Advances in Differential Equations and Control Processes
© 2024 Pushpa Publishing House, Prayagraj, India
https://pphmjopenaccess.com
https://doi.org/10.17654/0974324324022
Volume 31, Number 3, 2024, Pages 397-415 **P-ISSN: 0974-3243**

FRACTIONAL THERMAL RESPONSE IN A THERMOSENSITIVE RECTANGULAR PLATE DUE TO THE ACTION OF A MOVING SOURCE OF HEAT

V. R. Manthena, V. B. Srinivas, N. K. Lamba and G. D. Kedar*

Department of Mathematics
Priyadarshini J. L. College of Engineering
Nagpur, India

Department of Mathematics
Anurag University
Venkatapur, Ghatkesar
Medchal-Malkajgiri District
Telangana, India

Department of Mathematics
Shri Lemdeo Patil Mahavidyalaya
Mandhal, Nagpur, India

Received: March 28, 2024; Revised: June 19, 2024; Accepted: July 1, 2024

2020 Mathematics Subject Classification: 35A09, 44A10, 44A20, 74A10, 74A15.

Keywords and phrases: fractional heat conduction, moving heat source, thermal deflection, thermal stresses.

*Corresponding author

How to cite this article: V. R. Manthena, V. B. Srinivas, N. K. Lamba and G. D. Kedar, Fractional thermal response in a thermosensitive rectangular plate due to the action of a moving source of heat, Advances in Differential Equations and Control Processes 31(3) (2024), 397-415. https://doi.org/10.17654/0974324324022

Published Online: July 6, 2024

Department of Mathematics
RTM Nagpur University
Nagpur, India

Abstract

The fractional order theory explores over the field of mathematics and physical sciences as it provides the generalization of non-integer order for derivative and integration. Numerical methods are used to study temperature fields with heat transfer in homogeneous bodies whose thermophysical characteristics depend on the temperature. However, analytical solutions of such problems are needed for qualitative analysis to solve the corresponding problems of thermoelasticity. This study focuses on examining the thermoelastic behavior of a rectangular plate, incorporating time dependent fractional order derivative. Moving line heat source in x-direction is considered for heat conduction analysis. The nonlinearity of the heat conduction equation is dealt using Kirchhoff's variable transformation. The solution of fractional heat conduction equation (FHCE) is obtained using finite Fourier cosine transform and Laplace transform methods. The obtained solution in transformed domain is expressed in terms of Mittag-Leffler function, trigonometric functions and hypergeometric functions. The effect of time fractional order parameter and velocity on temperature profile and thermal profile is analyzed graphically. During the analysis, it is observed that the inhomogeneous material properties cause the magnitude of profile of thermal characteristics to increase on comparison to that of homogeneous case. Smaller magnitudes of temperature, deflection and stresses are seen for larger values of velocity.

1. Introduction

In thermoelasticity, the heat diffusion process is heterogeneous and non-regular in nature and does not obey laws of mechanics. Hence, it is essential to introduce fractional order derivative in diffusion equation. In this paper, the theory of fractional calculus is used to modify the existing model of physical processes which includes the fields of heat conduction, diffusion, viscoelasticity and solids mechanics.

Initially, the investigation of time fractional derivative was carried out by Caputo and Mainardi [1-3]. Noda [4] studied different cases of temperature dependent thermal conductivity. Luchko and Gorenflo [5] and Mainardi and Gorenflo [6] used Caputo derivatives and M-L functions and solved fractional differential equations. Povstenko [7, 8] solved thermoelastic problems involving fractional derivatives. Tarasov [9] proved chain rule for fractional derivatives. Manthena et al. [10] studied the thermal behaviour of a rectangular plate with thermally sensitive material properties. Considering moving heat source in fractional order context, Bassiouny and Youssef [11] discussed thermoelastic behaviour of a thin layered plate. Manthena et al. [12, 13] considered temperature dependent material properties and analyzed thermoelasticity of a plate. The effect of fractional parameter for thermoelastic half space subjected to a moving heat source is studied by Hussein [14].

Yi et al. [15] developed a comprehensive analytic thermal model taking moving heat source and the superposition principle of heat source. Sur et al. [16] investigated thermal effect on skin tissues due to the influence of the Caputo-Fabrizio moving heat source. Kumar and Kamdi [17] solved a two-dimensional finite hollow cylinder problem using fractional thermoelasticity. Geetanjali and Sharma [18] discussed spherical cavity with generalized thermoviscoelastic diffusion. Chaurasiya and Singh [19] numerically investigated a non-linear moving boundary problem with temperature-dependent conductivity. Singh and Mukhopadhyay [20] investigated the effect of strain rate and temperature rate factors on an elastic medium originating due to continuous line heat source. Rahimi et al. [21] analyzed the vibrational behaviour of the double-layered micro-nanosphere. Several authors [22-27] discussed the heat conduction, thermal stresses and fractional thermoelasticity in different solids.

During the previous three decades, due to the utilization of basic materials at high temperatures, a pattern of examination of thermoelasticity is made in which the impact of temperature and mechanical properties of the structure is mulled over, and subsequently the investigation of thermoelasticity in solids with material properties dependent on temperature received attention. Even though numerical methods are used to solve such problems, analytical solutions are still needed for qualitative analysis of thermoelastic problems. This paper investigates an uncoupled problem for a finite rectangular plate. Finite Fourier cosine-transform and Laplace transform are used for solving the heat conduction equation (HCE) and the solution is expressed using trigonometric and Mittag-Leffler (M-L) function. The temperature distribution profile and thermal profile are analyzed graphically due to the effect of moving velocity.

