequential process is divided into multiple rounds, with each round encompassing two core stages: fire state evolution and emergency decision-making response. The specific procedure is as follows.

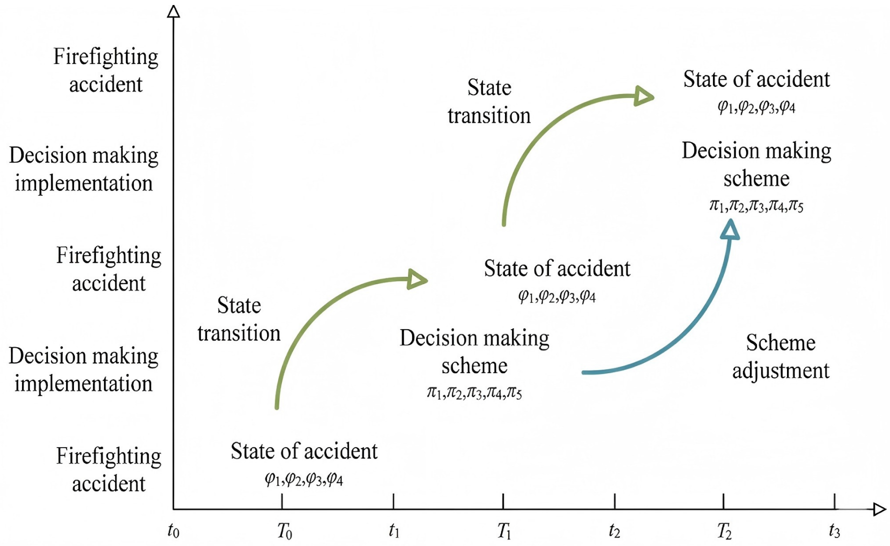

Figure 3. Multi-stage process of urban fire control.

(a) At the commencement of each round, the system receives the current fire incident state information, which is converted into multi-granularity uncertain linguistic terms by the decision-maker, aligned with the same level of granularity as the state information. Simultaneously, the data within the database is segmented into corresponding Datasets, serving as support for subsequent modeling and decision analysis.

(b) The Random Forest algorithm is utilized for training and feature extraction on the datasets, enabling preliminary identification and classification of fire risks. By randomly partitioning the datasets, the model's generalization ability and robustness are enhanced.

(c) A Decision Tree model is constructed for each of the partitioned datasets to facilitate comparison and further analyze the key factors influencing fire development and their evolutionary paths, thereby forming interpretable decision logic.

(d) Based on the Decision Tree analysis, the optimal reference point that best matches the input is determined. The result of this reference point is output, along with the calculated difference between each alternative plan and the reference point.

(e) Integrating the Value Function and Weighting Function, the feasible emergency plans for the current stage are evaluated for their value, and their weights are adjusted. This leads to the determination of the best plan for the current stage, reflecting the decision-maker's subjective perception of risk and return.

(f) Ultimately, guided by the principle of maximizing expected utility, the optimal plan for the current stage is outputted and immediately implemented. Concurrently, the system continuously monitors the fire's development, collects new data, and enters the next decision-making cycle.

(g) The aforementioned process iterates repeatedly until the fire is completely controlled, its impact is eliminated, and the emergency rescue operation is concluded.

4. Illustrative example in urban firefighting

We selected a representative fire case to verify the feasibility of the model M-PS proposed in this paper.

At around 14:28 on August 8, 2020, a fire broke out in the factory building at No. 95 Jinhou Road, Xiban Village, Chendai Town, Jinjiang City, resulting in 8 deaths and a direct economic loss of approximately 8.3075 million yuan. The factory building is a 7-story frame structure that has not obtained relevant approval and acceptance procedures, and is sub-leased to 6 different production and operation units, with many problems such as illegal construction and inadequate firefighting facilities. The cause of the fire was illegal welding and cutting during the construction of the elevator shaft, and the welding slag ignited combustibles, leading to the fire.

Initial Stage:

Step 1: A decision-making group composed of cross-disciplinary experts in fire protection, engineering, emergency management, disaster economics, etc. $\{E_1, E_2, E_3, E_4, E_5\}$ is selected. Due to the different experiences of experts in various dimensions, the dimension weight vector is $\lambda = \{0.25, 0.3, 0.2, 0.15, 0.1\}$. The following is the scoring **Table 4** of five experts in five dimensions combined with actual situations.

Table 4. Expert scoring table for five-dimensional professional competencies.

Expert	Professional qualification	Practical experience	Technical ability	Coordination ability	Innovative thinking
E_1	8	7	6	8	5
E_2	9	8	7	6	7
E_3	7	9	8	7	6
E_4	6	7	9	8	8
E_5	8	6	7	9	7

Step 2: The weights of each expert group calculated according to AHP-EWM are shown in **Table 5**.

Table 5. Basic weights of experts based on AHP-EWM.

Expert	E_1	E_2	E_3	E_4	E_5
Weight	7	7.65	7.7	7.4	7.25

Normalize the calculated weights, and the normalized weight vector is:

$$w = \{0.1892, 0.2068, 0.2081, 0.2, 0.1959\}$$

Step 3: Combined with historical statistical data, the disaster situation of the "8·8" fire in Xiban Village, Chendai Town, Jinjiang City, and the demand for rescue, the decision-making subject formulates different early warning decision-making plans for different levels of fire emergency management needs in accordance with the disposal points in the plan database. Considering the complexity and uncertainty of the fire scene, the following alternative plans are formed:

Plan A_1: Priority Control for Construction Area.

Fire source: 4th-floor elevator shaft (welding sparks ignited sponge waste, with residual steel bars/waste). Core measures: Cut power (15 min) and isolate shaft passageways (20 min); protect 10 ribbon processing equipment units (3rd/5th floors); cover shaft opening with fireproof cloth + spray water-based extinguishing agent; dismantle scaffolding to block fire spread; guide evacuation via east staircase (no return); remote power cutoff; water curtain sprinklers at 3rd/5th-floor shafts; deploy 2 high-mobility fire trucks + 4-person assault team.

Plan A_2: Pre-evacuation for High-risk Areas.

Deflagration risks: 2nd-floor PVC film warehouse (5 t) + 1st-floor glue storage. Core measures: Evacuate 1st/2nd floors (10 min, check for patrols); protect 3 printing equipment units (2nd floor) + transfer to temporary storage; activate "red warning" broadcast; security checks for stragglers; transfer 20 glue barrels off-site; spray fireproof coating on PVC stacks; coordinate temporary storage + dispatch 3 forklifts + 5 trucks.

Plan A_3: Construction Party Responsibility Tracing and Information Sharing.

Key issue: Lack of elevator shaft drawings. Core measures: Obtain oral renovation details from supervisor (10 min); draw simplified cross-sections to mark collapse risks; collect informal drawings from tenants; integrate into 3D model (red: uncut steel bars; yellow: temporary supports); push model to rescuers via AR glasses for high-risk alerts.

