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OPTIMIZING MAINTENANCE STRATEGIES OF COIL SHOP: A DIFFERENTIAL EQUATION APPROACH

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Abstract

This paper presents a control process of reliability models for key subsystems in a coil shop by advanced differential equations approach.

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The goal is to optimize the maintenance scheduling process for the critical subsystems, enabling the system to operate at maximum efficiency. Maintenance strategies significantly influence this outcome, and selecting the right strategy is not trivial. The designed control process model can be applied by administrative setup in manufacturing concerns. The 3-state models here are developed for the dynamic behavior of the system under the impact of preventive maintenance strategies. Both maintenance and repair of the units are perfect. The numerical analysis of the system is also discussed to compare the behavior of the present model and the proposed models. The comparison helps in findings the production-affecting factors and addresses maintenance planning gaps for critical subsystems. This approach aims to optimize the entire manufacturing system, potentially increasing profitability.

1. Introduction

Maintenance strategies are split into two types, corrective (post-failure) and preventive (pre-failure). Corrective actions fix issues after they arise, while preventive measures stop issues before they start. In the face of growing global competition, it is essential for businesses to prioritize performance enhancements. This can be achieved by studying system wear using control process for maintaining equipment consistently, leading to increased productivity and cost savings. Maintenance primarily ensures that equipment remains optimal and addresses underperforming components. By strategically choosing when and what to maintain, focusing on crucial subsystems and sidelining less vital ones, system reliability improves. This optimized reliability directly contributes to higher profits.

Over time, numerous system models have been presented across diverse industries, emphasizing the significance of maintenance in achieving optimal reliability. Gnedenko's studies [8, 9] provided foundational insights by investigating the parameters impacting reliability and availability, tailored to aid production sectors. Meanwhile, Mehta and colleagues [11] applied the supplementary variable technique to analyze the availability of an industrial setup. Aggarwal and Kumar [1] tackled systems focusing on unit replacements. In contrast, Dijkhuizen and Heijden [4] further contributed to this domain with a series of mathematical models, dissecting maintenance intervals and deriving availability using optimization strategies. A groundbreaking perspective was introduced by Todinav [14], proposing a novel methodology for system optimization by reducing the total associated costs. Furthermore, Husaini et al. [6] outlined methods for reliability-centered maintenance (RCM) and risk-based maintenance (RBM), tools vital for contemporary industries. Ebrahimi's work [5] centered on scheduling preventive maintenance to bolster equipment reliability, emphasizing timely interventions for extended operational longevity. Several other researchers, such as Kumar et al. [10], Garg et al. [7], Mohamed et al. [12], Bahl et al. [3], Oskadi [13], and Andalib and Sarker [2] have made notable contributions. They implemented diverse reliability techniques on a range of industrial system models, yielding critical insights beneficial for the broader research community and industries. Their collective efforts spotlight the essence of reliability in system designs, the underlying techniques to achieve it, and the varied applications across industrial landscapes.

By considering all the above, the present paper delves models for the maintenance strategies of most critical subsystem of the unit using differential equations approach by control process for maintenance point of view with various state optimizing the availability under transient state. The system in question is intricate, comprising multiple components. Our thorough examination seeks to enhance its availability. The predictive approach we utilize for understanding the coil production dynamics is grounded in non-Markov modeling techniques. Results of numerical analysis have been showcased in tables. Compared to existing state, the proposed state suggests a more reliable system given the same failure traits.

2. System's Analysis

The working position of coil shop production system is shown in Figure 1.

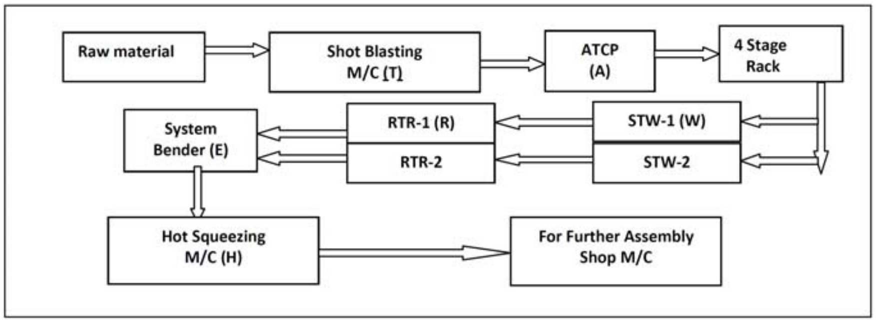


Figure 1. Process flow chart of coil shop.

The raw material, a tube (12 m) is shot blasted for cleaning the outer surface paint and scales in a shot blasting machine (*one unit with no failure*). Shot blasted tubes are fed to automatic tube cutting and preparation m/c (*one unit and fails only by the failure of boiler*) for sizing. Here they are cut according to required size, the ends are prepared (beveled) and cleaned by this machine. These are then put into multistage rack for storage with the help of conveyor system. Straight tube welder machine (*two identical units working in parallel*) is used for the welding of tubes to get desired length. The system can work with one unit in reduced capacity for some time after the failure of this subsystem. The welding is done by TIG (tungsten inert gas) welding process. Real time radiography (*two identical working in parallel and never fails*) is done to check the weld. After getting a length of 60 m bending process is carried out in serpentine shape with the help of system bender (*one unit*), which is a CNC machine. The system can work in reduced state for some time after the failure of this subsystem. The bend is then hot squeezed (*one units with never failure*) to close bending radius, as the system bender does cold bending. The rest of the assembly is done through manual bending. Multi layer bends are manually welded one above the other. The final finishing is done manually.

The following assumptions have been made for the modelling of the system:

- Initially, all the units are in operative state.
- After a unit fails, the repair process immediately starts.

- Resorted unit works as a new one.

- The preventive and corrective maintenance rates of the considered units are considered as arbitrarily distributed. On the other hand, transition rates which transit these units to degraded and failed states are taken as constant and arbitrarily distributed, respectively.

- There is no simultaneous failure amongst subsystems.

- Independent repair facilities are available to handle preventive and corrective maintenance.

