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ANALYSIS AND PREDICTION OF COVID-19 SPREAD USING NUMERICAL METHOD

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Abstract

In this paper, we have discussed the impact of Coronavirus variants in a phase of 2021-22 along with a previous phase of 2020-21 in Italy. We analyse and compare the Covid-19 scenario in Italy for the period from October 04, 2020 to January 16, 2021 with a period from October 04, 2021 to January 16, 2022. For this study, we have used repeated multi-step differential transform method (RMsDTM). Also, we have predicted the number of active cases for 10 days following the period of study.

1. Introduction

Covid-19 is a highly communicable disease, known to be caused by the SARS CoV-2 virus. The disease largely has mild to moderate effect on people, though mortality has also been a significant outcome of the infection. Covid-19, first reported in China in December 2019, was declared Public Health Emergency of International concern by World Health Organization on January 30, 2020 [29]. Subsequently, on March 11, 2020 it was officially declared to be a pandemic. The disease spread to almost all countries and impacted them in various measures and ways. Besides loss of lives, livelihoods were also lost. In order to contain spread of the disease, restrictions were implemented on unnecessary movement of individuals. Social distancing was maintained through imposing curbs on gatherings at weddings, parties and funerals. Non-essential businesses were shut down.

To cut the costs, some businesses altered their mode of work by either shifting their office to a smaller area or through shifting to work from home. Restaurant industries also switched over from dine-ins to takeaways and home deliveries. Tourism industry saw a major decline. Such restrictions affected the economy of many countries. This also disturbed the mental health of the population globally. The health systems of all countries were stressed out due to unforeseen number of patients. Various measures had to be taken to overcome the economic and medical challenges caused due to the pandemic.

The pandemic that saw its initiation in early 2020, continued to re-emerge in all countries. Trends of the pandemic were studied globally to understand the measures that need to be taken to prevent the systems getting adversely affected [8, 9, 10, 17]. The spread and behaviour of the virus in its subsequent waves was varying from that of the first wave. By the end of December 2020, vaccination for Covid-19 had started in many countries. UK was the first to start the immunization process [22]. Europe saw widespread effect of the virus across the continent in all the waves of the pandemic. Italy represented unique study as it was badly hit during the first wave, counting amongst one of the worst affected countries not only in Europe but worldwide. Northern regions of Italy, including Lombardia, Emilia-Romagna and Veneto Strict were amongst the worst affected. Lockdown in Italy was enforced (one of the longest during the first wave of Covid-19 [24]). During the lockdown period, starting from March 9 to May 4, 2020, restrictions were implemented to a smaller area of initial spread and were gradually intensified to the complete nation with relaxation only for emergent services. With strict measures in place, Italy ultimately controlled the pandemic in the first wave. Italy was less affected during the second wave, that began in October 2020 [6], in comparison to the neighbouring countries. Subsequently, Italy saw a third wave starting March 2021 and is currently experiencing a fourth wave. Till date, Italy is amongst the countries with highest number of Coronavirus cases and high mortality rate. This is despite the fact that the healthcare system in Italy is considered to be amongst the top worldwide and the life expectancy is second highest in Europe. The healthcare system in this country is a regionally based National Health Service providing universal coverage, mostly free of cost at the point of delivery [26].

Vaccination against the Covid-19 causing virus is a major tool in controlling the pandemic. In September 2021, Italy implemented the concept of ‘Green Pass’ as a preventive measure other than lockdown to stop the rapid spread of the disease [25]. The Covid-19 Vaccination Passport or Green Pass for Italy is issued to those who have received any dose of the vaccine, or those with negative report of RTPCR/antigen test taken in the last 48 hours. Besides, those who have recovered from the disease over the last six months may also get the document. Restrictions in different regions of Italy are also quantified in terms of allowing only people with Green Pass at public places. The vaccination process started in Italy in the end of December 2020 and at present more than 81 percent population of Italy is vaccinated. Vaccination has resulted in significant decrease in number of hospitalization and deaths in all subsequent waves of the pandemic [27].

As the pandemic resulted in huge causalities, researchers’ attention got focused towards dealing with Covid-19 situations. The purpose of such research was helping the system by providing data analysis, predictions, effect of measures like masking, isolation, vaccination etc. This could help in better preparedness for future needs like number of beds required, supply of oxygen and other medical supplies etc. With similar objective, we have studied the Covid-19 scenario in Italy for a chosen period.

