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DECOUPLING OF NONLINEAR CONTROL SYSTEM BASED ON LIE SYMMETRY METHOD

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Abstract

The Lie symmetry method is introduced for the decoupling problem of nonlinear control system. Firstly, the key technology of Lie symmetry theory for differential equations is introduced; secondly, a kind of nonlinear control system model is established, and the conditions and properties of Lie symmetry are given in detail. Finally, the decoupling global and local forms of the system are given through the derived distribution of infinitesimal generators. The numerical results show the effectiveness of Lie symmetry method. As long as the infinitesimal generator is constructed, the cascade decoupling form of nonlinear control system could be obtained.

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1. Introduction

Mathematically, the nonlinear control system with finite degrees of freedom is usually described by ordinary differential equations in the form of state space. The decoupling control refers to the matrix of transfer function of the closed-loop system which is a diagonal matrix through a certain control law such as state feedback [1]. Thus a coupling multivariable system is transformed into an independent single variable system. At present, a variety of control methods have been developed: modern frequency domain method, adaptive decoupling control method, robust decoupling control, intelligent decoupling control, predictive decoupling and disturbance decoupling [2-7]. It should be said that each of the above decoupling methods has its own unique advantages and limitations, such as the lack of complete use of the system's own characteristics such as symmetry and similarity before the design of precise control laws. As we all know, symmetric nonlinear control systems are widely used in practical engineering [8], we have some advantages to study the control problems by using symmetry. In the 19th century, Sophie Lie put forward the concept of symmetry of differential equation in the theory of continuous groups, and gave the general integration method of equation, namely, method by Lie group analysis [9, 10]. This method is equivalent to linear and nonlinear, constant coefficient and variable coefficient equations, and is applicable to both ordinary and partial differential equations. In fact, the theory of Lie group analysis has become the only general and effective method to solve differential equations.

Some achievements have been made in the study of nonlinear control problems by using the Lie symmetry method. Xiaoxin et al. [11, 12] had studied the structural decomposition and controllability of symmetric nonlinear control system; Palazoglu et al. [13, 14] proposed the idea of the control of distributed parameter system by using symmetry group; Frederico and Torres [15] and Torres [16] studied the optimal control problem and obtained the controller conservation laws by using symmetry group. On one hand, these works made the original complex and profound problems in nonlinear control that can be solved creatively, and on the other hand, they promote the further development of the theory of Lie symmetry. However, there is not a mature theory about using Lie symmetry method to study the decoupling problem of nonlinear control system.

In this paper, the conditions and forms of decoupling for a kind of nonlinear control problems by using Lie symmetry are discussed. The approach of decoupling is given by using the infinitesimal generator of Lie symmetry, and the results are illustrated numerically.

2. Theory of Lie Symmetry of First-order Ordinary Differential Equations

For the first-order ordinary differential equations:

$$\frac{dx^{\alpha}}{dt} = f_{\alpha}(t, \boldsymbol{x}) \; (\alpha = 1, 2, ..., n), \tag{1}$$

a continuous transformation group with single parameter is

$$G = \{T_a \,|\, \bar{t} = \varphi(t, \boldsymbol{x}, a), \; \bar{x}^{\alpha} = \psi^{\alpha}(t, \boldsymbol{x}, a)\},$$

its tangent vector field is $\frac{\partial T_a}{\partial a}|_{a=0} = (\xi(t, \boldsymbol{x}), \boldsymbol{\eta}(t, \boldsymbol{x}))$, the infinitesimal generator $X^{(0)}$ and first-order extension $X^{(1)}$ of G are, respectively,

$$X^{(0)} = \xi \frac{\partial}{\partial t} + \eta^i \frac{\partial}{\partial x^i}, \quad X^{(1)} = X^{(0)} + (\dot{\eta}^j - p^j \dot{\xi}) \frac{\partial}{\partial p^j},$$

$$p^j = \frac{dx^j}{dt} \quad (i, j = 1, 2, ..., n). \tag{2}$$

The sufficient and necessary conditions for the form invariance (Lie symmetry) of equations (1) under G are as follows:

$$X^{(1)}\left[\frac{dx^{\alpha}}{dt} - f_{\alpha}(t, \boldsymbol{x})\right]\Bigg|_{\frac{dx^{\alpha}}{dt} - f_{\alpha}(t, \boldsymbol{x}) = 0} = 0. \tag{3}$$

The group G is also called the *symmetric group* of equations.

