Advances in Differential Equations and Control Processes
© 2024 Pushpa Publishing House, Prayagraj, India
https://pphmjopenaccess.com
https://doi.org/10.17654/0974324324030
Volume 31, Number 4, 2024, Pages 583-607 **P-ISSN: 0974-3243**

A PROBLEM IN FRACTIONAL ORDER THERMO-VISCOELASTICITY THEORY FOR A POLYMER MICRO-ROD WITH AND WITHOUT ENERGY DISSIPATION

Mohamed H. Hendy[1,2,*], **Magdy A. Ezzat**[3], **Esraa M. Al-lobani**[4] **and Ahmed S. Hassan**[5]

[1]Department of Mathematics
Faculty of Science
Northern Border University
Arar, Saudi Arabia
e-mail: hendy442003@yahoo.com

[2]Department of Mathematics
Faculty of Science
Al Arish University
Al Arish, Egypt

Received: August 1, 2024; Revised: September 25, 2024; Accepted: October 14, 2024
2020 Mathematics Subject Classification: 76XX.
Keywords and phrases: size-dependent thermo-viscoelastic coupling, generalized thermoelasticity with LS and without energy dissipation (GN-II), polymer micro-rod, fractional calculus.

*Corresponding author

Communicated by K. K. Azad

How to cite this article: Mohamed H. Hendy, Magdy A. Ezzat, Esraa M. Al-lobani and Ahmed S. Hassan, A problem in fractional order thermo-viscoelasticity theory for a polymer micro-rod with and without energy dissipation, Advances in Differential Equations and Control Processes 31(4) (2024), 583-607. https://doi.org/10.17654/0974324324030

Published Online: October 25, 2024

[3]Department of Mathematics
Faculty of Education
Alexandria University
Egypt

[4]Department of General Courses
Applied College
Northern Border University
Rafha, Saudi Arabia

[5]Applied College
Northern Border University
Arar, Saudi Arabia

Abstract

A new study has been developed that considers the size-dependent interaction between viscoelastic deformation and thermal fields, incorporating the fractional heat conduction law with and without energy dissipation. The model is used for a particular one-dimensional problem involving a polymer micro-rod of arbitrary length experiencing three different types of thermal loading without the presence of any heat source. The study uses Laplace transforms and numerical inversion to examine how fractional order, nonlocal elasticity, and nonlocal thermal conduction impact thermal dispersion and thermo-viscoelastic response. Comparative numbers demonstrate the effects of various parameters. Findings demonstrate that nonlocal thermal and viscoelastic characteristics have a significant impact on all recorded field values, offering possible suggestions for the creation and assessment of thermal-mechanical attributes in nanoscale devices.

List of Symbols

λ, μ	Lame's constants
ρ	Density
t	Time

T	Absolute temperature
T_0	Reference temperature so that $\lvert T - T_0 \rvert / T_0 \ll 1$
C_E	Specific heat at constant strain
σ_{ij}	Components of stress tensor
ε_{ij}	Components of strain tensor
e_{ij}	Components of strain deviator tensor
u_i	Components of displacement vector
q_i	Components of heat flux vector
k	Thermal conductivity
e	Dilatation
α_t	Coefficient of linear thermal expansion
Q	Strength of the heat source
C_0^2	$= K_0/\rho$
γ	$= 2K_0\alpha_t$
K_0	$= \lambda + \frac{2}{3}\mu$, bulk modulus
T_0	$= \rho C_0^2 \delta_0/\gamma$
η_0	$= \rho C_E/k$
ε	$= \delta_0\gamma/\rho C_E$, thermal coupling parameter
$\tau_0,\ \tau_\upsilon$	Relaxation times
δ_0	Non-dimensional constant
$R(t)$	Relaxation function

δ_{ij}	Kronecker delta function
α^*, β^*, A	Empirical constants
ξ, ζ	Nonlocal parameters
α, β	Fractional orders

1. Introduction

Nanostructural materials exhibit a high degree of versatility across a range of temperatures. However, the challenge of engineering at scales involving significant geographical and temporal dimensions is frequently linked to heat conduction processes. According to the classical theory of uncoupled thermoelasticity, two phenomena have been postulated that do not conform to empirical observations: a parabolic heat equation characterized by infinite propagation rates of heat waves, and a conduction equation devoid of elastic terms, as suggested by Nowinski [1].

To address the incongruity between the classical theory and empirical observations, Biot [2] introduced the concept of coupled thermoelasticity theory. Despite this advancement, the theory encountered a second issue due to the presence of mixed parabolic-hyperbolic heat equations. Various generalizations have since been proposed to overcome these challenges. Notably, the Cattaneo [3] heat conduction equation stands out for its simplicity, offering a finite propagation speed based on the Fourier law, which it successfully replaces. Lord and Shulman (LS) [4] advanced the theory with an extended form known as homogeneous elastic media theory, incorporating a relaxation time (LS) to modify the Fourier law, as outlined by Cattaneo's framework.

The application of these generalized theories to practical scenarios is explored in Kaminski's [5] publication. Theoretical research has also made significant contributions to the field, such as the demonstration of uniqueness theorems under different conditions by Ignaczak [6], Sherief and Dhalival [7], Ezzat and El-Karamany [8], and Sur [9]. The Green and Naghdi

(GN-II) paper, published in [10], delves into the delineation of criteria necessary for the application of unequal entropy output in the formulation of governing equations, subsequently examining the outcomes across various categories of classical thermoelasticity. Within the realm of thermoelastic theory devoid of energy dissipation, Chandrasekharaiah [11] presented a theorem establishing uniqueness. Furthermore, El-Karamany and Ezzat [12] alongside Lata [13], and Lata and Kaur [14] have made significant contributions to the advancement of a generalized Green-Naghdi theory of thermoelasticity, alluding to its application without energy dissipation.

In recent years, there has been an expanding array of physical phenomena that have been described utilizing fractional calculus. Ezzat [15, 16] has employed the Taylor’s series expansion of time-fractional order, as introduced by Jumarie [17], to develop a fractional model for heat conduction within magneto-thermoelasticity theories and magnetohydrodynamics. Within the realm of continuum mechanics, an increasing number of fractional models have emerged from scholars such as Yu et al. [18], Ezzat et al. [19, 20], El-Attar et al. [21], Amin et al. [22], and Yang [23]. This presentation aims to provide a comprehensive review of the presentation of general fractional derivatives as a means to facilitate understanding within this area of research.