The paper has been arranged as follows. In the second section, differential transform method (DTM) and multi-step DTM (MsDTM) have been discussed. We have applied repeated multi-step DTM to solve the system of differential equations to study the Covid-19 situation in Italy for a period of 105 days starting from October 4, 2021. In Section 3, after analysing the data and the parameters, we have attempted to estimate and predict the extent of spread of the disease for the next ten days. Analysis of the pandemic conditions in 2021 in comparison to those of 2020 and conclusions drawn from the study are presented in Section 4.

2. Methodology

Mathematical modelling is an important tool to study the spread of epidemics. Modern epidemiologists apply dynamic mathematical models to forecast and predict the impact of a disease so that various measures can be adopted by the government and medical professionals to contain the disease.

Our objective is to study the spread of Covid-19 in Italy over the span of 105 days (October 4, 2021 to January 16, 2022). We have also made a prediction about the extent of the spread of the disease, for 10 days post the above-mentioned period. For this, the model we consider is SIR model which was first introduced by Kermack and McKendrick [11]. SIR model is a basic compartmental model in which the whole population (n) is considered to be constant over a short period of time. In this model, the population is divided into three compartments - $s(t)$ or susceptible - these are those individuals who can catch the infection, $i(t)$ or infectives - these are those people who have caught the infection and can infect other people, i.e., it consists of active infected cases and third compartment is $r(t)$ which comprises of those individuals who have been removed from the system because they have either recovered or died. Since the population (n) is considered to be constant over this period, so $n = s(t) + i(t) + r(t)$. It is also assumed that within the span of study, an individual does not get infected again which means that the person who has been removed from the system cannot become susceptible again. It is also assumed that the population is homogeneously mixed [5]. The model can be represented as the following set of differential equations:

$$\frac{ds}{dt} = -\beta si, \frac{di}{dt} = \beta si - \gamma i, \frac{dr}{dt} = \gamma i, \tag{1}$$

where β is the rate of infection and γ is the rate of recovery.

2.1. Repeated multi-step differential transform method (RMsDTM)

The differential transform method (DTM) is an iterative method employed for solving various types of initial value differential equations, on a small domain. This numerical method, introduced by Zhou [20], is based on Taylor series and gives solution in terms of convergent series. The solution is an analytical solution, in the form of a finite degree polynomial, which is an approximation to the exact solution. The polynomial is obtained by truncated Taylor series whose coefficients can be easily obtained by recursive relations given by DTM. This method differs from the traditional Taylor series method which involves complex computational procedures. The DTM has many useful applications in diverse areas of Engineering, Physics and Science.

We now give a brief introduction of the DTM [4, 15]. Suppose, we have an ordinary differential equation

$$P(y, y', y'', \ldots) = 0, \tag{2}$$

where P is a polynomial in terms of y and its derivatives, $y(t_0) = c$ for some constant c and $y(t)$ is the solution of the differential equation (2) in domain $[t_0, t_N]$. We describe the differential transform method to find the approximate solution of equation (2). The differential transformation of $y(t)$ is $Y(k)$ which is expressed at the point t_0 as follows:

$$Y(k) = \frac{1}{k!}\left[\frac{d^k y(t)}{dt^k}\right]_{t=t_0}. \tag{3}$$

The inverse differential transformation of $Y(k)$ is $y(t)$ which is defined as

$$y(t) = \sum_{k=0}^{\infty} Y(k)(t - t_0)^k. \tag{4}$$

For the purpose of obtaining solution, $y(t)$ is approximated by the finite series given as:

$$y_N(t) = \sum_{k=0}^{N} \frac{(t - t_0)^k}{k!}\left[\frac{d^k y(t)}{dt^k}\right]_{t=t_0}, \tag{5}$$

for some positive integer N.

Some rules for the differential transforms are given in Table 1.

