Advances in Differential Equations and Control Processes
© 2024 Pushpa Publishing House, Prayagraj, India
https://pphmjopenaccess.com
https://doi.org/10.17654/0974324324016
Volume 31, Number 3, 2024, Pages 299-315 **P-ISSN: 0974-3243**

HOMOTOPY PERTURBATION METHOD TO SOLVE DUFFING-VAN DER POL EQUATION

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Abstract

In this paper, the Homotopy Perturbation Method (HPM) and the Regular Perturbation Method (RPM) are used to study Duffing-Van der Pol equation. Then we compare the solutions obtained by these two methods.

Received: February 15, 2024; Accepted: April 13, 2024

2020 Mathematics Subject Classification: 74H10, 74H15, 35A35, 41A10, 65Yxx, 55PXX, 47F05.

Keywords and phrases: Duffing-Van der Pol equation, homotopy perturbation method, regular perturbation method.

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How to cite this article: BAGAYOGO Moussa, MINOUNGOU Youssouf, NEBIE Abdoul Wassiha and PARE Youssouf, Homotopy perturbation method to solve Duffing-Van der Pol equation, Advances in Differential Equations and Control Processes 31(3) (2024), 299-315. https://doi.org/10.17654/0974324324016

Published Online: May 15, 2024

1. Introduction

The Duffing-Van der Pol equation which we examine is nonlinear and it contains two perturbation parameters μ and λ. The general form of this equation is:

$$\frac{d^2u(t)}{dt^2} - \mu(1 - u^2(t))\frac{du(t)}{dt} + u(t) + \lambda u^3(t) = f(t); \quad t > 0. \tag{1}$$

We here examine the following initial value problem:

$$\begin{cases} \dfrac{d^2u(t)}{dt^2} - \mu(1 - u^2(t))\dfrac{du(t)}{dt} + u(t) + \lambda u^3(t) = 0 & 0 < t < T \\ u(0) = 1; & \dfrac{du(0)}{dt} = 0, \end{cases} \tag{2}$$

where $0 < \mu \ll 1$, $0 < \lambda \ll 1$.

First, we construct the solution of (2) by the HPM [4-6]. We then apply the regular perturbation method [2, 3] and finally we compare the two methods. We will be interested in the cases $\mu = \frac{1}{4}$ and $\mu = \frac{1}{8}$.

2. Application of the HPM to Solve the Duffing-Van der Pol Equation

In order to apply the HPM, we construct a homotopy $H(v, p)$ which satisfies:

$$H(v, p) = (1 - p)[L(v) - L(u_0)] + p[L(v) + N(v) - f(t)] = 0$$

with

$$L(v) = \frac{d^2v}{dt^2} - \mu\frac{dv}{dt} + v, \; L(u_0) = \frac{d^2u_0}{dt^2} - \mu\frac{du_0}{dt} + u_0, \; N(v) = \mu v^2\frac{dv}{dt} + \lambda v^3$$

and $f(t) = 0$.

Assuming

$$\frac{d^2 u_0}{dt^2} = \frac{du_0}{dt} = u_0 = 0,$$

we have

$$\frac{d^2 v}{dt^2} - \mu \frac{dv}{dt} + v + p\mu v^2 \frac{dv}{dt} + p\lambda v^3 = 0. \tag{3}$$

Assume that the solution of equation (2) can be given by a power series in p:

$$v = v_0 + pv_1 + p^2 v_2 + \dots. \tag{4}$$

By substituting (4) into (3) and equating the terms with the identical powers of p, we have

$$p^0 : \begin{cases} \dfrac{d^2 v_0}{dt^2} - \mu \dfrac{dv_0}{dt} + v_0 = 0 \\ v_0(0) = 1; \qquad \dfrac{dv_0(0)}{dt} = 0, \end{cases} \tag{5}$$

