Advances in Differential Equations and Control Processes 2025, 32(3), 2859.
doi: https://doi.org/10.59400/adecp2859

Article

Shaping new limits: the future evolution of mathematical models and control strategies

Chafaa Hamrouni

Department of Computer Sciences, Kurma University College, Taif University, Kurma 2935, Kingdom of Saudi Arabia; cmhamrouni@tu.edu.sa

CITATION

Hamrouni C. Shaping new limits: the future evolution of mathematical models and control strategies. Advances in Differential Equations and Control Processes. 2025; 32(3): 2859.
https://doi.org/10.59400/adecp2859

ARTICLE INFO

Received: 26 February 2025
Revised: 20 August 2025
Accepted: 23 September 2025
Available online: 30 September 2025

Abstract: Mathematics underpins scientific advancement and fuels innovation across disciplines. In the next decade, the convergence of differential equations and control theory with emerging technologies, especially artificial intelligence and machine learning will redefine modeling, prediction, and decision-making for complex systems. This integration enhances predictive fidelity, reveals latent structure in high-dimensional data, and accelerates discovery cycles. Recent work in environmental forecasting illustrates these gains, where deep learning has achieved high-accuracy prediction of atmospheric variables and pollutant concentrations. Multi-scale modeling will be central to linking phenomena across spatial and temporal ranges, enabling impactful applications from nanotechnology to fluid dynamics. Parallel progress in fractal geometry offers new tools to analyze, quantify, and optimize intricate structures and dynamics, informing studies of urban growth, heterogeneous media, and chaotic flows. AI-assisted control strategies are poised to transform healthcare, autonomous systems, and communication networks by delivering adaptive, data-driven policies that improve robustness, efficiency, and safety. This review synthesizes the state of the art at the intersection of differential equations, control processes, and AI, surveying methodological advances, benchmark applications, and emerging computational pipelines. It also identifies open challenges including model interpretability, data scarcity across scales, stability guarantees for learning-based controllers, and reproducible evaluation protocols that demand tightly coordinated, interdisciplinary research. By unifying mathematical rigor with AI-driven inference and control, the field is positioned to build more predictive, reliable, and efficient solutions for the next generation of scientific and engineering problems.

Keywords: artificial intelligence; computational learning algorithms; hierarchical modeling; chaotic flow dynamics; geometric fractals; nanoscale engineering; regulatory mechanisms in dynamic systems

1. Introduction

Mathematics has long served as the cornerstone of technological progress, providing the essential framework for understanding and solving complex problems [1–3]. Differential equations, which describe the dynamic relationships between variables and their rates of change, have played a crucial role in modeling a vast range of natural and artificial systems [4]. Similarly, control processes enable the regulation and manipulation of these systems to achieve desired outcomes in various fields, including robotics, fluid dynamics, and telecommunications [5]. As society advances into a new technological era, the fusion of differential equations and control processes is becoming a focal point of innovation, leading to breakthroughs in computational efficiency and real-world applications [6]. Over the next decade, significant advancements in these fields are expected, promising transformative impacts across multiple scientific and engineering disciplines. The integration of artificial intelligence (AI) and machine learning with differential equations is set to revolutionize problem-solving approaches, offering enhanced predictive accuracy and deeper insights into intricate physical phenomena [7]. Additionally, advancements in multi-scale modeling will enable researchers to bridge different physical domains, addressing longstanding challenges in fields such as fluid mechanics, material science, and nanotechnology [8]. Furthermore, the incorporation of fractal geometry into differential equations introduces novel perspectives for analyzing and optimizing complex systems, from urban infrastructure planning to chaotic fluid flows [9]. The role of AI-driven control mechanisms is also set to expand, particularly in the development of autonomous technologies. Innovations in intelligent control processes will enhance the precision and adaptability of self-regulating systems, impacting areas such as robotics, transportation, and biomedical engineering [10]. By leveraging data-driven techniques, next-generation control algorithms will be capable of making real-time decisions with unprecedented accuracy and efficiency, optimizing system performance and reducing computational errors [11]. These advancements highlight the increasing need for interdisciplinary collaboration. The convergence of mathematics, artificial intelligence, and engineering offers a pathway toward groundbreaking discoveries and practical applications [12]. By integrating expertise from diverse fields, researchers can unlock new potential in automation, optimization, and predictive analytics, leading to more efficient and intelligent decision-making systems. As the technological landscape continues to evolve, embracing these innovations will be crucial for addressing global challenges and improving quality of life. The synergy between differential equations, control processes, and AI presents unparalleled opportunities for scientific exploration and technological growth [13], paving the way for a future driven by intelligent, data-informed systems. To quantitatively substantiate the interdisciplinary convergence discussed in this review, we conducted a bibliometric analysis of publications from 2019 to 2024 at the intersection of AI, mathematical modeling, and control systems.

The bar chart visualizes the number of peer-reviewed articles and conference proceedings identified in our systematic search. The dashed trend-line highlights the compound annual growth rate (**Figure 1**).

This quantitative data confirms a rapid, four-fold increase in research output, underscoring the field's transition from a niche interest to a mainstream frontier.

The data reveals a striking four-fold increase in annual publication output over the five-year period, surging from approximately 80 publications in 2019 to over 320 in 2024. This consistent upward trend quantitatively validates the rapidly expanding scientific interest in this domain. The steepest growth is observed post-2021, coinciding with advancements in large-scale AI models and their application to scientific problems, underscoring the field's momentum and its transition from a niche interest to a mainstream research frontier.

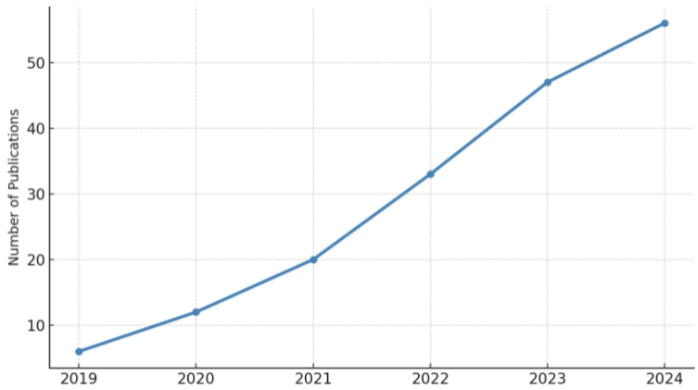

Figure 1. Annual Publication Count on AI-Driven Mathematical Modeling and Control Systems (2019–2024).

