



THERMAL BEHAVIOUR OF A CIRCULAR PLATE UNDER CAPUTO-FABRIZIO FRACTIONAL IMPACT WITH SECTIONAL HEATING

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Abstract

Recent advances in the understanding of the precise physical thermal behaviour of various solids under the effect of fractional-order

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derivatives have boosted the study of thermoelasticity, which is primarily important in various industrial designs of usable structural materials. We investigated a thin circular plate that was subjected to additional sectional heating on its top and lower surfaces while creating thermal insulation around its outside border. In this work, we maintained the heat transfer equation while accounting for the impact of Caputo-Fabrizio fraction-order derivatives. According to specified boundary constraints, the integral transformation approach is used to assess the analytical solution of the displacement, temperature change, and thermal stresses. Furthermore, various functions and fractional parameters are computed using the material properties of aluminium metal plates for numerical purposes.

1. Introduction

The area of thermoelasticity has a long history and is expanding quickly because of its many possibilities in the creation of realistic structural designs. The domains of aeronautics, nuclear fields, nuclear reactors, and contemporary propulsive technologies like jet and rocket engines are all directly correlated with temperature. The field of thermal stresses originates from the high temperatures connected to combustion processes. Additionally, somewhat intense thermal stresses are frequently linked to technology such as shipbuilding, fracturing, spacecraft and missiles. A remarkable number of theoretical and experimental research articles have been written about different facets of thermal stresses in engineering structures. Authors in [1] examined the bending and stresses caused by thermal influences in a thin circular plate problem that was simulated with an axisymmetric heating element and restrained and insulated edges. The article [2] used the integral transformed approach to ascertain the impact of thermal strains and bending in a thin circular plate caused by internal heat accumulation. Analytical and numerical investigations were conducted by [3] to solve the heating issue for temperature-dependent stress in a thin circular elastic-plastic plate. The fundamental transformation method is used to solve the problem of temporary transfer of heat in a thin circular plate under various boundary circumstances, yielding an infinite series [4]. A thin

circular plate with homogeneous internal heat production and a parabolic variation in temperature was the subject of a transient thermoelastic stress measurement in [5].

Using the Laplace and Hankel transform methods, a general approach for the 2-dimensional issue of a thick circular plate with sources of heat in altered couple stresses and thermoelastic dispersion in the setting of one and two relaxed durations has been established in [6]. Under the specified thermal operation and residual strains in the thin circular elastoplastic plate, the dimensional problem is defined and investigated in [7]. A computational investigation of the impact of two thermal parameters for axisymmetric distortion in a 2-dimensional isotropic thick circular plate with no dissipating energy is performed in [8] by using Laplace and Hankel transforms. Analytically, computationally, and through experimentation, the thermally generated oscillations of merely supported and clamped circular plates have been investigated [9]. In order to calculate the conductive temperature, movement sections, and stress conceptually and numerically, the article [10] took into consideration a thick circular plate with an axisymmetric supply of heat and traction-free below and top surfaces.

Instead of using the Fourier law and conventional heat transfer formulas, nonclassical models make use of broader equations that have caught the attention of many scholars recently. To ascertain the effects of the fractional ordered gradient of a few other parameters on the contours of temperature, deviation, and tension got observed on the thin circular plate [11]. In [12], the variations in order of fraction strain were investigated for a uniformly thick circular sheet with hyper-two-temperature ring pressure. By using the fractional ordering method for thermoelastic diffusion, a 2-dimensional thermoelastic situation involving a thick circular plate with finite thickness has been examined to look at the impact of frequency [13]. A thorough frame-invariant fractional-order structure to support the analysis of physically nonlinear bends and postbuckling of not local plates under coupled mechanical and heat loads is provided in [14]. Using the integral transform approach, the duration of a fractional thermal reaction in a thick plate when it becomes heated internally is calculated in [15]. While solving

the time-fractional phase heat transfer equation for an axisymmetric functional grading temperature-sensitive hollow cylinder, the effects of convective boundary variables and internal heat production have been considered in [16].