3. Lie Symmetry of Nonlinear Control System

A nonlinear control system with single input and output is a quintuple $\sum(M \times U, M, \boldsymbol{f}, Y, h)$, where $\boldsymbol{f}$ is a smooth mapping and $\pi_M(\boldsymbol{f}) = \pi$, $\pi_M : TM \to M$ is the tangent bundle projection, $\pi : M \times U \to M$ is the smooth fiber bundle, $h : M \to Y$ is a smooth mapping, M, U, Y are smooth state input and output manifolds. If the coordinates of M are $\boldsymbol{x}$, the coordinate of U is $\boldsymbol{u}$, the coordinate of Y is y, then the local coordinates of the nonlinear system are:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, u),$$

$$y = h(\boldsymbol{x}). \tag{4}$$

The Lie symmetry of the nonlinear control system (4) under fixed control quantity can be described as follows:

$$X^{(1)}[\dot{\boldsymbol{x}} - \boldsymbol{f}(\boldsymbol{x}, u)]|_{\dot{\boldsymbol{x}} - \boldsymbol{f}(\boldsymbol{x}, u) = 0} = 0,$$

$$X^{(0)}[h(\boldsymbol{x})] = 0. \tag{5}$$

Equation (5) indicates that the system has both state symmetry and output symmetry.

Using the Lie bracket [,] in the differential operator space $X^{(0)}$, we get the corresponding Lie algebra structure $\{X^{(0)}\}$:

$$[X_1^{(0)}, X_2^{(0)}] = X_1^{(0)} X_2^{(0)} - X_2^{(0)} X_1^{(0)}. \tag{6}$$

Let $\Phi : G \times M \to M$ to be a smooth action of G to M. Then the nonlinear control system $\sum$ is called *Lie symmetry* with (G, Φ). The symmetric nonlinear control system has some special properties, which will bring some advantages to the solution of control problems, such as decoupling control, noninteracting control and optimum control.

4. The Decoupling of System

The global decoupling of symmetric nonlinear control system $\sum$ refers to: (1) Φ is free and normal; (2) M is diffeomorphic to $(M/G) \times G$, where M/G is the quotient manifold, that is, there is a smooth cross section $s : M/G \to M$ and for the projection mapping $p : M \to M/G$, $p \circ s$ the identity mapping, so that $\sum$ can be decomposed in the large:

$$\dot{z}^{(1)} = \boldsymbol{f}^{(1)}(z^{(1)}, u), \quad z^{(1)} \in M/G,$$

$$\dot{z}^{(2)} = \boldsymbol{f}^{(2)}(\mathbf{z}^{(1)}, \dot{z}^{(2)}, u), \quad z^{(2)} \in G,$$

$$y = h(z^{(1)}, z^{(2)}). \tag{7}$$

Equation (7) shows that the symmetric nonlinear control system can be expressed as a cascade decoupled system with quotient system and independent subsystem. If the solution of the quotient system can be obtained and the solution of the independent subsystem can be expressed by the solution of the quotient system, then the entire system will be solved. The proof can be found in [17].

The local decoupling of symmetric nonlinear control system $\sum$ refers to: If the action Φ is nondegenerate at the point of p, then there are local coordinates $(z_1, z_2, ..., z_n)$ of $p \in M$, namely, the coordinate transformation $z = \boldsymbol{W}(\boldsymbol{x})$ which transforms the system to

$$\dot{z}^{(1)} = \boldsymbol{f}^{(1)}(z^{(1)}, u),$$

$$\dot{z}^{(2)} = \boldsymbol{f}^{(2)}(\mathbf{z}^{(1)}, \dot{z}^{(2)}, u),$$

$$\boldsymbol{y} = h(z^{(1)}). \tag{8}$$

Here $\boldsymbol{z}^{(1)} = (z_1, ..., z_{n-k})^T$, $\boldsymbol{z}^{(1)} = (z_{n-k+1}, ..., z_n)^T$, $k = \dim G$.

The coordinate transformation satisfies:

$$z_i = L_{\boldsymbol{f}}^{i-1} h(\boldsymbol{x}) \quad (i = 1, ..., r),$$

$$z_j = \lambda_{j-r}(\boldsymbol{x}) \quad (j = r+1, \ldots, n-k),$$

$$z_q = \sigma_{q-n+k}(\boldsymbol{x}) \quad (q = n-k+1, \ldots, n). \tag{9}$$

Here r is the correlation degree of M and M/G,

$$L_{\boldsymbol{f}}^{i-1}(h(\boldsymbol{x}) = d^{i-1}(h(\boldsymbol{x}))[\boldsymbol{f}(\boldsymbol{x})])$$

is Lie derivative operation, $\lambda \in C^{\infty}(M/G)$, $\sigma \in C^{\infty}(M)$. The proof can be found in [18].

