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Review

Stochastic differential equation model for detecting digital currency market manipulation: A systematic review

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Abstract: Digital currency markets exhibit extreme volatility, heavy tails, and bursty order-flow that complicate surveillance and heighten manipulation risk. This systematic review synthesizes the state of the art in stochastic differential equation (SDE)–based approaches for detection and monitoring of abusive practices in cryptocurrencies. Following PRISMA 2020 and PICOS, an Elsevier-indexed search (2015–2025) yielded 273 records; after screening and eligibility assessment, 20 primary studies (2018–2025) were included. The literature clusters into six methodological families: (i) stochastic volatility and jump–diffusion models for heavy-tailed returns and volatility smiles; (ii) Hawkes and related point-process models for clustered order arrivals, spoofing, pump-and-dump, and wash trading; (iii) Markov and regime-switching diffusions that delineate latent "fair" versus manipulated regimes; (iv) hybrid SDE–machine learning frameworks for high-frequency prediction; (v) change-point and sequential detection methods grounded in likelihood ratios and optimal stopping; and (vi) meta-studies consolidating performance trends. Across studies, Hawkes-type intensities consistently outperform Poisson and threshold baselines in event detection; regime-switching models align with known market breaks; and hybrid neural–SDE systems achieve the strongest forecasting but at reduced interpretability. We formalize a taxonomy linking model structure to surveillance objective, and we delineate a practical trade-off between transparency (classical SDEs) and predictive accuracy (hybrid models). The review highlights open needs in explainable hybrid designs, reproducible datasets, and real-time deploy ability for exchanges and regulators. By connecting applied probability, financial engineering, and control, the paper clarifies how SDE frameworks can underpin robust, auditable market-integrity tools for digital assets.

Keywords: change-point detection; Cryptocurrency markets; market manipulation; stochastic differential equations; volatility modelling

1. Introduction

Over the past decade, digital currencies have evolved from niche technological experiments into significant components of the global financial system. Bitcoin, Ethereum, and a vast ecosystem of altcoins now trade on exchanges worldwide, attracting retail investors, institutions, and algorithmic traders alike. The rapid pace of this expansion has been marked by remarkable innovation in finance, as well as significant regulatory concerns. While these markets promise decentralization and new opportunities, they also present vulnerabilities that can undermine trust and stability. One of the most pressing challenges is the prevalence of market manipulation. Practices such as spoofing, wash trading, layering, and pump-and-dump schemes have been documented in cryptocurrency exchanges [1]. These tactics exploit the

decentralized and often lightly regulated environment of digital assets, taking advantage of information asymmetries and fragmented oversight [2]. For individual investors, manipulation erodes confidence and increases risk exposure; for regulators and exchanges, it creates obstacles to building transparent and fair markets [3,4].

Detecting manipulative behaviour in cryptocurrency markets is complex. Cryptocurrency markets generate vast amounts of high-frequency data that are volatile, non-stationary, and often exhibit heavy tails. Price movements and order-flow patterns are influenced by normal trading activity, as well as abrupt external shocks and speculative frenzies [5]. Standard statistical methods frequently fail to differentiate between "ordinary" volatility and distortions caused by market manipulation [6]. This necessitates the development of more sophisticated mathematical frameworks that effectively capture the randomness and structure of these market dynamics while offering reliable tools for detection [7].

Stochastic differential equations (SDEs) provide a framework for modelling systems that evolve under uncertainty, combining deterministic trends and random fluctuations. They have long been utilized in mathematical finance to represent a wide range of market behaviours, including the continuous drift and volatility of traditional price series as well as the sudden jumps and clustering observed in digital assets [8,9]. Various models have been adapted to financial data over time, including geometric Brownian motion, Ornstein–Uhlenbeck processes, stochastic volatility frameworks, jump-diffusion models, and Hawkes processes [10]. These models link probabilistic theory with observable market patterns, offering interpretable frameworks for regulators and researchers [11]. In addition to their descriptive power, SDEs are compatible with detection and control methods. Techniques such as change-point analysis, likelihood ratio tests, and sequential monitoring strategies can be integrated into SDE-based models, enabling the identification of sudden deviations that may indicate market manipulation [12]. This dual role—capturing realistic dynamics and facilitating surveillance—renders SDEs particularly promising for application in cryptocurrency markets.

Despite this potential, the literature applying stochastic differential equations (SDEs) to manipulation in digital currencies remains fragmented. Relevant studies are dispersed across finance, econometrics, probability, and computer science, each using different assumptions, data sources, and evaluation metrics [13]. Some focus narrowly on volatility estimation, others on jump detection, and others on modelling clustered order-flow. As a result, the field lacks a coherent synthesis that would allow scholars and practitioners to compare methods, evaluate performance, and identify the gaps where future work is most needed. This study addresses that gap by conducting a systematic review of SDE-based approaches for detecting market manipulation in digital currency markets. Using the PRISMA 2020 framework, we identify and assess relevant studies according to the PICOS criteria: the population of interest (digital currency markets), the intervention (SDE-based modelling approaches), the comparators (alternative econometric or machine-learning methods), the outcomes (accuracy and effectiveness of manipulation detection), and the study designs (empirical analyses, simulations, and back tests) [14].