$$p^1 : \begin{cases} \dfrac{d^2 v_1}{dt^2} - \mu \dfrac{dv_1}{dt} + v_1 + \mu v_0^2 \dfrac{dv_0}{dt} + \lambda v_0^3 = 0 \\ v_1(0) = 0; \qquad \dfrac{dv_1(0)}{dt} = 0, \end{cases} \tag{6}$$

$$p^2 : \begin{cases} \dfrac{d^2 v_2}{dt^2} - \mu \dfrac{dv_2}{dt} + v_2 + \mu v_0^2 \dfrac{dv_1}{dt} + 2\mu v_0 v_1 \dfrac{dv_0}{dt} + 3\lambda v_0^2 v_1 = 0 \\ v_2(0) = 0; \qquad \dfrac{dv_2(0)}{dt} = 0. \end{cases} \tag{7}$$

We solve the systems (5), (6), (7) for $\mu = \frac{1}{4}$ and $\mu = \frac{1}{8}$.

Case: $\mu = \frac{1}{4}$

From (5), we have

$$v_0(t) = e^{\frac{t}{8}}\left[\cos\left(\frac{3\sqrt{7}}{8}t\right) - \frac{1}{3\sqrt{7}}\sin\left(\frac{3\sqrt{7}}{8}t\right)\right].$$

From (6), we have

$$v_1(t) = \left(-\frac{4355\lambda}{95634\sqrt{7}} - \frac{1843}{95634\sqrt{7}}\right)e^{\frac{3t}{8}}\sin\left(\frac{9\sqrt{7}t}{8}\right)$$

$$+\left(\frac{289\lambda}{10626} - \frac{109}{31878}\right)e^{\frac{3t}{8}}\cos\left(\frac{9\sqrt{7}t}{8}\right)$$

$$+\left[-\left(\frac{5}{126\sqrt{7}} + \frac{248\lambda}{63\sqrt{7}}\right)e^{\frac{3t}{8}} + \left(\frac{6541\lambda}{1518\sqrt{7}} + \frac{48}{253\sqrt{7}}\right)e^{\frac{t}{8}}\right]\sin\left(\frac{3\sqrt{7}t}{8}\right)$$

$$+\left[-\left(\frac{8\lambda}{21} + \frac{17}{126}\right)e^{\frac{3t}{8}} + \left(\frac{35}{253} + \frac{179\lambda}{506}\right)e^{\frac{t}{8}}\right]\cos\left(\frac{3\sqrt{7}t}{8}\right).$$

From (7), we have

$$v_2(t) = \left(-\frac{10638623\lambda^2}{4586989176\sqrt{7}} - \frac{1938703\lambda}{1720120941\sqrt{7}} + \frac{2718725}{13760967528\sqrt{7}}\right)e^{\frac{5t}{8}}\sin\left(\frac{15\sqrt{7}t}{8}\right)$$

$$+\left(\frac{38851\lambda^2}{72809352} - \frac{306961\lambda}{573373647} - \frac{350885}{4586989176}\right)e^{\frac{5t}{8}}\cos\left(\frac{15\sqrt{7}t}{8}\right)$$

$$+\left[\left(-\frac{17200306\lambda^2}{67278519\sqrt{7}} + \frac{28226119\lambda}{403671114\sqrt{7}} + \frac{223532807}{25834951296\sqrt{7}}\right)e^{\frac{5t}{8}}\right.$$

$$\left.+\left(-\frac{5387645\lambda^2}{129042144\sqrt{7}} - \frac{235\lambda}{4554\sqrt{7}} + \frac{1666318497}{308\sqrt{7}}\right)e^{\frac{3t}{8}}\right]\sin\left(\frac{9\sqrt{7}t}{8}\right)$$

$$+\left[\left(-\frac{12582\lambda^2}{118657}-\frac{1865485\lambda}{44852346}+\frac{1864883}{2870550144}\right)e^{\frac{5t}{8}}\right.$$

$$\left.+\left(\frac{1000959\lambda^2}{14338016}+\frac{25\lambda}{506}+\frac{8278270321}{21252}\right)e^{\frac{3t}{8}}\right]\cos\left(\frac{9\sqrt{7}t}{8}\right)$$