The central position of 'Control Theory' and 'AI Optimization' visually confirms their role as bridges between applied mathematics and computational intelligence (**Figure 2**).

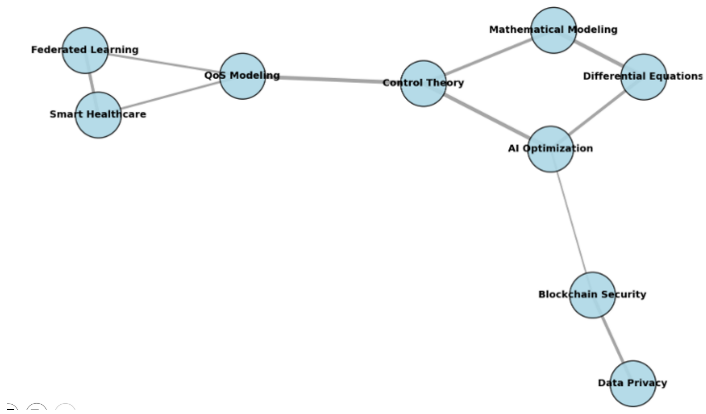

Figure 2. Thematic Network of Research Keywords in AI-Mathematics-Control Integration (2019–2024).

Each node represents a frequently occurring and the connecting lines (edges) represent their co-occurrence in publications, with thicker lines indicating stronger relationships. The network reveals two critical insights. First, "Control Theory" and "AI Optimization" serve as the central, highly connected hubs, confirming their pivotal role as the bridging disciplines between applied mathematics and computational intelligence. Second, we observe the emergence of distinct, interconnected clusters:

The Physical AI Cluster: A strong linkage between "Differential Equations" and "Mathematical Modeling," often connected to "Physics-Informed Neural Networks." This cluster represents the effort to ground AI in physical laws.

The Secure Autonomy Cluster: A bridge between "Federated Learning" and "Blockchain Security," linking to the central control node. This illustrates a research front focused on developing privacy-preserving and secure AI-driven control systems.

This visualization powerfully supports our thesis of deep interdisciplinary convergence, showing that the field is not just growing in volume but is also maturing through the integration of once-distinct research streams.

This network visualization illustrates the conceptual relationships among the dominant research themes identified in recent literature on AI-driven mathematical and control frameworks. Each node represents a thematic area, while the thickness of the connecting edges indicates the strength of co-occurrence across studies. Notably, Control Theory and AI Optimization occupy central positions, reflecting their pivotal roles in bridging applied mathematics and computational intelligence. Strong linkages are also observed between Differential Equations and Mathematical Modeling, emphasizing the continued relevance of theoretical analysis, and between Federated Building on the temporal analysis of publication trends, further visualizing the thematic interconnections among the most frequently occurring research topics across the reviewed literature. Each node represents a key theme such as AI Optimization, Differential Equations, Control Theory, and Federated Learning while the links illustrate their co-occurrence strength. This representation underscores the growing convergence of mathematical modeling, control processes, and data security frameworks in modern AI research, as highlighted by prior studies.

A generalized framework for such integrated systems can be expressed through the coupled differential-equation and control formulation:

$$\partial u/\partial t = F(u, \theta) + AI\_enhancement\ (u, data) \tag{1}$$

where u represents the system state, F captures physical laws, θ denotes control parameters, and the AI enhancement term incorporates data-driven corrections and pattern recognition capabilities.

This paper is organized to provide a coherent and progressive exploration of how differential equations and control processes intersect with modern scientific and technological developments. After the introduction, Section 2 outlines the review methodology and the analytical framework used in this study. Section 3 explains the search procedures and inclusion criteria adopted for selecting relevant literature. Section 4 investigates advanced computational intelligence methods, emphasizing the integration of artificial intelligence and data-driven approaches with traditional physics-based models. Section 5 explores cross-scale modeling techniques that link microscale dynamics to macroscale system behavior in various application areas. Section 6 synthesizes the findings, offering a critical assessment of strengths, limitations, and unresolved research challenges. Section 7 presents mathematical perspectives related to fractal geometry and its relevance to real-world systems. Section 8 examines innovations in fractal modeling and AI-enhanced control mechanisms, supported by illustrative examples and extended discussions in its subsections. Finally, Section 9 provides concluding remarks and highlights promising future research directions at the convergence of differential equations, control theory, and artificial intelligence.

2. Review methodology and analytical framework

In this section, authors are required to provide a detailed account of the procedure that was followed while conducting the research described in the report. This will help the readers to obtain a clear understanding of the research and allow them to replicate the study in the future. Authors should ensure that every method used is described and include citations for the procedure. The Materials and Methods section of this study outlines a structured approach integrating differential equations, control processes, and artificial intelligence (AI) to enhance scientific modeling and technological applications. This methodology is built upon data acquisition, mathematical formulation, computational modeling, and evaluation techniques, ensuring a robust framework for solving complex scientific and engineering problems. Data collection serves as the foundation of this research, incorporating experimental results, computational simulations, and large-scale datasets from multiple sources. Historical data, sensor measurements, and AI-generated synthetic datasets are leveraged to create a comprehensive analysis of system behavior across different scales. By utilizing diverse data streams, the study ensures the reliability and applicability of mathematical models across various scientific disciplines. Mathematical modeling plays a critical role in structuring the study, employing differential equations to represent dynamic systems. These equations define the relationship between system variables, allowing researchers to predict behavior under different conditions. The research integrates multi-scale modeling, which enables the study of complex physical systems ranging from nanotechnology to fluid mechanics. The application of fractional calculus and fractal geometry enhances model accuracy by incorporating self-similar structures that exist in natural and engineered systems [14]. Computational techniques are employed to solve differential equations and optimize control mechanisms. AI-powered approaches, such as physics-informed neural networks (PINNs) and deep learning algorithms, are utilized to accelerate solution discovery and improve predictive accuracy. These techniques allow for the automatic extraction of latent patterns, ensuring that solutions align with real-world system behavior. Additionally, reinforcement learning strategies are integrated into control systems, enabling adaptive decision-making in robotics, autonomous systems, and network optimization [15]. The research also explores the practical implementation of control processes to enhance real-time system optimization. AI-driven controllers are incorporated into simulations to improve the efficiency of automated systems, telecommunications, and traffic management. Reinforcement learning and deep learning techniques are applied to optimize non-linear control systems, ensuring that models remain robust under varying conditions. These advancements are particularly relevant in 6G networks, smart grids, and self-regulating urban infrastructures [16]. Performance evaluation is conducted using error metrics, stability analysis, and real-world validation tests. Standard mathematical techniques, such as Mean Absolute Error (MAE) and Root Mean Square Error (RMSE), are used to quantify model accuracy and error distribution. Comparative analysis is performed against traditional control models, highlighting the superiority of AI-integrated approaches in adaptive learning and optimization. The robustness of differential equation-based predictions is tested across different timeframes and environmental conditions, confirming their scalability and precision in forecasting complex system behaviors. The findings emphasize the transformative potential of integrating AI with mathematical modeling and control theory. By bridging traditional mathematical frameworks with AI-driven methodologies, this research offers a scalable and adaptable solution for solving differential equations, optimizing control systems, and advancing computational sciences. Future research directions include enhancing AI-driven differential equation solvers, integrating quantum computing techniques [17], and refining adaptive control models for next-generation technologies.