Notations	Descriptions
-/m/r	The subsystem (unit) is operative/under maintenance/under repair
$P_i(t)$	State probability when the system is in ith state at time 't' $(i = 0$ represents good state$)$
$F_j(t)$	Probability density of the failed state of the system at time with respect to different models $(j = 1, 2, 3)$
$W_{i,3-i}^{b,c}$	Working status of the subsystem W. The respective ordered pair $\begin{pmatrix} b \\ i \end{pmatrix}$ and $\begin{pmatrix} c \\ 3-i \end{pmatrix}$ represents the functioning of the unit with respect to 'b' and 'c' $(i = 1, 2;\ b = c = -, m, r)$
E^b	Functioning status of the subsystem E with respect to 'b' $(b = -, m, r)$
α_i	Constant transition rates for reduced state of the subsystems E and W $(i = 1, 3)$
$\alpha_j(y)$	Failure rates of any one of the subsystems E and W, with elapsed failure time 'y' $(j = 2, 4)$
$\eta_i(x)$	Preventive maintenance rate of the subsystems E and W, with elapsed repair time 'x' $(i = 1, 3)$
$\eta_j(x)$	Corrective repair rates of any one of the subsystems E and W to return it from failed to normal state with elapsed repair time 'x' $(j = 2, 4)$
$R_1(t)$	Reliability function under stated assumption
$M_2(\tau_1, \tau_3, t)$	Mission reliability function under stated assumption

Mathematical analysis of the system

The transition diagram gives the following differential equations associated with the different states of the system.

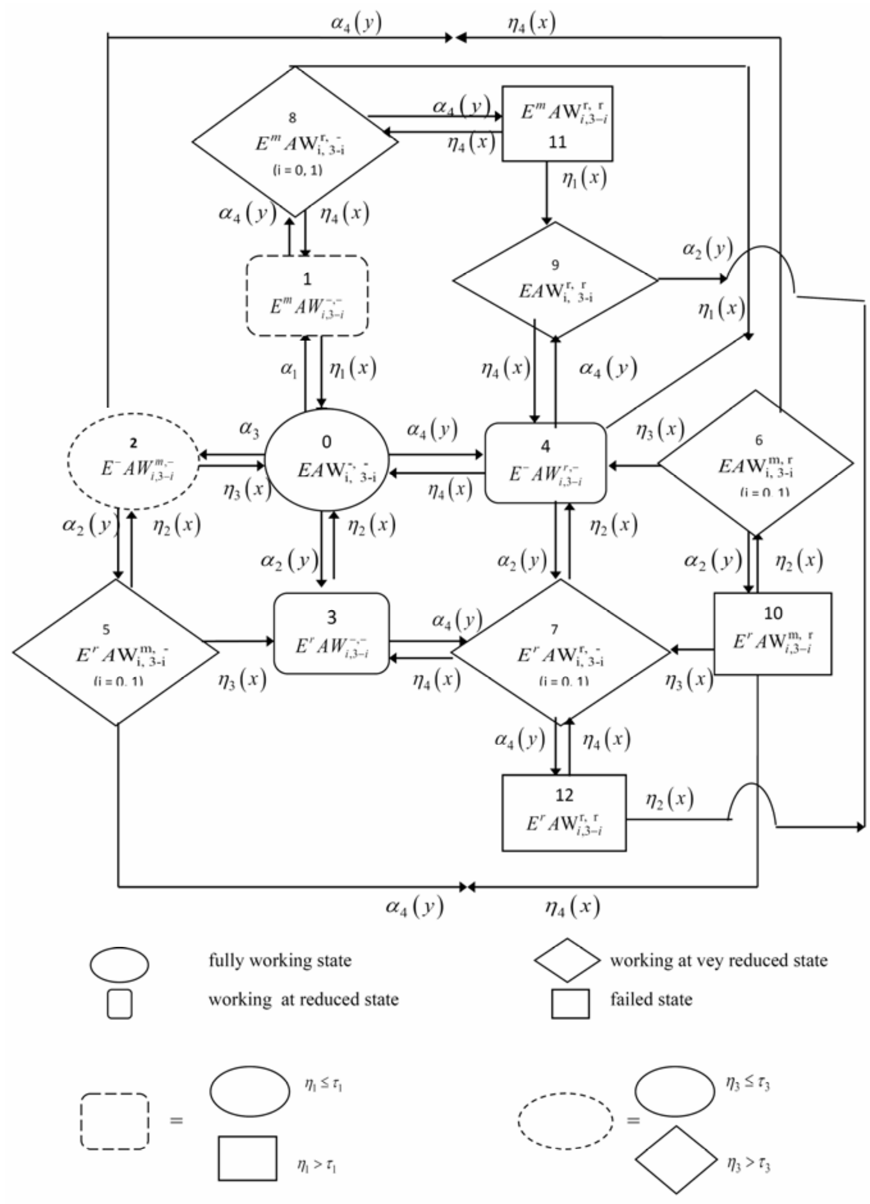


Figure 2. Transition diagram.

$$\left(\frac{d}{dt} + \alpha_1 + \alpha_3 + \alpha_2(y) + \alpha_4(y)\right)P_0(t) = H_0(t), \tag{1}$$

$$(\partial_1 + \eta^l(x) + \alpha^l(y))P_l(x, t) = H_l(x, t),\ l = 1, 2, \tag{2}$$

$$(\partial_2 + \eta^m(x) + \alpha^m(y))P_m(x, y, t) = H_m(x, y, t),\ m = 3, 4, ..., 12, \tag{3}$$

where

$$\partial_1 = \frac{\partial}{\partial x} + \frac{\partial}{\partial t},\ \partial_2 = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial t},$$

$$\eta^1(x) = \eta_1(x), \quad \eta^2(x) = \eta_3(x),$$

$$\eta^3(x) = \eta_2(x),\ \eta^4(x) = \eta^8(x) = \eta^9(x) = \eta_4(x),$$

$$\eta^5(x) = \eta_2(x) + \eta_3(x),\ \eta^6(x) = \eta_3(x) + \eta_4(x),$$

$$\eta^7(x) = \eta^{12}(x) = \eta_2(x) + \eta_4(x),$$

$$\eta^{10}(x) = \eta_2(x) + \eta_3(x) + \eta_4(x),\ \eta^{11}(x) = \eta_1(x) + \eta_4(x),$$

$$\alpha^1(y) = \alpha^3(y) = \alpha^5(y) = \alpha^7(y) = \alpha^8(y) = \alpha_4(y),$$

$$\alpha^2(y) = \alpha^4(y) = \alpha_2(y) + \alpha_4(y),$$

$$\alpha^6(y) = \alpha^9(y) = \alpha_2(y),\ \alpha^{10}(y) = \alpha^{11}(y) = \alpha^{12}(y) = 0$$

with

$$P_i(0) = 1,\ (i = 0),\ \ P_j(x, 0) = 0,\ j = 1, 2,$$

$$P_k(x, y, 0) = 0, ..., (3 \le k \le 12),\ \ P_i(0, t) = a^i P_0(t),\ i = 1, 2,$$