$$+\left[\left(\frac{140948480\lambda^2}{67278519\sqrt{7}}+\frac{214983680\lambda}{201835557\sqrt{7}}+\frac{4426960}{201835557\sqrt{7}}\right)e^{\frac{5t}{8}}\right.$$

$$-\left(\frac{1636\lambda^2}{483\sqrt{7}}+\frac{145\lambda}{66\sqrt{7}}+\frac{15803}{340032\sqrt{7}}\right)e^{\frac{3t}{8}}+\left(\frac{155643712503\lambda^2}{78361354016\sqrt{7}}\right.$$

$$\left.\left.+\frac{2788901\sqrt{7}\lambda}{19358042}-\frac{61289336254718701}{3716744064\sqrt{7}}\right)e^{\frac{t}{8}}\right]\sin\left(\frac{3\sqrt{7}t}{8}\right)$$

$$+\left[\left(-\frac{975872\lambda^2}{1067913}+\frac{6568960\lambda}{67278519}+\frac{1869232}{67278519}\right)e^{\frac{5t}{8}}\right.$$

$$+\left(\frac{4044\lambda^2}{1771}-\frac{25\lambda}{506}-\frac{17071}{340032}\right)e^{\frac{3t}{8}}+\left(\frac{104530270821\lambda^2}{78361354016}\right.$$

$$\left.\left.+\frac{1074589\lambda}{19358042}+\frac{482593173192685}{1238914688}\right)e^{\frac{t}{8}}\right]\cos\left(\frac{3\sqrt{7}t}{8}\right).$$

Finally, the approximate solution of (2) is given by

$$u(t)\simeq \lim_{p\to 1}[v_0(t)+pv_1(t)+p^2v_2(t)]$$

$$\simeq v_0(t)+v_1(t)+v_2(t).$$

Case: $\mu = \frac{1}{8}$

From (5), we have

$$v_0(t) = e^{\frac{t}{16}}\left[\cos\left(\frac{\sqrt{255}}{16}t\right) - \frac{1}{\sqrt{255}}\sin\left(\frac{\sqrt{255}}{16}t\right)\right].$$

From (6), we have

$$v_1(t) = -\left(\frac{72707\lambda}{520710\sqrt{255}} + \frac{31939}{520710\sqrt{255}}\right)e^{\frac{3t}{16}}\sin\left(\frac{3\sqrt{255}t}{16}\right)$$

$$+\left(\frac{5249\lambda}{173570} - \frac{89}{104142}\right)e^{\frac{3t}{16}}\cos\left(\frac{3\sqrt{255}t}{16}\right)$$

$$+\left[-\left(\frac{4064\lambda}{85\sqrt{255}} + \frac{7\sqrt{3}}{170\sqrt{85}}\right)e^{\frac{3t}{16}}\right.$$

$$\left.+\left(\frac{99901\lambda}{2042\sqrt{255}} + \frac{192\sqrt{3}}{1021\sqrt{85}}\right)e^{\frac{t}{16}}\right]\sin\left(\frac{\sqrt{255}t}{16}\right)$$

$$+\left[-\left(\frac{32\lambda}{85} + \frac{13}{102}\right)e^{\frac{3t}{16}} + \left(\frac{707\lambda}{2042} + \frac{131}{1021}\right)e^{\frac{t}{16}}\right]\cos\left(\frac{\sqrt{255}t}{16}\right).$$