Viscoelastic materials are becoming a popular topic in engineering due to their outstanding rheological properties by nature Meyers and Chawla [24]. Significant progress has been made thus far in the viscoelastic theory of the static and dynamic reactions of viscoelastic structures, we refer to Ezzat [25, 26] and El Sherif et al. [27] for details. Viscoelastic nanomaterials, characterized by their exceptional mechanical, thermal, and chemical properties, have been identified as a prime candidate for the development of nanodevices by Shi et al. [28]. Concurrently, their diminutive size and robustness have led to their extensive application as resonators in micro/nano structures by Eom et al. [29], Currano et al. [30], and others. Furthermore, these materials have played a crucial role in the advancement of micro/nano-electromechanical systems (MEMSs/NEMSs), as evidenced by the contributions of Toril et al. [31] and Abouelregal and Marin [32]. Within the realm of micro/nano engineering, viscoelastic micro/nano structures have been employed in MEMSs/NEMSs for the purpose of dissipating vibrational energy. To date, a plethora of research has been conducted on the viscoelastic behaviors of micro/nano structures, encompassing dynamic response (Lyu et al. [33]), bending (Sobhy and Zenkour [34]), and vibration (Attia and Abdel Rahman [35]).

The challenge of maintaining precise control at the nanoscale level has led to a common occurrence of experimental findings on viscoelastic nanomaterials being marred by notable limitations. Furthermore, there is a notable inconsistency among experimental reports regarding the performance of materials under slightly varied test conditions. To mitigate these issues, it is recommended that theoretical modeling approaches be employed to offer fresh perspectives and foundational principles for the thermal management of viscoelastic nanocomposites. Additionally, a comprehensive understanding of thermo-viscoelastic interactions at the nanoscale is essential, as demonstrated by Yang and Chen [36].

In the pursuit of enhanced performance within engineering applications, subjected to increasingly severe loads and climatic conditions, there is a significant demand for materials based on polymers. For instance, polyimides, renowned for their exceptional mechanical properties across a wide temperature range, are regarded as among the most critical materials in the aerospace sector, as highlighted by Ferry [37]. Conversely, the phenomenon of thermoelastic coupling, particularly in the context of heat conduction and deformation, is seldom observed. Consequently, there is an urgent need for a comprehensive theoretical framework that elucidates the interaction between these two fields, specifically in terms of displacement and temperature, or stress and heat flow. The primary objective of this research is to address this gap, as evidenced by the references provided (Lata and Singh [38], and Ezzat et al. [39]).

2. Mathematical Modeling

The governing equations for a thermo-viscoelastic media, devoid of any heat source, can be delineated as follows, incorporating both the spatial nonlocal effects of viscoelastic deformation and heat transmission:

(1) Equation of motion:

$$\sigma_{ij,\,j} = \rho \frac{\partial^2 u_i}{\partial t^2}. \tag{1}$$

(2) Kinematic relation:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,\,j} + u_{j,\,i}). \tag{2}$$

(3) Nonlocal stress-strain-temperature relations, Yang and Chen [36]:

$$(1 - \xi^2\nabla^2)\sigma_{ij} = R(t)\left(\varepsilon_{ij} - \frac{\varepsilon_{kk}}{3}\delta_{ij}\right) + K_0\varepsilon_{kk}\delta_{ij} - \gamma\theta\delta_{ij}, \tag{3}$$

where $\theta = |T - T_0|$ and $\frac{\theta}{T_0} \ll 1$,

$$R(t) = 2\mu\left[1 - A\int_0^t e^{-\beta^* t} t^{\alpha^* - 1} dt\right], \quad R(0) = 2\mu, \tag{4}$$

and

$$0 < \alpha^* < 1, \ \ \beta^* > 0, \ \ 0 \le A < \frac{\beta^*}{\Gamma(\alpha^*)}, \ \ R(t) > 0, \ \ \frac{d}{dt}R(t) < 0.$$

(4) Nonlocal fractional heat equation with and without energy dissipation (Hassaballa et al. [40]):

$$k_{ij}\theta_{,ii} = \tau_\upsilon^\beta \frac{\partial^\beta}{\partial t^\beta}(1 - \zeta^2\nabla^2)\left(1 + \frac{\tau_0^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)\left[\rho C_E \frac{\partial\theta}{\partial t} + \gamma T_0 \frac{\partial e}{\partial t}\right], \ 0 < \alpha, \ \beta \le 1. \tag{5}$$

(5) The fractional heat flux:

$$\tau_\upsilon^\beta \frac{\partial^\beta}{\partial t^\beta}\left(1+\frac{\tau_0^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)(1-\zeta^2\nabla^2)q_i = -k_{ij}T_{,j}, \quad 0<\alpha, \beta \le 1. \tag{6}$$

The equations use a comma for material derivatives, following the summation convention.

The preceding equations form an elaborate framework encompassing the nonlocal fractional thermo-viscoelasticity model, including scenarios with and without the presence of energy dissipation.

The research delves into the study of thermo-viscoelastic materials, concentrating on one-dimensional scenarios where the characteristics of features are influenced by factors such as space (x) and time (t). It explores various aspects related to the displacement components:

$$u_x = u(x, t), \quad u_y = 0, \quad u_z = 0. \tag{7}$$

The strain-displacement relation:

$$e = \frac{\partial u}{\partial x}. \tag{8}$$

The displacement equation:

$$\frac{\partial \sigma}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2}. \tag{9}$$

The constitutive equation:

$$\left(1-\xi^2\frac{\partial^2}{\partial x^2}\right)\sigma = \left(\frac{2}{3}R+K_0\right)\frac{\partial u}{\partial x} - \gamma\theta. \tag{10}$$

The energy equation:

$$k\frac{\partial^2\theta}{\partial x^2} = \tau_\upsilon^\beta\frac{\partial^\beta}{\partial t^\beta}\left(1-\zeta^2\frac{\partial^2}{\partial x^2}\right)\left(1+\frac{\tau_0^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)$$

$$\cdot\left(\rho C_E\frac{\partial T}{\partial t}+\gamma T_0\frac{\partial u}{\partial x \partial t}\right), \quad 0<\alpha, \beta\le 1. \tag{11}$$

The heat flux equation:

$$\tau_\upsilon^\beta \frac{\partial^\beta}{\partial t^\beta}\left(1+\frac{\tau_0^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)\left(1-\zeta^2\frac{\partial^2}{\partial x^2}\right)q=-\frac{\partial\theta}{\partial x},\quad 0<\alpha,\beta\le 1. \tag{12}$$