By using the fractional-order framework for thermoelasticity, the authors in [17] were able to find traction-free thermoelastic half-spaces with specified axisymmetric distributions of temperatures and determined the thermoelastic interaction. Reference [18] examined the temperature, displacement, and stress field caused by the interior heat source and then resolved a fractional ordering thermoelastic issue involving a thin hollow circular disc under an axisymmetric energy source. In order to obtain a solution using the Laplace transform approach, authors in [19] applied the fraction thermal elasticity concept to a two-dimensional problem for a sphere with an axisymmetric distribution of heat. In order to examine the transitory reactions brought on by a moving heat origin, the unifying fractional thermoelastic model is created in [20]. Atangana-Baleanu, tempered-Caputo type, and Caputo-Fabrizio are three novel definitions of fractional derivatives that theoretically bring new insights into fractional thermoelasticity.

Numerous scholars have been interested in the Caputo-Fabrizio fractional derivative for many years due to its occurrence in a wide range of applications and the fact that its kernel is non-local and nonsingular of convolution. Utilizing derivatives with fractional order in the Caputo-Fabrizio sense, this kind of fractional derivative model is helpful not only in illustrating thermoelastic challenges but also for research including the exponential kernel in the human liver [21, 22]. After successfully describing the many uses of the Caputo-Fabrizio fraction derivative of the function, the article [23] analyses it in both traditional and distributional situations. By using the Caputo-Fabrizio fractal derivative operation with no singular kernel, further numerically distinct characteristics got suggested in [24]. A numerical approximation has also been made of it using the classical initial derivative's two-point limit forward difference equation.

In source [25], a weighted Caputo-Fabrizio fractional derivation of the Caputo notion was explained. Additionally, the associated nonlinear and linear fractional differential formulas were examined, along with their corresponding applications. The nonlinear partial differential model of fractional equations has been modified to include Caputo-Fabrizio derivatives of fraction order as well [26]. It was demonstrated that this new derivative meets the requirements of the mixed partial and the expanded equation. Furthermore, the analysis verifies the presence and exclusivity of the precise result [27]. A time-fractional Caputo derivative and a time-fractional Caputo-Fabrizio derivative enabled to solve the advection partial differential equation. Considering the definitions of the two differential operators, an observation was made in [28]. Atangana, Baleanu, and Caputo-Fabrizio derivatives are compared by [29]. The non-singular Caputo-Fabrizio differential formulation for the 1D infinite potential problem was used in [30] to address fractional Schrödinger difficulties for time, space, and time-space. The phase delays and a heat transfer formula with a fractional derivative were proposed by [31] to solve the thermoelastic problem using the Caputo-Fabrizio fractional derivative operation.

In order to ascertain the analytical outcomes for thermal stress measurements by using integral transform methods, the article [32] applied convection boundary circumstances to the curving surface of the cylinder, where sources of heat are created as a linear function of temperature. For an epoxy matrix composite material reinforced with graphite fibre, the integral transform approach was used by [33] to explore the coupling and uncoupling effects of temperature, moisture, and thermal stresses. Additionally, the related study conducted by a few additional researchers is referenced in [34] to [42].

From the review of the literature, it can be seen that no quasi-static approach problem of thermoelasticity has been studied in relation to the Caputo-Fabrizio derivative in the equation of heat transfer. In order to study the change in temperature and stress function, it is anticipated that a thin circular plate with additional sectional heating at the bottom and higher

surfaces is going to be used in this problem to know the overall thermal response.

2. Statement of the Problem

A novel definition of a fractional derivative with a smoother kernel that considers the temporal and spatial variables was introduced by Caputo and Fabrizio [42]. It has been found that the Laplace transform technique works well for explaining temporal modelling analysis. This new method with a regular kernel sparked interest because it may be possible to characterize a class of non-local platforms that can better explain material heterogeneities and fluctuations across various scales than classical local theories or fractional models with a singular kernel.

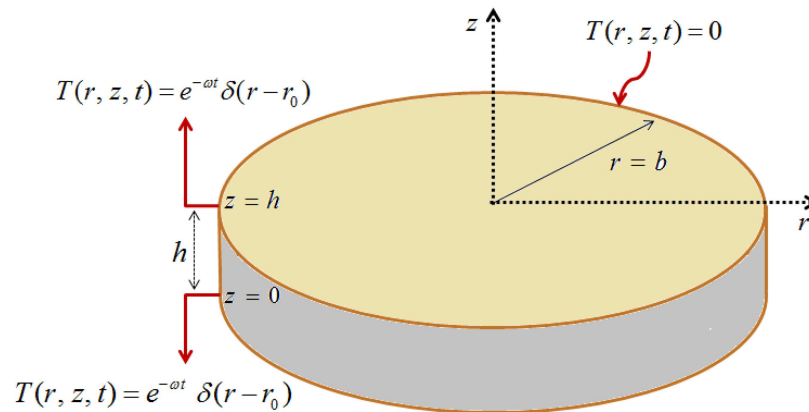


Figure 1. Geometry of problem in context of Caputo-Fabrizio derivatives.