From (7), we have

$$v_2(t) = \left(-\frac{3141581903\lambda^2}{407018178600\sqrt{255}} - \frac{723890191\lambda}{152631816975\sqrt{255}}\right.$$

$$\left.+\frac{37776433}{244210907160\sqrt{255}}\right)e^{\frac{5t}{16}}\sin\left(\frac{5\sqrt{255}t}{16}\right)$$

$$+\left(\frac{117014853\lambda^2}{135672726200} - \frac{21801073\lambda}{152631816975}\right.$$

$$- \frac{342689}{14365347480}\Bigg) e^{\frac{5t}{16}} \cos\left(\frac{5\sqrt{255}t}{16}\right)$$

$$+ \Bigg[\Bigg(- \frac{675884312\lambda^2}{163763295\sqrt{255}} + \frac{1449273611\lambda}{6878058390\sqrt{255}}$$

$$+ \frac{42452231179}{1760782947840\sqrt{255}}\Bigg) e^{\frac{5t}{16}}$$

$$+ \left(\frac{1523267\lambda^2}{347140\sqrt{255}} - \frac{25021\lambda}{173570\sqrt{255}} - \frac{2879\sqrt{3}}{347140\sqrt{85}}\right) e^{\frac{3t}{16}}\Bigg] \sin\left(\frac{3\sqrt{255}t}{16}\right)$$

$$+ \Bigg[\left(- \frac{31203544\lambda^2}{272938825} - \frac{174224063\lambda}{3821143550} + \frac{4564663}{34525155840}\right) e^{\frac{5t}{16}}$$

$$+ \left(\frac{32829\lambda^2}{347140} + \frac{8253\lambda}{173570} + \frac{61}{347140}\right) e^{\frac{3t}{16}}\Bigg] \cos\left(\frac{3\sqrt{255}t}{16}\right)$$

$$+ \Bigg[\Bigg(\frac{42519101440\lambda^2}{163763295\sqrt{255}} + \frac{41970122752\lambda}{3439029195\sqrt{255}}$$

$$+ \frac{227918144}{3439029195\sqrt{255}}\Bigg) e^{\frac{5t}{16}}$$

$$- \left(\frac{1172528\lambda^2}{86785\sqrt{85}} + \frac{1422529\sqrt{3}\lambda}{173570\sqrt{85}} + \frac{2987201}{22216960\sqrt{255}}\right) e^{\frac{3t}{16}}$$

$$+ \Bigg(\frac{3051545301\sqrt{3}\lambda^2}{694794584\sqrt{85}} + \frac{4877576339\sqrt{3}\lambda}{1215890522\sqrt{85}}$$

$$+ \frac{20650444499}{311267973632\sqrt{255}}\Bigg) e^{\frac{t}{16}}\Bigg] \sin\left(\frac{\sqrt{255}t}{16}\right)$$

$$+\left[\left(-\frac{1169752064\lambda^2}{272938825}+\frac{1528152064\lambda}{17195145975}+\frac{4954816}{202295835}\right)e^{\frac{5t}{16}}\right.$$

$$+\left(\frac{783504\lambda^2}{86785}-\frac{8253\lambda}{173570}-\frac{1059519}{22216960}\right)e^{\frac{3t}{16}}$$

$$+\left(\frac{3281824383\lambda^2}{694794584}+\frac{52445677\lambda}{1215890522}\right.$$

$$\left.\left.-\frac{1018853333}{44466853376}\right)e^{\frac{t}{16}}\right]\cos\left(\frac{\sqrt{255}t}{16}\right).$$

Finally, the approximate solution of (2) is given by

$$u(t)\simeq \lim_{p\to 1}[v_0(t)+pv_1(t)+p^2v_2(t)]$$

$$\simeq v_0(t)+v_1(t)+v_2(t).$$

3. The Regular Perturbation Method

Let us suppose that the solution $u(t)$ of the initial value problem (2) has the following form [2]:

$$u(t)=\sum_{n=0}^{+\infty}\lambda^n u_n(t)+R_N(t,\lambda), \tag{8}$$

where $R_N(t, \lambda)$ is the remainder of the series.