Table 1. Properties of differential transform

	Function	**Transformed function**
1	$y(t) = u(t) \pm v(t)$	$Y(k) = U(k) \pm V(k)$
2	$y(t) = u(t)v(t)$	$Y(k) = \sum_{j=0}^{k} U(j)V(k-j)$
3	$y(t) = \alpha u(t)$	$Y(k) = \alpha U(k)$
4	$y(t) = \frac{du(t)}{dt}$	$Y(k) = (k+1)U(k+1)$
5	$y(t) = \frac{d^j u(t)}{dt^j}$	$Y(k) = (k+1)(k+2), \ldots, (k+j)U(k+j)$

The numerical solution obtained by DTM is an approximate solution which converges to the exact solution with the approximated error given by

$$|y(t) - y_N(t)| \leq \frac{M|t - t_0|^{N+1}}{(N+1)!}, \tag{6}$$

where $\left|\frac{d^{N+1}y(t)}{dt^{N+1}}\right| \leq M$. We observe that DTM gives good approximation since the error is small, however the solution converges only in a small region around the initial point. It can be seen from (6) that the error increases when $|t - t_0|$ increases for a fixed value of N. As a result the convergence is within a small region and convergence rate is also slow [1].

In order to overcome this drawback and to improve the outcome of the numerical approximation for the solution, multi-step differential transform method (MsDTM) was introduced [19].

MsDTM can be applied to obtain numerical solution of non-linear system (1) of differential equations as follows [4, 14, 16, 18]: In this method, the interval under consideration is $[0, T]$ which is divided into M equal sub-intervals $[t_{m-1}, t_m]$, where $m = 1, 2, 3, ..., M$; $t_0 = 0$ and $t_M = T$. The length of each sub-interval is $h = T/M$ and the nodal points $t_m = mh$. The algorithm for the MsDTM is as follows: We first find the approximate solution of the differential equation (2) by applying the DTM in the sub-interval $[0, t_1]$. The approximate solution so obtained is given as:

$$y_1(t) = \sum_{k=0}^{N} Y(k)(t-0)^k, \; t \in [0, t_1], \tag{7}$$

where N is the order of approximation of the power series. $y_1(t_1)$ is the solution obtained at the point t_1 which is taken as initial condition when DTM is applied to the second interval $[t_1, t_2]$. Similarly for all $m \geq 2$, and at each sub-interval $[t_{m-1}, t_m]$ we apply DTM, where the initial condition is taken as $y_m(t_{m-1}) = y_{m-1}(t_{m-1})$. Proceeding like this, DTM is applied for all the sub-intervals which generate a sequence of approximate solutions $y_m(t)$, $m = 1, 2, \ldots, M$, where $y_m(t) = \sum_{k=0}^{N} Y(k)(t - t_{m-1})^k$, $t \in [t_{m-1}, t_m]$. So, MsDTM generates the following approximate solution:

$$y(t) = \begin{cases} y_1(t) \\ y_2(t) \\ \vdots \\ y_M(t), \end{cases}$$

where

$$y_m(t) = \sum_{k=0}^{N} Y(k)(t - t_{m-1})^k, \; t \in [t_{m-1}, t_m]$$

for $m = 1, \ldots, M$.

The SIR model under consideration has two very important parameters namely β, the rate of transmission and γ, the rate of recovery. These parameters play a very crucial role in the study of epidemics and help us to analyse the spread of the disease. These can be considered to be constant over a very short duration of time. But, over a span of more than three months (the period we are studying) these parameters tend to change depending upon factors like lockdown, travel restrictions, herd immunity, density of population, nature of the Covid variant, overall immunity, intermingling of the population etc. It is therefore imperative to improve the MsDTM further, using varying values of the parameters especially under the current scenario. For this purpose, we have considered repeated multi-step differential transform method [3, 12]. In this algorithm, the time under consideration is divided into smaller periods and the parameters are altered according to the conditions prevailing over that period. The parameters thus chosen are applied over the sub-intervals. It has been observed that the RMsDTM gives better approximation to the solution.

2.2. Numerical simulation

In this subsection, we apply RMsDTM to SIR model in (1) to study the Covid-19 situation in Italy over a period of 105 days starting from October 4, 2021. The time under study is divided into 5 parts of 15 days each and 3 parts of 10 days each. The phases were taken to be of 15 days length initially but observing the rapid variation of numbers in the later period, the last three phases were taken to be of shorter length, that of 10 days each. Using the properties of transform function of the original function as given in Table 1, we obtain the recurrence relation for the system of equations in (1) as [2]:

$$\begin{cases} S(k+1) = \frac{1}{k+1}\left(-\beta\sum_{m=0}^{k}(S(m)I(k-m))\right), \\ I(k+1) = \frac{1}{k+1}\left(\beta\sum_{m=0}^{k}(S(m)I(k-m) - \gamma I(k))\right), \\ R(k+1) = \frac{1}{k+1}\gamma I(k). \end{cases} \tag{8}$$

As mentioned earlier, the parameters of the model vary with change in circumstances, so we considered different values of parameters for different time periods as given in Table 2. The parameters were found by analysing the actual data [23]. With initial conditions $s(0) = 55645746, i(0) = 93307$, $r(0) = 11022$ and parameters $\beta = 5.4130272 \times 10^{-10}$, $\gamma = 0.042374389$; we solved the non-linear system of equations (1) for s, i and r, using RMsDTM.