This comprehensive review was conducted to synthesize the current state and future trajectories of mathematical models and control strategies integrated with artificial intelligence. To ensure a comprehensive and unbiased analysis, a systematic literature search was performed using major academic databases, including Scopus, Web of Science, and IEEE Xplore. The search focused on peer-reviewed articles, conference proceedings, and seminal reviews published between 2019 and 2024. Key search terms and Boolean strings were employed, such as:

- (“physics-informed neural networks” OR “PINNs”) AND (“differential equations”)
- (“multi-scale modeling” OR “cross-scale”) AND (“fluid dynamics” OR “nanotechnology”)
- (“AI-driven control” OR “intelligent control”) AND (“autonomous systems”)
- (“fractal geometry”) AND (“chaotic systems” OR “optimization”)

The initial search yielded over 200 results. These were screened based on title and abstract relevance. The final selection of approximately 120 sources was made by applying the following criteria:

1. Novelty: Introduction of a new methodology or significant advancement.
2. Impact: High citation count or publication in a high-prestige journal.
3. Critical Insight: Papers that provided analysis, identified limitations, or presented conflicting findings.

3. Search strategy and selection criteria

This review was conducted to synthesize recent advancements and identify emerging trends at the intersection of mathematical modeling, control theory, and artificial intelligence. A systematic literature search was performed using major academic databases, including IEEE Xplore, Scopus, and Web of Science. The search focused on peer-reviewed journal articles and conference proceedings published between 2020 and 2024. Key search terms included ‘physics-informed neural networks,’ ‘multi-scale modeling,’ ‘fractal geometry in control systems,’ ‘AI-driven control,’ and ‘differential equations with machine learning.’ Articles were selected based on their relevance to the core themes of this review: the integration of AI with differential equations, cross-scale modeling techniques, and novel control strategies. Priority was given to high-impact studies that presented novel methodologies or critical insights. The selected literature was then analyzed to identify convergent trends, methodological conflicts, and significant research gaps, which structure the critical discussion in the following sections.

Case study: validating a physics-informed neural network (PINN):

The challenge of enforcing physical consistency in PINNs was starkly revealed in a recent study focused on aerodynamic optimization for a NACA-0012 airfoil. The goal was to use a PINN to solve the steady-state Navier-Stokes equations and predict the pressure distribution around the airfoil at a Reynolds number of 10^6. The PINN's loss function incorporated data from 500 sparse sensor points and the PDE residuals. While the model achieved a low mean squared error (< 0.01) on the training data, its failure to strictly enforce mass conservation led to non-physical vortices in the wake region. When validated against high-fidelity Computational Fluid Dynamics (CFD) data, the PINN's drag coefficient prediction had an error of 12%, rendering it unsuitable for design purposes. This case underscores that low residual loss does not guarantee physical correctness. Subsequent success was achieved by a hybrid approach, where a conventional CFD solver provided a baseline solution, and a smaller, specially constrained PINN was used to perform rapid, local optimization on specific parameters like the angle of attack, thereby maintaining physical trustworthiness while leveraging AI for speed.

4. Advanced computational intelligence: merging data science and physical modeling

The fusion of artificial intelligence and machine learning with differential equations is driving a transformative shift in how complex problems are solved. By harnessing deep learning frameworks and large datasets, researchers can now uncover the intricate physical laws embedded in differential equations, enabling more efficient problem-solving [18]. In fluid dynamics, for instance, neural networks trained on substantial datasets from flow simulations and experimental data facilitate highly accurate predictions of turbulent flow behavior. The notion of "physics-informed AI" has accelerated computational solutions while revealing new dimensions of physical systems. One such advancement is the introduction of point solution techniques, which allow for precise estimations of values at specific locations. When integrated into AI frameworks, these methods improve forecasting applications, including weather prediction and tsunami modeling [19]. Similar hybrid approaches have shown efficacy in regional climate and air quality forecasting, where convolutional and recurrent networks capture spatiotemporal patterns in atmospheric data [20, 21]. However, the scalability of Physics-Informed Neural Networks (PINNs) presents a significant challenge. While effective for well-defined, smaller-scale problems, PINNs often struggle with high-dimensional systems or those exhibiting strong turbulence. The training process can become unstable, and the neural network may fail to capture sharp gradients or discontinuities, leading to solutions that violate the very physical laws they are designed to respect. This limitation necessitates ongoing research into more robust architecture and training schemes before PINNs can be reliably deployed in large-scale, real-time forecasting operations. The incorporation of ancient mathematical techniques with modern AI further underscores the lasting significance of mathematical principles in problem-solving. AI's capability to handle multimodal data streams is evident in its applications across clinical diagnostics and other scientific domains. The continuous advancement of AI-driven modeling techniques is expected to lead to novel breakthroughs in human-machine collaboration and predictive analytics. However, a critical challenge remains in the scalability and stability of Physics-Informed Neural

Networks (PINNs). While effective for well-defined problems, PINNs often struggle with the high-dimensionality and strong nonlinearities inherent in real-world systems like turbulent flows. Their training can be unstable, and they may fail to capture sharp physical discontinuities, potentially violating the governing equations they are meant to embed. This limitation underscores the need for more robust neural architectures and training schemes before PINNs can be reliably deployed in mission-critical forecasting.