2. Heat Conduction Equation and its Solution

A rectangular plate occupying the space defined as $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$ is considered. We considered an rth order FHCE in context with Caputo derivatives and prepared a mathematical model.

Following Caputo and Mainardi [1-3], we have

$$\frac{\partial^r F(t)}{\partial t^r} = \frac{1}{\Gamma(h-r)} \int_0^t (t-\tau)^{h-r-1} \frac{d^h F(\tau)}{d\tau^h} d\tau, \quad h-1 < r < h, \tag{1}$$

$$L\left[\frac{\partial^r F(t)}{\partial t^r}\right] = s^r \overline{F}(s) - \sum_{k=0}^{h-1} F^{(k)}(0^+) s^{r-1-k}, \quad h-1 < r < h. \tag{2}$$

FHCE of a rectangular plate with heat source is

$$\frac{\partial}{\partial x}\left(k(T)\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k(T)\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k(T)\frac{\partial T}{\partial z}\right) + H(x, y, z, t)$$

$$= \rho[C(T)]^r \frac{\partial^r T}{\partial t^r}. \tag{3}$$

The boundary and initial conditions are

$$\frac{\partial T}{\partial x} = 0 \text{ at } x = 0, a, \quad \frac{\partial T}{\partial y} = 0 \text{ at } y = 0, b, \quad \frac{\partial T}{\partial z} = 0 \text{ at } z = 0, c, \tag{4}$$

$$T = 0 \text{ at } t = 0, 0 < r \leq 2, \quad \frac{\partial T}{\partial t} = 0 \text{ at } t = 0, 1 < r \leq 2, \tag{5}$$

where $k(T)$ and $C(T)$ are heat conductivity and heat capacity, $H(x, y, z, t)$ is the moving heat source along x-direction, and ρ is the density.

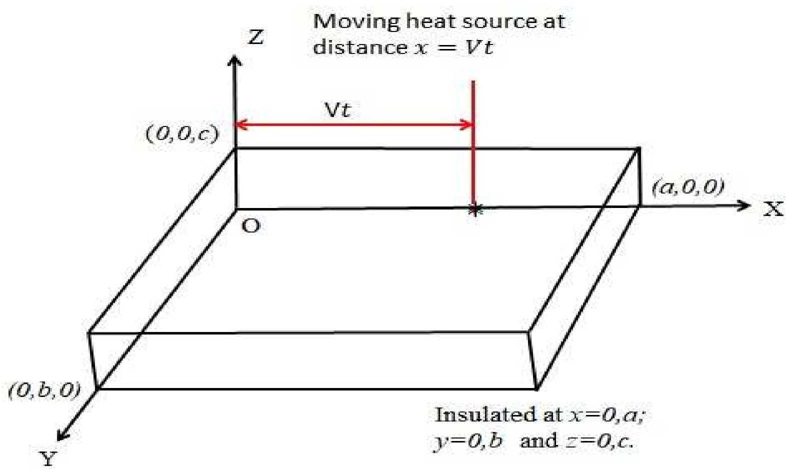

Figure 1. Geometry of the problem with moving heat source.

We introduce the following dimensionless parameters:

$$T^* = \frac{T}{T_0}, \quad (x^*, y^*, z^*) = \frac{(x, y, z)}{a}, \quad (a^*, b^*, c^*) = \frac{(a, b, c)}{a},$$

$$t^* = \sqrt[r]{\kappa/a^2}\,t, \quad E_0^* = \frac{E_0}{E_1}, \quad \alpha_0^* = \frac{\alpha_0}{\alpha_1}, \quad \rho^* = \frac{\rho}{\rho_0}, \quad k_0^* = \frac{k_0}{k_1},$$

$$\varpi_1^* = \frac{\varpi_1 a^2}{\kappa}, \quad \varpi_2^* = \frac{\varpi_2 a^2}{\kappa}. \tag{6}$$

Here T_0 is the ambient temperature, κ is the thermal diffusivity, ρ_0, E_0, E_1, α_0, α_1 are reference values of density, Young's modulus, thermal-expansion coefficient, and ϖ_1, ϖ_2 are the frequencies.

We define $k(T), C(T), H(x, y, z, t)$ as

$$k(T) = k_0 k*(T*), \quad [C(T)]^r = C_0[C*(T*)]^r,$$

$$H(x, y, z, t) = h_0 H*(x*, y*, z*, t*). \tag{7}$$

Using equations (6-7), equations (3-5) become (ignoring asterisks for convenience):

$$\frac{\partial}{\partial x}\left(k(T)\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k(T)\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k(T)\frac{\partial T}{\partial z}\right) + P_0 H(x, y, z, t)$$

$$= \rho[C(T)]^r \frac{\partial^r T}{\partial t^r} \tag{8}$$

with boundary and initial conditions

$$\frac{\partial T}{\partial x} = 0 \text{ at } x = 0, a, \quad \frac{\partial T}{\partial y} = 0 \text{ at } y = 0, b, \quad \frac{\partial T}{\partial z} = 0 \text{ at } z = 0, c, \tag{9}$$

$$T = 0 \text{ at } t = 0, \; 0 < r \le 2, \quad \frac{\partial T}{\partial t} = 0 \text{ at } t = 0, 1 < r \le 2, \tag{10}$$

where $P_0 = \frac{h_0 a^2}{k_0 T_0}$ is the dimensionless Pomerantsev reference number.