In this article, we examine a straightforward issue involving a thin, circular, thermoelastic plate with a thickness of h that is located in space $D : 0 \leq r \leq b, 0 \leq z \leq h$. The materials that make up the plates are isotropic and homogeneous. Further sectional heating is placed on the upper and lower surfaces, while the outer border is kept thermally insulated, in order to increase the problem's significance and use. This kind of material-property-based structural design is most helpful when analyzing temperature swings and stress-bearing capacity for a variety of industrial designs. It

may also be applicable in space technology, where extreme heat might be damaging.

2.1. Equation of heat transfer

The following differential formula can be satisfied by the heat transfer equation expressed using the Caputo-Fabrizio fractional derivative method:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k} {}_0^{CF} D^{(\alpha)} T(r, z, t), \quad (1)$$

where Caputo-Fabrizio [42], differential operator of order $\alpha \in (0, 1)$ for an absolutely continuous function $T(t)$ is defined as below:

$${}_0^{CF} D^{(\alpha)} T(t) = \frac{1}{1-\alpha} \int_0^t T'(\tau) \exp\left(-\alpha \frac{(t-\tau)}{1-\alpha}\right) d\tau, \quad 0 \leq \alpha \leq 1. \quad (2)$$

The modified Caputo-Fabrizio definition mentioned above has the benefit of having a nonsingular kernel.

Moreover, as stated in equation (1), the Laplace transform that occurs of the Caputo-Fabrizio derivative is defined as [42]:

$$L[{}_0^{CF} D^{(\alpha)} T(t)] = \frac{sL[T(t)] - T(0)}{s + \alpha(1-s)}, \quad (3)$$

$$L[T(t)] = \hat{T}(s) = \int_0^\infty e^{-st} T(t) dt. \quad (4)$$

2.2. Boundary constraints

The heat transfer equation (1) is subjected to the following significant initial and boundary constraints as below:

Initial constraint

$$T(r, z, 0) = 0 \text{ for all } 0 \leq r \leq b, 0 \leq z \leq h. \quad (5)$$

Boundary constraints

$$T(b, z, t) = 0 \text{ for all } 0 \leq z \leq h, t > 0, \quad (6)$$

$$T(r, 0, t) = e^{-\omega t} \delta(r - r_0) \text{ for all } 0 \leq r \leq b, t > 0, \quad (7)$$

$$T(r, h, t) = e^{-\omega t} \delta(r - r_0) \text{ for all } 0 \leq r \leq b, t > 0, \quad (8)$$

where $\delta(r - r_0)$ is the Dirac delta function, $\omega > 0$ is a constant, and $e^{-\omega t} \delta(r - r_0)$ is the additional sectional heat available on its surface at $z = 0, h$. Also, $0 \leq r_0 \leq b$.

2.3. Stress-displacement relationship

The displacement component $U(r, z, t)$ is governed by the problem of differential equations:

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r} \frac{\partial U}{\partial r} = (1 + \nu) a_t T \quad (9)$$

with

$$U = 0 \text{ at } r = b, \quad (10)$$

where ν and a_t stand for the Poisson's ratio and the plate material linear coefficient of thermal expansion, respectively.

The stress functions σ_{rr} and $\sigma_{\theta\theta}$ are given by

$$\sigma_{rr} = -2\mu \frac{1}{r} \frac{\partial U}{\partial r}, \quad (11)$$

$$\sigma_{\theta\theta} = -2\mu \frac{\partial^2 U}{\partial r^2}, \quad (12)$$

where the stress components σ_{rr} , σ_{zz} and $\sigma_{\theta z}$ are all zero inside the plate in the plane state of stress, and μ is the Lamé's constant.

The subject under investigation is mathematically formulated by the equations (1) through (12).