Step 4: Five experts give multi-granularity decision matrices for the above three plans, respectively:

$$x^1 = \begin{bmatrix} \{s^2_{1/2}\} & \{s^1_{2/1}\} & \{s^2_{3/3}\} & \{s^3_{4/2}\} & \{s^2_{5/3}\} & \{s^1_{6/1}\} \\ \{s^1_{1/3}\} & \{s^1_{2/2}\} & \{s^2_{3/3}\} & \{s^2_{4/2}\} & \{s^2_{5/3}\} & \{s^1_{6/1}\} \\ \{s^3_{1/3}\} & \{s^2_{2/2}\} & \{s^3_{3/3}\} & \{s^2_{4/2}\} & \{s^2_{5/3}\} & \{s^1_{6/1}\} \end{bmatrix}$$

$$x^2 = \begin{bmatrix} \{s^3_{1/3}\} & \{s^1_{2/3}\} & \{s^1_{3/1}\} & \{s^4_{4/3}\} & \{s^2_{5/2}\} & \{s^1_{6/2}\} \\ \{s^1_{1/2}\} & \{s^1_{2/2}\} & \{s^2_{3/3}\} & \{s^3_{4/3}\} & \{s^1_{5/1}\} & \{s^1_{6/3}\} \\ \{s^2_{1/2}\} & \{s^2_{2/3}\} & \{s^2_{3/2}\} & \{s^1_{4/2}\} & \{s^2_{5/2}\} & \{s^1_{6/1}\} \end{bmatrix}$$

$$x^3 = \begin{bmatrix} \{s^1_{1/2}\} & \{s^1_{2/3}\} & \{s^2_{3/3}\} & \{s^4_{4/3}\} & \{s^2_{5/2}\} & \{s^1_{6/2}\} \\ \{s^1_{1/2}\} & \{s^1_{2/2}\} & \{s^2_{3/3}\} & \{s^3_{4/3}\} & \{s^1_{5/1}\} & \{s^1_{6/3}\} \\ \{s^2_{1/2}\} & \{s^2_{2/3}\} & \{s^2_{3/2}\} & \{s^1_{4/2}\} & \{s^2_{5/2}\} & \{s^1_{6/1}\} \end{bmatrix}$$

$$x^4 = \begin{bmatrix} \{s^1_{1/2}\} & \{s^1_{2/3}\} & \{s^2_{3/3}\} & \{s^4_{4/3}\} & \{s^1_{5/2}\} & \{s^1_{6/2}\} \\ \{s^1_{1/2}\} & \{s^1_{2/3}\} & \{s^2_{3/3}\} & \{s^2_{4/2}\} & \{s^1_{5/2}\} & \{s^2_{6/3}\} \\ \{s^3_{1/3}\} & \{s^2_{2/2}\} & \{s^3_{3/2}\} & \{s^2_{4/3}\} & \{s^2_{5/3}\} & \{s^1_{6/2}\} \end{bmatrix}$$

$$x^5 = \begin{bmatrix} \{s^1_{1/1}\} & \{s^1_{2/1}\} & \{s^2_{3/3}\} & \{s^3_{4/3}\} & \{s^1_{5/1}\} & \{s^1_{6/1}\} \\ \{s^2_{1/3}\} & \{s^1_{2/2}\} & \{s^2_{3/3}\} & \{s^2_{4/2}\} & \{s^2_{5/2}\} & \{s^1_{6/3}\} \\ \{s^1_{1/2}\} & \{s^1_{2/3}\} & \{s^1_{3/3}\} & \{s^1_{4/3}\} & \{s^1_{5/1}\} & \{s^1_{6/3}\} \end{bmatrix}$$

Step 5: Retrieve pre-stored data from the tuple semantic model

Step 6: Convert to homogeneous granularity matrix

$$Z^1 = \begin{bmatrix} \{s_{1/3}^2\} & \{s_{2/3}^1\} & \{s_{3/3}^2\} & \{s_{4/3}^4\} & \{s_{5/3}^2\} & \{s_{6/3}^1\} \\ \{s_{1/3}^1\} & \{s_{2/3}^1\} & \{s_{3/3}^2\} & \{s_{4/3}^2\} & \{s_{5/3}^2\} & \{s_{6/3}^1\} \\ \{s_{1/3}^3\} & \{s_{2/3}^2\} & \{s_{3/3}^3\} & \{s_{4/3}^2\} & \{s_{5/3}^2\} & \{s_{6/3}^1\} \end{bmatrix}$$

$$Z^2 = \begin{bmatrix} \{s_{1/3}^3\} & \{s_{2/3}^1\} & \{s_{3/3}^1\} & \{s_{4/3}^4\} & \{s_{5/3}^2\} & \{s_{6/3}^1\} \\ \{s_{1/3}^1\} & \{s_{2/3}^1\} & \{s_{3/3}^2\} & \{s_{4/3}^3\} & \{s_{5/3}^1\} & \{s_{6/3}^1\} \\ \{s_{1/3}^2\} & \{s_{2/3}^2\} & \{s_{3/3}^2\} & \{s_{4/3}^1\} & \{s_{5/3}^2\} & \{s_{6/3}^1\} \end{bmatrix}$$

$$Z^3 = \begin{bmatrix} \{s_{1/3}^2\} & \{s_{2/3}^1\} & \{s_{3/3}^2\} & \{s_{4/3}^4\} & \{s_{5/3}^2\} & \{s_{6/3}^1\} \\ \{s_{1/3}^1\} & \{s_{2/3}^1\} & \{s_{3/3}^2\} & \{s_{4/3}^3\} & \{s_{5/3}^1\} & \{s_{6/3}^1\} \\ \{s_{1/3}^2\} & \{s_{2/3}^2\} & \{s_{3/3}^2\} & \{s_{4/3}^1\} & \{s_{5/3}^2\} & \{s_{6/3}^1\} \end{bmatrix}$$

$$Z^4 = \begin{bmatrix} \{s_{1/3}^2\} & \{s_{2/3}^1\} & \{s_{3/3}^2\} & \{s_{4/3}^4\} & \{s_{5/3}^1\} & \{s_{6/3}^1\} \\ \{s_{1/3}^1\} & \{s_{2/3}^1\} & \{s_{3/3}^2\} & \{s_{4/3}^2\} & \{s_{5/3}^1\} & \{s_{6/3}^2\} \\ \{s_{1/3}^3\} & \{s_{2/3}^2\} & \{s_{3/3}^4\} & \{s_{4/3}^2\} & \{s_{5/3}^2\} & \{s_{6/3}^1\} \end{bmatrix}$$

$$Z^5 = \begin{bmatrix} \{s_{1/3}^1\} & \{s_{2/3}^1\} & \{s_{3/3}^2\} & \{s_{4/3}^3\} & \{s_{5/3}^1\} & \{s_{6/3}^1\} \\ \{s_{1/3}^2\} & \{s_{2/3}^1\} & \{s_{3/3}^2\} & \{s_{4/3}^2\} & \{s_{5/3}^2\} & \{s_{6/3}^1\} \\ \{s_{1/3}^1\} & \{s_{2/3}^1\} & \{s_{3/3}^1\} & \{s_{4/3}^1\} & \{s_{5/3}^1\} & \{s_{6/3}^1\} \end{bmatrix}$$

Step 7: Obtain reference points

The homogeneous granularity language of the expected value of the reference point is obtained according to the fire area $R = 750\ m^2$, population density 0.015 people$/m^2$ and asset density, and the weight of each dimension $w_k = \{0.3, 0.15, 0.1, 0.15, 0.2, 0.1\}$.