Using Kirchhoff's transformation [26] from the following equation (11), equation (8) transforms to equation (12),

$$\Theta(T) = \int_0^T k(T)dT, \tag{11}$$

$$\frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2} + \frac{\partial^2 \Theta}{\partial z^2} + P_0 H(x, y, z, t) = \rho \frac{\partial^r \Theta}{\partial t^r}. \tag{12}$$

The boundary and initial conditions (9) and (10) become

$$\frac{\partial \Theta}{\partial x} = 0 \text{ at } x = 0, a, \quad \frac{\partial \Theta}{\partial y} = 0 \text{ at } y = 0, b, \quad \frac{\partial \Theta}{\partial z} = 0 \text{ at } z = 0, c, \tag{13}$$

$$\Theta = 0 \text{ at } t = 0, 0 < r \le 2, \quad \frac{\partial \Theta}{\partial t} = 0 \text{ at } t = 0, 1 < r \le 2. \tag{14}$$

We consider $H(x, y, z, t)$ as a line heat source moving with dimensionless velocity V along positive x direction defined by [14, 16, 20]:

$$H(x, y, z, t) = \delta(x - Vt)\delta(y)\delta(z). \tag{15}$$

Using equation (15) in equation (12) and applying Laplace transform (LT), we arrive at

$$\frac{\partial^2 \overline{\Theta}}{\partial x^2} + \frac{\partial^2 \overline{\Theta}}{\partial y^2} + \frac{\partial^2 \overline{\Theta}}{\partial z^2} + P_0 V e^{-xs/\vartheta} \delta(y)\delta(z) = \rho s^r \overline{\Theta}. \tag{16}$$

Implementing finite Fourier cosine transform (FCT) on equation (16) over the variables *x*, *y* and *z*, we obtain

$$-(\eta_m^2 + \beta_n^2 + \gamma_l^2)\overline{\overline{\overline{\Theta}}} + P_0 V \overline{\overline{F}}\{e^{-xs/V} \delta(y)\delta(z)\} = \rho s^r \overline{\overline{\overline{\Theta}}}, \tag{17}$$

where $\eta_m = m\pi x/a$, $\beta_n = n\pi y/b$, $\gamma_l = l\pi z/c$ and $\cos(m\pi x/a)$, $\cos(n\pi y/b)$, $\cos(l\pi z/c)$ are the kernels of finite FCT and $\overline{\overline{F}}\{e^{-xs/V} \delta(y)\delta(z)\}$ represents the finite Fourier transform with respect to variables *x*, *y* and *z*.

On simplification, the above equation (17) leads to

$$\overline{\overline{\overline{\Theta}}} = \frac{P_0 V^2}{\rho} \times \left\{ \frac{e^{-x's/V} s \cos(m\pi)}{(s^2 + (m^2\pi^2 V^2/a^2))} + \frac{m\pi V \cos(m\pi)}{a(s^2 + (m^2\pi^2 V^2/a^2))} \right.$$

$$\left. - \frac{sV}{(s^2 + (m^2\pi^2 V^2/a^2))} \right\} \times \frac{1}{(s^r + ((\eta_m^2 + \beta_n^2 + \gamma_l^2)/\rho))}. \tag{18}$$

Taking inverse LT of the above equation (18), we obtain

$$\overline{\overline{\overline{\Theta}}} = (P_0 V^2/\rho) \sum_{\varepsilon=0}^{\infty} \{[\zeta^{\varepsilon}/\Gamma(\Phi(\varepsilon + 1))]\xi_1(t)\}, \tag{19}$$

where

$$\zeta = -(\eta_m^2 + \beta_n^2 + \gamma_l^2)/\rho^2,$$

$$\xi_1(t) = [t - (a/V)]^{r\varepsilon + r - 1}(\cos(m\pi))^2 + (m\pi V \cos(m\pi)/a)$$

$$\times \phi_1(r(\varepsilon + 1), O) - \phi_2(r(\varepsilon + 1), O),$$

and $\phi_1(r(\varepsilon+1), O)$, $\phi_2(r(\varepsilon+1), O)$ are the hypergeometric functions, pG_q is the generalized hypergeometric series defined as

$$\phi_1(r(\varepsilon+1), O)$$

$$= \frac{t^{r(\varepsilon+1)}}{\Gamma(r(\varepsilon+1)+1)} \times {}_1G_2(1, (r(\varepsilon+1)+1)/2, (r(\varepsilon+1)+2)/2, -(O^2t^2)/4),$$

$$\phi_2(r(\varepsilon+1), O)$$

$$= \frac{t^{r(\varepsilon+1)}}{\Gamma(r(\varepsilon+1)+1)} \times {}_1G_2(1, (r(\varepsilon+1)+2)/2, (r(\varepsilon+1)+3)/2, -(O^2t^2)/4),$$

$$pG_q(a_1, \ldots, a_p, b_1, \ldots, b_q; O)$$

$$= \frac{\Gamma(b_1)\cdots\Gamma(b_q)}{\Gamma(a_1)\cdots\Gamma(a_p)} \sum_{i=0}^{\infty} \frac{\Gamma(a_1+i)\cdots\Gamma(a_p+i)}{\Gamma(b_1+i)\cdots\Gamma(b_q+i)} \frac{O^i}{i!}, \quad O = m\pi V/a.$$