Our contributions are fourfold. First, we consolidate a fragmented body of research to provide the first systematic overview of stochastic differential equation

(SDE) models applied to cryptocurrency market manipulation. Second, we propose a taxonomy of approaches, classifying models according to their stochastic structure and detection mechanisms. Third, we critically evaluate the relative strengths and limitations of these models with respect to interpretability, robustness, and empirical performance. Fourth, we highlight areas for future research, particularly the integration of SDEs with modern machine learning and the design of real-time detection systems suitable for regulatory or exchange deployment. By combining perspectives from applied mathematics, financial engineering, and regulatory studies, this review contributes both theoretically and practically. Theoretically, it demonstrates how SDEs can be extended beyond traditional equity markets to address the unique features of digital assets. Practically, it provides evidence to guide regulators, exchanges, and market participants in selecting effective tools for market surveillance and fraud prevention.

2. Theoretical framework

The detection of manipulation in digital currency markets requires models capable of capturing both the continuous fluctuations of asset prices and the abrupt distortions induced by abusive practices. Stochastic differential equations (SDEs) provide a versatile framework for this purpose. This section reviews the principal classes of SDEs applied in financial modelling, their extensions to incorporate jumps and order-flow effects, and their integration with control-theoretic detection methods.

2.1. Classical SDEs in finance

The canonical stochastic model for asset prices is the **geometric Brownian motion** (GBM)**:**

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{1}$$

where S_t is the asset price at time t, μ is the drift parameter, σ is volatility, and W_t is a standard Brownian motion [15].

An alternative is the **Ornstein–Uhlenbeck** (OU) process**,** which captures mean reversion:

$$dX_t = \theta (\mu - X_t)dt + \sigma dW_t \tag{2}$$

where θ is the speed of reversion and μ is the long-term equilibrium [16].

Stochastic volatility models extend GBM by allowing volatility itself to follow a mean-reverting process. The **Heston model** is a standard formulation:

$$dS_t = \mu S_t dt + \sqrt{v_t} S_t dW_t^S \tag{3}$$

$$dv_t = \kappa (\theta - v_t)dt + \xi \sqrt{v_t} dW_t^v \tag{4}$$

with correlated Brownian motions W_t^S and W_t^v. This captures volatility clustering and leverage effects commonly observed in crypto markets. This modelling approach captures volatility clustering and leverage effects commonly observed in financial markets [17]. Employing these models allows for better prediction and understanding

of asset price movements, especially in volatile environments such as digital currency markets where traditional models often fall short [18].

2.2. Jump and Lévy Extensions

To account for abrupt discontinuities such as pump-and-dump events or regulatory shocks, jump-diffusion models are applied. The **Merton jump-diffusion model** is given by:

$$dS_t = \mu S_t dt + \sigma S_t dW_t + S_t(J_t - 1)dN_t \tag{5}$$

where N_t is a Poisson process with intensity λ, and J_t is the jump size distribution [19]. More general Lévy processes (Variance Gamma, Normal Inverse Gaussian) allow heavy-tailed increments and capture skewness and kurtosis in return distributions. However, specific references that support this claim were not identified.

2.3. Order-flow and point processes

Order-book manipulation can be modelled using **Hawkes processes**, which capture self-exciting arrivals:

$$\lambda(t) = \mu + \int_0^t \phi(t - s)dN(s) \tag{6}$$

where $\lambda(t)$ is the intensity of arrivals, μ is the baseline intensity, and $\phi(\cdot)$ is a kernel describing self-excitation. Hybrid models combining Hawkes's intensities with diffusion-based price dynamics enable joint monitoring of price and order-flow anomalies [20]. Regime-switching diffusions, in which parameters change according to a hidden Markov process, further distinguish between "fair" and manipulated regimes [21].

2.4. Control-theoretic detection

SDEs integrate naturally with sequential detection and optimal stopping theory. Manipulation can be conceptualized as a change-point in the stochastic process [22]. The **CUSUM procedure** is widely used:

$$G_t = max\left(0, G_{\{t-1\}} + log\left(\frac{f_1(X_t)}{f_0(X_t)}f_0(X_t)\right)\right) \tag{7}$$

where f_0 and f_1 are likelihoods under the "fair" and "manipulated" hypotheses [23]. The stopping time is the first t such that G_t exceeds a threshold.

Bayesian alternatives such as the Shiryaev–Roberts's procedure and robust extensions such as the generalized likelihood ratio (GLR) test also fit within this framework. Collectively, they formalize financial surveillance as an optimal stopping problem under uncertainty [24].

3. Methods

3.1. Design

This study was conducted as a systematic review in accordance with the PRISMA 2020 guidelines. The objective was to identify, screen, and synthesize research applying stochastic differential equation (SDE) models to cryptocurrency market manipulation detection.

3.2. Search strategy

The primary search was conducted in Elsevier's Scopus and ScienceDirect databases using the following Boolean string, chosen for their comprehensive coverage of applied mathematics, control theory, and financial engineering literature,

("stochastic differential equation" OR "jump diffusion" OR "stochastic volatility" OR "hawkes process")
AND
("cryptocurrency" OR "bitcoin" OR "Ethereum" OR "digital currency")
AND
("manipulation" OR "spoofing" OR "wash trading" OR "pump and dump")

The search string combined terms related to stochastic modelling, cryptocurrencies, and manipulation, and was restricted to articles published within the past ten years (2015–2025). Only original research articles and were included. The search yielded 273 records. After restricting by year, document type, and language (English only articles), 92 records remained.