Step 8: Plan value evaluation results

$$\mathrm{d}^1 = \begin{bmatrix} 0 & 0 & 0 & 0.25 & 0.25 & 0 \\ -0.25 & 0 & 0 & -0.25 & 0.25 & 0 \\ 0.25 & 0.25 & 0.25 & -0.25 & 0.25 & 0 \end{bmatrix}$$

$$\mathrm{d}^2 = \begin{bmatrix} 0.25 & 0 & -0.25 & 0 & 0.25 & 0 \\ -0.25 & 0 & 0 & 0.25 & 0 & 0 \\ 0 & 0.25 & 0 & -0.5 & 0.25 & 0 \end{bmatrix}$$

$$\mathrm{d}^3 = \begin{bmatrix} 0 & 0 & 0 & 0.25 & 0.25 & 0 \\ -0.25 & 0.25 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.5 & 0.25 & 0 \end{bmatrix}$$

$$\mathrm{d}^5 = \begin{bmatrix} -0.25 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.25 & 0.25 & 0 \\ -0.25 & 0 & -0.25 & -0.5 & 0 & 0 \end{bmatrix}$$

Step 9: Calculate values

$$\mathbf{v}^1 = \begin{bmatrix} 0 & 0 & 0 & -0.63 & -0.63 & 0 \\ 0.29 & 0 & 0 & 0.29 & -0.63 & 0 \\ -0.63 & -0.63 & -0.63 & 0.29 & -0.63 & 0 \end{bmatrix}$$

$$\mathbf{v}^2 = \begin{bmatrix} -0.63 & 0 & 0.29 & -0.63 & -0.63 & 0 \\ 0.29 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.63 & 0 & 0.54 & -0.63 & 0 \end{bmatrix}$$

$$\mathbf{v}^3 = \begin{bmatrix} 0 & 0 & 0 & -0.63 & -0.63 & 0 \\ 0.29 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.63 & 0 & 0.54 & -0.63 & 0 \end{bmatrix}$$

$$\mathbf{v}^4 = \begin{bmatrix} 0 & 0 & 0 & -0.63 & 0 & -0.63 \\ 0.29 & 0 & 0 & 0.29 & 0 & 0 \\ -0.63 & -0.63 & -1.19 & 0.29 & -0.63 & 0 \end{bmatrix}$$

$$\mathbf{v}^5 = \begin{bmatrix} 0.29 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.29 & 0.29 & -0.63 & 0 \\ 0.29 & 0 & 0 & 0.54 & 0 & 0 \end{bmatrix}$$

Step 10: Calculate prospect values

$$V(A_1) = -0.1099605$$
$$V(A_2) = 0.02058$$
$$V(A_3) = -0.1183$$

$$V(A_2) > V(A_1) > V(A_3)$$

The ranking of prospect values is $V(A_2) > V(A_1) > V(A_3)$. According to the results, the final perceived value of plan 2 meets the expectations of the decision-making group, while the losses brought by plan 1 and plan 3 exceed the expectations of the decision-making group. Therefore, the decision-making group finally selects plan 2 as the final plan.

Fully developed stage:

As the fire accident continues, the on-site fire rescue personnel re-judge or revise the first-phase decision according to the on-site fire development degree, occurrence time, building type, weather conditions, and the feedback on the number of casualties and trapped persons from other departments, combined with the opinions exchanged by various departments and based on the emergency rescue results of the first phase, and by analyzing the collected new information.

Step 1: At this time, the decision-maker adjusts the new plan based on the decision plan of the previous phase, and modifies plan 2 to obtain three new rescue plans:

Plan A_1': The core is to prioritize dredging rescue channels. A team composed of security guards and firefighters conducts a segmented search and rescue for stranded persons and expands the early warning range; uses 2 forklifts to clear the stacks to open a 1.5-m channel and suspend equipment transfer; first transfers the glue barrels, and then transfers the uncollapsed PVC film after the channel is clear to rationally allocate vehicles.

Plan A_2': Focus on strengthening safety protection. Set up a 20-m warning zone, rescuers are equipped with protective equipment, and use UAVs and manual inspection to check for personnel; double the amount of fireproof coating sprayed on the uncollapsed PVC film, and forklifts cooperate with water cannons to cool down the covered equipment before clearing debris to rescue the equipment; transfer materials in 3 batches, keep forklifts on standby, and contact fire trucks to be on standby.

Plan A_3': The key lies in multi-department collaboration. Establish a command center for division of labor: fire department for obstacle removal and firefighting, public security for guidance and evacuation, and medical department for setting up medical points; conduct two-way clearing to open 2 channels and use jacks to move the covered equipment; communicate with storage sites in advance, synchronously transfer with 5 vehicles and 3 forklifts within 2 h, and the fire department will conduct a thorough site clearance afterwards.

Step 2: Five experts give multi-granularity decision matrices for the above three plans, respectively:

$$X^1 = \begin{bmatrix} \{s^3_{1/3}\} & \{s^1_{2/3}\} & \{s^1_{3/1}\} & \{s^4_{4/3}\} & \{s^2_{5/2}\} & \{s^1_{6/2}\} \\ \{s^1_{1/3}\} & \{s^1_{2/3}\} & \{s^1_{3/3}\} & \{s^2_{4/2}\} & \{s^1_{5/2}\} & \{s^1_{6/2}\} \\ \{s^2_{1/2}\} & \{s^2_{2/3}\} & \{s^2_{3/2}\} & \{s^1_{4/2}\} & \{s^2_{5/2}\} & \{s^1_{6/1}\} \end{bmatrix}$$

$$X^2 = \begin{bmatrix} \{s^1_{1/1}\} & \{s^1_{2/1}\} & \{s^2_{3/3}\} & \{s^3_{4/3}\} & \{s^1_{5/1}\} & \{s^1_{6/1}\} \\ \{s^2_{1/2}\} & \{s^1_{2/2}\} & \{s^2_{3/2}\} & \{s^2_{4/3}\} & \{s^2_{5/3}\} & \{s^1_{6/2}\} \\ \{s^1_{1/2}\} & \{s^1_{2/3}\} & \{s^1_{3/3}\} & \{s^1_{4/3}\} & \{s^1_{5/1}\} & \{s^1_{6/3}\} \end{bmatrix}$$

$$X^3 = \begin{bmatrix} \{s^2_{1/3}\} & \{s^1_{2/2}\} & \{s^1_{3/3}\} & \{s^3_{4/3}\} & \{s^1_{5/3}\} & \{s^1_{6/3}\} \\ \{s^1_{1/3}\} & \{s^1_{2/2}\} & \{s^2_{3/3}\} & \{s^2_{4/2}\} & \{s^2_{5/3}\} & \{s^1_{6/1}\} \\ \{s^3_{1/3}\} & \{s^2_{2/2}\} & \{s^3_{3/3}\} & \{s^2_{4/2}\} & \{s^2_{5/3}\} & \{s^1_{6/1}\} \end{bmatrix}$$