$$P_j(0, y, t) = a^j(y)P_0(t),\ j = 3, 4,$$

$$P_k(0, y, t) = \int a^k(y)P_2(x, t)dx,\ k = 5, 6,$$

$$a^1 = \alpha_1,\ a^2 = \alpha_3,\ a^3(y) = a^5(y) = \alpha_2(y),$$

$$a^4(y) = a^6(y) = \alpha_4(y),$$

$$P_7(0, y, t) = \int (\alpha_2(y) P_4(x, y, t) + \alpha_4(y) P_3(x, y, t)) dx,$$

$$P_8(0, y, t) = \int \alpha_4(y) P_1(x, t) dx,$$

$$P_9(0, y, t) = \int \alpha_4(y) P_4(x, y, t) dx,$$

$$P_{10}(0, y, t) = \int (\alpha_4(y) P_5(x, y, t) + \alpha_2(y) P_6(x, y, t)) dx,$$

$$P_{11}(0, y, t) = \int (\alpha_4(y) P_8(x, y, t)) dx,$$

$$P_{12}(0, y, t) = \int (\alpha_4(y) P_7(x, y, t) + \alpha_2(y) P_9(x, y, t)) dx,$$

$$H_0(t) = \int \sum_{i=1}^{2} \eta^i(x) P_i(x, t) dx + \int \sum_{j=3}^{4} \eta^j(x) P_j(x, y, t) dx dy,$$

$$H_1(x, t) = \alpha_1 P_0(t) + \int (\eta_4(x) P_8(x, y, t)) dy,$$

$$H_2(x, t) = \alpha_3 P_0(t) + \int (\eta_2(x) P_5(x, y, t) + \eta_4(x) P_6(x, y, t)) dy,$$

$$H_3(x, y, t) = \alpha_2(y) P_0(t) + \eta_4(x) P_7(x, y, t) + \eta_3(x) P_5(x, y, t),$$

$$H_4(x, y, t) = \alpha_4(y) P_0(t) + \eta_3(x) P_6(x, y, t) + \eta_2(x) P_7(x, y, t) + \eta_4(x) P_9(x, y, t) + \eta_1(x) P_8(x, y, t),$$

$$H_5(x, y, t) = \alpha_2(y) P_2(x, t) + \eta_4(x) P_{10}(x, y, t),$$

$$H_6(x, y, t) = \alpha_4(y) P_2(x, t) + \eta_2(x) P_{10}(x, y, t),$$

$$H_7(x, y, t) = \alpha_2(y) P_4(x, y, t) + \alpha_4(y) P_3(x, y, t) + \eta_3(x) P_{10}(x, y, t) + \eta_4(x) P_{12}(x, y, t),$$

$$H_8(x, y, t) = \alpha_4(y)P_1(x, t) + \eta_4(x)P_{11}(x, y, t),$$

$$H_9(x, y, t) = \alpha_4(y)P_4(x, y, t) + \eta_2(x)P_{12}(x, y, t) + \eta_1(x)P_{11}(x, y, t),$$

$$H_{10}(x, y, t) = \alpha_4(y)P_5(x, y, t) + \alpha_2(y)P_6(x, y, t),$$

$$H_{11}(x, y, t) = \alpha_4(y)P_8(x, y, t),$$

$$H_{12}(x, y, t) = \alpha_4(y)P_7(x, y, t) + \alpha_2(y)P_9(x, y, t).$$

Solution of equations

$$P_0(t) = e^{-K_0 t}\left[1 + \int H_0(t)e^{K_0 t}dt\right],$$

$$P_m(x, t) = e^{-\int K_m(x,y)dx}\left[c^m P_0(t-x) + \int H_m(x, t)e^{\int K_m(x,y)dx}dx\right],$$

$$m = 1, 2,$$

$$P_n(x, y, t) = e^{-\int K_n(x,y)dx}\left[c^n P_0(t-x) + \int H_n(x, y, t)e^{\int K_n(x,y)dx}dx\right],$$

$$n = 3, 4,$$

$$P_s(x, y, t) = e^{-\int K_s(x,y)dx}\left[\int\left\{c^s P_3(0, t-x) + H_s(x, y, t)e^{\int K_s(x,y)dx}\right\}dx\right],$$

$$s = 5, 6,$$

$$P_7(x, y, t) = e^{-\int K_7(x,y)dx}\Big[\int[\alpha_4(y-x)P_3(0, y-x, t-x)$$

$$+\alpha_3(y-x)P_4(0, y-x, t-x)]dx + \int H_7(x, y, t)e^{\int K_7(x,y)dx}dx\Big],$$

$$P_8(x, y, t) = e^{-\int K_8(x,y)dx}\Big[\int \alpha_4(y-x)P_1(0, t-x)dx$$

$$+\int H_8(x, y, t)e^{\int K_8(x,y)dx}dx\Big],$$

$$P_9(x,\ y,\ t) = e^{-\int K_9(x,\,y)\,dx}\left[\int \alpha_4(y-x)P_4(0,\ y-x,\ t-x)dx\right.$$

$$\left. + \int H_9(x,\ y,\ t)e^{\int K_9(x,\,y)\,dx}dx\right],$$

$$P_{10}(x,\ y,\ t) = e^{-\int K_{10}(x)\,dx}\left[\int [\alpha_4(y-x)P_9(0,\ y-x,\ t-x) + \alpha_2(y-x)\right.$$

$$\left. \times P_6(0,\ y-x,\ t-x)]dx + \int H_{10}(x,\ y,\ t)e^{\int K_{10}(x)\,dx}dx\right],$$

$$P_v(x, y, t) = e^{-\int K_v(x)\,dx}\left[\int\left\{H_v(x, y, t)e^{\int K_v(x)\,dx} + H_v(0, y-x, t-x)\right\}dx\right],$$

$$v = 11,\ 12,$$

$$c^1 = \alpha_3,\ c^2 = \alpha_1,\ c^3 = c^6 = \alpha_2(y-x),\ c^4 = c^5 = \alpha_4(y-x),$$

where

$K_0 = \alpha_1 + \alpha_3 + \alpha_2(y) + \alpha_4(y),\qquad K_1(x,\ y) = \eta_1(x) + \alpha_4(y),$

$K_2(x,\ y) = \eta_3(x) + \alpha_2(y) + \alpha_4(y),\qquad K_3(x,\ y) = \eta_2(x) + \alpha_4(y),$

$K_4(x,\ y) = \eta_4(x) + \alpha_4(y) + \alpha_2(y),\qquad K_5(x,\ y) = \eta_3(x) + \eta_2(x) + \alpha_4(y),$

$K_6(x,\ y) = \eta_3(x) + \eta_4(x) + \alpha_2(y),\qquad K_7(x,\ y) = \eta_4(x) + \alpha_4(y) + \eta_2(x),$