3.3. Study selection and screening

After removal of duplicates, 92 records proceeded to screening. Title and abstract screening excluded 52 studies that were not directly related to SDE models or cryptocurrency applications. The remaining 40 full-text articles were assessed for eligibility.

3.4. Eligibility criteria

Studies were considered eligible if they examined cryptocurrencies or digital assets such as Bitcoin, Ethereum, or DeFi tokens, and applied stochastic differential equations or closely related models, including stochastic volatility, jump-diffusion, Hawkes processes, or Markov switching frameworks. To be included, articles had to employ these models in the context of market manipulation detection, volatility modelling, or risk estimation, and provide empirical validation through comparison with baseline approaches such as GARCH, Poisson, or threshold-based detectors. Only original research articles were included, and the search was restricted to publications from 2015 to 2025 to ensure contemporary relevance. Articles were excluded if they were published prior to 2015, if they were reviews or systematic reviews rather than primary research, or if they lacked direct application of SDE-based methods to cryptocurrency markets or market abuse detection.

3.5. Quality appraisal

The methodological quality of the included studies was assessed with a focus on model specification, validation, and reproducibility. Because the review encompassed a diverse set of approaches, ranging from classical stochastic volatility models to hybrid machine learning–SDE frameworks, no single standardized appraisal tool was applied across all studies. Instead, the evaluation emphasized whether each study provided a clear definition of the stochastic model, transparent parameter estimation methods, and adequate validation procedures such as out-of-sample testing or

comparison with established baselines. Studies that lacked methodological transparency or failed to demonstrate empirical rigor were excluded during the eligibility stage, thereby ensuring that the final synthesis was based on contributions of sufficient scientific quality and relevance.

3.6. Study selection flow

The study selection process followed the PRISMA 2020 framework. An initial search in Elsevier databases identified 273 records. After applying filters to restrict the timeframe to 2015–2025 and including review and systematic review papers, 92 records remained. Following duplicate removal, these records underwent title and abstract screening, during which 52 studies were excluded for not directly addressing SDE models or cryptocurrency applications. The remaining 40 articles were retrieved in full text and evaluated for eligibility. Of these, 20 were excluded because they either did not implement explicit SDE formulations, relied solely on conventional econometric techniques, or lacked relevance to market manipulation detection. Ultimately, 20 studies met all inclusion criteria and were incorporated into the final review. The flow of studies from identification to inclusion is presented in the PRISMA 2020 diagram (**Figure 1**).

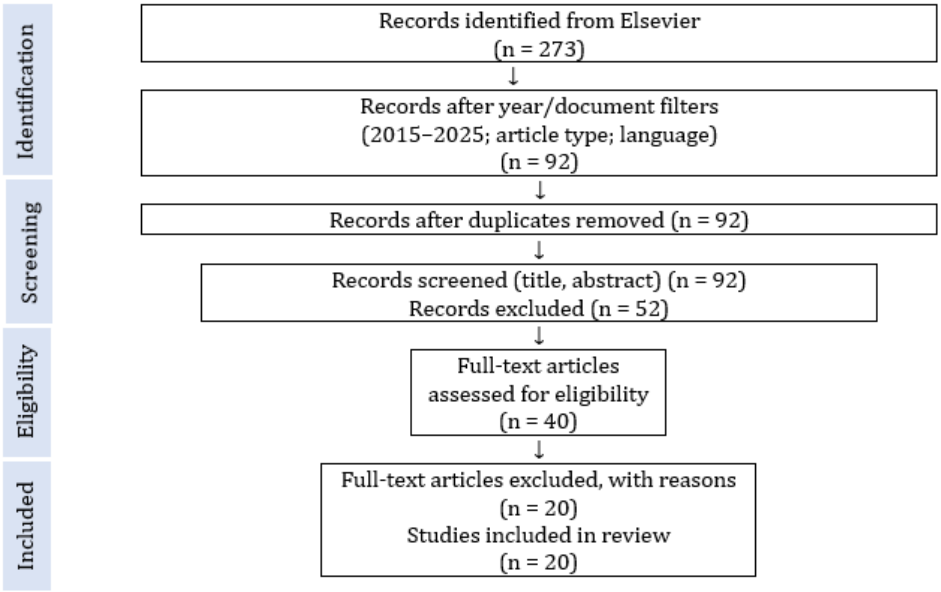

Figure 1. PRISMA 2020 flow diagram illustrating the study selection process, from the initial identification of 273 records in Elsevier databases to the final inclusion of 20 studies in the systematic review.

4. Results

4.1. Study selection

Following PRISMA guidelines, a total of 20 studies published between 2018 and 2025 were included. The majority focused on Bitcoin, with several extending to Ethereum, DeFi tokens, and low-cap altcoins. Data frequency ranged from daily series to minute-level high-frequency order-book records.

4.2. Study characteristics

Table 1 summarizes the included studies, which collectively demonstrate a broad methodological spectrum consistent with the theoretical framework introduced in Section 2. Classical stochastic volatility and jump–diffusion models (Section 2.1–2.2) remain widely employed for volatility estimation and option pricing, particularly in capturing heavy tails, volatility clustering, and abnormal jumps. Order-flow and point-process models (Section 2.3), especially Hawkes processes and their extensions, dominate in the detection of manipulation patterns such as spoofing, wash trading, and pump-and-dump events. Markovian and regime-switching models also appear frequently, reflecting their ability to capture illiquidity, hidden states, and macroeconomic regime shifts. More recent studies integrate SDEs with machine learning methods, producing hybrid models that improve predictive accuracy in high-frequency settings but often reduce interpretability. In parallel, change-point detection methods and statistical transforms (Section 2.4) have been applied to identify volatility persistence and structural breaks relevant to forensic investigations of illicit activity. Finally, one systematic review synthesized nearly one hundred studies and confirmed the shift from traditional GARCH baselines toward stochastic volatility, regime-switching, and hybrid deep learning models.