$$X^4 = \begin{bmatrix} \{s^2_{1/2}\} & \{s^1_{2/3}\} & \{s^2_{3/3}\} & \{s^4_{4/3}\} & \{s^2_{5/2}\} & \{s^1_{6/2}\} \\ \{s^1_{1/2}\} & \{s^1_{2/2}\} & \{s^2_{3/3}\} & \{s^3_{4/3}\} & \{s^1_{5/1}\} & \{s^1_{6/3}\} \\ \{s^2_{1/3}\} & \{s^2_{2/2}\} & \{s^2_{3/3}\} & \{s^1_{4/3}\} & \{s^2_{5/3}\} & \{s^1_{6/2}\} \end{bmatrix}$$

$$X^5 = \begin{bmatrix} \{s^2_{1/2}\} & \{s^2_{2/3}\} & \{s^1_{3/3}\} & \{s^3_{4/3}\} & \{s^1_{5/3}\} & \{s^1_{6/3}\} \\ \{s^1_{1/2}\} & \{s^1_{2/3}\} & \{s^2_{3/3}\} & \{s^2_{4/2}\} & \{s^1_{5/2}\} & \{s^2_{6/3}\} \\ \{s^3_{1/3}\} & \{s^2_{2/2}\} & \{s^3_{3/2}\} & \{s^2_{4/3}\} & \{s^2_{5/3}\} & \{s^1_{6/2}\} \end{bmatrix}$$

Step 3: Convert to homogeneous granularity matrix

$$Z^1 = \begin{bmatrix} \{s^3_{1/3}\} & \{s^1_{2/3}\} & \{s^1_{3/3}\} & \{s^4_{4/3}\} & \{s^2_{5/3}\} & \{s^1_{6/3}\} \\ \{s^1_{1/3}\} & \{s^1_{2/3}\} & \{s^1_{3/3}\} & \{s^2_{4/3}\} & \{s^1_{5/3}\} & \{s^1_{6/3}\} \\ \{s^2_{1/3}\} & \{s^2_{2/3}\} & \{s^2_{3/3}\} & \{s^1_{4/3}\} & \{s^2_{5/3}\} & \{s^1_{6/3}\} \end{bmatrix}$$

$$Z^2 = \begin{bmatrix} \{s^1_{1/3}\} & \{s^1_{2/3}\} & \{s^2_{3/3}\} & \{s^3_{4/3}\} & \{s^1_{5/3}\} & \{s^1_{6/3}\} \\ \{s^2_{1/3}\} & \{s^1_{2/3}\} & \{s^2_{3/3}\} & \{s^2_{4/3}\} & \{s^2_{5/3}\} & \{s^1_{6/3}\} \\ \{s^1_{1/3}\} & \{s^1_{2/3}\} & \{s^1_{3/3}\} & \{s^1_{4/3}\} & \{s^1_{5/3}\} & \{s^1_{6/3}\} \end{bmatrix}$$

$$Z^3 = \begin{bmatrix} \{s^3_{1/3}\} & \{s^1_{2/3}\} & \{s^1_{3/3}\} & \{s^4_{4/3}\} & \{s^2_{5/3}\} & \{s^1_{6/3}\} \\ \{s^1_{1/3}\} & \{s^1_{2/3}\} & \{s^1_{3/3}\} & \{s^2_{4/3}\} & \{s^1_{5/3}\} & \{s^1_{6/3}\} \\ \{s^2_{1/3}\} & \{s^2_{2/3}\} & \{s^2_{3/3}\} & \{s^1_{4/3}\} & \{s^2_{5/3}\} & \{s^1_{6/3}\} \end{bmatrix}$$

$$Z^4 = \begin{bmatrix} \{s^2_{1/3}\} & \{s^1_{2/3}\} & \{s^2_{3/3}\} & \{s^4_{4/3}\} & \{s^2_{5/3}\} & \{s^1_{6/3}\} \\ \{s^1_{1/3}\} & \{s^1_{2/3}\} & \{s^2_{3/3}\} & \{s^3_{4/3}\} & \{s^1_{5/3}\} & \{s^1_{6/3}\} \\ \{s^2_{1/3}\} & \{s^2_{2/3}\} & \{s^2_{3/3}\} & \{s^1_{4/3}\} & \{s^2_{5/3}\} & \{s^1_{6/3}\} \end{bmatrix}$$

$$Z^5 = \begin{bmatrix} \{s^4_{1/3}\} & \{s^1_{2/3}\} & \{s^2_{3/3}\} & \{s^3_{4/3}\} & \{s^1_{5/3}\} & \{s^1_{6/3}\} \\ \{s^1_{1/3}\} & \{s^1_{2/3}\} & \{s^2_{3/3}\} & \{s^2_{4/3}\} & \{s^1_{5/3}\} & \{s^2_{6/3}\} \\ \{s^3_{1/3}\} & \{s^2_{2/3}\} & \{s^4_{3/3}\} & \{s^2_{4/3}\} & \{s^2_{5/3}\} & \{s^1_{6/3}\} \end{bmatrix}$$

Step 4: The expected value of the reference point is obtained by the random forest model, and the weight of each dimension remains unchanged, still $w_k = [0.3, 0.15, 0.1, 0.15, 0.2, 0.1]$.

Step 5: Calculate deviation values

$$\mathrm{d}^1 = \begin{bmatrix} 0.25 & 0 & -0.25 & 0.5 & 0.25 & -0.25 \\ -0.25 & 0 & -0.25 & 0 & 0 & -0.25 \\ 0 & 0.25 & 0 & -0.25 & 0.25 & 0.25 \end{bmatrix}$$

$$\mathrm{d}^2 = \begin{bmatrix} -0.25 & 0 & 0 & 0.25 & 0 & -0.25 \\ 0 & 0 & 0 & 0 & 0.25 & -0.25 \\ -0.25 & 0 & -0.25 & -0.25 & 0 & -0.25 \end{bmatrix}$$

$$\mathrm{d}^3 = \begin{bmatrix} 0 & 0 & -0.25 & 0.25 & 0.25 & -0.25 \\ -0.25 & 0 & 0 & 0 & 0.25 & -0.25 \\ 0.25 & 0.25 & 0.25 & 0 & 0.25 & -0.25 \end{bmatrix}$$

$$\mathrm{d}^4 = \begin{bmatrix} 0 & 0 & 0 & 0.5 & 0 & -0.25 \\ -0.25 & 0 & 0 & 0.25 & 0.25 & 0 \\ 0.5 & 0.25 & 0 & -0.25 & 0.25 & -0.25 \end{bmatrix}$$

$$\mathrm{d}^5 = \begin{bmatrix} 0 & 0 & 0 & 0.25 & 0 & -0.25 \\ -0.25 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0.25 & 0.5 & 0 & 0.25 & -0.25 \end{bmatrix}$$