Applying inverse FCT on equation (19), we obtain

$$\Theta(x, y, z, t) = [[\overline{\Theta}(\eta_m, y, z, t)]_{m=0}/a] + (2/a)\sum_{m=1}^{\infty}\{\overline{\Theta}(\eta_m, y, z, t)\times\cos(m\pi x/a)\}, \tag{20}$$

where

$$\overline{\Theta}(\eta_m, y, z, t) = [[\overline{\overline{\Theta}}(\eta_m, \beta_n, z, t)]_{n=0}/b] + (2/b)\sum_{n=1}^{\infty}\{\overline{\overline{\Theta}}(\eta_m, \beta_n, z, t)\times\cos(n\pi y/b)\},$$

$$\overline{\overline{\Theta}}(\eta_m, \beta_n, z, t) = [[\xi_2(t)]_{l=0}/c] + (2/c)\sum_{l=1}^{\infty}\{\xi_2(t)\times\cos(l\pi z/c)\},$$

$$\xi_2(t) = (P_0V^2/\rho)\sum_{\varepsilon=0}^{\infty}\{[\zeta^{\varepsilon}/\Gamma(r(\varepsilon+1))]\xi_1(t)\}.$$

By employing the inverse Kirchhoff's transform on equation (20), the solution of temperature is derived as

$$T(x, y, z, t) \cong [[\overline{\Theta}(\eta_m, y, z, t)]_{m=0}/k_0 a] + (2/k_0 a)\sum_{m=1}^{\infty}\{\overline{\Theta}(\eta_m, y, z, t) \times \cos(m\pi x/a)\}. \quad (21)$$

3. Thermoelastic Analysis

In rectangular coordinates for a rectangular plate with support at its ends, the deflection equation is [25]:

$$\nabla^2\nabla^2 w = \frac{-1}{(1-\nu(T))D(T)}\nabla^2 M_T, \quad (22)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, \quad D(T) = \frac{E(T)c^3}{12(1-\nu(T)^2)} \quad (23)$$

with

$$w = 0, \quad \frac{\partial^2 w}{\partial x^2} = \frac{-1}{(1-\nu(T))D(T)} M_T \text{ at } x = 0, a,$$

$$w = 0, \quad \frac{\partial^2 w}{\partial y^2} = \frac{-1}{(1-\nu(T))D(T)} M_T \text{ at } y = 0, b, \quad (24)$$

where w is the deflection, M_T is the resultant moment, $D(T)$, $E(T)$, $\nu(T)$ are flexural-rigidity, elastic-modulus and Poisson's ratio, respectively.

The net-forces are

$$N_x = N_y = N_{xy} = 0. \quad (25)$$

The force-resultants are

$$M_x = -D(T)\left(\frac{\partial^2 w}{\partial x^2} + \nu(T)\frac{\partial^2 w}{\partial y^2}\right) - \frac{1}{1-\nu(T)} M_T,$$

$$M_y = -D(T)\left(\nu(T)\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) - \frac{1}{1-\nu(T)} M_T,$$

$$M_{xy} = (1 - \nu(T))D(T)\frac{\partial^2 w}{\partial x \partial y}. \quad (26)$$

The stress elements are [25]:

$$\sigma_{xx} = \frac{1}{c}N_x + \frac{12z}{c^3}M_x + \frac{1}{1-\nu(T)}\left(\frac{1}{c}N_T + \frac{12z}{c^3}M_T - \alpha(T)E(T)T\right),$$

$$\sigma_{yy} = \frac{1}{c}N_y + \frac{12z}{c^3}M_y + \frac{1}{1-\nu(T)}\left(\frac{1}{c}N_T + \frac{12z}{c^3}M_T - \alpha(T)E(T)T\right),$$

$$\sigma_{xy} = \frac{1}{c}N_{xy} - \frac{12z}{c^3}M_{xy}, \quad (27)$$

$$M_T = \int_0^c \alpha(T)E(T)Tz\,dz, \quad N_T = \int_0^c \alpha(T)E(T)T\,dz. \quad (28)$$

Here $\alpha(T)$ is the thermal-expansion coefficient.

For calculating moments M_T, N_T, let

$$E(T) = E_0 \exp(\varpi_1 T), \quad \alpha(T) = \alpha_0 \exp(\varpi_2 T),$$

$$\nu(T) = \nu_0 \exp(\varpi_2 T), \quad \varpi_1 \le 0, \varpi_2 \ge 0. \quad (29)$$

Applying equations (29) and (21) on equation (28), we get

$$M_T = \sum_{m=1}^{\infty}\left\{L_1 \times \left\{(2/b)\sum_{n=1}^{\infty}[[\xi_2(t)]_{l=0}/c] + (2/c)\sum_{l=1}^{\infty}[\xi_2(t)\times\cos(n\pi y/b)]\right\}\times\cos(m\pi x/a)\right\}, \quad (30)$$

$$N_T = \sum_{m=1}^{\infty}\left\{L_2 \times \left\{(2/b)\sum_{n=1}^{\infty}[[\xi_2(t)]_{l=0}/c] + (2/c)\sum_{l=1}^{\infty}[\xi_2(t)\times\cos(n\pi y/b)]\right\}\times\cos(m\pi x/a)\right\}, \quad (31)$$