3. Solution of the Problem

3.1. Solution of heat transfer equation

Applying the finite Hankel transform to equation (1) and imposing corresponding initial and boundary constraints from (5) to (8), we get

$$k \left[-\xi_n^2 T^*(\xi_n, z, t) + k \frac{\partial^2 T^*(\xi_n, z, t)}{\partial z^2} \right] = {}_0^CF D^{(\alpha)} T^*(\xi_n, z, t) \quad (13)$$

with transformed boundaries as

$$T^*(\xi_n, z, 0) = 0, \quad (14)$$

$$T^*(\xi_n, 0, t) = e^{-\omega t} r_0 f_0(\xi_n, r_0), \quad (15)$$

$$T^*(\xi_n, h, t) = e^{-\omega t} r_0 f_0(\xi_n, r_0). \quad (16)$$

For the finite Hankel transform, which is defined as follows, the sign (*) indicates the function of the transformed domain and nucleus

$$f_0(\xi_n, r) = \frac{-\sqrt{2}}{b} \left(\frac{J_0(\xi_n r)}{\xi_n J_0(\xi_n b)} \right).$$

Next, we present another integral transform that reacts to types (15) and (16) boundary conditions as

$$\bar{Q}(m) = \int_0^h Q(z) \sin \frac{m\pi z}{h} dz, \quad Q(z) = \frac{2}{h} \sum_{m=1}^{\infty} \bar{Q}(m) \sin \frac{m\pi z}{h}. \quad (17)$$

Further analyzing equation (13), using equations (15) and (16) in combination with the transform given in equation (17), we get

$$\begin{aligned} & -k \left(\xi_n^2 + \frac{m^2 \pi^2}{h^2} \right) \bar{T}^*(\xi_n, m, t) + \frac{km\pi}{h} [(-1)^{m+1} + 1] e^{-\omega t} r_0 f_0(\xi_n, r_0) \\ & = {}_0^CF D^{(\alpha)} \bar{T}^*(\xi_n, m, t) \end{aligned} \quad (18)$$

with transformed initial condition as

$$\bar{T}^*(\xi_n, m, 0) = 0, \quad (19)$$

where \bar{T}^* is the transformed function of T^* and m is the transform parameter.

The symbol $(-)$ means a function of transformed domain and the nucleus given in the interval $0 \leq z \leq h$ with operational property:

$$\int_0^h \frac{\partial^2 Q}{\partial z^2} \sin \frac{m\pi z}{h} dz = \frac{m\pi}{h} [(-1)^{m+1} Q(h) - Q(0)] - \frac{m^2 \pi^2}{h^2} \bar{Q}(m).$$

After performing calculations on equation (18), the reduction is made to the following differential equation:

$${}_0^{\text{CF}} D^{(\alpha)} \bar{T}^*(\xi_n, m, t) + k \left(\xi_n^2 + \frac{m^2 \pi^2}{h^2} \right) \bar{T}^*(\xi_n, m, t) = H_0(m, n) e^{-\omega t}, \quad (20)$$

where

$$H_0(m, n) = [(-1)^{m+1} + 1] \frac{km\pi}{h} r_0 f_0(\xi_n, r_0). \quad (21)$$

Next, on applying Laplace transform defined in equation (3) with initial condition (20) and further on taking its corresponding inversion, we get

$$\bar{T}^*(\xi_n, m, t) = H_0(m, n) \left\{ \frac{l_1[l_2 - \omega]}{[l_3 - \omega l_4]} e^{-\omega t} + \frac{l_1[l_2 l_4 - l_3]}{l_4[\omega l_4 - l_3]} e^{-\frac{l_3}{l_4} t} \right\},$$

where

$$(1 - \alpha) = l_1, \quad \frac{\alpha}{(1 - \alpha)} = l_2, \quad k \left(\xi_n^2 + \frac{m^2 \pi^2}{h^2} \right) l_1 l_2 = l_3$$

and

$$\left(1 + k \left(\xi_n^2 + \frac{m^2 \pi^2}{h^2} \right) l_1 \right) = l_4.$$

Finally, on inverting for Hankel transform and transformed stated in (18), we obtain the desired expression of temperature distribution as below:

$$T(r, z, t) = \frac{2}{h} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} H_0(m, n) \left\{ \frac{l_1[l_2 - \omega]}{[l_3 - \omega l_4]} e^{-\omega t} + \frac{l_1[l_2 l_4 - l_3]}{l_4[\omega l_4 - l_3]} e^{-\frac{l_3}{l_4} t} \right\} \cdot \sin \frac{m\pi z}{h} f_0(\xi_n, r). \quad (22)$$