Step 6: Calculate values

$$\mathbf{v}^1 = \begin{bmatrix} -0.63 & 0 & 0.29 & -1.1963 & -0.63 & 0.29 \\ 0.29 & 0 & 0 & -0.63 & 0 & 0.29 \\ 0 & 0.29 & -0.63 & 0 & -0.63 & 0.29 \end{bmatrix}$$

$$\mathbf{v}^2 = \begin{bmatrix} 0 & 0 & 0 & -0.29 & 0 & 0.29 \\ 0 & 0 & 0 & 0 & -0.63 & 0.29 \\ 0.29 & 0 & 0.29 & 0.29 & 0 & 0.29 \end{bmatrix}$$

$$\mathbf{v}^3 = \begin{bmatrix} 0 & 0 & 0.29 & -0.63 & -0.63 & 0.29 \\ 0.29 & 0 & 0 & 0 & -0.63 & 0.29 \\ -0.63 & -0.63 & -0.63 & 0.29 & -0.63 & 0.29 \end{bmatrix}$$

$$\mathbf{v}^4 = \begin{bmatrix} 0 & 0 & 0 & -1.19 & -0.63 & 0.29 \\ 0.29 & 0 & 0 & -0.63 & 0 & 0.29 \\ 0 & -0.63 & 0 & 0.54 & -0.63 & 0.29 \end{bmatrix}$$

$$\mathbf{v}^5 = \begin{bmatrix} 0.29 & 0 & 0 & -0.63 & 0 & 0.29 \\ 0.29 & 0 & 0 & 0 & 0 & 0 \\ -0.63 & -0.63 & -1.19 & 0 & -0.63 & 0.29 \end{bmatrix}$$

Step 7: Calculate prospect values

$$\begin{aligned} V(A_1') &= -0.17433625 \\ V(A_2') &= -0.014555 \\ V(A_3') &= -0.128 \end{aligned}$$

$$V(A_2') > V(A_3') > V(A_1')$$

The ranking of prospect values is $V(A_2') > V(A_3') > V(A_1')$. According to the results, the final prospect value of plan 2 is the largest, while the losses brought by plan 1 and plan 3 are greater. Therefore, the decision-making group finally selects plan 2 as the final plan.

5. Comparison and discussion

Building on the case analysis in the preceding section, this section adopts distinct methodologies to conduct a comparative analysis with the aforementioned case, aiming to verify the effectiveness of the proposed M-PS model.

5.1. Traditional static multiple attribute decision-making method TOPSIS

5.1.1. Experimental procedure

In this part, the TOPSIS method is used as the comparison benchmark, which is based on the core logic of "approximate ideal solution" and determines the priority of alternatives by calculating the Euclidean distance between each alternative and the positive and negative ideal solution. Its notable feature is that it only relies on objective index data to make decisions, and does not consider the psychological behavior characteristics of decision makers in high-risk scenarios.

Step 1: The 6 core dimensions are used, and the weight vector determined by the AHP-EWM method is $w_k = [0.3, 0.15, 0.1, 0.15, 0.2, 0.1]$.

Step 2: A two-stage alternative is presented: Phase 1 (A_1, A_2, A_3), Phase 2

(A'_1, A'_2, A'_3).

The stage-one weighted normalization matrix is shown in **Table 6**.

Table 6. Weighted normalized indicator values of alternatives in Phase 1.

Plan	F_1 (0.3)	F_2 (0.15)	F_3 (0.1)	F_4 (0.15)	F_5 (0.2)	F_6 (0.1)
A_1	0.1050	0.0630	0.0680	0.0825	0.0600	0.0400
A_2	0.0450	0.0300	0.0750	0.0930	0.0900	0.0580
A_3	0.1500	0.0825	0.0420	0.0570	0.1440	0.0320

The stage-two weighted normalization matrix is shown in **Table 7**.

Table 7. Weighted normalized indicator values of alternatives in Phase 2.

Plan	F_1 (0.3)	F_2 (0.15)	F_3 (0.1)	F_4 (0.15)	F_5 (0.2)	F_6 (0.1)
A'_1	0.0660	0.0450	0.0650	0.0870	0.0800	0.0450
A'_2	0.0300	0.0270	0.0820	0.1050	0.1100	0.0650
A'_3	0.1140	0.0630	0.0500	0.0675	0.1360	0.0380

Step 3: Standardization method: Range standardization is adopted.

Indicator types: F_1 (casualties) and F_2 (economic loss) are negative indicators, and the rest are positive indicators.

The normalization formula for the positive index is given by Equation (15):

$$x'_{ij} = \frac{x_{ij} - \min(x_j)}{\max(x_j) - \min(x_j)} \tag{15}$$

The normalization formula for the negative index is given by Equation (16):

$$x'_{ij} = \frac{\max(x_j) - x_{ij}}{\max(x_j) - \min(x_j)} \tag{16}$$

Step 4: Determine the positive and negative ideal solutions

Positive ideal solution (Z^+): The maximum value of the weighted standardized values in each dimension, as shown in Equation (17).

$$Z_j^+ = \max(Z_{1j}, Z_{2j}, Z_{3j}) \tag{17}$$

Negative ideal solution (Z^-): The minimum value among the weighted standardized values of each dimension, according to Equation (18).

$$Z_j^- = \min(Z_{1j}, Z_{2j}, Z_{3j}) \tag{18}$$

Positive and negative ideal solutions in Stage one are shown in **Table 8**.

Table 8. Positive and negative ideal solutions of TOPSIS method in Phase 1.

Dimension	F_1	F_2	F_3	F_4	F_5	F_6
Z^+	0.1500	0.0825	0.0750	0.0930	0.1440	0.0580
Z^-	0.0450	0.0300	0.0420	0.0570	0.0600	0.0320

Positive and negative ideal solutions in Stage two are shown in **Table 9**.

Table 9. Positive and negative ideal solutions of TOPSIS method in Phase 2.

Dimension	**F$_1$**	**F$_2$**	**F$_3$**	**F$_4$**	**F$_5$**	**F$_6$**
Z^+	0.1140	0.0630	0.0820	0.1050	0.1360	0.0650
Z^-	0.0300	0.0270	0.0500	0.0675	0.0800	0.0380

Step 5: Euclidean distance is calculated in Equation (19).

$$d = \sqrt{\sum_{j=1}^{6}(Z_{ij} - Z_j)^2} \tag{19}$$

Table 10 shows the calculation results of the Euclidean distance in stage 1.

Table 10. Euclidean distances between alternatives and TOPSIS ideal solutions in Phase 1.

Plan	**d$^+$ (distance from Z^+)**	**d$^-$ (distance from Z^-)**
A_1	0.0997	0.0780
A_2	0.1292	0.0694
A_3	0.0553	0.1305

Table 11 shows the calculation results of the Euclidean distance in stage 2.

Table 11. Euclidean distances between alternatives and TOPSIS ideal solutions in Phase 2.

Plan	**d$^+$ (distance from Z^+)**	**d$^-$ (distance from Z^-)**
A_1'	0.0648	0.0531
A_2'	0.0953	0.0598
A_3'	0.0465	0.1098

Step 6: Relative closeness and ranking

The relative closeness is in Equation (20).

$$C_i = \frac{d_i^-}{d_i^+ + d_i^-} \tag{20}$$

The final results of stage one are shown in **Table 12**.

Table 12. Relative closeness and ranking of alternatives via TOPSIS in Phase 1.

Plan	**Relative closeness C_i**	**Ranking**
A_1	0.441	2
A_2	0.347	3
A_3	0.703	1

The final results of stage two are shown in **Table 13**.