The above equations allow us to present the non-dimensional variables that follow:

$$(x^*, u^*, \xi^*, \zeta^*)=C_0\eta_0(x, u_1, \xi, \zeta),\quad t^*=C_0^2\eta_0 t,$$

$$\sigma^*=\frac{1}{K_0}\sigma,\quad \theta^*=\frac{\gamma}{K_0}\theta,\quad q^*=\frac{\gamma}{kK_0C_0\eta_0}q,\quad R^*=\frac{2}{3K_0}R.$$

In non-dimensional form, equations (8)-(12) become

$$e=\frac{\partial u}{\partial x}, \tag{13}$$

$$\frac{\partial\sigma}{\partial x}=\frac{\partial^2 u}{\partial t^2}, \tag{14}$$

$$\left(1-\xi^2\frac{\partial^2}{\partial x^2}\right)\sigma=(1+R)\frac{\partial u}{\partial x}-\theta, \tag{15}$$

$$\frac{\partial^2\theta}{\partial x^2}=\tau_\upsilon^\beta\frac{\partial^\beta}{\partial t^\beta}\left(1-\zeta^2\frac{\partial^2}{\partial x^2}\right)\left(1+\frac{\tau_0^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)$$

$$\cdot\left(\frac{\partial\theta}{\partial t}+\varepsilon\frac{\partial u}{\partial x\partial t}\right),\quad 0<\alpha,\beta\le 1, \tag{16}$$

$$D\theta=-\tau_\upsilon^\beta\frac{\partial^\beta}{\partial t^\beta}\left(1+\frac{\tau_0^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)\left(1-\zeta^2\frac{\partial^2}{\partial x^2}\right)q,\quad 0<\alpha,\beta\le 1, \tag{17}$$

$$R(t)=\frac{4\mu}{3K_0}\left(1-A\int_0^t e^{-\beta t}t^{\alpha^*-1}dt\right). \tag{18}$$

3. Limitative Situations of a Nonlocal Systematical Formulation

(i) The system of equations (13)-(18) in the limiting case $R = 2\mu$, and $\alpha, \beta, \xi, \zeta \to 0$, transforms to the work of Biot [2] in the coupled thermoelasticity theory.

(ii) The system of equations (13)-(18) in the limiting case $R > 0$, and $\alpha, \beta, \zeta, \xi \to 0$, coupled thermo-viscoelasticity (CTV) transforms to the work of Gross [41] in the coupled thermo-viscoelasticity theory ignoring the rheological volume properties.

(iii) The system of equations (13)-(18) in the limiting case $R = 2\mu$, $\alpha = 1$, $\beta = 0$, and $\xi, \zeta \to 0$ transforms to the work of Lord and Shulman [4] in the generalized thermoelasticity theory with thermal relaxation.

(iv) The system of equations (13)-(18) in the limiting case $R = 2\mu$, $\beta = 1$, and $\alpha, \xi, \zeta \to 0$, transforms to the work of Green and Naghdi [10] in the generalized thermoelasticity theory without energy dissipation.

(v) The system of equations (13)-(18) in the limiting case $R = 2\mu$, $\alpha, \beta \to 0$ and $\xi, \zeta > 0$, transforms to the work of Yu et al. [18] in the nonlocal thermoelasticity theory.

(vi) The system of equations (13)-(18) in the limiting case $\alpha = \beta = 0$, $R > 0$, and $\xi, \zeta > 0$, transforms to the work of Yang and Chen [36] in nonlocal thermo-viscoelasticity theory.

(vii) The system of equations (13)-(18) in the case, $0 < \alpha, \beta, \xi, \zeta \le 1$, provides result for the nonlocal fractional thermo-viscoelasticity theories with and without energy dissipation.

4. Model Construction in the Laplace Transforms Domain

The expression that specifies the parameter s for the Laplace transform is

$$\left.\begin{aligned} L\{g(t)\} &= \bar{g}(s) = \int_0^\infty e^{-st} g(t)\,dt \\ L\{D^n g(t)\} &= s^n L\{g(t)\} \end{aligned}\right\}, \; s > 0,$$

for equations (13)-(18), and with the uniform starting conditions, we obtain the following interconnected system of equations:

$$\bar{e} = D\bar{u}, \tag{19}$$

$$D\bar{\sigma} = s^2\bar{u}, \tag{20}$$

$$(1 - \xi^2 D^2)\bar{\sigma} = (1 + \bar{R})D\bar{u} - \bar{\theta}, \tag{21}$$

$$D^2\bar{\theta} = \omega(1 - \zeta^2 D^2)(\bar{\theta} + \varepsilon D\bar{u}), \tag{22}$$

$$D\bar{\theta} = -\omega(1 - \zeta^2 D^2)\bar{q}, \tag{23}$$

$$\bar{R}(s) = \frac{4\mu}{3sK_0}\left(1 - \frac{A\Gamma(\alpha^*)}{(s + \beta)^{\alpha^*}}\right), \tag{24}$$

where $D = \dfrac{d}{dx}$ and $\omega(s) = \tau_\upsilon^\beta s^\beta \left(1 + \dfrac{\tau_0^\alpha}{\alpha!} s^\alpha\right)$. All starting functions must be equal to zero.

Merging the previous equations results in a system made up of the two equations provided below:

$$D^2\bar{u} = \beta_1\bar{u} + \beta_2 D\bar{\theta}, \tag{25}$$

$$D^2\bar{\theta} = \beta_3\bar{\theta} + \beta_4 D\bar{u}, \tag{26}$$

where

$$\beta_1 = \frac{s^2}{1 + \bar{R} + s^2\xi^2}, \quad \beta_2 = \frac{1}{1 + \bar{R} + s^2\xi^2}, \quad \beta_3 = \frac{\omega}{1 + \omega\zeta^2(1 + \varepsilon\beta_2)}$$

and

$$\beta_4 = \frac{\omega\varepsilon(1 - \zeta^2\beta_1)}{1 + \omega\zeta^2(1 + \varepsilon\beta_2)}.$$

5. Application: A Polymer Micro-rod Subjected to Different Types of Thermal Loading

Consider a homogeneous, isotropic, thermo-viscoelastic polymer micro-rod with a small length ℓ_0 with a left end surface free of traction and a base supporting its right end surface. The coordinate axes are chosen with an x-directional action on the mid-plane at $x = 0,$ as shown in Figure 1.

Figure 1. Schematic of a polymer micro-rod problem.