$K_8(x,\ y) = \eta_1(x) + \eta_4(x) + \alpha_4(y),\qquad K_9(x,\ y) = \eta_4(x) + \alpha_2(y),$

$K_{10}(x) = \eta_3(x) + \eta_4(x) + \eta_2(x),\qquad K_{11}(x) = \eta_1(x) + \eta_4(x),$

$K_{12}(x) = \eta_4(x) + \eta_2(x).$

Transient behavior of the system

For the transient behavior with all the constant transition rates, we get the following equations:

$$\left(\frac{d}{dt} + C_i\right)P_i(t) = Z_i,\ \ 0 \le i \le 12$$

where

$C_0 = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4,\ Z_0 = \eta_1 P_1(t) + \eta_4 P_4(t) + \eta_3 P_2(t) + \eta_4 P_3(t),$

$C_1 = \alpha_4 + \eta_1,\ Z_1 = \eta_4 P_8(t) + \alpha_1 P_0(t),$

$C_2 = \eta_3 + \alpha_2 + \alpha_4,\ Z_2 = \alpha_3 P_0(t) + \eta_2 P_5(t) + \eta_4 P_6(t),$

$C_3 = \eta_2 + \alpha_4,\ Z_3 = \alpha_2 P_0(t) + \eta_4 P_7(t) + \eta_3 P_5(t),$

$C_4 = \eta_4 + \alpha_2 + \alpha_4,\ Z_4 = \alpha_4 P_0(t) + \eta_3 P_6(t) + \eta_2 P_7(t) + \eta_4 P_9(t) + \eta_1 P_8(t),$

$C_5 = \alpha_4 + \eta_3 + \eta_2,\ Z_5 = \alpha_2(y) P_2(x, t) + \eta_4(x) P_{10}(x, y, t),$

$C_6 = \alpha_2 + \eta_3 + \eta_4,\ Z_6 = \alpha_4 P_2(t) + \eta_2 P_{10}(t),$

$C_7 = \eta_4 + \eta_2 + \alpha_4,\ Z_7 = \alpha_4 P_3(t) + \alpha_2 P_4(t) + \eta_3 P_{10}(t) + \eta_4 P_{12}(t),$

$C_8 = \eta_1 + \eta_4 + \alpha_4,\ Z_8 = \alpha_4 P_1(t) + \eta_4 P_{11}(t),$

$C_9 = \eta_4 + \alpha_2,\ Z_9 = \alpha_4 P_4(t) + \eta_2 P_{12}(t) + \eta_1 P_{11}(t),$

$C_{10} = \eta_3 + \eta_4 + \eta_2,\ Z_{10} = \alpha_4 P_5(t) + \alpha_2 P_6(t),$

$C_{11} = \eta_1 + \eta_4,\ Z_{11} = \alpha_4 P_8(t),$

$C_{12} = \eta_2 + \eta_4,\ Z_{12} = \alpha_4 P_7(t) + \alpha_2 P_9(t).$

Reliability function $R_1(t)$ for the system is given by

$$R_1(t) = 1 - F_1(t), \text{ where } F_1(t) = \sum_{i=1}^{12} P_i(t).$$

Availability function of presented model

The availability function $Av_1(t)$ of the system with running at full capacity is

$$Av_1(t) = P_0(t).$$

Renewal frequency function of the system $N_1(t)$ is given by

$$N_1(t) = \int \sum_{i=1}^{2} \eta^i(x) P_i(x, t) dx + \int \sum_{j=3}^{4} \eta^j(x) P_j(x, y, t) dx dy.$$

Availability function for the proposed model

The two subsystems E and W are very critical machines in the series. We can reschedule the said systems only after achieving a particular buffer stock limit for running of the complete system without interruption.

- The buffer stock limit of system E is for 16 hours.
- The buffer stock limit of system W is for 8 hours.

The PM is time bound and must be completed in the specified time $\tau_1 = 16\,\text{hrs}$ with $\tau_3 = 8\,\text{hrs}$ failing, the systems will transit to failed state and reduced state, respectively.

Taking into account the above noted limits, additional assumptions and notations, the reliability function and availability function have been derived for the new proposed model which has been incorporated in Figure 2.

Reliability function

$M_2(\tau_1, \tau_3, t) = 1 - F_2(t)$, where

$$F_2(t) = \begin{cases} 0 & \text{for } t \le \tau_1 \text{ and } t \le \tau_3 \\ \sum_{j=2}^{12} P_j(t) & \text{for } t \le \tau_1 \text{ and } t > \tau_3 \\ P_1(t) + \sum_{j=3}^{12} P_j(t) & \text{for } t > \tau_1 \text{ and } t \le \tau_3 \\ \sum_{j=3}^{12} P_j(t) & \text{for } t > \tau_1 \text{ and } t > \tau_3 \end{cases}.$$

Availability function

$$\left.\begin{cases} Av_{21}(t) = P_0(t) + P_1(t) \\ \qquad \text{if } (\eta_1)^{-1} \le \tau_1 \text{ when PM of } E \text{ is carried within target time} \\ Av_{21}(t) = P_0(t) \\ \qquad \text{if } (\eta_1)^{-1} > \tau_1 \text{ when PM of } E \text{ is not carried within target time} \end{cases}\right\},$$

$$\left.\begin{cases} Av_{22}(t) = P_0(t) + P_2(t) \\ \qquad \text{if } (\eta_3)^{-1} \le \tau_3 \text{ when PM of } W \text{ is carried within target time} \\ Av_{22}(t) = P_0(t) \\ \qquad \text{if } (\eta_3)^{-1} > \tau_3 \text{ when PM of } W \text{ is not carried within target time} \end{cases}\right\},$$

$$\left.\begin{cases} Av_3(t) = P_0(t) + P_1(t) + P_2(t) \\ \qquad \text{if } (\eta_1)^{-1} \le \tau_1 \text{ and } (\eta_3)^{-1} \le \tau_3 \\ \qquad \text{when PM of } E \text{ and } W \text{ is carried within target time} \\ Av_3(t) = P_0(t) \\ \qquad \text{if } (\eta_1)^{-1} > \tau_1 \text{ and } (\eta_3)^{-1} > \tau_3 \\ \qquad \text{when PM of } E \text{ and } W \text{ is not carried within target time} \end{cases}\right\}.$$

3. Numerical Results

With respect to the presented and proposed models, the numerical analysis of transient state availability for the system of differential equations w.r.t. possible combinations of transition rates of the subsystems can be predicted from Tables 3.1 to 3.8.