Taking (8) into (2), and collecting equal powers of λ we obtain a system of recurrent initial value problems for $u_n(t)$, $n = 0, 1, 2, \ldots$

$$\lambda^0:\begin{cases}\dfrac{d^2u_0}{dt^2}+u_0-\mu(1-u_0^2)\dfrac{du_0}{dt}=0\\ u_0(0)=1; \qquad\qquad \dfrac{du_0(0)}{dt}=0,\end{cases} \tag{9}$$

$$\lambda^1 : \begin{cases} \dfrac{d^2u_1}{dt^2} + u_1 - \mu\left[(1-u_0^2)\dfrac{du_1}{dt} - 2u_0u_1\dfrac{du_0}{dt}\right] + u_0^3 = 0 \\ u_1(0) = 0; \qquad \dfrac{du_1(0)}{dt} = 0, \end{cases} \tag{10}$$

$$\lambda^2 : \begin{cases} \dfrac{d^2u_2}{dt^2} + u_2 - \mu\left[(1-u_0^2)\dfrac{du_2}{dt} - 2u_0u_1\dfrac{du_1}{dt} \right.\\ \qquad\qquad \left. - (u_1^2 + 2u_0u_2)\dfrac{du_0}{dt}\right] + 3u_0^2u_1 = 0 \\ u_2(0) = 0; \qquad \dfrac{du_2(0)}{dt} = 0. \end{cases} \tag{11}$$

Consider the initial value problems (9), (10) and (11), and suppose that the solution of each one of them is of the following form:

$$u_k(t) = \sum_{n=0}^{+\infty} \mu^n u_{kn}(t), \tag{12}$$

where $k = 0, 1, 2.$

Taking (12) into (9), (10) and (11), and collecting equal powers of μ, we obtain a linear system of recurrent initial value problems for $u_{kn}(t)$, $k = 0, 1, 2$ and $n = 0, 1, \ldots.$

For $k = 0$

$$\mu^0 : \begin{cases} \dfrac{d^2u_{00}}{dt^2} + u_{00} = 0 \\ u_{00}(0) = 1; \qquad \dfrac{du_{00}(0)}{dt} = 0, \end{cases} \tag{13}$$

$$\mu^1 : \begin{cases} \dfrac{d^2u_{01}}{dt^2} + u_{01} - (1-u_{00}^2)\dfrac{du_{00}}{dt} = 0 \\ u_{01}(0) = 1; \qquad \dfrac{du_{01}(0)}{dt} = 0. \end{cases} \tag{14}$$

For $k = 1$

$$\mu^0 : \begin{cases} \dfrac{d^2 u_{10}}{dt^2} + u_{10} + u_{00}^3 = 0 \\ u_{10}(0) = 0; \qquad \dfrac{du_{10}(0)}{dt} = 0, \end{cases} \tag{15}$$

$$\mu^1 : \begin{cases} \dfrac{d^2 u_{10}}{dt^2} + u_{11} - \left[(1 - u_{00}^2) \dfrac{du_{10}(0)}{dt} \right. \\ \qquad\qquad \left. - 2u_{00}u_{10} \dfrac{du_{00}(0)}{dt} - 3u_{00}^2 u_{01} \right] = 0 \\ u_{11}(0) = 0; \qquad \dfrac{du_{11}(0)}{dt} = 0. \end{cases} \tag{16}$$

For $k = 2$

$$\mu^0 : \begin{cases} \dfrac{d^2 u_{20}}{dt^2} + u_{20} + 3u_{00}^3 u_{10} = 0 \\ u_{20}(0) = 0; \qquad \dfrac{du_{20}(0)}{dt} = 0, \end{cases} \tag{17}$$

$$\mu^1 : \begin{cases} \dfrac{d^2 u_{21}}{dt^2} + u_{21} - \left[(1 - u_{00}^2) \dfrac{du_{20}}{dt} - 2u_{00}u_{10} \dfrac{du_{10}}{dt} \right. \\ \qquad \left. - (u_{10}^2 + 2u_{00}u_{20}) \dfrac{du_{00}}{dt} + 3u_{00}(2u_{01}u_{10} + u_{00}u_{11}) \right] = 0 \\ u_{20}(0) = 0; \qquad \dfrac{du_{20}(0)}{dt} = 0. \end{cases}$$

(18)