For this, the tools of Mathematica software were employed [13]. It was observed that the solution so obtained gives us good approximation to the number of susceptible, infectives and recovered cases on different days during the above mentioned time period starting from October 4, 2021.

Table 2. Parameters used in various phases

Phase	β	γ
I	$5.4130272 \times 10^{-10}$	0.042374389
II	$8.9711717 \times 10^{-10}$	0.044634442
III	$1.1812829 \times 10^{-09}$	0.039242804
IV	$1.3069543 \times 10^{-09}$	0.039548767
V	$1.2700563 \times 10^{-09}$	0.038886656
VI	$1.5369238 \times 10^{-09}$	0.032016475
VII	$2.3610134 \times 10^{-09}$	0.019850156
VIII	$1.7033072 \times 10^{-09}$	0.036456222
Prediction	$1.5976418 \times 10^{-09}$	0.04645202

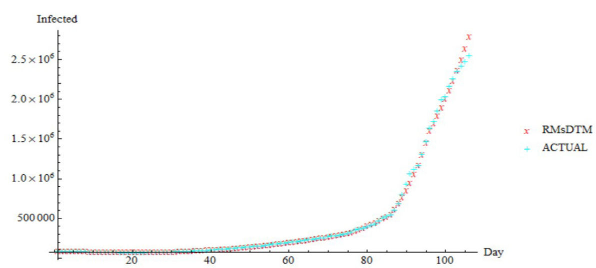

Figure 1. Comparison of estimated infectives over the period of 105 days with actual data for the same period.

As can be seen from Figure 1, the actual infectives vs infectives (i) derived from the SIR model using RMsDTM show very good proximity. Similarly, the actual recovered vs recovered (r) in the period under study is also showing very good convergence (Figure 2). The error table (Table 3) shows the accuracy of the method. The error can be seen to be small in this period.

Table 3. Error table showing error in estimated infectives and estimated recovered

Day	Error (I)	Error (R)	Day	Error (I)	Error (R)	Day	Error (I)	Error (R)
1	261	1446	36	387	4851	71	2061	10216
2	413	1067	37	621	4088	72	946	7212
3	1359	386	38	813	2718	73	2212	4041
4	1340	608	39	374	2208	74	256	2307
5	1501	1136	40	1266	1634	75	4223	178
6	1055	811	41	2788	1896	76	13195	9731
7	493	67	42	3952	2906	77	9215	11555
8	162	1452	43	2574	3925	78	3028	14682
9	689	965	44	1831	3337	79	7041	12928
10	1766	385	45	2285	1662	80	11966	6190
11	1846	659	46	3532	1461	81	7787	3022
12	1698	801	47	3791	702	82	1418	2056
13	1170	844	48	5605	773	83	12870	1510
14	710	439	49	6358	1927	84	2503	6192
15	960	65	50	4372	3946	85	4726	13018
16	3172	955	51	2442	2829	86	7728	7867
17	4462	2039	52	2133	1293	87	2598	986
18	4794	2237	53	4151	1117	88	32806	668
19	5232	2607	54	4743	53	89	84467	6536
20	5362	2677	55	5491	91	90	126507	14999
21	5023	2075	56	7157	1633	91	65506	22468
22	5568	1144	57	4331	4058	92	933	30911
23	5604	1200	58	2744	3352	93	7361	24592
24	5713	1853	59	1890	1456	94	20074	17749
25	4731	1662	60	4379	1615	95	38202	714
26	3295	1465	61	5100	53	96	26323	31858
27	1995	924	62	5892	796	97	64401	33105
28	363	319	63	4980	1173	98	97810	79536
29	468	2495	64	3868	4217	99	36179	77121
30	756	4129	65	1573	2629	100	54678	59681
31	659	3122	66	2211	2455	101	20451	30406
32	880	2737	67	364	5026	102	5954	30896
33	271	2470	68	77	3167	103	79125	21358
34	638	2644	69	2427	3201	104	159499	50057
35	1447	3806	70	4795	5604	105	235256	32911