where

$$L_1 = (2\alpha_0 E_0/abk_0\pi^2 l^2)((1/288\pi^2 l^2)c^2(3(-112A_2^3\varpi_1\varpi_2$$
$$+ 96A_2(-1 + 3A_1^2\varpi_1\varpi_2) + 3A_2^2(-4\varpi_2 + \varpi_1(-2 + 3A_1(-5$$
$$+ 32\pi^2 l^2)\varpi_2)) + 24\pi^2 l^2 \times (3\varpi_1 + 2\varpi_2 + A_1(2 + 5A_1^2\varpi_1\varpi_2$$
$$+ 2A_1(\varpi_1 + \varpi_2))) - 4A_2(9(16A_1 + A_2)\varpi_1 + ((432A_1^2\varpi_1$$
$$+ 18A_1(8 + 3A_2\varpi_1) + A_2(9 + 112A_2\varpi_1))\varpi_2) + 48A_2(2$$
$$+ 9A_1^2\varpi_1\varpi_2 + 4A_1(\varpi_1 + \varpi_2))\cos(l\pi)$$
$$+ 2A_2^2\varpi_1\varpi_2(27A_1\cos(2l\pi) + 2A_2\cos(3l\pi)))))),$$

$$L_2 = (c\alpha_0 E_0/2abk_0)(14A_1^3\varpi_1\varpi_2 + 8A_1^2(\varpi_1 + \varpi_2)$$
$$+ 2A_2^2(\varpi_1 + \varpi_2) + A_1(4 + 9A_2^2\varpi_1\varpi_2)$$
$$+ A_1\varpi_1\varpi_2(2A_1^2\cos(2l\pi) + 3A_2^2(4\cos(l\pi) + \cos(3l\pi)))),$$

$$A_1 = [[\xi_2(t)]_{l=0}/c], \quad A_2 = [2\xi_2(t)/c].$$

4. Numerical Results and Discussion

Using the thermally induced resultant moments, thermal deflection, stress resultants and the corresponding stress components are obtained with Mathematica software. In numerical analysis, we examined a model characterized by Copper material with thermo-elastic properties as given below:

$$T_0 = 320\text{K}, \quad a = 4\text{m}, \quad b = 2\text{m}, \quad c = 1\text{m}, \quad k_0 = 386\,\text{W/mK},$$

$$k_1 = 370\,\text{W/mK}, \quad \rho = 8954\text{kg/m}^3, \quad \rho_0 = 8952\text{kg/m}^3,$$

$$C_0 = 388\text{J/kgK}, \quad \kappa = 11.1\times 10^{-5}\text{m}^2/\text{sec}, \quad E_0 = 133\times 10^9\,\text{kg/m sec}^2,$$

$E_1 = 121 \times 10^9 \,\text{kg/m sec}^2, \quad \alpha_0 = 17.9 \times 10^{-6}/\text{K},$

$\alpha_1 = 16.8 \times 10^{-6}/\text{K}, \quad t = 2\,\text{sec}.$

The following Figures 2 to 6 are plotted by taking the fractional order parameter $r = 0.5, 1, 1.5, 2.$ Figures 2 to 6 represent temperature, deflection and stress distribution. Figures 3 to 6 on the left represent homogeneous case, while on the right represent nonhomogeneous case.

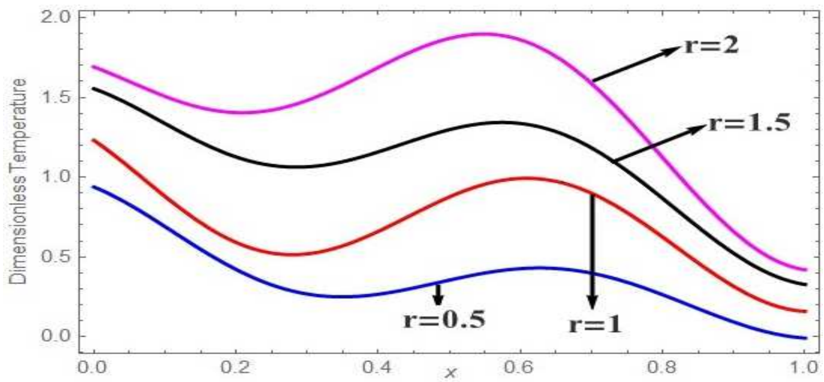

Figure 2. Plot of temperature.

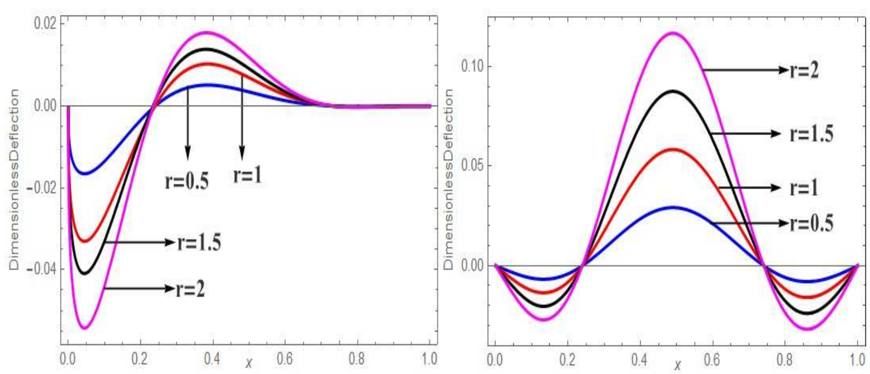

Figure 3. Plot of deflection.