Table 3.1. Availability/failure rates (α_2) corresponding to 3 state model

	$Av_1(t)$ $\eta_1 = .03$ $\eta_3 = .04$	$Av_{21}(t)$ $\eta_1 = .06$ $\eta_3 = .04$	$Av_3(t)$ $\eta_1 = .06$ $\eta_3 = .125$	$\alpha_1 = .003$ $\eta_2 = .05$	$\alpha_3 = .004$ $\eta_4 = .04$	$\alpha_4 = .0025$
$\alpha_2 \rightarrow$	**.0012**			**.0011**		
↓Days	$Av_1(t)$	$Av_{21}(t)$	$Av_3(t)$	$Av_1(t)$	$Av_{21}(t)$	$Av_3(t)$
25	.8438425732	.889623067	.9449138922	.845004179	.8908350052	.9462188986
50	.7937145047	.8552776487	.9257504737	.7950475286	.8567222836	.9273626928
75	.7761843423	.8436561954	.9183496082	.7775226974	.8451428913	.9200342878
100	.769557362	.8395068997	.9153090898	.7708823401	.8409987768	.9170095813
125	.766872113	.8379672507	.9140154097	.7681874709	.8394584013	.9157185661
150	.7657198061	.8373782361	.9134543899	.7670299671	.838868413	.9151575493
175	.7652032052	.8371470783	.9132087031	.7665107915	.8386366993	.9149115421
200	.7649642477	.837054379	.9131006241	.7662705966	.8385437374	.9148032335
225	.7648513303	.8370165239	.9130530075	.7661570898	.8385057674	.9147554917
250	.7647972086	.8370008323	.9130320323	.7661026876	.8384900273	.9147344549
275	.7647710247	.8369942489	.9130228025	.76607637	.8384834238	.9147251963
300	.7647582793	.8369914604	.9130187472	.7660635605	.8384806271	.914721128
325	.7647520501	.8369902705	.9130169686	.7660573007	.8384794337	.9147193435
350	.7647489975	.8369897598	.9130161898	,7660542334	.8384789217	.9147185622
375	.7647474988	.8369895398	.9130158495	.7660527277	.8384787011	.9147182207
400	.7647467621	.8369894447	.913015701	.7660519877	.8384786057	.9147180717

Table 3.2. Availability/failure rates (α_2) corresponding to 3 state model

	$Av_1(t)$ $\eta_1 = .03$ $\eta_3 = .04$	$Av_{21}(t)$ $\eta_1 = .06$ $\eta_3 = .04$	$Av_3(t)$ $\eta_1 = .06$ $\eta_3 = .125$	$\alpha_1 = .003$ $\eta_2 = .05$	$\alpha_3 = .004$ $\eta_4 = .04$	$\alpha_4 = .0025$
$\alpha_2 \rightarrow$	**.0009**			**.0008**		
↓Days	$Av_1(t)$	$Av_{21}(t)$	$Av_3(t)$	$Av_1(t)$	$Av_{21}(t)$	$Av_3(t)$
25	.8473342814	.8932660076	.9488365986	.8485027882	.8944850823	.9501493036
50	.7977255882	.8596244974	.930601682	.7990706545	.8610821088	.9322284887
75	.7802130859	.8481315744	.9234212103	.7815651626	.8496336097	.9251235085
100	.7735462972	.843998603	.9204292184	.7748853245	.8455066081	.9221484291
125	.7708321566	.8424570033	.9191438873	.7721615344	.833964514	.9208661214
150	.7696641833	.8418651263	.918582981	.7709882884	.8433717231	.9203053246
175	.7691398037	.8416323129	.9183363608	.7704612794	.8431383662	.9200584121
200	.7688971029	.8415388267	.9182275989	.7702173099	.8430446186	.9199494268
225	.7687824013	.8415006262	.9181796071	.7701020028	.8430063026	.9199013103
250	.7687274303	.8414847886	.9181584466	.7700467432	.8429904159	.9198800877
275	.7687008412	.8414781446	.91814913	.7700200166	.8429837514	.9198707419
300	.7686879021	.8414753311	.9181450353	.7700070117	.8429809294	.9198666339
325	.7686815802	.8414741309	.918143239	.7700006583	.8429797257	.9198648317
350	.7686784831	.841473616	.9181424525	.7699975462	.8429792094	.9198640425
375	.7686769631	.8414733942	.9181421088	.769996019	.842978987	.9198636977
400	.7686762163	.8414732983	.9181419588	.7699952687	.8429788908	.9198635472

Table 3.3. Availability/failure rates (α_4) corresponding to 3 state model

	$Av_1(t)$ $\eta_1 = .03$ $\eta_3 = .04$	$Av_{22}(t)$ $\eta_1 = .06$ $\eta_3 = .04$	$Av_3(t)$ $\eta_1 = .06$ $\eta_3 = .125$		$\alpha_1 = .003$ $\eta_2 = .05$	$\alpha_3 = .004$ $\eta_4 = .04$	$\alpha_2 = .001$
$\alpha_4 \rightarrow$	**.01**				**.007**		
↓Days	$Av_1(t)$	$Av_{22}(t)$	$Av_3(t)$		$Av_1(t)$	$Av_{22}(t)$	$Av_3(t)$
25	.7517652391	7999142844	.839771901		.7882809983	.838332608	8814046585
50	.6755364846	.7299877509	.782477713		.7219036167	.7812079646	.8384977206
75	.6469981805	.701019893	.758167562		.6974867339	.757420524	.8205277166
100	.6349327313	.687625005	.747126325		.6874216094	.7465266516	.8124810262
125	.6294591119	.681155276	.742003024		.6829435931	.7412870423	.8087816792
150	.6268720021	.6779722745	.739617605		.6808512893	.7387064772	.8070670248
175	.6256187248	.676393894	.738510514		.6798430087	.7374210597	.806271968
200	.6250021771	.6756083167	.737999581		.6793475171	.7367772449	.805904363
225	.6246957993	.6752167157	.737765301		.679100922	.7364538813	.8057350991
250	.6245424987	.675021316	.737658598		.6789771565	.7362912193	.8056575191
275	.6244654089	.6749237621	.737610333		.6789146728	.7362093188	.8056221276
300	.6244264943	.6748750389	.737588657		.6788829918	.7361680551	.8056060569
325	.6244067889	.6748506962	.737578980		.6788668753	.7361472547	.8055987924
350	.6243967834	.6748385307	.737574697		.6788586548	.7361367648	.8055955231
375	.6243916904	.674832449	.739572815		.678854452	.7361314723	.8055940581
400	.6243890918	.6748294077	.737571994		.678852299	.7361288011	80559340441