From (13), we have

$$u_{00}(t) = \cos t.$$

From (14), we have

$$u_{01}(t) = -\frac{1}{32} \sin(3t) - \frac{9}{32} \sin t + \frac{3}{8} t \cos t.$$

From (15), we have

$$u_{10}(t) = \frac{1}{32}\cos(3t) - \frac{3}{8}t\sin t - \frac{1}{32}\cos t.$$

From (16), we have

$$u_{11}(t) = -\frac{1}{384}\sin(5t) - \frac{9}{256}\sin(3t) + \left(-\frac{9}{32}t^2 + \frac{295}{768}\right)\sin t - \frac{17}{64}t\cos t.$$

From (17), we have

$$u_{20}(t) = \frac{1}{1024}\cos(5t) - \frac{9}{256}t\sin(3t) - \frac{3}{128}\cos(3t)$$

$$+ \frac{3}{32}t\sin t + \left(-\frac{9}{128}t^2 + \frac{23}{1024}\right)\cos t.$$

From (18), we have

$$u_{21}(t) = -\frac{7}{49152}\sin(7t) + \frac{11}{49152}\sin(5t) - \frac{25}{8192}t\cos(5t)$$

$$+ \left(-\frac{135}{4096}t^2 + \frac{361}{4096}\right)\sin(3t) - \frac{717}{8192}t\cos(3t)$$

$$+ \left(\frac{915}{4096}t^2 - \frac{4209}{8192}\right)\sin t + \left(-\frac{81}{1024}t^3 + \frac{87}{256}t\right)\cos t.$$

We give the approximate solution of (2) for $\mu = \frac{1}{4}$ and $\mu = \frac{1}{8}$.

Case: $\mu = \frac{1}{4}$

$$u(t) \simeq u_0(t) + \lambda u_1(t) + \lambda^2 u_2(t)$$

$$\simeq \left[u_{00}(t) + \frac{1}{4}u_{01}(t)\right] + \left[u_{10}(t) + \frac{1}{4}u_{11}(t)\right]\lambda + \left[u_{20}(t) + \frac{1}{4}u_{21}(t)\right]\lambda^2.$$

Case: $\mu = \frac{1}{8}$

$$u(t) \simeq u_0(t) + \lambda u_1(t) + \lambda^2 u_2(t)$$

$$\simeq \left[u_{00}(t) + \frac{1}{8}u_{01}(t)\right] + \left[u_{10}(t) + \frac{1}{8}u_{11}(t)\right]\lambda + \left[u_{20}(t) + \frac{1}{8}u_{21}(t)\right]\lambda^2.$$

4. Solutions Analysis

In this section, we analyze the approximate solutions of (2) obtained by the two numerical methods (HPM and RPM). We vary the perturbation parameters μ and λ in order to observe their incidence on the approximate solutions.

Tables 1 and 2 give some values of the solutions approached for $\mu = \frac{1}{4}$, $\mu = \frac{1}{8}$, $\mu = \frac{1}{16}$ and $\mu = \frac{1}{32}$.

Table 1. Numerical analysis for $\lambda = 0.002$

(a) $\mu = \frac{1}{4}$

t	U_{hpm}	U_{rpm}	$\lvert U_{hpm} - U_{rpm} \rvert$
0	1.0000	1.0000	0
0.1	0.9950	0.9950	0
0.2	0.9800	0.9800	0
0.3	0.9552	0.9552	0
0.4	0.9208	0.9208	0
0.5	0.8770	0.8770	0
0.6	0.8241	0.8241	0
0.7	0.7625	0.7626	0.0001
0.8	0.6927	0.6927	0
0.9	0.6149	0.6150	0.0001
1	0.5297	0.5299	0.0002

(b) $\mu = \frac{1}{8}$

t	U_{hpm}	U_{rpm}	$\lvert U_{hpm} - U_{rpm} \rvert$
0	1.0000	1.0000	0
0.1	0.9950	0.9950	0
0.2	0.9800	0.9800	0
0.3	0.9552	0.9552	0
0.4	0.9208	0.9208	0
0.5	0.8772	0.8772	0
0.6	0.8246	0.8246	0
0.7	0.7635	0.7635	0
0.8	0.6944	0.6944	0
0.9	0.6180	0.6180	0
1	0.5347	0.5348	0.0001