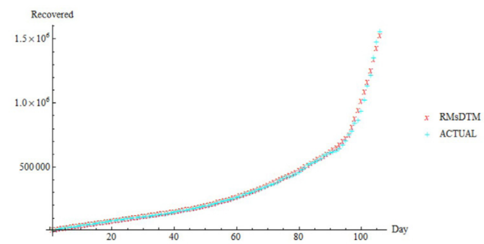

Figure 2. Comparison of estimated recovered over the period of 105 days with actual data for the same period.

3. Prediction

In an epidemic, extent and pace of the spread of disease determines how taxed the medical systems are. If there is a pre-estimate or prediction of the numbers of infectives, the systems can prepare themselves better and be well equipped to handle a situation that requires higher number of resources. Prediction thus is an important tool that could lead to better recovery rate and lesser causalities. Mathematical modelling is a tool employed for prediction using the current data. Using the last parameters given in Table 2, and the estimated number of susceptible, infectives and recovered on January 16, 2022 as initial conditions, we predict the values of active Covid-19 cases for the next 10 days, using RMsDTM. The predicted values are depicted in Figure 3. The graph shows the predicted infectives after the 105th day of the period under consideration. We can see from the graph that the method predicts a rise in the number of active cases in the coming days.

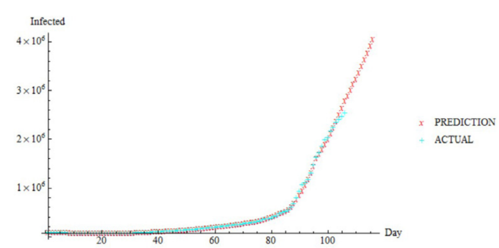

Figure 3. Prediction of infectives for a period of 10 days starting from January 17, 2022, using RMsDTM.

The basic reproduction number, R_0, is defined as the number of susceptible that will get infected through one infected person in a completely susceptible population till the time the person is infectious [5, 7]. Thus, at the onset of the disease, the quantity R_0 can be found using the formula $R_0 = \frac{\beta \times s(0)}{\gamma}$. As the number of susceptible and the parameters change with time, the reproduction number also changes. The number of infectives increases when the reproduction number is more than 1 and when it is less than 1, then the number of infectives starts decreasing and we can expect that the disease would die out. It may be noted that this value of R_0 is not constant or fixed for a particular disease or disease causing pathogens. This quantity is estimated using different mathematical models, so its value also depends upon the model used. This number represents an estimation for the 'Epidemiological threshold' [7]. Thus, the concept of reproduction number plays very important role in epidemiology. It helps in predicting the trend of the disease and keeping the spread of the disease in check.

For Italy, the values of R_0 for various phases of the study were calculated using the parameters given in Table 2 and are tabulated in Table 4.

Table 4. Reproduction number

Phase	I	II	III	IV	V	VI	VII	VIII	Prediction
R_0	0.72	1.16	1.68	1.84	1.81	2.73	7.24	2.75	1.80

Using the β, γ and $s(0)$ used for prediction, $R_0 = 1.80$, which is greater than 1. Hence, we can conclude that the number of infectives will increase.

4. Discussion and Conclusion

To compare the impact of the virus in the considered phase of 2021-22 with a previous wave, we analysed the Covid-19 scenario in Italy for the period from October 4, 2020 to January 16, 2021 with October 4, 2021 to January 16, 2022. This was chosen as the climatic conditions of the country would be similar in these periods for both the years. For the study of this pandemic in 2020, we used RMsDTM, determining the parameters from the available data [30]. Figure 4 shows the efficiency of RMsDTM. We can see that the trend shown by the estimated values is similar to the actual trend of the spread of the disease.