Table 13. Relative closeness and ranking of alternatives via TOPSIS in Phase 2.

Plan	Relative closeness C_i	Ranking
A'_1	0.450	2
A'_2	0.385	3
A'_3	0.701	1

5.1.2. Experimental result analysis

Limitations of the TOPSIS method: Although the weight of F_1 is increased to 0.3, the TOPSIS method still prioritizes A_3/A'_3 because it only focuses on the "distance to the optimal solution" of objective indicators, which does not reflect the principle of prioritizing life safety and ignores the decision-maker's loss aversion psychology.

Overall differences with the proposed M-PS model proposed in this paper are shown in **Table 14**.

Table 14. Comparison of decision results between TOPSIS method and proposed M-PS model.

Comparison dimension	TOPSIS method	Proposed M-PS model
Phase 1 Ranking	$A_3 > A_1 > A_2$	$A_2 > A_1 > A_3$
Phase 2 Ranking	$A'_3 > A'_1 > A'_2$	$A'_2 > A'_1 > A'_3$
Consideration of Psychological Preferences	No	Yes
Dynamic Adaptability	No	Yes

5.1.3. Experimental conclusions

Even if the weight of the casualty dimension is increased, the TOPSIS method still has a significant deviation in the ranking results in urban firefighting decision-making due to the lack of consideration of the decision-maker's psychological behavior and dynamic scene adaptability, and cannot prioritize avoiding high-risk hidden dangers. In contrast, the M-PS model in this paper quantifies subjective preferences through multi-granularity language, depicts irrational decisions through prospect theory, and updates reference points sequentially in multiple stages, resulting in significantly better decision accuracy and scene adaptability than the TOPSIS method.

5.2. Single prospect theory approach to decision making

5.2.1. Experimental premise and core settings

Multi-granularity uncertain linguistic term sets are not used, and experts directly use unified 3-granularity numerical values for evaluation, without the need for 2-tuple conversion and cross-granularity consistency processing.

Step 1: Expert Evaluation Data Collection Decision-making group: The same 5 cross-disciplinary experts as in this paper.

Evaluation method: Experts need to give numerical scores for the 6 evaluation dimensions of each plan based on a unified 5-granularity standard, without the freedom of term selection.

Data integration: The weighted average of expert scores is directly used to form a homogeneous granularity decision matrix for each stage.

$$Z^1=\begin{bmatrix}\{s^1_{1/3}\} & \{s^1_{2/3}\} & \{s^2_{3/3}\} & \{s^1_{4/3}\} & \{s^2_{5/3}\} & \{s^1_{6/3}\}\\ \{s^2_{1/3}\} & \{s^1_{2/3}\} & \{s^2_{3/3}\} & \{s^2_{4/3}\} & \{s^2_{5/3}\} & \{s^1_{6/3}\}\\ \{s^3_{1/3}\} & \{s^2_{2/3}\} & \{s^3_{3/3}\} & \{s^2_{4/3}\} & \{s^2_{5/3}\} & \{s^1_{6/3}\}\end{bmatrix}$$

$$Z^2=\begin{bmatrix}\{s^1_{1/3}\} & \{s^1_{2/3}\} & \{s^1_{3/3}\} & \{s^1_{4/3}\} & \{s^2_{5/3}\} & \{s^1_{6/3}\}\\ \{s^2_{1/3}\} & \{s^1_{2/3}\} & \{s^2_{3/3}\} & \{s^3_{4/3}\} & \{s^1_{5/3}\} & \{s^1_{6/3}\}\\ \{s^2_{1/3}\} & \{s^2_{2/3}\} & \{s^2_{3/3}\} & \{s^1_{4/3}\} & \{s^2_{5/3}\} & \{s^1_{6/3}\}\end{bmatrix}$$

$$Z^3=\begin{bmatrix}\{s^1_{1/3}\} & \{s^1_{2/3}\} & \{s^1_{3/3}\} & \{s^2_{4/3}\} & \{s^2_{5/3}\} & \{s^1_{6/3}\}\\ \{s^1_{1/3}\} & \{s^1_{2/3}\} & \{s^2_{3/3}\} & \{s^3_{4/3}\} & \{s^1_{5/3}\} & \{s^1_{6/3}\}\\ \{s^2_{1/3}\} & \{s^2_{2/3}\} & \{s^2_{3/3}\} & \{s^1_{4/3}\} & \{s^2_{5/3}\} & \{s^1_{6/3}\}\end{bmatrix}$$

$$Z^4=\begin{bmatrix}\{s^2_{1/3}\} & \{s^1_{2/3}\} & \{s^2_{3/3}\} & \{s^1_{4/3}\} & \{s^1_{5/3}\} & \{s^1_{6/3}\}\\ \{s^1_{1/3}\} & \{s^1_{2/3}\} & \{s^2_{3/3}\} & \{s^2_{4/3}\} & \{s^1_{5/3}\} & \{s^2_{6/3}\}\\ \{s^2_{1/3}\} & \{s^2_{2/3}\} & \{s^4_{3/3}\} & \{s^2_{4/3}\} & \{s^2_{5/3}\} & \{s^1_{6/3}\}\end{bmatrix}$$

$$Z^5=\begin{bmatrix}\{s^1_{1/3}\} & \{s^1_{2/3}\} & \{s^2_{3/3}\} & \{s^1_{4/3}\} & \{s^1_{5/3}\} & \{s^1_{6/3}\}\\ \{s^2_{1/3}\} & \{s^1_{2/3}\} & \{s^2_{3/3}\} & \{s^2_{4/3}\} & \{s^2_{5/3}\} & \{s^1_{6/3}\}\\ \{s^1_{1/3}\} & \{s^1_{2/3}\} & \{s^1_{3/3}\} & \{s^1_{4/3}\} & \{s^1_{5/3}\} & \{s^1_{6/3}\}\end{bmatrix}$$

Step 2: The reference point $\phi_q = (s^2_{1,3}, s^1_{2,3}, s^2_{2,3}, s^4_{4,3}, s^1_{5,3}, s^1_{6,3})$ of each dimension in Phase 1 is obtained through the random forest model.

Step 3: Deviation value calculation: Calculate the deviation value of each plan from the reference point in 6 dimensions according to the formula.

$$\mathrm{d}^1=\begin{bmatrix}-0.25 & 0 & 0 & -0.5 & 0.25 & 0\\ 0 & 0 & 0 & -0.25 & 0.25 & 0\\ 0.25 & 0.25 & 0.25 & -0.25 & 0.25 & 0\end{bmatrix}$$

$$\mathrm{d}^2=\begin{bmatrix}-0.25 & 0 & -0.25 & 0 & 0.25 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0.25 & 0 & -0.5 & 0.25 & 0\end{bmatrix}$$

$$\mathrm{d}^3=\begin{bmatrix}-0.25 & 0 & -0.25 & -0.25 & 0.25 & 0\\ -0.25 & 0 & 0 & 0 & 0 & 0\\ 0 & 0.25 & 0 & -0.5 & 0.25 & 0\end{bmatrix}$$

$$\mathrm{d}^4=\begin{bmatrix}0 & 0 & 0 & -0.25 & 0 & 0.25\\ -0.25 & 0 & 0 & -0.25 & 0 & 0.25\\ 0.25 & 0.25 & 0.5 & -0.25 & 0.25 & 0\end{bmatrix}$$

$$\mathrm{d}^5=\begin{bmatrix}-0.25 & 0 & 0 & -0.5 & 0 & 0\\ 0 & 0 & 0 & -0.25 & 0.25 & 0\\ -0.25 & 0 & -0.25 & -0.5 & 0 & 0\end{bmatrix}$$