The mechanical boundary conditions can be expressed as:

$$\sigma(0,\ t) = 0,\ \text{or}\ \overline{\sigma}(0,\ s) = 0, \tag{27}$$

$$u(\ell_0,\ t) = 0,\ \text{or}\ \overline{u}(\ell_0,\ s) = 0. \tag{28}$$

The thermal boundary conditions are assumed to be

$$\theta(0,\ t) = g(t),\ \text{or}\ \overline{\theta}(0,\ s) = \overline{g}(s), \tag{29}$$

$$q(\ell_0,\ t) = 0,\ \text{or}\ \overline{q}(\ell_0,\ s) = 0, \tag{30}$$

where q represents the components of the heat flux vector perpendicular to the surface pole. Condition (29) shows the thermal load at the left end at time $t = 0,$ while condition (30) shows the heat input at the right end.

From equation (23), condition (30) reduces to:

$$\overline{\theta}'(\ell_0,\ s) = 0. \tag{31}$$

By substituting equations (27) and (29) into equation (21), we have

$$\overline{u}'(0,\ s) = \beta_2 \overline{g}(s). \tag{32}$$

For a bounded region, the general solution of equation (25) is considered to be

$$\overline{u}(x,\ s) = A\cosh k_1 x + B\sinh k_1 x + C\cosh k_2 x + D\sinh k_2 x, \tag{33}$$

where the parameters A, B, C and D are influenced by x and s.

From equations (25) and (33), we obtain

$$\overline{\theta}(x,\ s) = \frac{(k_1^2 - \beta_1)}{\beta_2 k_1}(A \sinh k_1 x + B \cosh k_1 x)$$

$$+ \frac{(k_2^2 - \beta_1)}{\beta_2 k_2}(C \sinh k_2 x + D \cosh k_2 x). \tag{34}$$

Using the boundary conditions (27)-(32) in equations (33) and (34), we get

$$A = -\frac{\beta_2 k_1}{k_1^2 - k_2^2}\,\overline{g}(s) \tanh k_1 \ell_0, \quad B = \frac{\beta_2 k_1}{k_1^2 - k_2^2}\,\overline{g}(s),$$

$$C = \frac{\beta_2 k_2}{k_1^2 - k_2^2}\,\overline{g}(s) \tanh k_2 \ell_0, \quad D = -\frac{\beta_2 k_2}{k_1^2 - k_2^2}\,\overline{g}(s). \tag{35}$$

By using equation (25), equations (33) and (34) yield to

$$\overline{u}(x,\ s) = -\frac{\beta_2 \overline{g}(s)}{k_1^2 - k_2^2}\left[k_1 \frac{\sinh k_1(\ell_0 - x)}{\cosh k_1 \ell_0} - k_2 \frac{\sinh k_2(\ell_0 - x)}{\cosh k_2 \ell_0}\right], \tag{36}$$

$$\overline{\theta}(x,\ s) = \frac{\overline{g}(s)}{k_1^2 - k_2^2}\left[(k_1^2 - \beta_1)\frac{\cosh k_1(\ell_0 - x)}{\cosh k_1 \ell_0} - (k_2^2 - \beta_1)\frac{\cosh k_2(\ell_0 - x)}{\cosh k_2 \ell_0}\right]. \tag{37}$$

The form of the stress field can be obtained from equations (20) and (36), as

$$\overline{\sigma}(x,\ s) = \frac{\beta_2 s^2 \overline{g}(s)}{k_1^2 - k_2^2}\left[\frac{\cosh k_1(\ell_0 - x)}{\cosh k_1 \ell_0} - \frac{\cosh k_2(\ell_0 - x)}{\cosh k_2 \ell_0}\right]. \tag{38}$$

To obtain complete solutions in the Laplace transform domain, we must determine the function $\overline{g}(s)$ and consider the following three types of thermal loading:

Case (i). Thermal shock

$$g(t) = H(t - \omega),\ t \geq \omega \ \text{ or } \ \overline{g}(s) = \frac{\theta_0 e^{-\omega s}}{s}. \tag{39}$$

Case (ii). Uniformly laser pulse irradiation

$$g(t) = \frac{t^2}{16t_p^3} e^{-\frac{t}{t_p}}, \quad t \geq 0 \text{ or } \bar{g}(s) = \frac{1}{8(st_p + 1)^3}. \tag{40}$$

Case (iii). Harmonic thermal heat

$$g(t) = \theta_0 \sin wt, \ t \geq 0 \text{ or } \bar{g}(s) = \frac{\theta_0}{s^2 - w^2}, \tag{41}$$

where ω is the thermal shock parameter, t_p is the time duration of a laser pulse, w is the angular thermal parameter, and θ_0 is a constant.

6. Numerical Results

This study employs a nonlocal systematical formulation to establish the size-dependent relationship between viscoelastic deformation and thermal fields in a thermo-viscoelastic solid setting.

The Laplace transform in equations (36)-(38) is inverted using a Fourier series expansion method proposed by Honig and Hirdes [42]. Five-digit accuracy was ensured by using the Fortran 77 computer language to construct the numerical code.

We take into consideration the qualities of a polymethyl methacrylate (Plexiglas) material in order to comprehend the numerical computations. The values of physical constants are displayed in the following table:

Table 1. The parameters of polymethyl methacrylate (Plexiglas) are given as in Ezzat [43]

$\rho = 2.65 \times 10^3 \text{kg/m}^3$	$\gamma = 1.16 \times 10^6 \text{N/m}^2\text{K}$	$k = 1.4 \text{W/mK}$
$K_0 = 3.7 \times 10^{10} \text{Pa}, T_0 = 370\text{K}$	$\lambda = 15.87$ GPa	$\mu = 31.26$ GPa
$\varepsilon = 0.392 \times 10^{-10}$ F/m	$C_E = 670 \text{J/(kgK)}$	$\alpha_T = 5.5 \times 10^{-7}$ 1/K

The calculations were done for multiple nonlocal parameters (ξ, ζ), fractional order (α, β), speed of a moving heat source (v), thermal shock time parameter (ω), time duration of a laser pulse (t_p) and angular thermal parameter (w). The numerically estimated non-dimensional temperature, displacement and stress at different points of x are displayed in Figures 2-8 for Problem (I) and Figures 9-10 for problem (II).

6.1. Verification procedure

The model predictions are compared with El-Karamany and Ezzat [12] in the absence of nonlocal effects for Case (i) in Figure 2. The starting and stopping conditions are identical to those in that reference. The results due to the classical theory (CTV) with the generalized Green-Naghdi theory of Type II (GN-II) are illustrated. Figure 2 demonstrates that the solution for $\beta \approx 1$ resembles the generalized thermo-viscoelasticity theory, indicating that the new theory may maintain the finite wave velocity advantage (Sherief and Abd El-Latief [44]).