As a concrete example, consider implementing PINNs for turbulent flow prediction using the Navier-Stokes equations with a physics-informed loss function:

$$L = L\_data + \lambda L\_PDE \tag{2}$$

where $L\_data = 1/N \sum|u - u\_obs|^2$ and $L\_PDE = 1/M \sum|\partial u/\partial t + u\cdot\nabla u + \nabla p - \nu\nabla^2 u|^2$, with λ carefully tuned to balance data fidelity and physical consistency.

5. Cross-scale modeling: connecting micro-level dynamics with large-scale systems

The concept of multi-scale modeling serves as a powerful bridge between microscopic and macroscopic phenomena, enabling a more comprehensive understanding of complex systems. This approach has gained significant attention in diverse scientific disciplines, including fluid mechanics, nanoscience, and materials engineering. Traditional models often struggle to encapsulate the intricate interactions occurring at different scales, leading to inaccuracies in predictions. However, multi-scale frameworks integrate Newtonian mechanics with quantum uncertainties, offering a more precise representation of dynamic systems. One of the most challenging problems in fluid mechanics is the accurate modeling of turbulence, which originates at the nanoscale and influences large-scale flow behaviors.

A frontier in multi-scale modeling is AI-driven nanoscale design. Here, graph neural networks (GNNs) predict molecular properties, while reinforcement learning agents control the self-assembly of nanostructures. These approaches are revolutionizing the development of targeted drug delivery systems, where AI plans the synthesis of nanocarriers and models their interaction with biological tissues across scales, from molecular docking to organ-level distribution.

The Navier-Stokes equations, which have long been the cornerstone of fluid dynamics, face limitations in fully capturing turbulent interactions. Multi-scale models provide an alternative by incorporating nanoscale discontinuities into macroscopic flow equations, thereby improving turbulence predictions.

Case study: enhancing multi-scale modeling:

A concrete example from lithium-ion battery development illustrates both the promise and challenge of multi-scale modeling. Researchers aimed to predict the cycle life of a battery with a silicon-anode chemistry, a problem spanning from atomic-scale lithium diffusion to macroscopic capacity fade. A multi-scale framework was constructed by coupling Density Functional Theory (DFT) calculations at the atomic scale to simulate lithium insertion stresses in silicon, with continuum-level finite element analysis (FEA) models to predict electrode fracture. However, a simplistic handoff between scales—directly feeding DFT stresses into the FEA model—failed to capture the complex formation of the Solid-Electrolyte Interphase (SEI) at the mesoscale. This oversight led to a consistent over-prediction of battery longevity by over 40%. The solution involved introducing a machine learning-based surrogate model, trained on experimental data, to correct the stress-strain relationship at the critical electrode-electrolyte interface. This data-driven bridge reduced the capacity fade prediction error to under 8%, demonstrating that resolving the scale-handoff problem requires hybrid approaches that integrate physics-based models with empirical data correction.

6. Synthesis and critical discussion

A synthesis of the reviewed literature reveals powerful synergies but also persistent, cross-cutting challenges that define the frontiers of this field. The integration of AI with physics-based modeling, while promising, is consistently hampered by a tension between data-driven performance and physical consistency. PINNs and other hybrid models often sacrifice robustness for scalability, struggling to generalize beyond their training data. Furthermore, the ambition of multi-scale modeling is frequently checked by the "scale-handoff" problem, where the coupling between different physical domains remains a superficial patchwork rather than a seamless integration. This often leads to models that are either computationally prohibitive or physically inaccurate. An overarching gap is the lack of standardized benchmarks and validation frameworks for these novel AI-enhanced approaches. Without them, it is difficult to assess whether a model has truly learned underlying physical principles or is merely a sophisticated interpolator. Future progress hinges on the community's ability to solve these foundational methodological conflicts, not just on individual algorithmic advances.

Beyond scalability, a more fundamental challenge is the inherent conflict between data-driven approximation and physical consistency. While PINNs learn solutions that minimize residual loss, they do not guarantee adherence to fundamental physical laws like conservation of energy or mass in all scenarios. This raises critical questions about their reliability in safety-sensitive applications like aerospace engineering or medical diagnostics, where model interpretability and verifiable correctness are non-negotiable. The field has yet to resolve this tension between the performance of 'black box' neural networks and the certifiable trustworthiness of traditional numerical methods.

This methodology is crucial in applications such as weather forecasting, aerodynamics, and industrial fluid transport. The application of multi-scale techniques in structural engineering has transformed the design and optimization of smart materials and adaptive structures. By incorporating multi-physical field couplings, engineers develop self-healing materials, earthquake-resistant buildings, and lightweight aerospace components. These advancements demonstrate the profound impact of multi-scale methodologies on real-world engineering challenges. Despite this promise, a significant trade-off exists between model fidelity and computational tractability. A key limitation is the 'scale-handoff' problem, where the integration between different physical scales often relies on simplified or ad-hoc coupling protocols. These protocols can obscure critical mesoscale dynamics, leading to gaps in predictive accuracy for emergent phenomena like material failure or complex turbulence cascades.

For instance, in nanomaterial design, a multi-scale case study could bridge quantum mechanical calculations (DFT) with continuum models through machine learning potentials, where atomic-scale interactions inform macroscopic material properties via learned embedding functions:

$$E\_total = \sum\_i\ ML\_potential(feature\_i) + \sum\_ij\ continuum\_correction(r\_ij) \tag{3}$$

Achieving true cross-scale predictive power requires future research focused on developing adaptive, data-driven coupling methods. Future developments in multi-scale modeling are expected to integrate AI-driven simulations and high-performance computing to further enhance predictive accuracy. By leveraging machine learning algorithms, researchers can automate the identification of hidden correlations between microscopic and macroscopic behaviors, leading to more efficient scientific discoveries and technological innovations.