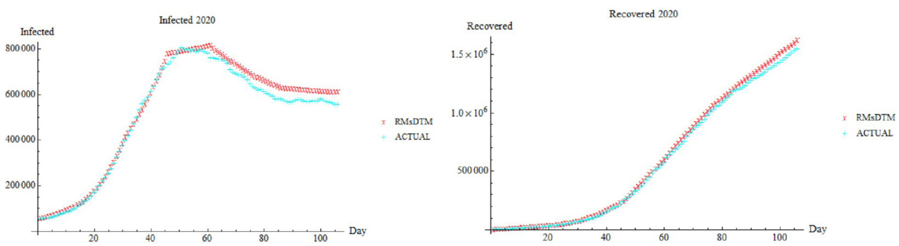

Figure 4. Comparison of estimated infectives and estimated recovered with actual data from October 4, 2020 to January 16, 2021.

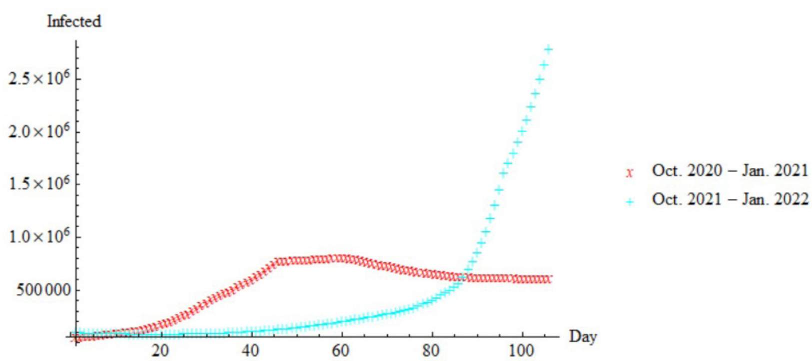

Figure 5. Comparison of estimated infectives of years 2020 and 2021 from October 4 to January 16.

Further, we compared the estimated number of infectives in the said period. During the initial phase of study, we observe that in 2021, in the infectives' graph (Figure 5), the slope of the curve is not so high, indicating that the number of active infected cases in Italy is quite less as compared to the corresponding period in 2020. The number is increasing slowly and gradually in 2021. However, after the 69th day, the number of cases starts rising fast, and the rise is even faster after the 75th day and the curve is still steeper after the 90th day, indicating that the number of infectives is increasing at a very high speed during this period. In Italy, the first case of Omicron variant of the virus was detected on November 28, 2021 [21]. It is evident from the graph as well that the number of cases started rising after that point as there is a steep rise in the graph during that period. Since this variant is reported to be highly contagious [28], the infection is spreading at a very fast speed, as can be seen from the graph.

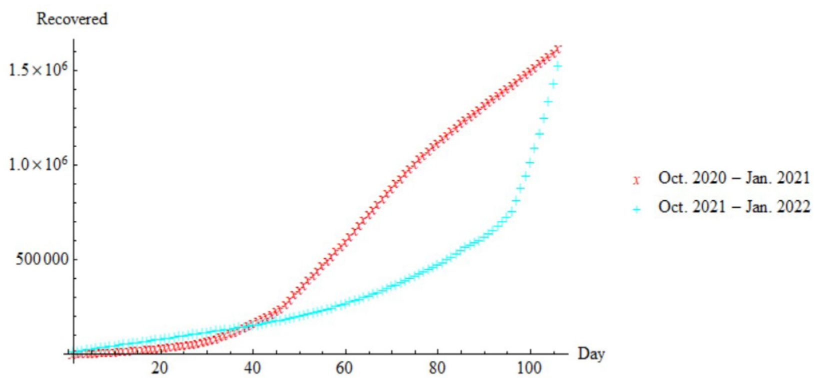

Figure 6. Comparison of estimated recovered of years 2020 and 2021 from October 4 to January 16.

If we look at the graph of the recovered cases (Figure 6), then we observe that throughout the period, the number of recovered in 2021-22 is less than those of the last year. This is because on an average, the recovery time for Covid-19 is approximately 14 days. So, in 2021-22, the number of recovered that we see at any point is those who entered the infective compartment 14 days back. As we see from Figure 5, around 90th day the number of infectives was comparable in both the years and in Figure 6, both are showing similar number of recovered cases on 105th day. Because of this connection, the number of recovered will improve much faster in 2021-22 as the number of infectives is higher in this period. Number of recovered cases depends on the number of infectives but this link can be seen after the recovery period. So, the impact of sharp increase of infective cases during the last 14 days will be reflected as the number of recovered cases will start rising after two weeks. Hence, we foresee that the curve of recovered cases of 2021-22 will eventually surpass the curve of recovered cases of 2020-21.

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