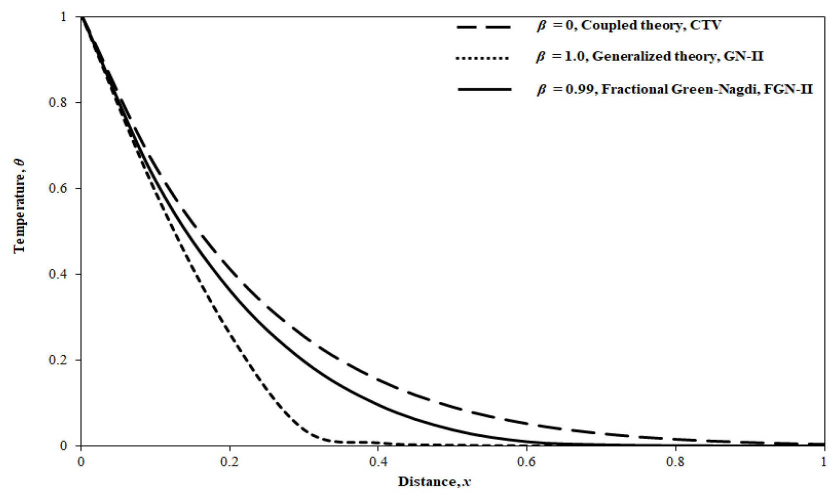

Figure 2. The variation of the temperature against distance for different theories.

6.2. The effect of fractional order (α, β) on all fields for Case (i)

Figures 3-5 display the spatial variations of temperature, displacement, and stress at different fractional order values. The solutions obtained from coupled thermo-viscoelasticity $(\alpha = \beta = 0,\ R > 0,\ \text{and}\ \xi = \zeta = 0,\ \text{CTV})$, and generalized theory of thermo-viscoelasticity without energy dissipation $(\alpha = 0,\ \beta = 1,\ R > 0,\ \text{and}\ \xi = \zeta = 0,\ \text{GN-II})$ are represented by dashed and dotted lines, respectively. The investigation results of the new model for fractional Green-Naghdi-II without energy dissipation in thermo-viscoelastic material $(\alpha = 0,\ \beta = 0.2,\ R > 0,\ \text{and}\ \xi = \zeta = 0,\ \text{FGN-II})$ are depicted using solid lines. The study revealed that temperature fields are influenced by β delay and decrease as parameter estimation increases in polymer micro-rod. The increase in fractional order leads to a greater displacement distribution and a decrease in the stress field at a certain distance x.

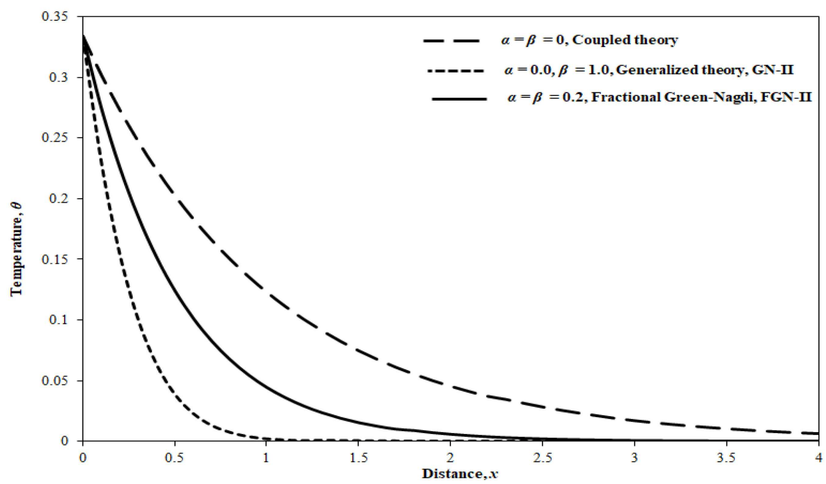

Figure 3. The variation of the temperature against distance for different theories.

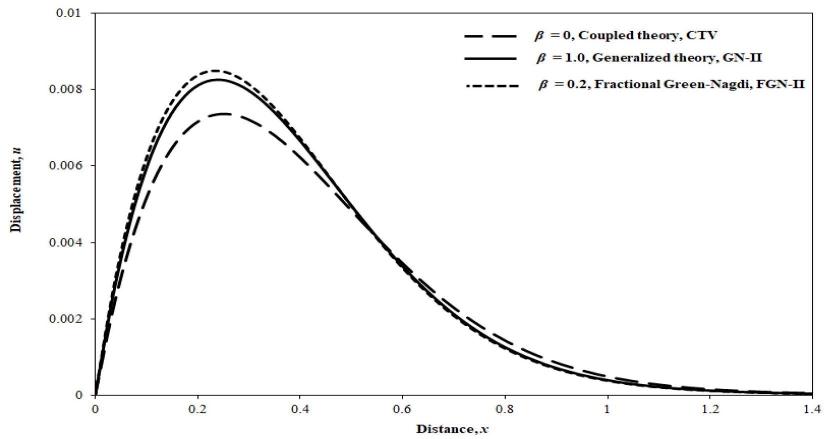

Figure 4. The variation of the displacement against distance for different theories.

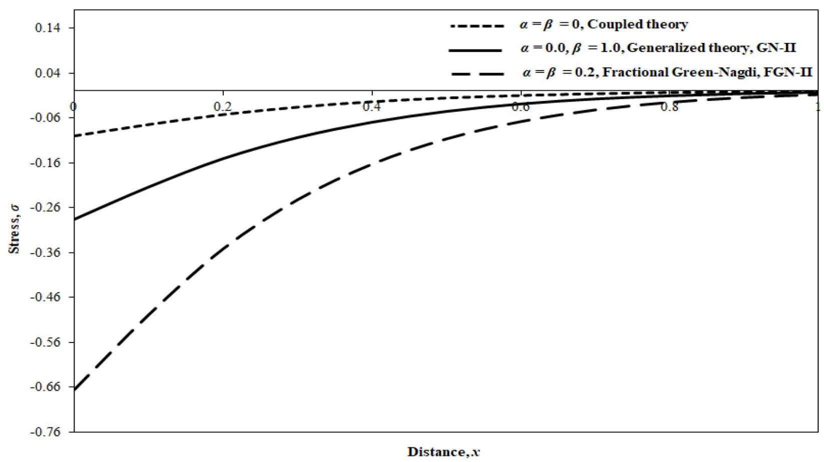

Figure 5. The variation of the stress against distance for different theories.