3.2. Determination of thermoelastic displacement

Substituting value of $T(r, z, t)$ from equation (22) in equation (9), we obtain the thermoelastic displacement function $U(r, z, t)$ as

$$U(r, z, t) = -(1 + \nu) a_t \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{H_0(m, n)}{\xi_n^2} \left\{ \frac{l_1[l_2 - \omega]}{[l_3 - \omega l_4]} e^{-\omega t} + \frac{l_1[l_2 l_4 - l_3]}{l_4[\omega l_4 - l_3]} e^{-\frac{l_3}{l_4} t} \right\} \cdot \sin \frac{m\pi z}{h} f_0(\xi_n, r). \quad (23)$$

3.3. Determination of stress functions

Simplifying equations (11) and (12) with the use of (23) yields the formula for radial and tangential stresses as:

$$\sigma_{rr} = \frac{2}{h} (1 + \nu) a_t \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{H_0(m, n)}{\xi_n^2} \left\{ \frac{l_1[l_2 - \omega]}{[l_3 - \omega l_4]} e^{-\omega t} + \frac{l_1[l_2 l_4 - l_3]}{l_4[\omega l_4 - l_3]} e^{-\frac{l_3}{l_4} t} \right\} \cdot \sin \frac{m\pi z}{h} \times \frac{2\mu}{r} \left[\frac{\sqrt{2}}{b} \frac{J_1(\xi_n r)}{J_1(\xi_n b)} \right], \quad (24)$$

$$\sigma_{\theta\theta} = \frac{2}{h} (1 + \nu) a_t \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{H_0(m, n)}{\xi_n^2} \left\{ \frac{l_1[l_2 - \omega]}{[l_3 - \omega l_4]} e^{-\omega t} + \frac{l_1[l_2 l_4 - l_3]}{l_4[\omega l_4 - l_3]} e^{-\frac{l_3}{l_4} t} \right\} \cdot \sin \frac{m\pi z}{h} \times \xi_n \left[\frac{J_0(\xi_n r)}{J_1(\xi_n b)} - \frac{J_1(\xi_n r)}{(\xi_n r) J_1(\xi_n b)} \right]. \quad (25)$$

4. Numerical Computations

The following aluminium metal material parameters are used for numerical computations and are emphasized in Table 1.

Table 1. Material properties

Modulus of elasticity, E	6.9×10^{11} (dynes/cm ²)
Shear modulus, G	2.7×10^{11} (dynes/cm ²)
Poisson ratio, ν	0.281
Thermal expansion coefficient, α_r	25.5×10^{-6} (cm/cm- ⁰ C)
Thermal diffusivity, k	0.86 (cm ² /sec)
Outer radius, b	2 (cm)
Thickness, h	0.2 (cm)
r_0	1.5
ω	0.5

4.1. Graphical analysis

The link between the Caputo fractional time derivative of order α and the temperature and thermal stress functions is examined in this subsection. These distributions in radial and axial directions are graphically shown in Figures 2 through 4, accounting for the additional sectional heating at the top and lower surfaces of the plates.

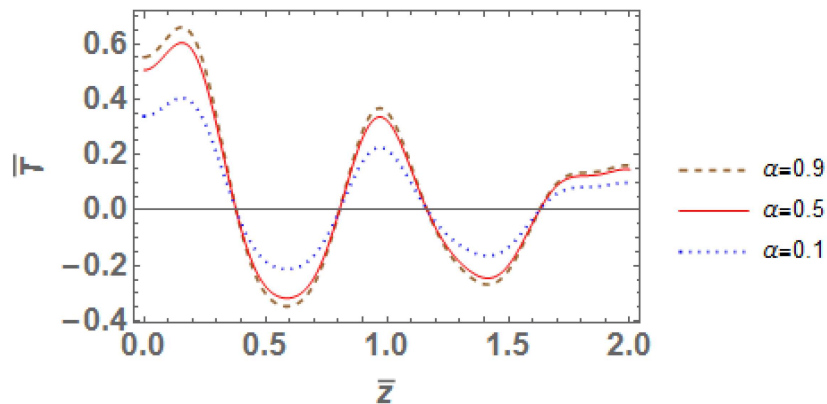


Figure 2(a). The spread of temperature in an axial direction.