Table 1. Summary of studies employing stochastic differential equation models for cryptocurrency market manipulation.

Study (Year)	Asset(s)	Focus / Manipulation	Model Used	Comparator	Main Outcomes
Fry [25]	BTC, ETH, XRP, BCH	Speculative bubbles, boom–bust cycles	Rational bubble SDE with Cauchy noise + collapse risk	Log-normal, Gaussian	Bubbles in BTC & ETH; none in XRP/BCH; liquidity risk explains heavy tails and booms.
Dipple et al. [26]	BTC, ETH, LTC, Monero, XRP + Twitter, Reddit, GitHub	Forecasting crypto + social media jointly	Correlated SDEs: GBM (prices), GOU (social/volume), correlated noise	Uncorrelated GBM/GOU	Reasonable predictive accuracy; MAPE ~0.03–0.07 for BTC/ETH; social media more predictable than prices.
Chen and Huang [27]	Bitcoin (daily, 2015–2018)	Jump risk, option pricing & hedging	Merton & Bates Jump-Diffusion with Lee–Mykland	Black–Scholes, pure diffusion	Jump intensities ($\lambda \approx 9.9$–16.8%); Bates captured volatility smiles/skews; improved hedging.
Kalariya et al. [28]	BTC, ETH, LTC (2017–2019)	Algorithmic trading under volatility	Stochastic Neural Networks (SNN) + trading rules	Buy-and-Hold, RSI	BTC +676%, ETH +886%, LTC +1053% returns; Sharpe ratios improved; outperformed classical.
Kurbucz et al. [29]	Bitcoin (BTC/USD)	Hidden anomaly detection	Linear laws of Markov Chains (categorical autocorrelation, eigenvalues)	None	Detected hidden Markov property pre-COVID crash and 2020–21 BTC surge; eigenvalue spikes = manipulation signals.
Ortu et al. [30]	BTC, ETH (+Libra, XRP)	Social media–driven volatility	Hawkes Process + LDA + sentiment metrics	Sentiment/topic baselines	Social/emotional signals predicted BTC/ETH moves; validated on WallStreetBets–GME.

El-Khatib & Hatemi-J [31]	BTC/USD (2021–2022)	Illiquidity, regime shifts	SDE with Regime Switching (Brownian motion + CTMC)	Illiquidity SDE (no regime switching)	Three regimes captured (μ, σ vary); simulated realistic BTC/USD bursts and clustering.
La Morgia et al. [32]	Low-cap cryptos; DOGE, XRP	Pump-and-dump, crowd pumps	Random Forest & AdaBoost (rush order features)	Kamps threshold model	Real-time detection in 25s (F1=94.5%); dataset of 900+ pump events released.
Theodosiadou et al. [33]	Bitcoin (wallet-level transactions)	Illicit activity (Ponzi, hack)	Multivariate CPD (E-Divisive + medoid clustering)	Feature-based classifiers	Structural breaks matched Pirate@40 Ponzi (2011–12) and MintPal hack (2014).
Lukić & Milošević [34]	BTC, ETH (Gemini hourly + min data, 2017–2023)	Change point detection	Empirical Hankel Transform (matrix-valued statistic)	CF- and CUSUM-based	Detected BTC/ETH regime shifts in 2017–23; high-frequency effective, weak long-term power.
Zournatzidou et al. [35]	Bitcoin (daily OHLC, 2014–2024)	Volatility persistence & signals	R/S (Hurst exponent) + SMA + RSI	Volatility estimators	Hurst = 0.59–0.64 (momentum tendency); SMA/RSI gave risk management signals.
Fabre & Muni Toke [36]	BTC-USD + 15 pairs (Coinbase, 2023)	Microstructure causality	Neural Hawkes (PINN solver)	Wiener–Hopf	Stable recovery in high-dim; BTC, ETH, XRP leaders; concave volume impact.
Harasheh & Bouteska [37]	BTC, ETH	Volatility anomalies, tail risk	GH-ASV-skw-st SV model (Bayesian MCMC)	GARCH	Captured clustering, leverage, heavy tails; improved VaR/ES; BTC higher tail risk.
Luo & Krishnamurthy [38]	41 DeFi coins (Binance, 2022–23)	Structural shifts, contagion	Multivariate Hawkes + Fréchet stats	GLR, CUSUM	Detected Terra-LUNA, FTX, BlockFi crises; robust on 41-dim dataset.
Fabre & Muni Toke [39]	SEI-USD (Coinbase, 2023–2024)	Wash trading, bursts	MMHP-δ (Markov-modulated Hawkes)	MMPP	AIC fit better than MMPP; suspicious regime = 0.14%-time, 24% volume.
Fabre & Challet [40]	BTC-USD, ETH-USD (Coinbase, Dec 2022)	Spoofing detection	Probabilistic Neural Network (PNN) with Hawkes features	Order imbalance heuristics	31% of large orders spoofable; layering patterns found; ran in <100μs/order.
Avordeh et al. [41]	BTC (1-min, Binance, 2025)	High-frequency forecasting	Hybrid Heston–LSTM (SV + DL) with blockchain data	Heston, LSTM, GARCH	MSE ↓43% vs Heston; Sharpe ratio 2.1; +18.5% return in March 2025 sim.
Pindza et al. [42]	Bitcoin (daily 2016–2021 + sentiment)	Option pricing with sentiment	Jump-Diffusion + NN solver	Black–Scholes, Merton JD	NN solved PDE robustly; ~3% error; sentiment-driven jumps captured.
Pakštaitė et al. [43]	Bitcoin (daily, 2016–2024)	Regime shifts, macro influence	HMM & NH-HMM with Bayesian MCMC	Bitcoin-only vs macro covariates	Early drivers: halving, mining; later: FX & DJI index; short-term MAPE 15–25%, weak long-term.