6.3. The effect of nonlocal parameters (ζ, ξ) on all fields for Case (i)

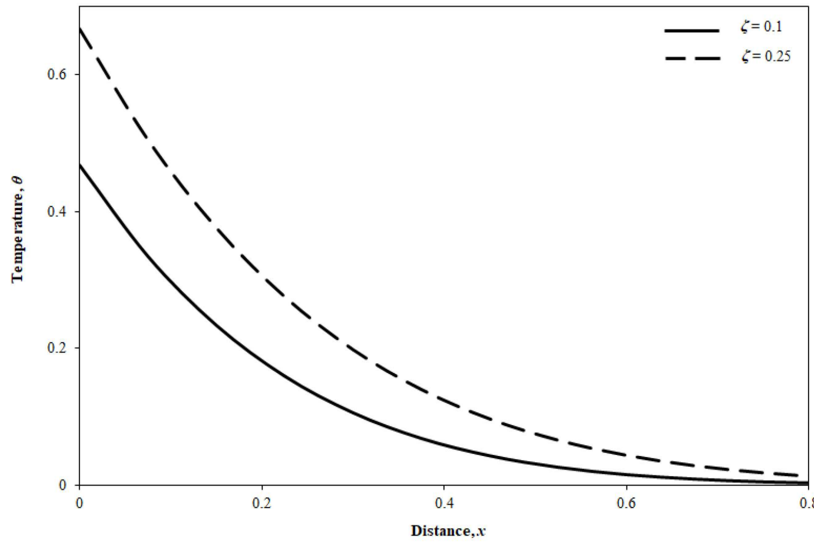

Figure 6. The effects of thermal nonlocal parameter on temperature distribution.

Figure 6 illustrates the influence of the nonlocal thermal parameter $(\zeta = 0.1, 0.25)$ on the temperature distribution. The increase in this parameter leads to a rise in temperature field. Thermal waves are reaching a stable state based on the values of the nonlocal thermal parameter ζ, this indicates that the particles rapidly exchange heat with one another, resulting in a faster cooling of the temperature than the rest. Additionally, we observed that at $\zeta = 0.1$ compared to $\zeta = 0.25$, the thermal waves in these photos appear to cut the x-axis faster. The displacement and stress field curves predicted by two distinct values of elastic nonlocal parameter $(\xi = 0.1, 0.2)$ are shown in Figures 7 and 8. The displacement and stress rise at first and then gradually decrease to zero as the distance x $(0 \leq x \leq 1.5)$ increases. In polymer micro-rod, when ξ is raised from 0.1 to 0.2, the displacement and stress field rise which are significantly influenced by the factor ξ.

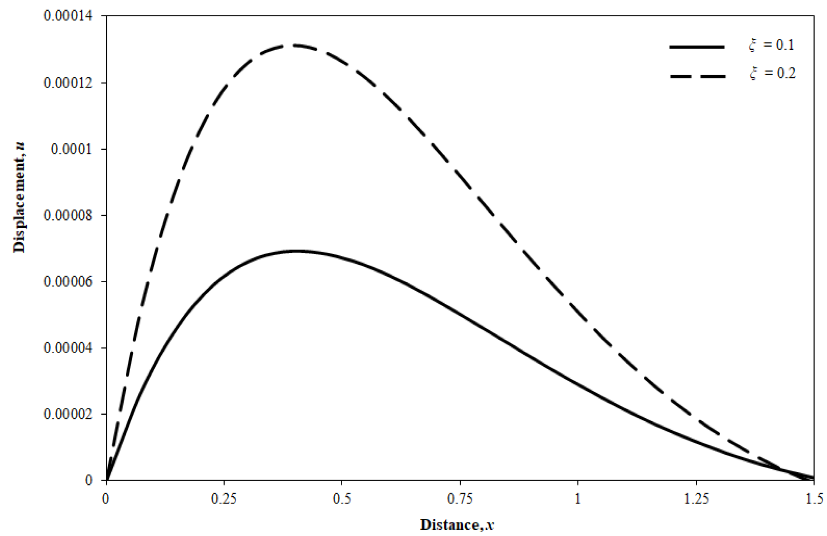

Figure 7. The effects of elastic nonlocal parameter on displacement distribution.

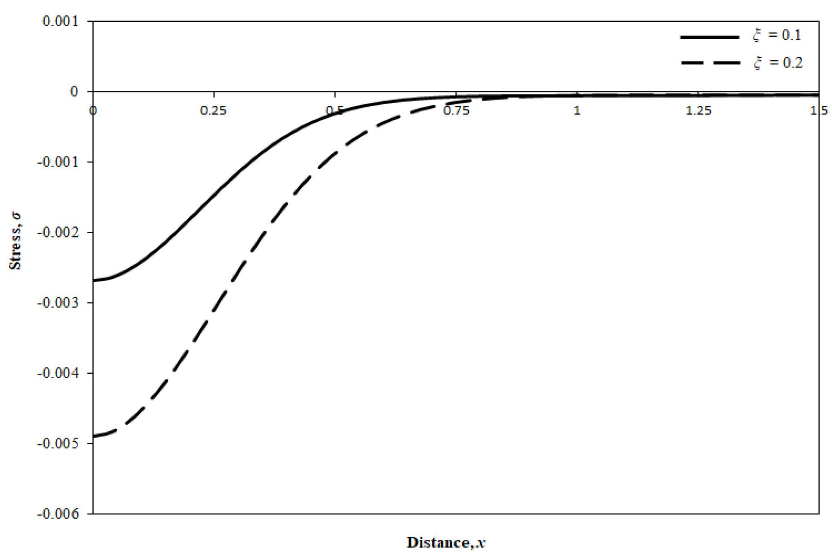

Figure 8. The effects of elastic nonlocal parameter on stress distribution.

6.4. The effect of time duration of a laser pulse t_p on temperature field for Case (ii)

Figure 9 shows the temperature distribution for $\alpha = 0.2,\ \beta = 0.3$ (FGN-II) and $\zeta = \xi = 0.15$ when $t = 0.15$ and $t_p = (0.02, 0.1)$. The figures illustrate the impact of the time duration of a laser pulse t_p. This field has been affected by the parameter of the time duration of a laser pulse t_p, where the increase in the value of the parameter t_p causes decrease in temperature. From this figure, we observe that the new model reveals thermal wave with finite propagation speed.