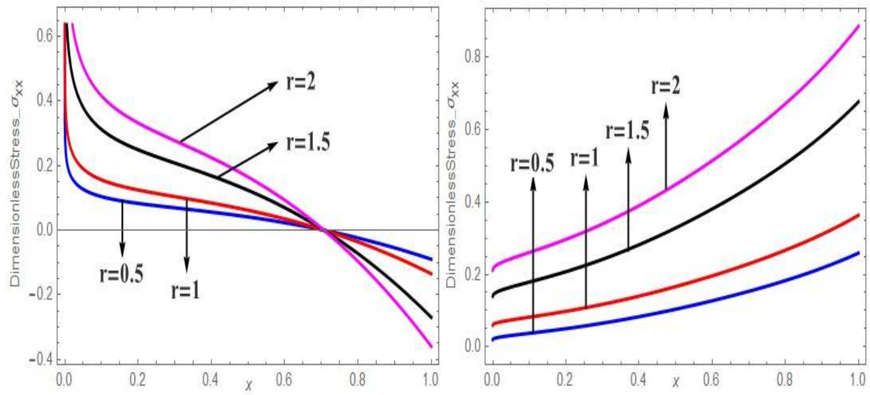

Figure 4. Plot of σ_{xx}.

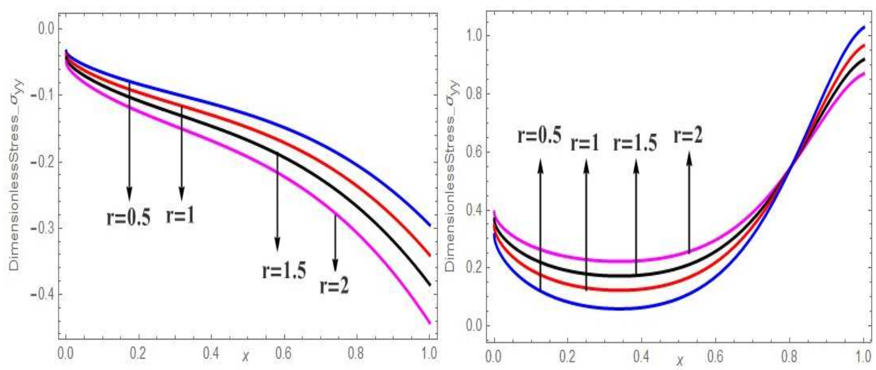

Figure 5. Plot of σ_{yy}.

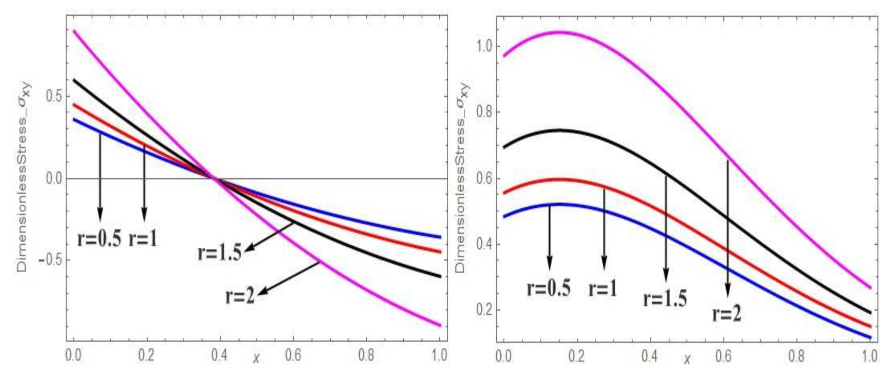

Figure 6. Plot of σ_{xy}.

From Figure 2, it is seen that the temperature assumes a uniform pattern for different r. It takes nonzero value at both extremities except for $r = 0.5$, where it becomes zero at the outer end. Due to thermal insulation at all ends, thermal energy gets accumulated at the middle region for larger value of r.

From Figure 3 it is seen that, deflection is positive in the middle portion, and negative at both the extremities.

From Figures 4-6, it is seen that in the homogeneous case, the components σ_{xx}, σ_{xy} are tensile in the regions $0 < x < 0.7$, $0 < x < 0.4$, respectively, while compressive towards the other end. The stress component σ_{yy} is compressive throughout. In the inhomogeneous scenario, all stress components exhibit tensile characteristics.

The following Figures 7 to 11 are plotted for different values of velocity $V = 1, 2, 3$ depicting temperature distribution, deflection and stresses. Figures 8-11 on the left represent homogeneous case, whereas that on the right represent nonhomogeneous case.

In Figures 7-11, as the velocity V increases, there is a decrease in the magnitude of temperature which consequently causes decrement in the magnitude of deflection and thermal stresses.

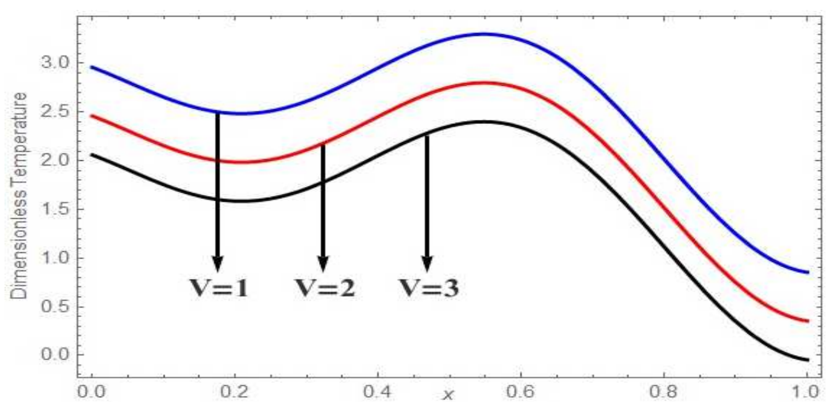

Figure 7. Plot of temperature.