Dote-Pardo & Espinosa-Jaramillo [44]	General crypto & DeFi (review of 96 studies, 2019–2024)	Pricing, risk, portfolio optimization	Systematic Review: SV, regime-switching, copulas, DL hybrids	Across reviewed studies	SV & MS-GARCH beat GARCH in crises; copulas modeled dependence; DL best for forecasting; gaps in ESG & DeFi-specific risks.

*Abbreviations: SDE = stochastic differential equation; SV = stochastic volatility; JD = jump-diffusion; HMM = hidden Markov model; NH-HMM = non-homogeneous hidden Markov model; ML = machine learning; DL = deep learning; MMHP = Markov-modulated Hawkes process; PNN = probabilistic neural network; RF = random forest; LSTM = long short-term memory; VaR = Value-at-Risk; ES = Expected Shortfall; CPD = change-point detection; BTC = Bitcoin; ETH = Ethereum; DOGE = Dogecoin; XRP = Ripple; BCH = Bitcoin Cash; SEI = Sei token; DeFi = decentralized finance.

4.3. SDE modelling approaches

The studies can be organized into six model families, consistent with the framework outlined in Section 2:

1. Stochastic Volatility and Jump-Diffusion Models (Section 2.1–2.2).

Fry [25], Chen & Huang [27], and Harasheh & Bouteska [37] employed jump-diffusion and stochastic volatility processes to capture volatility clustering, leverage effects, and heavy tails. Their findings confirm the inadequacy of Gaussian log-normal models and reinforce the relevance of Heston-type structures and Lévy-driven jumps.

2. Order-Flow and Point Processes (Section 2.3).

Ortu et al. [30], Luo & Krishnamurthy [38], and Fabre & Muni Toke [36,39] applied Hawkes-type processes to model order-flow bursts, contagion, and manipulation. Extensions such as MMHP-δ and Neural Hawkes capture both excitation and inhibition effects, providing real-time detection of spoofing and wash trading.

3. Markovian and Regime-Switching Models (Section 2.3).

Kurbucz et al. [29], El-Khatib & Hatemi-J [31], and Pakštaitė et al. [43] demonstrated that regime-switching diffusions and hidden Markov models detect structural breaks and macroeconomic dependencies. These findings align with Section 2.3's emphasis on distinguishing "fair" versus "manipulated" regimes through latent states.

4. Hybrid and Machine Learning–Enhanced Models.

Kalariya et al. [28], La Morgia et al. [32], Avordeh et al. [41], and Pindza et al. [42] integrated stochastic dynamics with machine learning (SNN, LSTM, neural solvers). These studies showed substantial performance gains in forecasting accuracy and trading profitability, but at the expense of interpretability.

5. Change-Point Detection and Control-Theoretic Methods (Section 2.4).

Lukić & Milošević [34], Zournatzidou et al. [35], and Theodosiadou et al. [33] applied Hankel transforms, Hurst analysis, and multivariate change-point detection to identify volatility persistence and illicit events. Luo & Krishnamurthy [38] explicitly combined change-point detection with Hawkes's intensities, paralleling the CUSUM/GLR approaches described in Section 2.4.

6. Systematic Review and Meta-Study.

Dote-Pardo & Espinosa-Jaramillo [44] synthesized 96 papers, concluding that stochastic volatility and regime-switching outperform GARCH during crises, while hybrid deep learning models dominate forecasting. This meta-study validates the general taxonomy set forth in Section 2.

4.4. Performance and validation

Three cross-cutting insights emerge. First, there is a clear trade-off between interpretability and accuracy: classical SDEs yield interpretable financial parameters directly linked to theory, while hybrid ML–SDE models provide superior predictive performance in high-frequency contexts but function largely as black boxes. Second, Hawkes-based methods consistently outperform Poisson and threshold baselines in detecting manipulation events such as spoofing, pump-and-dump, and wash trading, with recent variants (MMHP-δ, PNNs) achieving real-time detection. Finally, regime-switching and change-point models reliably align with known market crises, but reproducibility is limited, as only a few studies (e.g., La Morgia et al. [32]) made code or datasets publicly available.

In summary, the empirical evidence supports the theoretical framework presented in Section 2: SDEs, extended with jumps, regime shifts, and Hawkes-type intensities, provide powerful tools for modelling volatility and detecting manipulation in cryptocurrency markets, while hybridization with machine learning defines the current research frontier.