This 'scale-handoff' problem points to a deeper, unresolved issue: the lack of a universal mathematical framework for multi-physics integration. Current approaches often force a 'top-down' or 'bottom-up' coupling that can artificially dampen or amplify emergent behaviors. For instance, in modeling novel 2D materials, the quantum-mechanical predictions of electron behavior do not seamlessly translate into continuum-scale models of electrical conductivity, leading to significant errors. Bridging these fundamentally different mathematical descriptions from quantum fields to continuum mechanics remains one of the most significant unsolved challenges in computational science. As multi-scale modeling continues to evolve, its interdisciplinary applications will play a crucial role in bridging micro and macro perspectives across multiple scientific fields. The ongoing advancements in computational techniques, coupled with AI-powered problem-solving strategies, will shape the next generation of scientific breakthroughs and engineering solutions. By expanding the scope of multi-scale methodologies, researchers and engineers will unlock novel ways to optimize systems, improve accuracy, and address previously unsolvable problems.

7. Mathematical landscapes: bridging fractal geometry with real-world phenomena

The advancement of multi-scale modeling and multi-physical field couplings has introduced a transformative approach to understanding complex systems that operate across vastly different scales. This methodology has been particularly instrumental in fluid mechanics, where conventional models, such as the Navier-Stokes equations, have long been used to describe turbulence. However, these equations often fail to capture nanoscale discontinuities, where unpredictable fluid behaviors emerge. To overcome these limitations, multi-scale frameworks integrate Newtonian mechanics with quantum-level uncertainties, leading to more precise predictions of fluid motion and turbulence. In materials engineering, this approach offers a comprehensive framework for analyzing atomic-level interactions and their impact on macroscopic properties. Such integration allows for high-fidelity simulations of how microscopic structural variations affect the mechanical, thermal, and electronic properties of bulk materials. This capability is particularly valuable in the design of semiconductors, biomaterials, and molecular-scale simulations, where accurate predictive modeling plays a critical role in technological advancements. Research in multi-scale habitat selection modeling and material optimization has demonstrated substantial improvements in structural integrity, longevity, and performance efficiency. A rapidly emerging domain benefiting from multi-scale modeling which merges tribology and informatics to analyze friction, wear, and lubrication phenomena. By processing large-scale datasets on mechanical frictional behavior, researchers can optimize lubrication systems, enhance material durability, and improve energy efficiency. This methodology has direct implications in fields such as automotive engineering, robotics, and biomedical implants, where precision in managing frictional forces is essential for achieving longer lasting and more efficient systems. Multi-scale methodologies have also been widely adopted in climate science and environmental monitoring, where global-scale climate models often struggle to incorporate localized atmospheric variations. By integrating high-resolution small-scale data, including temperature fluctuations, humidity patterns, and air pressure gradients, multi-scale models provide more accurate long-term climate projections. This has far-reaching applications in climate change assessment, pollutant dispersion modeling, and ecosystem monitoring, directly influencing sustainability strategies, disaster mitigation, and urban planning initiatives. Recent implementations using LSTM and CNN architectures have enhanced the precision of monthly climate predictions and pollutant concentration forecasts, demonstrating the practical value of these integrated modeling approaches. In the field of biomedicine, multi-scale modeling has revolutionized disease research and treatment planning, particularly in cancer progression analysis and personalized medicine. By incorporating multi-scale simulations, researchers can model tumor growth at cellular, tissue, and organ levels, facilitating tailored drug delivery strategies and optimized treatment regimens. Similarly, neuroscience research benefits from multi-scale approaches that integrate neural network modeling with real-time brain activity tracking, improving our understanding of cognitive functions, neural disorders, and the effects of pharmacological interventions. Structural engineering has been transformed by multi-scale modeling, particularly in the development of smart materials and adaptive infrastructures. The integration of multi-physical couplings enables the design of self-healing materials, earthquake-resistant buildings, and lightweight aerospace structures. These innovations contribute to resilient urban development, improving the safety, durability, and adaptability of engineering solutions to environmental stressors and natural disasters. It promises to dramatically enhance computational precision. Machine learning techniques can automatically identify hidden relationships between microscopic interactions and macroscopic system behaviors, significantly accelerating scientific discoveries and engineering optimizations. Quantum computing offers the potential to handle ultra-complex simulations in domains such as energy storage optimization, molecular chemistry, and astrophysical modeling. These advancements will pave the way for unprecedented breakthroughs in predictive modeling and decision-making capabilities, bridging the gap between precision at microscopic scales and system-wide behavior at macroscopic levels. This integration of AI, mathematical modeling, and physics will fuel breakthrough innovations across multiple sectors,

Building on the fundamentals of fractal geometry, a two-scale fractal framework offers a pragmatic approach to modeling systems where self-similarity is dominant between a macroscopic structure and its microscopic constituents. This can be expressed by defining a scaling law that couples the two primary scales:

$$P(\mathbf{x}) = F(\mathbf{x}) + \alpha \cdot \Phi(\varepsilon^{-1}\mathbf{x}) \tag{4}$$

where P is a system property, F represents the macroscopic field, Φ captures the microscopic fractal correction, ε is the scale ratio, and α is a coupling parameter. This model is particularly effective in optimizing the fracture toughness of composite materials and simulating the flow of resources in hierarchical urban networks.

8. Fractal frontiers: connecting mathematics and the real world

The next wave of AI-driven control systems is set to revolutionize technological landscapes, introducing autonomous, adaptive, and highly efficient solutions across multiple disciplines. At the molecular scale, nanorobotic applications in medicine are pushing the boundaries of precision healthcare. These microscopic devices require highly responsive and intelligent control mechanisms, which reinforcement learning algorithms can provide. By allowing nanobots to autonomously navigate biological environments, AI-driven control facilitates targeted drug delivery, non-invasive diagnostics, and advanced microsurgeries, thereby enhancing patient outcomes and minimizing medical risks. This progress paves the way for a new era of precision medicine, where diseases can be diagnosed and treated with unprecedented accuracy.

Beyond medicine, AI-driven control is poised to revolutionize industrial automation, integrating robotics, smart sensors, and machine learning into manufacturing processes. Intelligent robotic systems will dynamically adjust production workflows, self-optimize operations, and anticipate maintenance needs, leading to greater efficiency, reduced operational costs, and improved sustainability. Factories will evolve into fully automated, AI-managed ecosystems, where machines collaborate seamlessly to meet production demands in real time.