Table 2. Numerical analysis for $\lambda = 0.002$

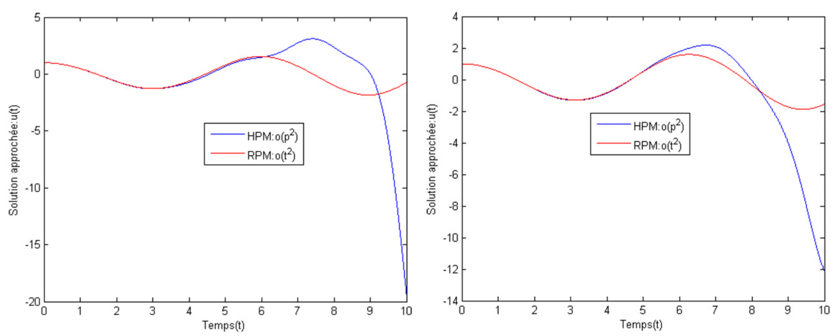
(a) $\mu = \frac{1}{16}$

t	U_{hpm}	U_{rpm}	$\lvert U_{hpm} - U_{rpm} \rvert$
0	1.0000	1.0000	0
0.1	0.9950	0.9950	0
0.2	0.9800	0.9800	0
0.3	0.9552	0.9552	0
0.4	0.9209	0.9209	0
0.5	0.8773	0.8773	0
0.6	0.8248	0.8248	0
0.7	0.7640	0.7640	0
0.8	0.6953	0.6953	0
0.9	0.6195	0.6195	0
1	0.5371	0.5372	0.0001

(b) $\mu = \dfrac{1}{32}$

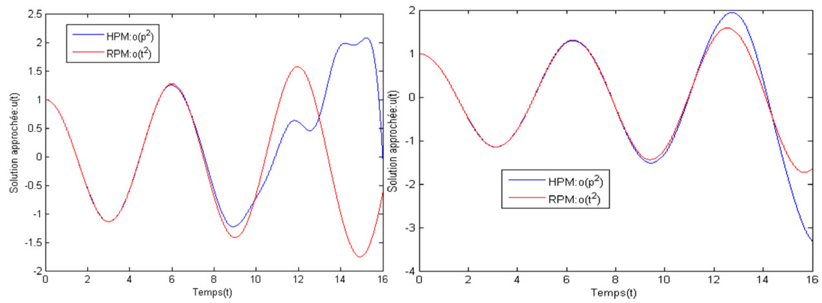
t	U_{hpm}	U_{rpm}	$\lvert U_{hpm} - U_{rpm} \rvert$
0	1.0000	1.0000	0
0.1	0.9950	0.9950	0
0.2	0.9800	0.9800	0
0.3	0.9552	0.9552	0
0.4	0.9209	0.9209	0
0.5	0.8773	0.8773	0
0.6	0.8249	0.8249	0
0.7	0.7642	0.7642	0
0.8	0.6958	0.6958	0
0.9	0.6202	0.6202	0
1	0.5384	0.5384	0

It is noted that the variation of the parameter λ does not influence the approximate solutions so much. However the variation of the parameter μ disturbs the approximate solutions. For small time interval the solutions obtained by the two methods are practically the same. But for large time interval the solutions obtained by the two methods are different. We can observe it on Figures 2-4.