5. Discussion

5.1. Theoretical implications

The synthesis of evidence confirms that stochastic differential equations (SDEs) provide a robust framework for modelling cryptocurrency markets. Classical stochastic volatility and jump–diffusion processes remain central, with studies showing that they outperform Gaussian log-normal models in capturing volatility clustering, leverage, and heavy tails [25,27,37]. Jump components are particularly important for risk management and hedging, where volatility smiles and abnormal returns are pronounced.

Order-flow and point-process formulations extend these foundations. Hawkes-type models have been widely applied to crypto markets, effectively modelling clustering of order arrivals and manipulation bursts [30,36,38–40]. These processes align closely with the theoretical representation of self-exciting activity described in Section 2.3, confirming their utility for detecting spoofing, pump-and-dump events, and wash trading.

Markov and regime-switching SDEs also feature prominently. Studies demonstrated that these models detect hidden states and structural shifts, reflecting macroeconomic influences, liquidity constraints, and periods of manipulation [29,31,43]. Collectively, these findings support the role of SDEs as mathematically interpretable yet practically relevant tools for market surveillance.

5.2. Strengths and weaknesses of approaches

Each model family offers distinct advantages and limitations (**Table 2**). Classical stochastic volatility and jump–diffusion models [25,27,37] provide interpretable risk parameters, directly supporting regulatory frameworks such as Value-at-Risk (VaR) and Expected Shortfall (ES). However, their flexibility in high-frequency contexts is limited.

Hawkes and related point-process models [30,36,40] excel in identifying manipulation patterns in real time, but parameter estimation can be unstable, and neural variants reduce interpretability. Markov and regime-switching models [29,31,43] are valuable for identifying hidden states and macroeconomic drivers, though their predictive performance weakens over long horizons. Hybrid SDE–machine learning models [28,32,41,42] consistently outperform classical baselines in predictive accuracy but act as black boxes. Change-point methods and statistical transforms [34,35,45] are highly sensitive to structural breaks but lack forecasting robustness.

Table 2. Strengths and weaknesses of stochastic differential equation–based model families.

Model family	Strengths	Weaknesses
Stochastic volatility/jump-diffusion	Interpretable parameters; aligned with VaR/ES	Limited in high-frequency anomaly detection
Hawkes processes & variants	Effective for order clustering and manipulation bursts; real-time detection	Estimation instability; reduced interpretability in neural forms
Markov/regime-switching	Captures hidden regimes, macro effects, illiquidity	Sensitive to covariates; weak long-term forecasts
Hybrid ML–SDE	High predictive accuracy; strong in HFT	Operates as black box; low theoretical transparency
Change-point/transforms	Sensitive to structural breaks; useful in forensic contexts	Limited forecasting robustness
Systematic reviews	Broad synthesis of approaches	No direct model development

5.3. Interpretability versus predictive accuracy

A recurring theme is the trade-off between interpretability and predictive accuracy. Classical SDEs [25,27] yield transparent financial parameters such as volatility and jump intensity, ensuring theoretical alignment with stochastic finance. In contrast, hybrid neural–SDE approaches [41,42] achieve the highest predictive accuracy in high-frequency trading simulations, but sacrifice interpretability. Hawkes processes [30,39] occupy an intermediate position, offering good predictive power with moderate interpretability. **Table 3** summarizes this trade-off.

Table 3. Interpretability and predictive accuracy of stochastic differential equation–based approaches.

Model type	Interpretability	Predictive accuracy	Example studies
Classical SV / Jump-Diffusion	High	Moderate	Fry [25]; Chen & Huang [27]
Regime-Switching SDE	Medium-High	Moderate	Kurbucz [29]; Pakštaitė [43]

Hawkes / Point Process	Medium	High	Ortu [30]; Fabre & Muni Toke [39]
Hybrid SDE–ML	Low	Very High	Avordeh [41]; Pindza [42]

5.4. Regulatory and practical implications

Several models reviewed have direct regulatory relevance. Stochastic volatility and jump–diffusion models improve VaR and ES estimation, strengthening risk compliance [27,37] Hawkes processes and regime-switching SDEs provide early-warning indicators for spoofing, wash trading, and illicit market activity [31,39]. Hybrid SDE–ML approaches offer powerful high-frequency monitoring tools [32,41], though their opacity presents challenges for auditability and enforcement.

5.5. Future directions

Future research should combine the interpretability of classical SDEs with the predictive power of modern machine learning. Explainable neural Hawkes processes [36,40] and macro-driven hidden Markov models [43] represent promising pathways. Reproducibility should also be prioritized: only a few studies [32] released datasets, limiting validation. Finally, the integration of environmental, social, and governance (ESG) dimensions into stochastic frameworks remains an open challenge, especially given the systemic implications of cryptocurrency markets.

6. Conclusion

This review synthesizes 20 studies published between 2018 and 2025 that apply stochastic differential equation (SDE) models to cryptocurrency markets, with emphasis on manipulation detection. The evidence spans six methodological families: stochastic volatility and jump–diffusion processes, Hawkes and point-process models, regime-switching frameworks, hybrid SDE–machine learning approaches, change-point methods, and systematic reviews. Together, these contributions demonstrate that SDEs remain essential for modelling volatility, capturing heavy tails, and identifying abnormal dynamics in digital assets.