5. Numerical Example

Consider the system with $M = \{\boldsymbol{x} = (x_1, \ldots, x_4) | x_i > 0,\ i = 1, 2, 3, 4\}$:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -x_1^2 x_3^{-1} \\ x_2 \\ x_4^{-1} x_3^2 \\ x_1^{-1} x_3 x_4 \end{bmatrix} + \begin{bmatrix} x_1^2 x_3^{-1} \\ x_1 \\ -x_3 \\ -x_4 \end{bmatrix} u,$$

$$y = x_1^{-1} x_2. \tag{10}$$

According to (5), the invariance of equation (10) under a Lie group satisfies:

$$\begin{cases} \dot{\eta}^1 - \dot{x}_1\dot{\xi} - \eta^1(-2x_1x_3^{-1} + 2x_1x_2^{-1}u) - \eta^2(-x_1^2x_2^{-2}u) - \eta^3(x_1^2x_3^{-2}) = 0, \\ \dot{\eta}^2 - \dot{x}_2\dot{\xi} - \eta^1(u) - \eta^2 = 0, \\ \dot{\eta}^3 - \dot{x}_3\dot{\xi} - \eta^3(2x_4^{-1}x_3 - u) - \eta^3(-x_4^{-2}x_3^2) = 0, \\ \dot{\eta}^4 - \dot{x}_4\dot{\xi} - \eta^1(-x_1^{-2}x_3x_4) - \eta^3(x_1^{-1}x_4) - \eta^4(x_1^{-1}x_3 - u) = 0, \\ \eta^1(-x_1^{-2}x_2) + \eta^2(x_1^{-1}) = 0. \end{cases} \tag{11}$$

It is easy to see that (11) has a set of solutions as

$$\xi = 1, \quad \eta^1 = \eta^2 = \eta^3 = \eta^4 = 0. \tag{12}$$

Therefore, Lie symmetric group corresponding to (10) is $G = (0, ..., +\infty)$, and the action Φ on M can be expressed as:

$$\Phi : G \times M \to M, \quad (g, \boldsymbol{x}) \to \Phi_g(\boldsymbol{x}) = g\boldsymbol{x}. \tag{13}$$

It is easy to check that $k = \dim G = 1$, $r = 2$, and thus

$$\begin{aligned} z_1 &= x_1^{-1} x_2, \\ z_2 &= x_1^{-1} x_2 + x_3^{-1} x_2, \\ z_3 &= x_3^{-1} x_4, \\ z_4 &= x_1^{-1} x_2 \ln x_2 + \ln x_4. \end{aligned} \tag{14}$$

From $\boldsymbol{z}^{(1)} = (z_1, z_2, z_3)^T \in M/G$, $\boldsymbol{z}^{(2)} = z_4 \in G$, the system with coordinate transformation is converted into

$$\begin{aligned} \dot{z}_1 &= z_2, \\ \dot{z}_2 &= 2z_2 - z_1 - (z_2 - z_1) z_3^{-1} + (z_2 - z_1)(z_1^{-1} + 1) u, \\ \dot{z}_3 &= z_1 z_3 (z_2 - z_1)^{-1} - 1, \\ \dot{z}_4 &= (z_1 + 1)^{-1} z_2 (z_4 + \ln(z_2 - z_1) z_3^{-1}) + z_1 + z_1 (z_2 - z_1)^{-1}, \\ y &= z_1. \end{aligned} \tag{15}$$

Obviously, the three equations in (15) are in the quotient space, and the fourth equation becomes an independent subsystem without control variables, which is in series with the quotient space, and the system is locally decoupled.

6. Conclusion

In this paper, the decoupling of a kind of nonlinear control system is obtained by using the classical Lie symmetry. A numerical example is given to provide the specific steps of calculation. The main conclusions are as follows: (1) the decoupling of symmetric nonlinear control system only needs to verify whether the derived distribution of infinitesimal generator satisfies the controllability and compatibility; (2) the decoupling form of symmetric nonlinear control system is equivalent to the quotient system with dimension $n - k$ (the difference between system and Lie group) and independent subsystem with dimension k (Lie group), which are in series.

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