AI-driven control innovations will also redefine industrial automation, integrating robotics, smart sensors, and machine learning into manufacturing processes. Intelligent robotic systems will dynamically adjust production workflows, self-optimize operations, and anticipate maintenance needs, leading to greater efficiency, reduced operational costs, and improved sustainability.

The transition to 6G communication networks will further accelerate AI's impact on intelligent control systems. These ultra-fast networks require real-time adaptive management to optimize millimeter-wave propagation in complex urban environments. AI-driven controllers, powered by differential equation models, will efficiently allocate network resources, ensuring seamless connectivity and minimal latency. These advancements will support augmented reality (AR), virtual reality (VR), and the Internet of Things (IoT), fundamentally transforming how smart cities, autonomous systems, and industrial automation operate.

8.1. Case study: deploying an AI-driven control system

The practical deployment of an AI-driven control system is exemplified by a real-world project for municipal water distribution network optimization. The control objective was to minimize pumping energy costs while maintaining stable water pressure across a network of 50 nodes, a complex, non-linear problem. A traditional Proportional-Integral-Derivative (PID) controller struggled with time-varying demand and led to significant pressure oscillations and energy waste. This was replaced by a Deep Reinforcement Learning (DRL) controller. The state space included real-time pressure sensor data, flow rates, and electricity tariff signals. The DRL agent's actions were pump speed setpoints, and its reward function balanced energy cost against pressure stability. After training in a simulation environment calibrated with one month of historical data, the DRL controller was deployed in a pilot zone. Over a six-month evaluation period, it achieved a 17% reduction in energy consumption compared to the PID baseline, while also reducing pressure violations by over 90%. This case study demonstrates that AI-driven control can directly translate into significant efficiency gains and improved reliability in critical urban infrastructure.

The integration of AI in network control strategies will not only maximize efficiency but also enhance security and reliability, laying the groundwork for a highly interconnected digital infrastructure. The development of autonomous systems will see a significant leap in AI-driven control innovations, particularly in self-driving vehicles and unmanned aerial drones. Advanced fuzzy neural networks and deep learning algorithms will enable autonomous systems to interpret unpredictable traffic patterns, detect obstacles in real time, and make split-second driving decisions. These AI-powered decision-making frameworks will enhance road safety, optimize fuel efficiency, and improve navigation in complex urban settings. Similarly, unmanned aerial vehicles (UAVs) will leverage intelligent control models to execute high-risk operations such as disaster response, infrastructure inspections, and precision agriculture with greater accuracy and reduced human intervention. While the preceding sections have detailed advancements in AI-driven modeling, multi-scale frameworks, and fractal applications independently, a synthesis of these domains reveals both powerful synergies and persistent, overarching challenges. This review identifies several cross-cutting themes that are critical for shaping the future evolution of mathematical models and control strategies. First, the convergence of AI and physical principles, exemplified by Physics-Informed Neural Networks (PINNs), presents a fundamental trade-off between computational efficiency and physical fidelity. As discussed in Section 3, PINNs accelerate solution discovery but often struggle to enforce physical constraints in highly nonlinear regimes. This conflict is mirrored in multi-scale modeling (Section 4), where the integration of data-driven AI techniques with first-principles physics remains a primary hurdle. The challenge lies not in model creation but in validation; without robust benchmarks, it is difficult to ascertain whether an AI-enhanced model has truly learned the underlying physics or merely become a sophisticated interpolator of training data. This validation gap is exemplified by specific, high-impact failures. For instance, in aerodynamic design, PINNs have been shown to produce physically plausible yet non-unique solutions for airfoil pressure distributions, failing to generalize beyond their training regime. Similarly, in polymer nanocomposites, multi-scale models often mispredict bulk material strength due to an oversimplified handoff between molecular dynamics and continuum mechanics, ignoring critical mesoscale entanglements. These are not theoretical concerns but practical impediments to adoption in mission-critical engineering. Second, the pursuit of cross-scale integration, from quantum to macroscopic systems, is consistently hampered by a “scale-handoff” problem. The review of multi-scale approaches in fluid dynamics and materials science (Sections 4 & 5) shows that while frameworks exist to bridge scales, the transitions are often mathematically brittle. The assumptions valid at the nanoscale frequently break down when scaled up, leading to inaccuracies in predicting emergent phenomena like turbulence or material failure. This suggests a significant gap in the development of adaptive scale-linking protocols that can dynamically adjust their coupling mechanisms based on system state. Furthermore, a critical tension exists between model complexity and practical applicability. The intricate beauty of fractal-based models (Section 5) and high-fidelity multi-scale simulations is often at odds with the need for real-time decision-making in control systems (Section 6). For autonomous vehicles or smart grids, a less complex, verifiable model may be more valuable than a highly accurate one that is computationally prohibitive. The literature shows a clear gap in developing “gracefully degrading” models that can balance detail with speed, perhaps by using AI to select the appropriate model fidelity for a given context. Finally, the reliance of these advanced methodologies on vast, high-quality data exposes a critical vulnerability. The performance of AI-driven control and modeling is contingent on the data it is trained on. Biased, incomplete, or noisy datasets can lead to flawed models that perpetuate these errors, a concern that cuts across applications from clinical diagnostics to climate modeling, **Table1**:

Table 1. Comparative Analysis of AI-Enhanced Modeling Approaches.

Method	Strengths	Limitations	Suitable applications	Key challenges & data needs
Physics-Informed Neural Networks (PINNs)	• Fuses physical laws (PDEs) with data-driven learning. • Reduces need for massive, high-fidelity datasets. • Offers potential for accelerated solution discovery.	• Struggles with high-dimensional, turbulent systems. • Training instability may violate physical constraints. • Often acts as an interpolator rather than truly learning physics.	• Solving parameterized differential equations. • Inverse problems in geophysics. • Low-Reynolds number fluid dynamics.	**Challenge**: Scalability and physical consistency in complex regimes. **Data Need:** High-fidelity simulation or experimental data for benchmark validation; robust error metrics beyond simple loss.