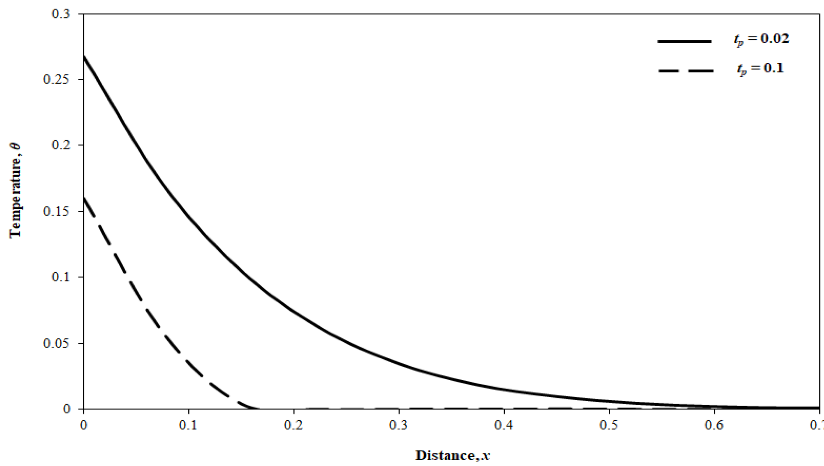

Figure 9. The variation of the temperature against distance for different values of time duration of a laser pulse.

6.5. The effect of on angular thermal parameter w on temperature field for Case (iii)

Figure 10 illustrates that the temperature distribution varies with distance for angular thermal parameters, $w = (10, 15)$, from this figure, we noticed that the thermal waves are smooth, continuous, and steady, influenced by angular thermal parameter, allowing particles to easily transport heat, resulting in a higher temperature decrease rate. The temperature increases with an increase in the angular thermal parameter.

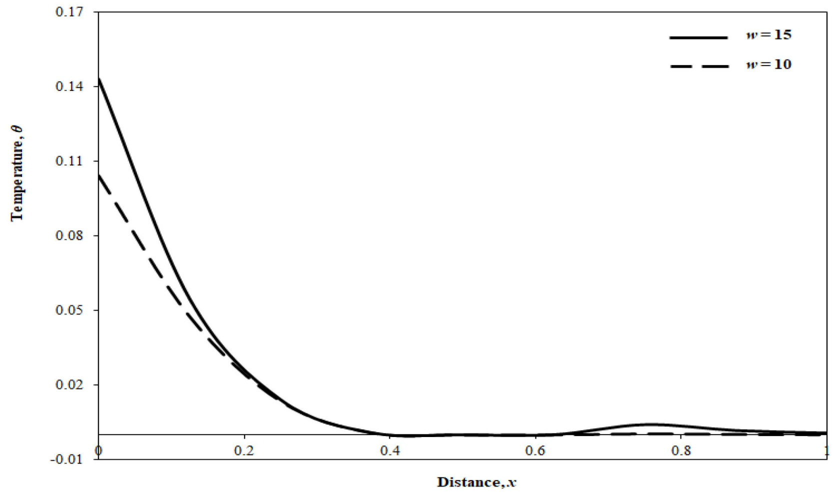

Figure 10. The variation of the displacement against distance for different values of angular thermal parameter.

7. Conclusion

For isotropic materials, a novel fractional theory for the Fourier law of heat conduction without energy dissipation has been introduced. This theory requires that materials should be classified based on their fractional characteristics, which is a novel measure for evaluating polymer heat-transport efficiency. This study might lead to a better understanding of polymer interactions as well as the creation of a novel fractional model with extensive applicability. With the use of modified GN-II and viscoelastic deformation, this work attempts to develop a comprehensive size-dependent thermo-viscoelastic coupling model that takes into account two different models governing fractional heat transfer with and without energy dissipation. The nonlocal parameter on the dimensionless displacement and stress has multiple degrees of weakening. This suggests that while creating and improving polymer microdevices for use in heat transfer situations, the nonlocal effect cannot be ignored (Guo et al. [45]).

Acknowledgements

The authors gratefully acknowledge the approval and the support of this research study by the Grant No. SCIA-2023-12-2056 from the Deanship of Scientific Research in Northern Border University, Arar, KSA.

Also, they are highly grateful to the referee for his careful reading, valuable suggestions and comments, which helped to improve the paper.

References

[1] J. L. Nowinski, Theory of Thermoelasticity with Applications, Sijthoff and Noordhoff International Publishers, 1978.

[2] M. A. Biot, Thermoelasticity and irreversible thermodynamics, Journal of Applied Physics 27(3) (1956), 240-253.

[3] C. Cattaneo, A form of heat conduction equation, which eliminates the paradox of instantaneous propagation, Comptes Rendus 247(3) (1958), 431-433.

[4] H. Lord and Y. Shulman, A generalized dynamical theory of thermoelasticity, Journal of the Mechanics and Physics of Solids 15(5) (1967), 299-309.

[5] W. Kaminski, Hyperbolic heat conduction equation for materials with a nonhomogeneous inner structure, ASME Journal Heat Transfer 112(3) (1990), 555-560.

[6] J. Ignaczak, Uniqueness in generalized thermoelasticity, J. Thermal Stresses 2(2) (1979), 171-175.

[7] H. H. Sherief and R. S. Dhalival, A uniqueness theorem and a variational principle for generalized thermoelasticity, J. Thermal Stresses 3(2) (1980), 223-230.

[8] M. A. Ezzat and A. S. El-Karamany, The uniqueness and reciprocity theorems for generalized thermoviscoelasticity for anisotropic media, J. Thermal Stresses 25(6) (2002), 507-522.

[9] A. Sur, Memory responses in a three-dimensional thermo-viscoelastic medium, Waves Random and Complex Media 32(1) (2022), 137-154.

[10] A. E. Green and P. M. Naghdi, Thermoelasticity without energy dissipation, J. Elasticity 31(3) (1993), 189-208.

[11] D. S. Chandrasekharaiah, A uniqueness theorem in the theory of thermoelasticity without energy dissipation, J. Thermal Stresses 19(3) (1996), 267-272.

[12] A. S. El-Karamany and M. A. Ezzat, Thermal shock problem in generalized thermoviscoelasticity under four theories, Internat. J. Engrg. Sci. 42(7) (2004), 649-671.

[13] P. Lata, Effect of energy dissipation on plane waves in sandwiched layered thermoelastic medium, Steel and Composite Structures, International Journal 27(4) (2018), 439-451.

[14] P. Lata and I. Kaur, Thermomechanical interactions in transversely isotropic magneto-thermoelastic solid with two temperatures and without energy dissipation, Steel and Composite Structures, International Journal 32(6) (2019), 779-793.