The comparative analysis shows clear trade-offs. Classical SDEs provide interpretability and regulatory alignment, Hawkes processes capture clustered manipulation events in real time, and hybrid models achieve the highest predictive accuracy in high-frequency contexts but reduce transparency. These findings have practical implications for market surveillance, risk management, and regulatory enforcement.

Looking ahead, the field should prioritize explainable hybrid models that integrate SDE structures with machine learning, as well as reproducibility through open datasets. Such advances will ensure both theoretical rigor and practical applicability in safeguarding the integrity of cryptocurrency markets.

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References

1. Mehta K, Chawla S. Illuminating the dark corners: a qualitative examination of cryptocurrency's risk. Digital Policy, Regulation and Governance. 2024; 26(2): 188-208. doi: 10.1108/DPRG-10-2023-0147
2. Fletcher E, Larkin C, Corbet S. Countering money laundering and terrorist financing: A case for bitcoin regulation. Research in International Business and Finance. 2021; 56. doi: 10.1016/j.ribaf.2021.101387
3. Jackon G. Cryptocurrency Adoption in Traditional Financial Markets in the United States. American Journal of Finance. 2024; 9(1): 40-50. doi: 10.47672/ajf.1810
4. Nguyen LTM, Nguyen PT. Do crypto investors wait and see during policy uncertainty? An examination of the dynamic relationships between policy uncertainty and exchange inflows of Bitcoin. Review of Behavioral Finance. 2024; 16(2): 234-247. doi: 10.1108/RBF-01-2023-0013
5. Donier J, Bouchaud JP. Why do markets crash? Bitcoin data offers unprecedented insights. PLoS One. 2015; 10(10). doi: 10.1371/journal.pone.0139356
6. Asafo-Adjei E, Owusu Junior P, Adam AM. Information Flow between Global Equities and Cryptocurrencies: A VMD-Based Entropy Evaluating Shocks from COVID-19 Pandemic. Complexity. 2021; 2021. doi: 10.1155/2021/4753753
7. Aslanidis N, Bariviera AF, López ÓG. The link between Bitcoin and Google Trends attention. arXiv. 2021. doi: 10.48550/arXiv.2106.07104
8. Erfanian HR, Hajimohammadi M, Abdi MJ. Using the euler-maruyama method for finding a solution to stochastic financial problems. International Journal of Intelligent Systems and Applications. 2016; 8(6): 48-55. doi: 10.5815/ijisa.2016.06.06
9. To Cheung K. Application and Empirical Analysis of Random Volatility Model in Financial Markets. Highlights in Business, Economics and Management. 2024; 41. doi: 10.54097/t8yke024
10. Houssam B. A Fractional Volatility Model: Estimation of the NASDAQ volatility parameter using Futures pricing. Research Square. 2025. doi: 10.21203/rs.3.rs-6547415/v1
11. Han J, Zhang XP, Wang F. Gaussian Process Regression Stochastic Volatility Model for Financial Time Series. IEEE Journal of Selected Topics in Signal Processing. 2016; 10(6): 1015-1028. doi: 10.1109/JSTSP.2016.2570738
12. Fang G, Ma H, Xia M, Zhang B. The FFBS Estimation of High Dimensional Panel Data Factor Stochastic Volatility Models. ArXiv; 2019. doi: 10.48550/arXiv.1901.10516
13. Hossain MJ, Ismail MT. Is there any influence of other cryptocurrencies on bitcoin? Asian Academy of Management Journal of Accounting and Finance. 2021; 17(1): 125-152. doi: 10.21315/aamjaf2021.17.1.5
14. He Y. Bitcoin Volatility in Web 3.0 and Revelation to Digital Currency. Highlights in Business Economics and Management. 2024; 24: 476-481. doi: 10.54097/9ms5px73
15. Kim J, Yoon J, Yu S. Multiscale Stochastic Volatility with the Hull–White Rate of Interest. Journal of Futures Markets. 2014; 34(9): 819-837. doi: 10.1002/fut.21625
16. Dhifaoui Z. Determinism and Non-linear Behaviour of Log-return and Conditional Volatility: Empirical Analysis for 26 Stock Markets. South Asian Journal of Macroeconomics and Public Finance. 2022; 11(1): 69-94. doi: 10.1177/2277978721995654
17. Burtnyak I, Malytska A. Cev Model with Stochastic Volatility. Journal of Vasyl Stefanyk Precarpathian National University. 2019; 6(3-4): 22-28. doi: 10.15330/jpnu.6.3-4.22-28
18. Billio M, Sartore D. Stochastic Volatility Models: A Survey with Applications to Option Pricing and Value at Risk. In: Applied Quantitative Methods for Trading and Investment. Wiley; 2003. pp. 239-291. doi: 10.1002/0470013265.ch8