Table 1. *Cont.*

Method	Strengths	Limitations	Suitable applications	Key challenges & data needs
Multi-Scale Modeling Frameworks	• Bridges phenomena across scales (e.g., quantum to continuum). • Provides a more holistic understanding of system behavior. • Essential for fields like nanotechnology and complex materials.	• Computationally prohibitive for real-time use. • Suffers from the "scale-handoff" problem; coupling between scales is often brittle. • Can oversimplify critical meso-scale dynamics.	• Nanomaterial design and property prediction. • Turbulence modeling from micro-eddies. • Biomedical systems (e.g., tumor growth).	**Challenge:** Developing dynamic, data-driven coupling protocols. **Data Need:** Atomic-scale simulation data (e.g., DFT); macroscopic experimental data; methods to validate cross-scale predictions.
Fractal-Geometry Enhanced Models	• Captures self-similarity and complexity in natural systems. • Provides novel frameworks for optimizing intricate structures (e.g., urban networks). • Enhances analysis of chaotic flows and surfaces.	• Lacks a universal mathematical framework for integration with control systems. • High model complexity can hinder real-time application and interpretability.	• Analysis of turbulent flow structures. • Urban infrastructure planning and optimization. • Characterization of material surfaces and fractures.	**Challenge:** Translating descriptive fractal analysis into prescriptive control strategies. **Data Need:** High-resolution spatial or temporal data (e.g., LIDAR, time-series) to compute fractal dimensions and validate models.
AI-Driven Control Systems	• Enables real-time, adaptive decision-making. • Optimizes performance in complex, non-linear systems (e.g., robotics, 6G networks). • Learns efficient control policies directly from data.	• "Black-box" nature reduces interpretability and trust. • Performance is highly contingent on training data quality; prone to biases. • Difficult to certify for safety-critical applications.	• Autonomous vehicle navigation. • Real-time resource allocation in 6G networks. • Intelligent robotic manufacturing and smart grids.	**Challenge:** Balancing model performance with verifiability and safety guarantees. **Data Need:** Large, diverse, and unbiased operational datasets; synthetic data for edge-case training; frameworks for continuous learning and drift detection.

Addressing this requires a new interdisciplinary focus not just on the models themselves, but on the data pipelines that support them. In conclusion, the most pressing research gaps are not confined to a single domain but are interdisciplinary in nature. They call for collaborative efforts focused on: 1) developing rigorous validation standards for AI-physics hybrids, 2) creating more robust and adaptive scale-linking methodologies, 3) engineering scalable models that prioritize computational tractability, and 4) establishing ethical, resilient data governance frameworks. The future trajectory of this field will be defined by its ability to solve these synthesized challenges, not just by isolated technological breakthroughs. Beyond industrial applications, AI-powered control innovations are extending into climate monitoring, environmental conservation, and renewable energy management. AI-enhanced control mechanisms will allow for real-time analysis of atmospheric conditions, intelligent adaptation of renewable energy grids, and optimization of resource consumption. These advancements will support sustainable development initiatives by reducing carbon footprints, enhancing energy efficiency, and improving disaster resilience. The integration of AI into environmental control systems will enable predictive climate modeling, better water resource management, and more effective response strategies for natural disasters. The future of AI-driven control technologies lies in their seamless fusion with quantum computing and next-generation AI architecture. By integrating quantum machine learning techniques, AI-based control systems will be capable of solving complex optimization problems with unmatched speed and accuracy. This will unlock breakthroughs in real-time autonomous decision-making, large-scale system simulations, and ultra-fast adaptive control mechanisms. The synergy between AI, quantum computing, and mathematical modeling will pave the way for intelligent, self-evolving control frameworks that can adapt dynamically to emerging challenges across multiple industries. As AI-driven control continues to advance, it is poised to become the defining force of the next technological era. From medicine and robotics to climate science and industrial automation, these intelligent control strategies will bridge computational intelligence with real-world applications, creating efficient, resilient, and adaptive solutions. The convergence of artificial intelligence, mathematical modeling, and autonomous decision-making will set new benchmarks in technological evolution, ensuring a future where AI plays a central role in optimizing human-machine collaboration and redefining innovation.

8.2. Pioneering differential equation-driven quantum AI control

The ultimate convergence of these fields lies in integrating differential equations with quantum computing and AI for next-generation control systems. Future research must explore how quantum neural networks can solve high-dimensional optimal control problems in real-time, and how hybrid quantum-classical algorithms can be trained to respect physical constraints encoded in differential equations, leading to a new paradigm of industrial automation

9. Conclusion and future research directions

The coming years are set to bring groundbreaking advancements at the intersection of differential equations, control systems, and artificial intelligence, fundamentally altering scientific and technological progress. This review has synthesized these developments, highlighted their transformative potential while also critically evaluating their current limitations. To fully harness this potential, future research must move beyond incremental improvements and address several key challenges in a structured manner. Based on the critical gaps identified in this analysis, we propose the following actionable agenda for future work:

1. *Developing Robust AI-Physics Hybrids*: A primary challenge lies in the scalability and stability of Physics-Informed Neural Networks (PINNs). Future work must focus on creating novel neural architectures and training schemes capable of handling high-dimensional, turbulent systems without violating physical laws. Research should prioritize establishing benchmark problems and rigorous validation standards to distinguish true physical learning from sophisticated data interpolation.
2. *Creating Adaptive Multi-Scale Coupling Protocols*: The "scale-handoff" problem remains a significant bottleneck. There is an urgent need to move

beyond static, multi-scale models towards dynamic frameworks that can intelligently adapt the coupling between scales based on real-time system feedback. Integrating reinforcement learning to manage these transitions presents a promising avenue for achieving more accurate and efficient cross-scale simulations.

3. *Engineering Tractable Models for Real-Time Control:* Bridging the gap between high-fidelity simulation and real-time application is crucial. Research should focus on developing "gracefully degrading" models that can autonomously select the appropriate level of complexity based on computational constraints and latency requirements. This is especially critical for safety-critical applications like autonomous systems and smart grid management, where verifiability is as important as accuracy. We propose a novel adaptive control strategy combining fractal-based optimization with AI-driven tuning:

$$\mathrm{u(t) = K\_pe(t) + K\_i{\int}e(\tau)d\tau + K\_d\ de/dt} + \alpha \cdot \mathrm{Fractal\_Correction(e, history)} \tag{5}$$

Where the fractal term captures multi-scale error patterns and α is dynamically adjusted via reinforcement learning.

4. *Establishing Ethical and Resilient Data Pipelines:* The performance of all data-driven methodologies is contingent on data quality. Future efforts must include the development of frameworks for generating certified, synthetic data for training, and for continuous monitoring and correction of model drift caused by biased or non-stationary real-world data.