[15] M. A. Ezzat, Theory of fractional order in generalized thermoelectric MHD, Applied Mathematical Modelling 35(10) (2011), 4965-4978.

[16] M. A. Ezzat, Fractional thermo-viscoelastic response of biological tissue with variable thermal material properties, J. Thermal Stresses 43(9) (2020), 1120-1137.

[17] G. Jumarie, Derivation and solutions of some fractional Black-Scholes equations in coarse-grained space and time, application to Merton's optimal portfolio, Comput. Math. Appl. 59 (2010), 1142-1164.

[18] Y. J. Yu, X. G. Tian and J. L. Tian, Fractional order generalized electro-magneto-thermoelasticity, European Journal of Mech. A/Solids 42 (2013), 188-202.

[19] M. A. Ezzat and A. A. El-Bary, Unified GN model of electro-thermoelasticity theories with fractional order of heat transfer, Microsystem Technologies 24(12) (2018), 4965-4979.

[20] M. A. Ezzat, A. A. El-Bary and M. A. Fayik, Fractional Fourier law with three-phase lag of thermoelasticity, Mechanics of Advanced Materials and Structures 20(8) (2013), 593-602.

[21] S. I. El-Attar, M. H. Hendy and M. A. Ezzat, Magneto-thermoelasticity Green-Naghdi theory with memory-dependent derivative in the presence of a moving heat source, Int. J. Advanced and Applied Sciences 9(7) (2022), 33-41.

[22] M. M. Amin, M. H. Hendy and M. A. Ezzat, On the memory-dependent derivative electric-thermoelastic wave characteristics in the presence of a continuous line heat source, Int. J. Advanced and Applied Sciences 9(8) (2022), 1-8.

[23] X. J. Yang, General Fractional Derivatives: Theory, Methods and Applications, CRC Press, 2019.

[24] M. A. Meyers and K. K. Chawla, Mechanical Behavior of Materials, Upper Saddle River, Prentice-Hall, NJ, Vol. 98, 1999, p. 103.

[25] M. A. Ezzat, Thermo-mechanical memory responses of biological viscoelastic tissue with variable thermal material properties, Int. J. Numerical Methods for Heat and Fluid Flow 31(1) (2020), 548-569.

[26] M. A. Ezzat, Bio-thermo-mechanics behavior in living viscoelastic tissue under the fractional dual-phase-lag theory, Archive of Applied Mechanics 91(9) (2021), 3903-3919.

[27] S. F. M. El Sherif, M. A. Ismail, A. A. El-Bary and H. M. Atef, Effect of magnetic field on thermos: viscoelastic cylinder subjected to a constant thermal shock, Int. J. Advanced and Applied Sciences 7(1) (2020), 117-124.

[28] X. Shi, M. L. Hassanzadeh-Aghdam and R. Ansari, Viscoelastic analysis of silica nanoparticle-polymer nanocomposites, Composites B 158 (2019), 169-178.

[29] K. Eom, H. S. Park, D. S. Yoon and T. Kwon, Nanomechanical resonators and their applications in biological/chemical detection: nanomechanics principles, Physics Reports 503(4/5) (2011), 115-163.

[30] L. J. Currano, M. Yu and B. Balachandran, Latching in a MEMS shock sensor: modeling and experiments, Sensors and Actuators A: Physical 159(1) (2010), 41-50.

[31] A. Toril, M. Sasaki, K. Hane and S. Okuma, Adhesive force distribution on microstructures investigated by an atomic force microscope, Sensors and Actuators A: Physical 44(2) (1994), 153-158.

[32] A. E. Abouelregal and M. Marin, The size-dependent thermoelastic vibrations of nanobeams subjected to harmonic excitation and rectified sine wave heating, Mathematics 8(7) (2020), 1128.

[33] Q. Lyu, N. H. Zhang, C. Y. Zhang, J. Z. Wu and Y. C. Zhang, Effect of adsorbate viscoelasticity on dynamical responses of laminated microcantilever resonators, Composites Structures 250 (2020), 112553.

[34] M. Sobhy and A. M. Zenkour, The modified couple stress model for bending, of normal deformable viscoelastic nanobeams resting on visco-Pasternak foundations, Mechanics of Advanced Materials and Structures 279(7) (2020), 525-538.

[35] M. A. Attia and A. A. Abdel Rahman, On vibrations of functionally graded viscoelastic nanobeams with surface effects, Internat. J. Engrg. Sci. 127 (2018), 1-32.

[36] W. Yang and Z. Chen, Nonlocal dual-phase-lag heat conduction and the associated nonlocal thermoviscoelastic analysis, International Journal of Heat and Mass Transfer 156 (2020), 119752.

[37] J. D. Ferry, Viscoelastic Properties of Polymers, Wiley, New York, 1970.

[38] P. Lata and S. Singh, Effect of nonlocal parameter on nonlocal thermoelastic solid due to inclined load, Steel and Composite Structures 33(1) (2019), 955-963.

[39] M. A. Ezzat, S. M. Ezzat and M. Y. Alkhrraz, State-space approach to nonlocal thermo-viscoelastic piezoelectric materials with fractional dual-phase lag heat transfer, Int. J. Numerical Methods for Heat and Fluid Flow 32(12) (2022), 3726-3750.

[40] A. A. Hassaballa, M. H. Hendy and M. A. Ezzat, A modified Green-Naghdi fractional-order model for analyzing thermoelectric semispace heated by a moving heat source, Mechanics of Time-Dependent Materials 28 (2024), 1815-1837.

[41] B. Gross, Mathematical Structure of the Theories of Viscoelasticity, Hermann, Paris, France, 1953.

[42] G. Honig and U. Hirdes, A method for the numerical inversion of the Laplace transform, J. Comput. Appl. Math. 10(1) (1984), 113-132.

[43] M. A. Ezzat, The relaxation effects of the volume properties of electrically conducting viscoelastic material, Mat. Sci. Eng. B 130(1/3) (2006), 11-23.

[44] H. H. Sherief and A. M. Abd El-Latief, Effect of variable thermal conductivity on a half-space under the fractional order theory of thermoelasticity, Int. J. Mech. Sci. 74(9) (2013), 185-189.

[45] H. Guo, L. Yaning, C. Li and T. He, Structural dynamic responses of layer-by-layer viscoelastic sandwich nanocomposites subjected to time-varying symmetric thermal shock loadings based on nonlocal thermo-viscoelasticity theory, Microsys. Tech. 28(5) (2022), 1143-1165.