Step 4: Value function calculation: Substitute the deviation value into the value function formula to obtain the value result of each dimension.

$$\mathbf{v}^1 = \begin{bmatrix} 0.29 & 0 & 0 & 0.54 & -0.63 & 0 \\ 0 & 0 & 0 & 0.29 & -0.63 & 0 \\ -0.63 & -0.63 & -0.63 & 0.29 & -0.63 & 0 \end{bmatrix}$$

$$\mathbf{v}^2 = \begin{bmatrix} 0.29 & 0 & 0.29 & 0.54 & -0.63 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.63 & 0 & 0.54 & -0.63 & 0 \end{bmatrix}$$

$$\mathbf{v}^3 = \begin{bmatrix} 0.29 & 0 & 0.29 & 0.29 & -0.63 & 0 \\ 0.29 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.63 & 0 & 0.54 & -0.63 & 0 \end{bmatrix}$$

$$\mathbf{v}^4 = \begin{bmatrix} 0 & 0 & 0 & 0.54 & 0 & 0 \\ 0.29 & 0 & 0 & 0.29 & 0 & -0.63 \\ -0.63 & -0.63 & -1.19 & 0.29 & -0.63 & 0 \end{bmatrix}$$

$$\mathbf{v}^5 = \begin{bmatrix} 0.29 & 0 & 0 & 0.54 & 0 & 0 \\ 0 & 0 & 0 & 0.29 & -0.63 & 0 \\ 0.29 & 0 & 0.29 & 0.54 & 0 & 0 \end{bmatrix}$$

Step 5: Prospect value aggregation: Aggregate the value and weight of each dimension according to formula (4) to obtain the prospect value of each plan in Phase 1:

$$V(A_1) = 0.089647$$
$$V(A_2) = -0.01945$$
$$V(A_3) = -0.1309$$

The ranking of prospect values is $V(A_1) > V(A_2) > V(A_3)$, and plan A_1 is selected.

Step 6: Plan update: Based on the implementation results of Phase 1, the 3 revised plans in Phase 2 of this paper are adopted. Data collection and calculation: Repeat the process of Phase 1, and the reference point is dynamically updated based on the fire evolution data in Phase 1.

Phase 2 prospect value results: Revised plan A_1 prospect value: 0.158, Revised plan A_2 prospect value: 0.103, Revised plan A_3 prospect value: 0.087.

The ranking of Phase 2 decision results is $V(A_1) > V(A_2) > V(A_3)$, and the revised plan A_1 is selected.

5.2.2. Experimental result comparison

The decision result pair is shown in **Table 15**.

Table 15. Comparison of optimal alternatives between single prospect theory and the proposed M-PS model.

Phase	Single prospect	Proposed M-PS model	Overall difference
Phase 1	A_1	A_2	Failure to identify the priority of risk proliferation
Phase 2	A_1'	A_2'	Ignoring secondary risks caused by insufficient protection

5.2.3. Limitations of the single prospect theory method

- Distortion of subjective evaluation quantification: Forcing unified granularity cannot adapt to the cognitive differences of experts and loses the semantic details of the original evaluation.
- Insufficient integration of expert preferences: Direct weighted average does not handle the heterogeneity of evaluations, resulting in the dilution of the importance of high-risk dimensions.
- Poor decision adaptability: The uncertainty information in the fire scene is not captured, which leads to the misalignment between the optimal plan and the actual high-risk point.

5.3. Overall comparison

To holistically and intuitively characterize the comprehensive performance disparities among alternative decision-making methods, a radar chart (**Figure 4**) is constructed to visualize their performance across six critical dimensions: Dynamic Adaptability, Psychological Preference Adaptability, Computational Complexity, Model Robustness, Decision Accuracy, and Subjective Evaluation Distortion.

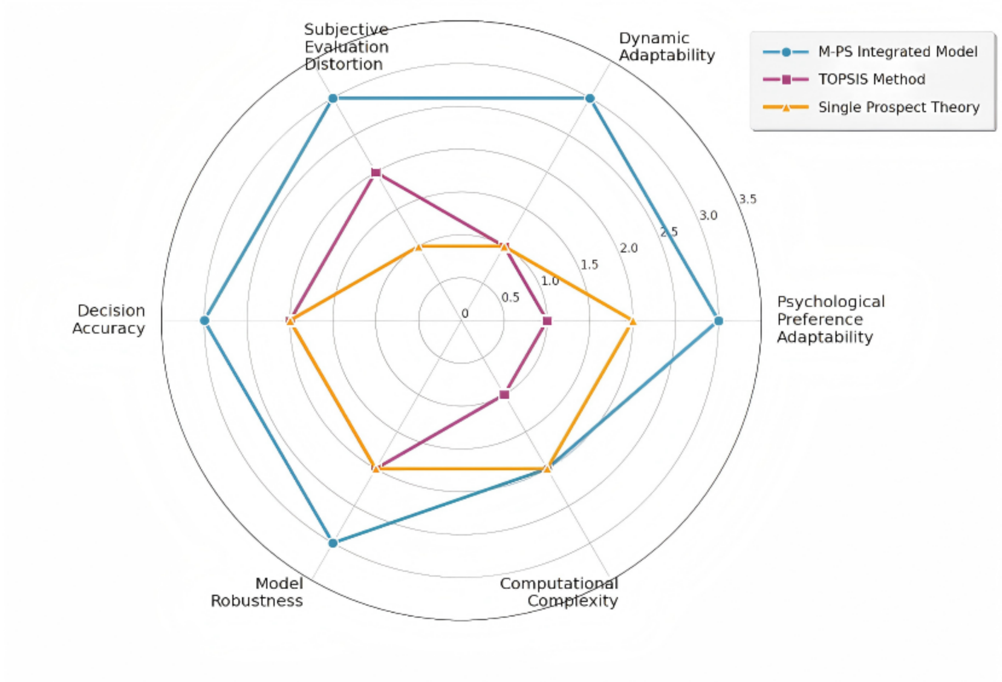

Figure 4. Performance comparison radar chart of different decision methods.

Table 16 systematically compares the core indicators of three representative existing methods and the proposed M-PS model. These indicators are selected to reflect the operational feasibility and practical effectiveness of each method in fire emergency decision-making scenarios. As shown in the table, most existing approaches only satisfy partial indicators: Wei and Liao [4] support Attribute Weight and Ranking but cannot optimize Decision Weight or ensure Stability; Yu et al. [37] address Decision Weight and Ranking but fail to adapt Attribute Weight to scenario demands; even Chen et al. [38] cover most dimensions, still do not guarantee Stable performance across multi-stage decision iterations. In contrast, the proposed M-PS model achieves full coverage across all five dimensions and demonstrates superiority in each.