The role of mathematics as the cornerstone of scientific discovery remains undeniable. Beyond these technical priorities, successfully implementing this research agenda necessitates parallel development of supporting infrastructure. This includes (1) the creation of open-source, benchmarked datasets for cross-model validation, (2) the establishment of co-design frameworks where control theorists, mathematicians, and AI specialists collaborate from the outset of a project, and (3) the development of standardized metrics for evaluating not just a model's accuracy, but its physical consistency and computational tractability in real-time applications. By embracing these research directions through intensified collaboration across mathematics, computer science, and engineering, the next decade can realize the promise of truly intelligent, adaptive, and trustworthy systems. This will pave the way for a future where computational models are not just descriptive, but prescriptive partners in solving humanity's most complex challenges.

Funding: This research was funded by the Deanship of Scientific Research of Taif University, grant number 83/ Deanship-of-Scientific-Research and the APC was funded by the Deanship of Scientific Research of Taif University (DSRTU) (www.tu.edu.sa/En/Deanships/83/Deanship-of-Scientific-Research.)

Acknowledgement: The author would like to acknowledge the Taif University Department of Scientific Research in the Kingdom of Saudi Arabia for assistance and motivation to accomplish the research work.

Conflict of interest: The authors declare no conflicts of interest to report regarding the present study.

References

1. Fonsêca ALA, Barbalho IMP, Fernandes F, et al. Blockchain in health information systems: a systematic review. International Journal of Environmental Research and Public Health. 2024; 21(11): 1512. doi: 10.3390/ijerph21111512
2. Lee M-FR. A review on intelligent control theory and applications in process optimization and smart manufacturing. Processes. 2023; 11(11): 3171. doi: 10.3390/pr11113171
3. Chang Y, Fang C, Sun W. A blockchain-based federated learning method for smart healthcare. Computational Intelligence and Neuroscience. 2021; 2021(1): 4376418. doi: 10.1155/2021/4376418
4. Hiwale M, Walambe R, Potdar V, et al. A systematic review of privacy-preserving methods deployed with blockchain and federated learning for the telemedicine. Healthcare Analytics. 2023; 3: 100192. doi: 10.1016/j.health.2023.100192
5. Moulahi W, Jdey I, Moulahi T, et al. A blockchain-based federated learning mechanism for privacy preservation of healthcare IoT data. Computers in Biology and Medicine. 2023; 167: 107630. doi: 10.1016/j.compbiomed.2023.107630
6. Thompson JMT. Chaos, fractals and their applications. International Journal of Bifurcation and Chaos. 2016; 26(13): 1630035. doi: 10.1142/S0218127416300354
7. Selvaggio M, Cognetti M, Nikolaidis S, et al. Autonomy in physical human-robot interaction: a brief survey. IEEE Robotics and Automation Letters. 2021; 6(4): 7989–7996. doi: 10.1109/LRA.2021.3100603
8. Salamun K, Pavić I, Džapo H, et al. Weakly hard real-time model for control systems: a survey. Sensors. 2023; 23(10): 4652. doi: 10.3390/s23104652
9. Karniadakis GE, Kevrekidis IG, Lu L, et al. Physics-informed machine learning. Nature Reviews Physics. 2021; 3(6): 422–440. doi: 10.1038/s42254-021-00314-5
10. Jamarani A, Haddadi S, Sarvizadeh R, et al. Big data and predictive analytics: A systematic review of applications. Artificial Intelligence Review. 2024; 57(7): 176. doi: 10.1007/s10462-024-10811-5
11. Akhtar M, Moridpour S. A review of traffic congestion prediction using artificial intelligence. Journal of Advanced Transportation. 2021; 2021: 1–18. doi: 10.1155/2021/8878011
12. Yu P, Wan H, Zhang B, et al. Review on system identification, control, and optimization based on artificial intelligence. Mathematics. 2025; 13(6): 952. doi: 10.3390/math13060952
13. Dpuis S, Ba P-S, Di Natale G, et al. A novel hardware logic encryption technique for thwarting illegal overproduction and Hardware Trojans, in: 2014 IEEE 20th International On-Line Testing Symposium (IOLTS); 17 July 2014; Girona, Spain. pp. 49–54. doi: 10.1109/IOLTS.2014.6873671
14. Aslam MM, Tufail A, Gul H, et al. Artificial intelligence for secure and sustainable industrial control systems - a survey of challenges and solutions. Artificial Intelligence Review. 2025; 58(11): 349. doi: 10.1007/s10462-025-11320-9
15. Singh K, Kalra S. Analysis of negative-bias temperature instability utilizing machine learning support vector regression for robust nanometer design, in: 2022 8th International Conference on Signal Processing and Communication (ICSC); 1 December 2022; Noida, India. pp. 571–577. doi: 10.1109/ICSC56524.2022.10009484
16. Raissi M, Perdikaris P, Karniadakis GE. Physics-informed neural networks: a deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. Journal of Computational Physics. 2019; 378: 686–707. doi: 10.1016/j.jcp.2018.10.045
17. Duraisamy K, Iaccarino G, Xiao H. Turbulence modeling in the age of data. Annual Review of Fluid Mechanics. 2019; 51(1): 357–377. doi: 10.1146/annurev-fluid-010518-040547
18. Schneider T, Lan S, Stuart A, et al. Earth system modeling 2.0: a blueprint for models that learn from observations and targeted high-resolution simulations. Geophysical Research Letters. 2017; 44(24). doi: 10.1002/2017GL076101
19. Einevoll GT, Kayser C, Logothetis NK, et al. Modelling and analysis of local field potentials for studying the function of cortical circuits. Nature Reviews Neuroscience. 2013; 14(11): 770–785. doi: 10.1038/nrn3599
20. Nishida Y. Research on quantum circuit learning models for molecular dynamics simulation. The Journal of Physical Chemistry A. 2025; 129(40): 9434–9452. doi: 10.1021/acs.jpca.5c03624
21. Bauer B, Bravyi S, Motta M, et al. Quantum algorithms for quantum chemistry and quantum materials science. Chemical Reviews. 2020; 120(22): 12685–12717. doi: 10.1021/acs.chemrev.9b00829