Table 3.4. Availability/failure rates (α_4) corresponding to 3 state model

	$Av_1(t)$ $\eta_1 = .03$ $\eta_3 = .04$	$Av_{22}(t)$ $\eta_1 = .06$ $\eta_3 = .04$	$Av_3(t)$ $\eta_1 = .06$ $\eta_3 = .125$		$\alpha_1 = .003$ $\eta_2 = .05$	$\alpha_3 = .004$ $\eta_4 = .04$	$\alpha_2 = .001$
$\alpha_4 \rightarrow$	**.004**				**.001**		
↓Days	$Av_1(t)$	$Av_{22}(t)$	$Av_3(t)$		$Av_1(t)$	$Av_{22}(t)$	$Av_3(t)$
25	.826448037	.8793240588	.9249841837		.866323636	.9221826703	9705842595
50	.7709029533	.835452113	.8979533856		.822515066	.8927226973	.9608740721
75	.7710231195	.8174098942	.8870044302		.8073718312	.886763639	.9573813846
100	.7432089603	.8093592521	.8822867205		.8018562991	.875672397	.9560490801
125	.7398976894	.8055827593	.8801914627		.7997408851	.8734296962	.9555209827
150	.7384136526	.8037603396	.8792481308		.7988894316	.8724184363	.9553065534
175	.7377219765	.8028674019	.878821232		.7985320694	.8719554645	.9552182039
200	.7373910424	.8024264124	.8786278736		.7983769175	.8717414935	.9551814885
225	.7372299664	.8022077545	.8785404028		.7983077959	.8716420436	.9551661575
250	.7371506842	.8020991284	.8785009217		.7982764166	.8715956739	.9551597402
275	.7371113733	.8020451191	.8784831487		.7982619808	.8715740178	.9551570514
300	.7370917854	.802018258	.8784751701		.7982552785	.8715638967	.9551559246
325	.737081992	.802004899	.8784715984		.7982521471	.8715591659	.9551554526
350	.7370770838	.8019982561	.8784700036		.798250678	.8715569551	.955155255
375	.7370746197	.8019949537	.8784692933		.7982499867	.8715559223	.9551551724
400	.7370733808	.8019933123	.8784689778		.7982496608	.8715554401	.9551551378

Table 3.5. Availability/repair rates (η_2) corresponding to 3 state models

	$Av_1(t)$ $\eta_1 = .03$ $\eta_3 = .04$	$Av_{21}(t)$ $\eta_1 = .06$ $\eta_3 = .04$	$Av_3(t)$ $\eta_1 = .06$ $\eta_3 = .125$		$\alpha_1 = .003$ $\alpha_4 = .0025$	$\alpha_2 = .001$ $\eta_4 = .04$	$\alpha_3 = .004$
$\eta_2 \rightarrow$	**.045**				**.055**		
↓Days	$Av_1(t)$	$Av_{21}(t)$	$Av_3(t)$		$Av_1(t)$	$Av_{21}(t)$	$Av_3(t)$
25	.8455354804	.8913982099	.9468419893		.8467545928	.8926533225	.9481620317
50	.7952006397	.8569146647	.9276214681		.797421911	.8592742113	.930174906
75	.7774541457	.8450967223	.9200358277		.78005888822	.8479383207	.9231610881
100	.7707293296	.8408493682	.9168922731		.7734418763	.8438655072	.9202383475
125	.768009001	.8392703821	.9155538352		.7707400283	.84234345	.9189775933
150	.7668459577	.8386671447	.9149753399		.7695736413	.8417572684	.9184246902
175	.7663266545	.8384311783	.9147232338		.7690486305	.8415259467	.9181805155
200	.7660872961	.8383369379	.9146128926		.7688052127	.841432795	.9180724556
225	.7659744922	.8382986137	.9145645043		.7686900506	.8413946462	.9180246494
250	.7659205262	.8382827876	.9145432735		.7686348271	.8413788046	.9180035313
275	.76589445	.8382761688	.9145339616		.7686081086	.8413721518	.917994221
300	.7658817669	.8382733723	.9145298809		.768595105	.8413693327	.9179901254
325	.7658755712	.8382721812	.9145280947		.7685887516	.8413681297	.9179883276
350	.7658725359	.8382716708	.914527314		.7685856392	.8413676136	.91798754
375	.7658710459	.838271451	.9145269732		.7685841118	.8413673912	.9179871958
400	.7658703136	.8382713561	.9145268246		.7685833613	.8413672951	.9179870455

Table 3.6. Availability/repair rates (η_2) corresponding to 3 state models

	$Av_1(t)$ $\eta_1 = .03$ $\eta_3 = .04$	$Av_{21}(t)$ $\eta_1 = .06$ $\eta_3 = .04$	$Av_3(t)$ $\eta_1 = .06$ $\eta_3 = .125$		$\alpha_1 = .003$ $\alpha_4 = .0025$	$\alpha_2 = .001$ $\eta_4 = .04$	$\alpha_3 = .004$
$\eta_2 \rightarrow$	**.065**				**.075**		
↓Days	$Av_1(t)$	$Av_{21}(t)$	$Av_3(t)$		$Av_1(t)$	$Av_{21}(t)$	$Av_3(t)$
25	.8478045528	.8937354921	.9493023715		.8487124023	.8946722463	.9502913999
50	.7991407276	.8611064717	.9321672336		.8004922901	.862552138	.9337465352
75	.7819508992	.8500126653	.9254566899		.7833698056	.8515750069	.9271954514
100	.7753532023	.8460006502	.9226218258		.7767615456	.8475794491	.9243936745
125	.7726414975	.8444903941	.9213832399		.7740359524	.8460684513	.923159937
150	.7714655382	.8439052752	.9208349978		.7728520102	.8454815226	.9226115665
175	.7709348636	.8436734026	.9205914125		.772317438	.8452486524	.9223674242
200	.770688528	.8435797858	.9204832091		.7720692822	.8451545955	.9222588674
225	.7705719388	.8435413925	.9204352315		.7719518478	.8451160225	.922210711
250	.7705160303	.8435254397	.9204140095		.7718955442	.8450999983	.9221894055
275	.7704889846	.8435187394	.9204046462		.7718683121	.8450932697	.9221800045
300	.7704758248	.8435159006	.9204005254		.7718550637	.8450904197	.922175865
325	.7704693965	.8435146895	.9203987161		.7718485931	.8450892041	.9221740503
350	.7704662482	.8435141701	.9203979234		.7718454245	.8450886828	.9221732544
375	.7704647036	.8435139463	.9203975769		.7718438702	.8450884583	.9221729065
400	.7704639448	.8435138497	.9203974256		.7718431067	.8450883613	.9221727546