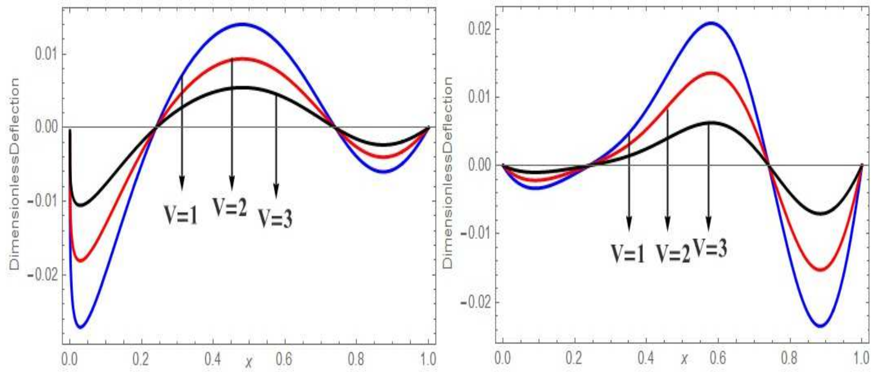

Figure 8. Plot of deflection.

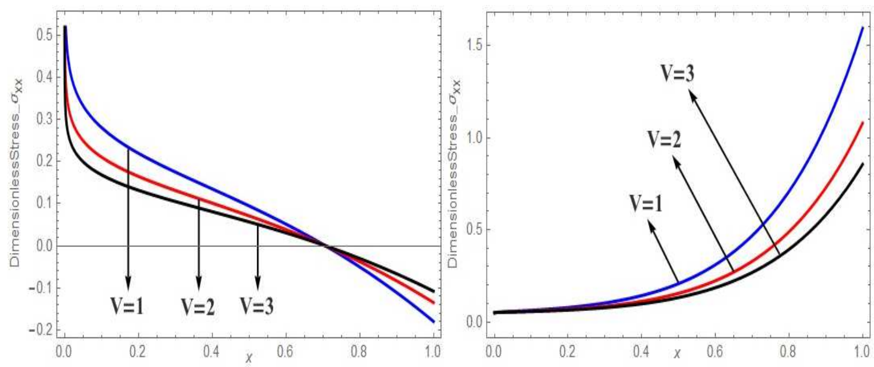

Figure 9. Plot of σ_{xx}.

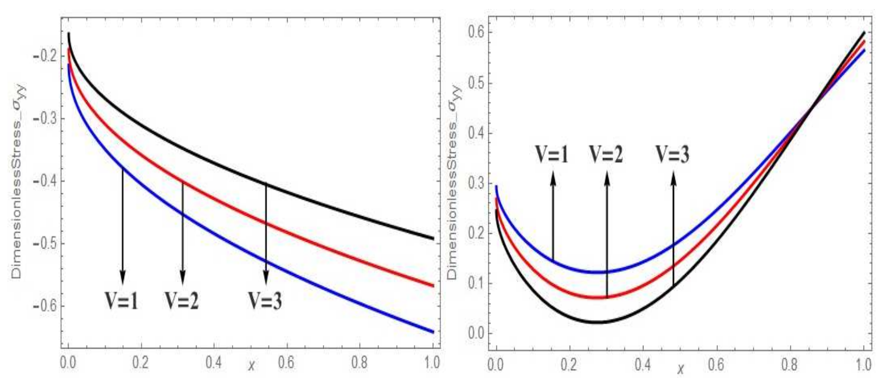

Figure 10. Plot of σ_{yy}.

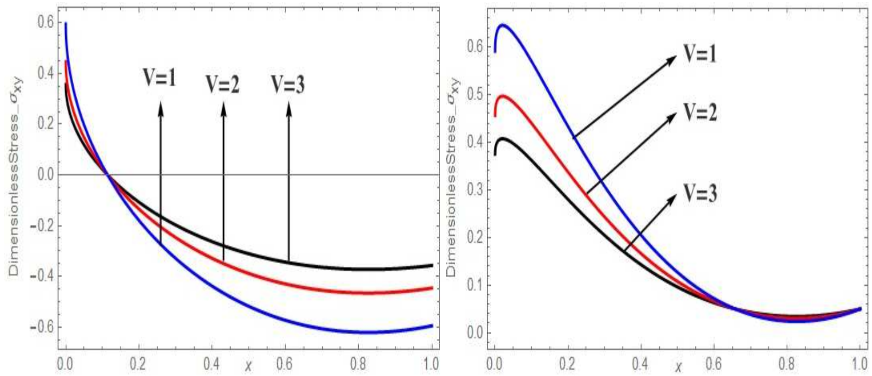

Figure 11. Plot of σ_{xy}.

5. Conclusion

In this paper, we have obtained the solution of fractional order heat conduction equation subjected to moving heat source for a thermally sensitive rectangular plate. The obtained solutions reduce to the solutions of classical HCE for $r = 1$.

From this study, it is concluded that

(1) The velocity V has a notable impact on the variations in temperature, deflection and stress profiles.

(2) For larger values of V, smaller magnitudes are obtained. It is physically reasonable that when the heat source accelerates, it lacks sufficient time for heat dissipation.

(3) A reduced heat release corresponds to a diminished temperature rise and vice-versa causing a thermal deformation, which generates stress in the rectangular plate. Hence, for smaller values of V, comparatively larger peak is observed.