Table 16. Comparison of performance indicators between different decision-making methods and the proposed M-PS model.

Method	Decision weight	Attribute weight	Ranking	Stability	Accuracy
Wei and Liao [4]	×	√	√	×	×
Yu et al. [37]	√	×	√	×	×
Chen et al. [38]	√	√	√	×	√
Proposed M-PS	√	√	√	√	√

Note: In this table, √ indicates that the corresponding method supports or performs well in the given performance indicator, while × indicates that the method does not support or performs poorly in that indicator.

6. Sensitivity analysis of experts' ability weights

To quantify the stability of experts' ability weights and verify their resistance to weight fluctuations, a sensitivity analysis experiment is designed to compare the decision-making robustness between the integrated weights and the AHP subjective weights. All other core parameters of the model remain unchanged, including the two-tuple linguistic conversion rules, the random forest reference point prediction model, prospect theory parameters, and the weights of six evaluation dimensions. Only the experts' ability weights are adjusted to focus on the fluctuations of decision-making results.

Weight disturbance rules

Three types of disturbance scenarios were designed for both the original AHP subjective weights and the optimized integrated weights. For each scenario, three groups of disturbance vectors were generated to eliminate random errors, and all weights were renormalized after disturbance to ensure the sum of weights equals 1:

Minor disturbance: ±5% perturbation for each expert's ability weight

Moderate disturbance: ±10% perturbation for each expert's ability weight

Major disturbance: ±15% perturbation for each expert's abilityE/ED weight

Sensitivity analysis was conducted separately based on the data from the initial stage and the spread stage of the Jinjiang "8·8" fire case, with the experimental results presented in **Tables 17** and **18**.

Table 17. Comparison of sensitivity analysis results in the initial stage.

Method	Disturbance scenario	Optimal alternative	Ranking change rate	Prospect value variation	Robustness coefficient
AHP	±0%	A_2	0%	0%	0.00
	±5%	A_2	0%	3.2%	0.016
	±10%	A_1/A_2	33.3%	7.2%	0.052
	±15%	A_1/A_3	66.7%	11.3%	0.089
AHP-EWM	±0%	A_2	0%	0%	0.00
	±5%	A_2	0%	2.1%	0.011
	±10%	A_2	10%	4.8%	0.029
	±15%	A_1/A_2	33.3%	8.5%	0.059

Table 18. Comparison of sensitivity analysis results in the spread stage.

Method	Disturbance scenario	Optimal alternative	Ranking change rate	Prospect value variation	Robustness coefficient
AHP	$\pm0\%$	A_2'	0%	0%	0.00
	$\pm5\%$	A_2'	0%	2.9%	0.015
	$\pm10\%$	A_1'/A_2'	33.3%	7.0%	0.051
	$\pm15\%$	A_1'/A_3'	66.7%	11.1%	0.089
AHP-EWM	$\pm0\%$	A_2'	0%	0%	0.00
	$\pm5\%$	A_2'	0%	2.0%	0.010
	$\pm10\%$	A_2'	10%	4.5%	0.028
	$\pm15\%$	A_1'/A_2'	33.3%	8.2%	0.058

- The robustness of the fused weights is significantly superior to the original AHP weights: in both stages, the robustness coefficient of the fused weights under moderate disturbance is only about 0.028, which is reduced by more than 45% compared with the original AHP weights, indicating a stronger resistance to subjective deviations.
- The fused weights exhibit higher decision-making stability: under moderate disturbance, the ranking change rate of the fused weights is only 10%, while the corresponding rate of the original AHP weights reaches 33.3%, demonstrating that the fused weights can effectively mitigate the impact of subjective fluctuations on decision-making results.
- The fluctuation of prospect values is smoother: the average fluctuation amplitude of prospect values of the fused weights is lower than that of the original AHP weights under all disturbance scenarios, indicating that the scheme evaluation results are more stable.
- The weight stability threshold is improved: the stability threshold of the fused weights is increased from $\pm8\%$ (original AHP weights) to $\pm12\%$, which allows for a larger tolerance of subjective deviations in practical applications.

7. Conclusion

Aiming at the core dilemmas in urban fire emergency decision-making, such as the uncertainty of subjective linguistic information, the bounded rationality of decision-makers, and the insufficient dynamic adaptability to fire scenarios, this paper constructs a multi-stage emergency decision-making model that integrates multi-granular uncertain linguistic information, prospect theory, and sequential reference point updating. To verify the effectiveness of the proposed model, an illustrative example is provided in the context of urban fire protection. The results demonstrate that the proposed M-PS model can accurately identify key risk points, and its decision outcomes are more aligned with the requirements of actual scenarios.

Through comparative experiments with the TOPSIS method, the single prospect theory method, as well as the methods proposed by Wei et al. [4], Yu et al. [37], and Chen et al. [38], the proposed M-PS model exhibits significant advantages in the following three aspects:

Firstly, a multi-granularity uncertain linguistic decision-making method is adopted to adapt to the uncertainty of the decision-making environment and the cognitive differences of decision-makers, and refine the analysis of risk weights.

Secondly, based on prospect theory, the irrational psychology of decision-makers, such as loss aversion, is objectively depicted.

Thirdly, a multi-stage sequential framework is introduced to dynamically adapt to the evolving situation of fires and realize real-time optimization of decisions.

Although this study has made significant progress in the integration of multi-granular uncertain linguistic information, prospect theory, and multi-stage decision-making models, there is still room for further expansion and optimization. Future research intends to carry out improvements in the following four aspects:

Firstly, this study mainly relies on the existing 2-tuple linguistic model when dealing with linguistic uncertainty. Future research can further expand the paradigm of uncertain information representation and explore the integration of more advanced mathematical tools, such as intuitionistic fuzzy sets, hesitant fuzzy sets, or fuzzy evidence theory. This will help to more comprehensively and finely characterize the hesitation, fuzziness, and evidential uncertainty in expert evaluations, thereby achieving a richer representation of decision-making information.

Secondly, in terms of the application of prospect theory, the current model is usually based on a shared set of risk preference parameters. However, in actual emergency decision-making scenarios, there may be significant differences in the risk aversion levels of different decision-makers and the risk attributes of specific fire incidents. Therefore, future research directions include dynamically calibrating the parameters of the value function and probability weighting function in prospect theory to provide more personalized and scenario-adaptive decision support.

Thirdly, the reference point prediction mechanism in the current research relies to a certain extent on the learning of historical data. To overcome this limitation, future work can focus on expanding and enriching the fire scenario database and actively integrating real-time fire monitoring data, thereby improving the universality of the model and the accuracy of predictions.

Finally, to achieve more precise and real-time strategy adjustment for the dynamic evolution process of urban fires, future research needs to explore a more fine-grained stage division of the fire evolution process. Meanwhile, it is necessary to strengthen the in-depth study of the information transmission mechanism between successive decision-making stages to ensure the continuity and coherence of the decision-making process and realize the iterative optimization of decisions.

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Informed consent statement: Not applicable.

Data availability statement: The data that support the findings of this study are available from the corresponding author (Nady Slam, nady128@xbmu.edu.cn) upon reasonable request. The data are not publicly available due to the inclusion of internal fire emergency management records that require confidentiality.

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