Table 3.7. Availability/repair rates (η_4) corresponding to 3 state models

	$Av_1(t)$ $\eta_1 = .03$ $\eta_3 = .04$	$Av_{22}(t)$ $\eta_1 = .06$ $\eta_3 = .04$	$Av_3(t)$ $\eta_1 = .06$ $\eta_3 = .125$		$\alpha_1 = .003$ $\alpha_4 = .0025$	$\alpha_2 = .001$ $\eta_2 = .05$	$\alpha_3 = .004$
$\eta_4 \rightarrow$		**.05**				**.1**	
↓Days	$Av_1(t)$	$Av_{22}(t)$	$Av_3(t)$		$Av_1(t)$	$Av_{22}(t)$	$Av_3(t)$
25	.8494995754	.9040169322	.9512042992		.8605730152	.9157171176	.9635381637
50	.8031154169	.8709749689	.9368903838		.8205674414	.8900361019	.9577671607
75	.7875504549	.8581992535	.932277508		.8071588668	.8799909275	.9565287065
100	.7819413397	.8528706155	.9307131609		.8023492326	.8757422634	.9562259207
125	.779789299	.8505376304	.9301686996		.8005420764	.8738887076	.9561477717
150	.7789175774	.8494850045	.9299768277		.7998355827	.8730672341	.9561270684
175	.7785481517	.8490011936	.9299088426		.7995495044	.87269985	.9561215009
200	.778385864	.8487762458	.9298847111		.799430105	.8725346428	.9561199888
225	.7783125889	.8486708859	.9298761472		.7993790115	.8724601009	.9561195752
250	.7782788279	.8486213003	.9298731117		.7993567101	.8724263976	.9561194614
275	.7782630448	.8485978883	.9298720379		.7993468269	.8724111394	.95611943
300	.7782555901	.8485868096	.9298716588		.7993423968	.8724042263	.9561194212
325	.7782520439	.8485815589	.9298715253		.7993403944	.8724010926	.9561194188
350	.7782503485	.8485790675	.9298714784		.7993394838	.8723996717	.9561194181
375	.7782495353	.8485778844	.929871462		.7993390679	.8723990273	.9561194179
400	7782491443	.8485773223	.9298714562		.7993388774	.8723987351	.9561194179

Table 3.8. Availability/repair rates (η_4) corresponding to 3 state models

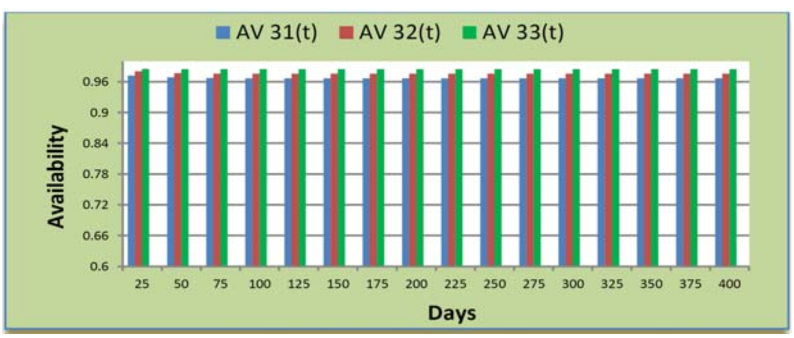
	$Av_1(t)$ $\eta_1 = .03$ $\eta_3 = .04$	$Av_{22}(t)$ $\eta_1 = .06$ $\eta_3 = .04$	$Av_3(t)$ $\eta_1 = .06$ $\eta_3 = .125$		$\alpha_1 = .003$ $\alpha_4 = .0025$	$\alpha_2 = .001$ $\eta_2 = .05$	$\alpha_3 = .004$
$\eta_4 \rightarrow$		**.125**				**.15**	
↓Days	$Av_1(t)$	$Av_{22}(t)$	$Av_3(t)$		$Av_1(t)$	$Av_{22}(t)$	$Av_3(t)$
25	.8638981879	.9192578427	.9672899093		.8663650781	.9218973737	.9700952308
50	.824568359	.8944805131	.9626688106		.8272578849	.8974927213	.9659992554
75	.8111922745	.8845557512	.9616186127		.8138566132	.887592472	.9650038665
100	.8063890703	.8803434652	.9613472563		.8090527453	.8833950972	.9647394952
125	.8045942923	.8785135076	.9612746689		.807265632	.8815785592	.9646679697
150	.803898465	.877707695	.9612549819		.8065764759	.8807820695	.9646484448
175	.8036193596	.87734983	.9612495984		.8063016319	.8804298739	.9646430801
200	.8035040025	.8771900432	.9612481173		.8061886997	.8802732951	9646415985
225	.8034551117	.8771184566	.961247708		.8061411121	.88020344454	.9646411876
250	8034339685	.877086317	.9612475944		.8061206466	.8801722185	.9646410733
275	.8034246807	.8770718685	.9612475628		.8061117039	.8801582396	.9646410414
300	8034205521	.8770653679	.961247554		.8061077487	.8801519765	.9646410325
325	8034187007	.8770624416	.9612475515		.8061659835	.8801491691	.96464103
350	8034178651	.8770611239	.9612475508		.8061051904	.8801479101	.9646410293
375	8034174862	.8770605305	.9612475506		.8061048324	.8801473455	.9646410291
400	8034173138	.8770602632	.9612475506		.8061046702	.8801470923	.9646410291

Availability/different combination of transition rates w.r.t. Figure 2

The numerical values of the availability further calculated by varying the repair rates of the important subsystems *E* and *W*. So, corresponding to different combinations of failure and repair rates of the subsystems *E* and *W*, the optimum availability can be seen in tabular and graphical way in Table 3.9 and Graph 3.1.

Table 3.9. Optimum value of availability corresponding to proposed model

$Av_{31}(t)$ $\alpha_4 = .0025$ $\eta_2 = .05$	$Av_{32}(t)$ $\alpha_4 = .001$ $\eta_2 = .05$	$Av_{33}(t)$ $\alpha_4 = .001$ $\eta_2 = .1$	$\alpha_1 = .003$ $\alpha_3 = .001$ $\alpha_2 = .001$ $\eta_1 = .06$ $\eta_3 = .125$ $\eta_4 = .175$
↓Days	$Av_{31}(t)$	$Av_{32}(t)$	$Av_{33}(t)$
25	.9722407977	.9806363232	.9855215649
50	.9683905435	.9769227584	.9848314706
75	.9674126889	.9759488387	.9848008602
100	.9671503132	.9756859371	.9848026757
125	.9670796592	.9756142686	.9848037067
150	.9670595584	.9755945889	.9848039761
175	.9670541889	.9755891544	.9848040371
200	.9670527034	.9755876471	.984804052
225	.9670522908	.9755872276	.9848040503
250	.9670521759	.9755871106	.9848090531
275	.9670521439	.9755870779	.9848040537
300	.9670521349	.9755870687	.9848040539
325	.9670521324	.9755870662	.9848040539
350	.9670521317	.9755870655	.9848040539
375	.9670521315	.9755870653	.9848040539
400	.9670521314	.9755870652	.9848040539



Graph 3.1. Availability/Table 3.9.