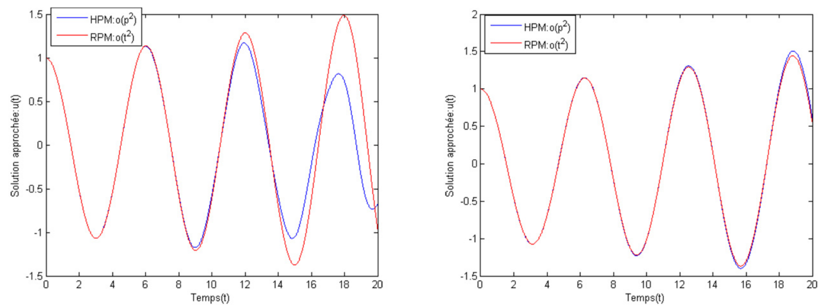
(a) HPM and RPM: $\lambda = 0.1$ (b) HPM and RPM: $\lambda = 0.002$

Figure 1. Comparison of the HPM solution with RPM solution for $\mu = \dfrac{1}{4}$.



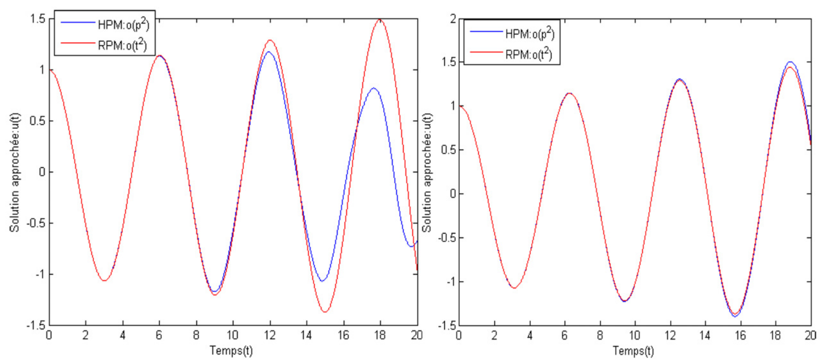
(a) HPM and RPM: $\lambda = 0.1$ (b) HPM and RPM: $\lambda = 0.002$

Figure 2. Comparison of the HPM solution with RPM solution for $\mu = \frac{1}{8}$.



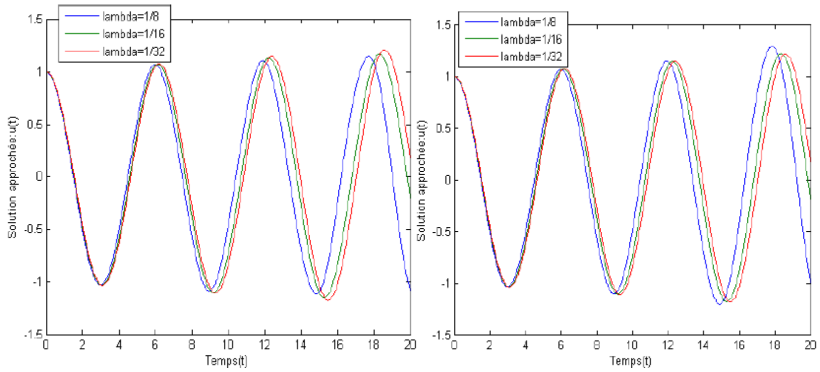
(a) HPM and RPM: $\lambda = 0.1$ (b) HPM and RPM: $\lambda = 0.002$

Figure 3. Comparison of the HPM solution with RPM solution for $\mu = \frac{1}{16}$.



(a) HPM and RPM: $\lambda = 0.1$ (b) HPM and RPM: $\lambda = 0.002$

Figure 4. Comparison of the HPM solution with RPM solution for $\mu = \frac{1}{16}$.



(a) Solution by HPM: $\mu = \dfrac{1}{32}$ (b) Solution by RPM $\mu = \dfrac{1}{32}$

Figure 5. Solution of the HPM and RPM solution for different values of λ.

5. Conclusion

In this paper, we applied HPM to obtain analytical solution of Duffing-Van der Pol equation. The results obtained from this method have been compared with those obtained from regular perturbation. A numerical comparison between HPM and regular perturbation method is depicted in Figures 2-4. The HPM converges in a small time interval and computationally takes long time for a large interval. The HPM series gives reasonable results in the small time interval.

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