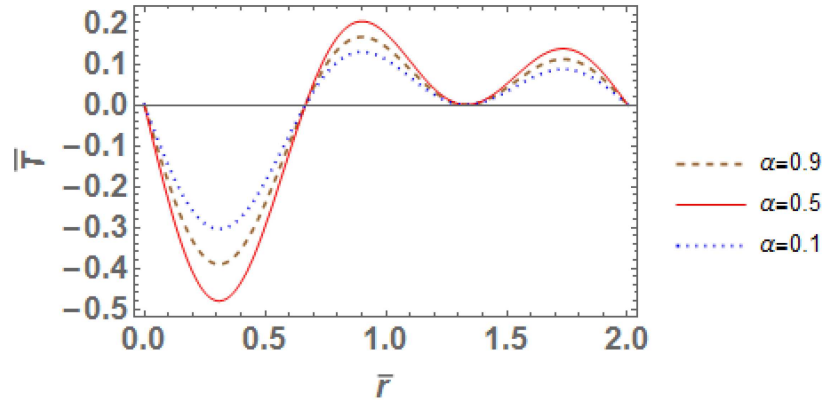


Figure 2(b). The spread of temperature in a radial direction.

As can be seen in Figure 2(a), for both $t = 0.5$ and $r = 1$, the dimensionless axial direction of the temperature distribution function is strongly influenced by the fractional parameters $\alpha = 0.1, 0.5, 0.9$. A more uniform temperature distribution is observed for large values of the fractional parameter; alternatively, one finds that the temperature rises as parameter α 's value rises. The temperature distribution on the bottom and upper surfaces of the plate varies as a result of sectional heating at both surfaces, meeting the given mathematical boundary limit.

The fluctuation of the dimensionless temperature distribution function for various fractional parameters $\alpha = 0.1, 0.5, 0.9$ is shown in Figure 2(b) for both $t = 0.5$ and $z = 0.2$. There are notable differences in the temperature curves for different factors, suggesting that these features could be useful for characterizing the material's properties and helping to build innovative structural stress-bearing designs for the sector. Additionally, it is discovered that the temperature flow variation at both the inner and outer edges is zero, satisfying the mathematically specified boundary and indicating the finite propagation of the wave within a limited range.

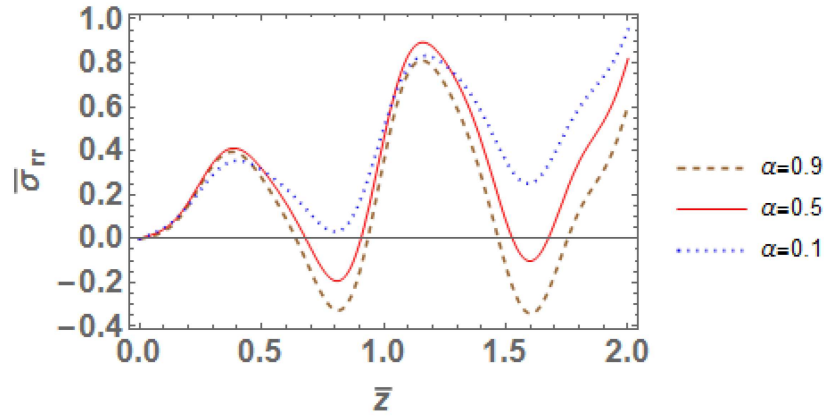


Figure 3(a). Spreading of radial stress in the axial direction.

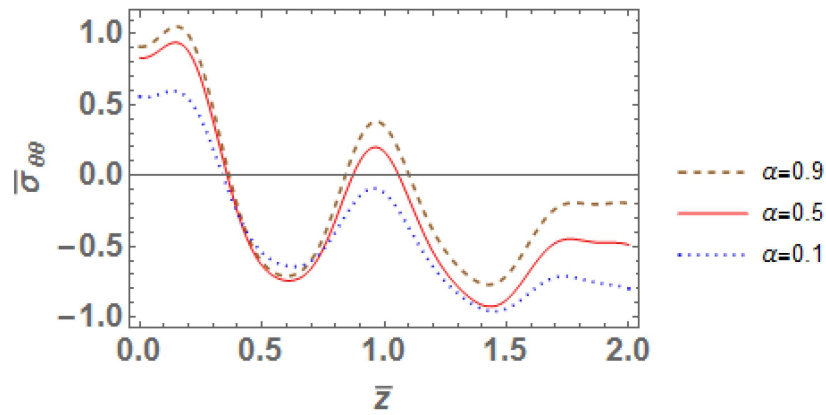


Figure 3(b). Spreading of tangential stress in the axial direction.