19. Withanawasam RM, Whigham PA, Crack TF, Premachandra IM. Simulating Trader Manipulation in a Limit-Order Driven Market. Mathematics and Computers in Simulation. 2011; 93(C): 43-52. doi: 10.1016/j.matcom.2012.09.012
20. Swishchuk A, Vadori N. A Semi-Markovian Modeling of Limit Order Markets. arXiv; 2016. doi: 10.48550/arXiv.1601.01710
21. Bacry E, Jaisson T, Muzy J. Estimation of slowly decreasing Hawkes kernels: application to high-frequency order book dynamics. Quant Finance. 2016; 16(8): 1179-1201. doi: 10.1080/14697688.2015.1123287
22. Bacry E, Mastromatteo I, Muzy JF. Hawkes processes in finance. arXiv; 2015. doi: 10.48550/arXiv.1502.04592
23. Cao Y, Li Y, Coleman S, Belatreche A. Mcginnity TM. Detecting Price Manipulation in the Financial Market. In: Proceedings of the 2014 IEEE Conference on Computational Intelligence for Financial Engineering & Economics (CIFEr); 27-28 March 2014; London, UK. pp. 77-84. doi: 10.1109/CIFEr.2014.6924057.
24. Kaj I, Caglar M. A buffer Hawkes process for limit order books. arXiv. 2017. doi: 10.48550/arXiv.1710.03506
25. Fry J. Booms, Busts and Heavy-Tails: The Story of Bitcoin and Cryptocurrency Markets? Economics Letters. 2018; 171. doi: 10.1016/j.econlet.2018.08.008
26. Dipple S, Choudhary A, Flamino J, et al. Using correlated stochastic differential equations to forecast cryptocurrency rates and social media activities. Applied Network Science. 2020; 5(1): 17. doi: 10.1007/s41109-020-00259-1
27. Chen KS, Huang YC. Detecting jump risk and jump-diffusion model for bitcoin options pricing and hedging. Mathematics. 2021; 9(20). doi: 10.3390/math9202567
28. Kalariya V, Parmar P, Jay P, et al. Stochastic Neural Networks-Based Algorithmic Trading for the Cryptocurrency Market. Mathematics. 2022; 10(9). doi: 10.3390/math10091456
29. Kurbucz MT, Pósfay P, Jakovác A. Linear Laws of Markov Chains with an Application for Anomaly Detection in Bitcoin Prices. arXiv. 2022. doi: 10.48550/arXiv.2201.09790
30. Ortu M, Vacca S, Destefanis G, Conversano C. Cryptocurrency ecosystems and social media environments: An empirical analysis through Hawkes' models and natural language processing. Machine Learning with Applications. 2022; 7: 100229. doi: 10.1016/j.mlwa.2021.100229
31. El-Khatib Y, Hatemi-J A. On a Regime Switching Illiquid High Volatile Prediction Model for Cryptocurrencies. Journal of Economic Studies. 2023; 51(2): 485-498. doi: 10.1108/JES-03-2023-0134
32. La Morgia M, Mei A, Sassi F, Stefa J. The Doge of Wall Street: Analysis and Detection of Pump and Dump Cryptocurrency Manipulations. ACM Transactions on Internet Technology. 2023; 23(1): 1-28. doi: 10.1145/3561300
33. Theodosiadou O, Koufakis AM, Tsikrika T, et al. Change Point Analysis of Time Series Related to Bitcoin Transactions: Towards the Detection of Illegal Activities. Journal of Risk and Financial Management. 2023; 16(9). doi: 10.3390/jrfm16090408
34. Lukić Ž, Milošević B. Change point analysis -- the empirical Hankel transform approach. arXiv. 2023. doi: 10.48550/arXiv.2401.00566
35. Zournatzidou G, Farazakis D, Mallidis I, Floros C. Stochastic Patterns of Bitcoin Volatility: Evidence across Measures. Mathematics. 2024; 12(11). doi: 10.3390/math12111719
36. Fabre T, Toke IM. Neural Hawkes: Non-Parametric Estimation in High Dimension and Causality Analysis in Cryptocurrency Markets. Quantitative Finance. 2024; 25(5): 671-698. doi: 10.1080/14697688.2025.2477673
37. Harasheh M, Bouteska A. Volatility estimation through stochastic processes: Evidence from cryptocurrencies. North American Journal of Economics and Finance. 2025; 75. doi: 10.1016/j.najef.2024.102320
38. Luo R, Krishnamurthy V. Detecting Structural Shifts in Multivariate Hawkes Processes with Fréchet Statistics. arXiv. 2025. doi: 10.48550/arXiv.2308.06769
39. Fabre T, Toke IM. High-Frequency Market Manipulation Detection with a Markov-modulated Hawkes process. arXiv. 2025. doi: 10.48550/arXiv.2502.04027
40. Fabre T, Challet D. Learning the Spoofability of Limit Order Books With Interpretable Probabilistic Neural Networks. arXiv. 2025. doi: 10.48550/arXiv.2504.15908
41. Avordeh TK, Arthur S, Quaidoo C. Hybrid machine learning and stochastic volatility models with blockchain data for high-frequency cryptocurrency trading. Research Square. 2025. doi: 10.21203/rs.3.rs-6352921/v1
42. Pindza E, Clement J, Mwambi S, Umeorah N. Neural Network for Valuing Bitcoin Options Under Jump-Diffusion and Market Sentiment Model. Computational Economics. 2024; 66(3): 2305-2342. doi: 10.1007/s10614-024-10792-1

43. Pakštaitė V, Filatovas E, Juodis M, Paulavičius R. Bitcoin Price Regime Shifts: A Bayesian MCMC and Hidden Markov Model Analysis of Macroeconomic Influence. Mathematics. 2025; 13(10). doi: 10.3390/math13101577
44. Dote-Pardo J, Espinosa-Jaramillo MT. Mathematical Models in Cryptocurrency Markets and Decentralized Finance (DeFi): Pricing, Risk, and Network Dynamics. SSRN. 2025. doi: 10.2139/ssrn.5248613