(4) The presence of thermosensitive material properties leads to a noticeable impact.

(5) The stress component σ_{xy} takes a high negative value in the homogeneous case as compared to rest of the stresses.

(6) In the inhomogeneous case, σ_{yy} takes highest magnitude as compared to σ_{xx}.

(7) This type of study may prove to be helpful for theoretical modelling of thermoelastic problems at micro/nanoscale and may be beneficial to the design of devices operated under high temperature environments.

Acknowledgement

We thank the anonymous referees for their comments and feedback on earlier version of this document.

References

[1] M. Caputo, Linear models of dissipation whose *Q* is almost frequency independent-II, Geophysical Journal International 13(5) (1967), 529-539.

[2] M. Caputo, Elasticita e dissipazione, Zanichelli, 1969.

[3] M. Caputo and F. Mainardi, Linear models of dissipation in an elastic solid, La Rivista del Nuovo Cimento 1(2) (1971), 161-198.

[4] N. Noda, Thermal stresses in materials with temperature dependent properties, Thermal Stresses I, North Holland, Amsterdam, 1986, pp. 391-483.

[5] Y. Luchko and R. Gorenflo, An operational method for solving fractional differential equations with the Caputo derivatives, Acta. Math. Vietnam. 24(2) (1999), 207-233.

[6] F. Mainardi and R. Gorenflo, On Mittag-Leffler-type functions in fractional evolution processes, J. Comput. Appl. Math. 118(2) (2000), 283-299.

[7] Y. Z. Povstenko, Fractional heat conduction equation and associated thermal stresses, Journal of Thermal Stresses 28(1) (2005), 83-102.

[8] Y. Z. Povstenko, Fundamental solutions to central symmetric problems for fractional heat conduction equation and associated thermal stresses, Journal of Thermal Stresses 31(2) (2008), 127-148.

[9] V. E. Tarasov, On chain rule for fractional derivatives, Commun. Nonlinear Sci. Numer. Simul. 30(1-3) (2016), 1-4.

[10] V. R. Manthena, N. K. Lamba, G. D. Kedar and K. C. Deshmukh, Effects of stress resultants on thermal stresses in a functionally graded rectangular plate due to temperature dependent material properties, International Journal of Thermodynamics 19(4) (2016), 235-242.

[11] E. Bassiouny and H. M. Youssef, Sandwich structure panel subjected to thermal loading using fractional order equation of motion and moving heat source, Canadian Journal of Physics 96(2) (2018), 174-182.

[12] V. R. Manthena, G. D. Kedar and K. C. Deshmukh, Thermal stress analysis of a thermosensitive functionally graded rectangular plate due to thermally induced resultant moments, Multidiscipline Modeling in Materials and Structures 14(5) (2018), 857-873.

[13] V. R. Manthena and G. D. Kedar, On thermoelastic problem of a thermosensitive functionally graded rectangular plate with instantaneous point heat source, Journal of Thermal Stresses 42(7) (2019), 849-862.

[14] E. M. Hussein, Effect of fractional parameter on thermoelastic half-space subjected to a moving heat source, International Journal of Heat and Mass Transfer 141 (2019), 855-860.

[15] J. Yi, T. Jin and Z. Deng, The temperature field study on the three-dimensional surface moving heat source model in volute gear form grinding, The International Journal of Advanced Manufacturing Technology 103(5-8) (2019), 3097-3108.

[16] A. Sur, S. Mondal and M. Kanoria, Influence of moving heat source on skin tissue in the context of two temperature memory dependent heat transport law, Journal of Thermal Stresses 43(1) (2020), 55-71.

[17] N. Kumar and D. B. Kamdi, Thermal behaviour of a finite hollow cylinder in context of fractional thermoelasticity with convection boundary conditions, Journal of Thermal Stresses 43(9) (2020), 1189-1204.

[18] G. Geetanjali and P. K. Sharma, Impact of fractional strain on medium containing spherical cavity in the framework of generalized thermoviscoelastic diffusion, Journal of Thermal Stresses 46(5) (2023), 333-350.

[19] V. Chaurasiya and J. Singh, Numerical investigation of a non-linear moving boundary problem describing solidification of a phase change material with temperature dependent thermal conductivity and convection, Journal of Thermal Stresses 46(8) (2023), 799-822.

[20] R. V. Singh and S. Mukhopadhyay, Mathematical significance of strain rate and temperature rate on heat conduction in thermoelastic material due to line heat source, Journal of Thermal Stresses 46(11) (2023), 1164-1179.

[21] Y. Rahimi, M. Ghadiri, A. Rajabpour and M. F. Ahari, Temperature-dependent vibrational behavior of bilayer doubly curved micro-nano liposome shell: simulation of drug delivery mechanism, Journal of Thermal Stresses 46(11) (2023), 1199-1226.

[22] K. B. Oldham and J. Spanier, The Fractional Calculus, Academic Press, New York, 1974.

[23] K. S. Miller and B. Ross, An Introduction to the Fractional Calculus and Fractional Differential Equations, Wiley, New York, 1993.

[24] I. Podlubny, Fractional Differential Equations, Academic Press, New York, 1999.

[25] N. Noda, R. B. Hetnarski and Y. Tanigawa, Thermal Stresses, 2nd ed., Taylor and Francis, New York, 2003.

[26] L. M. Jiji, Heat Conduction, 3rd ed., Springer, Berlin Heidelberg, 2009.

[27] Y. Povstenko, Fractional Thermoelasticity, Springer, New York, Vol. 219, 2015.