4. Discussion

Mathematical analysis/failure rate

The numerical results pertaining to Figure 2 have been enumerated in Tables 3.1 to 3.4 w.r.t. failure rates, an interesting observation has been made regarding the availability of the system over a year. Specifically, as the failure rate of the subsystem *E* increases from .0009 to 0012, there is a nominal decline in system availability by 76.9% to 76.5%. In the case of system *W*, when failure rate increases from 0.007 to 0.01, there is a significant decline in system availability settling at 62.5%, from 67.8%. However, this availability can be substantially improved, by as much as 77% and 79.8%, if the failure rate of subsystems *E* and *W* is successfully reduced and maintained below .0008 and 0.001, respectively under presented model.

Post rescheduling of the preventive maintenance for any one of the subsystems *E* or *W*; the data in Tables 3.1 to 3.4 present some significant findings. The availability of subsystem *E* experiences a nominal dip from 83.8% to 83.7%, and a dip from 73% to 67% for subsystem *W*. Yet, this can be reversed with an impressive increase of 84.3%/87% if efforts are made to regulate the failure rate to a mere .0008/0.001 for the subsystem *E*/*W*, respectively under proposed model 1.

After the subsequent rescheduling of the preventive maintenance for both subsystems *E* and *W*, this data accentuates that a year's availability can be boosted to a striking 91.5%, from 91.38%, if the failure rate of subsystem *E* is prudently scaled down from 0.0012 to 0.0011. Likewise, availability can be hiked up to 80.6%, from 73.8%, if the failure rate of *W* is prudently scaled down from 0.01 to 0.004. Further controlling the failure rate to 0.0008/.01 for the subsystem *E*/*W*, there is a possibility of making better the availability to 92% and 96%, respectively under proposed model 2.

Mathematical analysis/repair rate

Tables 3.5 to 3.8 bring into focus the impact of the repair rate on system availability for the proposed model. It has been found that by augmenting the repair rate of subsystem *E* from 0.045 to 0.065, the system availability for presented model witnesses an uplift, going from 76.6% to 77% and if the repair rate of subsystem *W* considered from 0.05 to 0.125, the system availability witnesses uplift, going from 78% to 80%. But beyond this point, any further augmentation in the repair rate yields only a marginal enhancement in the system's availability.

Tables 3.5 to 3.8 showcase the influence of the repair rate on system availability for proposed model 1. There is an increment from 83.8% to 84.4%, when the repair rate is corrected from 0.045 to 0.065 in case of subsystem *E*. Likewise, we get an increment in availability, from 85% to 87%, when the repair rate is rectified from 0.05 to 0.125 in case of subsystem *W*. Interestingly, maintaining the repair rate at 0.075 for the subsystem *E* and repair rate 0.15 for the subsystem *W* can further push this availability by another 84.5% or 88%, respectively.

Also, Tables 3.5 to 3.8 provide insights into the significance of varying the repair rate for the 2nd proposed model. By tweaking the repair rate of *E* from 0.045 to 0.065, the system's availability can be increased from 91.5% to 92%. And by tweaking the repair rate of *W* from 0.05 to 0.125, the system's availability can be increased from 92.9% to 96.1%. Any subsequent increase in the repair rate to 0.075 for the subsystem *E* and 0.15 for the subsystem *W* contributes only minimally to the system's availability.

Lastly, Table 3.9 which compares the failure and repair rates for the subsystems *E* and *W*, reveal that the system's availability can reach an impressive ceiling of 98.5%. This is achievable by enhancing (η_2) the repair rate to 0.1, equivalent to 10 hours, for the subsystem *E*. This tabular data provides a quantitative perspective on how each maintenance policy impacts the overall availability. For a more visual interpretation and to better discern the patterns and trends, one can refer to the graphical depictions provided in Graph 3.1.

5. Conclusion

Further, the detailed analysis has provided some insightful conclusions based on the effect of various rates on system availability.

Influence of failure rate:	**Influence of repair rate:**
The data from Table 3.1 reveals that, under the influence of the failure rate for the subsystem *E*, there is potential to achieve a system availability of up to 0.8785, up from an initial 0.9147. Furthermore, delving into Table 3.2, we discern that there is room for improvement in system availability, with the potential to reach as high as 0.9199. Again from Table 3.3 reveals that, under the influence of the failure rate for the subsystem *W*, there is potential to achieve a system availability of up to 0.8056, up from an initial 0.6244. Furthermore, delving into Table 3.4, we discern that there is room for improvement in system availability, with the potential to reach as high as 0.9552.	Again from Table 3.5 reveals that, under the influence of the repair rate for the subsystem *E*, there is potential to achieve a system availability of up to 0.9179, up from .7659. Furthermore, delving into Table 3.6, we discern that there is room for improvement in system availability, with the potential to reach as high as 0.9221. Turning our attention to the repair rate's impact on the subsystem *W*, Table 3.7 suggests that system availability can witness an uptick, potentially reaching up to 0.9562. Expanding on this, Table 3.8 offers a more refined view, highlighting the possibility of nudging the system availability even higher, up to 0.9646.

Table 3.9 illustrates how manipulating the repair rate can have a positive effect on system availability. Specifically, over the span of a year, there is a potential to elevate system availability up to 0.9848. By examining these trends, it becomes evident that both the failure and repair rates play pivotal roles in determining system availability. This knowledge can serve as a foundation for optimizing system performance in future endeavors.

Analyzing the data, following conclusions have been drawn about the system's availability in relation to its maintenance:

- Single subsystem maintenance: Maintenance of a single subsystem *E* or *W* leads to an enhancement in system availability by as much as 15%. This signifies the importance of even isolated subsystem maintenance on the system's performance.

- Maintenance of two subsystems: Taking a step further, when maintenance efforts are channeled into two subsystems concurrently, there is a more pronounced positive impact. Specifically, system availability sees an uplift of up to 34%, underscoring the compounded benefits of multi-subsystem maintenance.

- Repair policy for subsystem (E): Delving deeper, when the repair policy specifically for subsystem (E) is put into effect, an additional increase in system availability is observed, reaching up to 98.5%. Notably, this value represents the pinnacle of system availability enhancement when viewing the system holistically.

Thus, the insights gleaned from our analysis underscore the utility of the proposed models. They serve as a guiding tool in pinpointing the most optimal maintenance strategy for the critical subsystems. By leveraging these findings, decision-makers can make control processes to bolster both production rates and overall plant availability in the coil shop.

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