Figures 3(a) and 3(b) show the distribution of radial and tangential stresses in the axial direction for various fractional order parameters. It is shown in both plots that the additional sectional heating is causing a uniform growth in the flow of stresses at the upper and lower surfaces, and that the impact of various fractional parameters is significantly discriminating the curve. Therefore, the examination of these kinds of stresses could prove beneficial in the evaluation of a practical, highly-heated model that has a direct correlation with temperature.

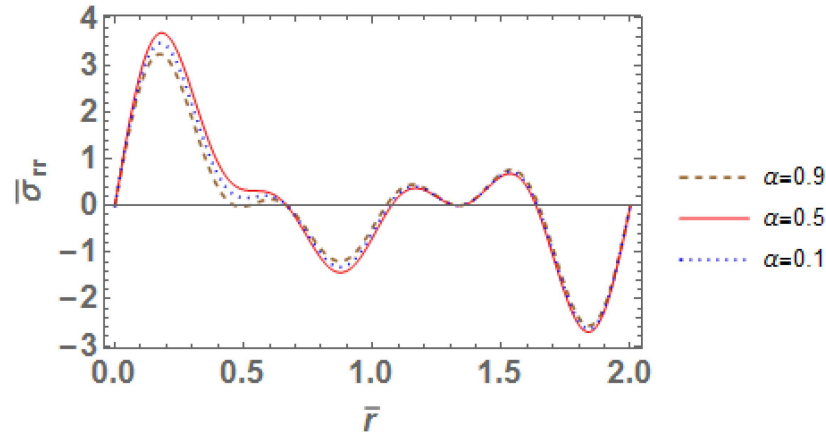


Figure 4(a). Spreading of radial stress in the radial direction.

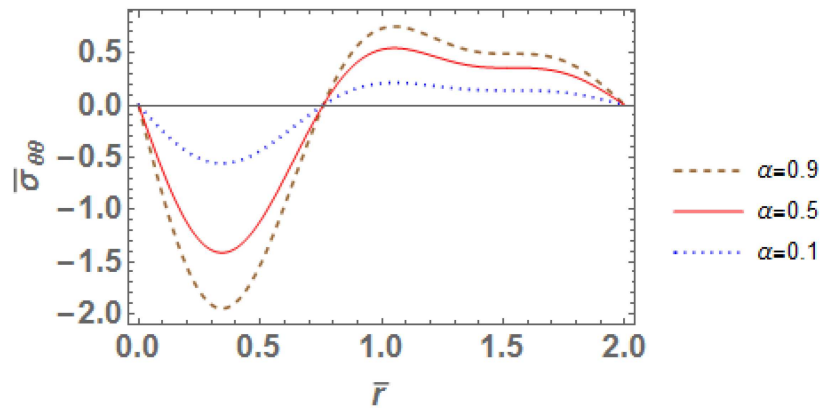


Figure 4(b). Spreading of tangential stress in the axial direction.

The radial and axial stress patterns in the radial direction are displayed graphically in Figures 4(a) and 4(b). The stress functions are zero at both the inner and outer borders. While tangential stress distribution exhibits tensile characteristics up until the midpoint of the radial direction, and then after it becomes compressive, radial stress distribution fluctuates in a compressive manner near the inner edge and a tensile manner at the outer edge, this fluctuation may also be the result of sectional heating responses.

5. Conclusion

Within the framework of Caputo-Fabrizio fraction order derivatives, the analytical expression of the displacement, temperature change, and thermal stresses of a thin circular plate are examined. Utilizing the finite integral transform approach, the heat transfer equation is solved, and all of the derived thermal expressions of stress and temperature are numerically computed using the aluminium metal plate's material parameters. Moreover, Mathematica software is used to graphically display the expressions. By giving the parameters and functions in the expressions along with appropriate values, any specific situation of special interest can be located.

The temperature and stress function fluctuations are significantly impacted by the various values of the Caputo-Fabrizio fractional parameter, according to graphical analysis. An important reaction to the additional segmental heating at both the top and bottom plate surfaces is obtained by thermal tracking of different curves under fractional response. The finite speed of wave propagation is demonstrated by the curve phenomenon. Lastly, different structural designs may benefit from the categorization of materials based on Caputo-Fabrizio fractional parameter features that can indicate the impacts of memory allocated on temperature and stress history.

Therefore, the Caputo-Fabrizio fractional parameter response in solid objects could be helpful in the construction of realistic structures or machine designs, as well as in engineering applications based on temperature changes for various parameters and functions.

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