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ON FINITE CHARACTER GEOMETRICAL PROPERTY OF THE DIFFERENTIAL REALIZATION OF NONSTATIONARY HYPERBOLIC SYSTEMS

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Abstract

Topological-algebraic investigation of the problem of existence of realization of finite-dimensional continuous dynamic processes in the class of second-order ordinary differential equations in a separable Hilbert space has been conducted. Simultaneously, analytical-geometric conditions of continuity of the process of constructing

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projections for the Rayleigh-Ritz nonlinear functional operator together with computation of the fundamental group of its image have been determined. The results may be applied to a posteriori modeling nonstationary hyperbolic systems.

0. Introduction

Despite the fact that rather efficient algorithms have been proposed and some important applications have been developed in the theory of realization/identification of finite-dimensional systems [1-4], one can state that there is the need to construct an integrated formal basement for the case (issue) of solvability for the problem of mathematical modeling for dissipative-wave systems on the basis of results of observing the processes of their functioning. On the other hand, the application efficiency of the qualitative approach, which is bound up with investigation of the problem of realization of behavioristic models (see e.g. [5-13]), has (in the variant of differential realization) substantial importance right for nonstationary hyperbolic systems [1], at least to the extent in which the mathematician involved in solving the applied problem may expect such importance and the related efficiency from the totally abstract topological-algebraic constructions. Within the given context, the present paper represents the results of development of previous investigations [11-13]. It has been ideologically adapted to the problem of a posteriori modeling of second-order controlled (program-positionally controlled) nonstationary differential systems in the real separable Hilbert space.

1. Principal Denotations and the Terminology

From now on, $(X, \|\cdot\|_X)$, $(Y, \|\cdot\|_Y)$, $(Z, \|\cdot\|_Z)$ represent real separable Hilbert spaces (pre-Hilbert characters are defined by norms $\|\cdot\|_X, \|\cdot\|_Y, \|\cdot\|_Z$), $L(Y, X)$ represents the Banach space (with the operator norm) of all linear continuous operators acting from space Y into X ($L(X, X)$, $L(X, Z)$, $L(Z, X)$ are defined similarly), T is an intercept of numerical line R with Lebesgue measure μ, $AC^1(T, X)$ represents the linear set of functions on T with the values in space X, whose first derivative is absolutely continuous in interval T (with respect to measure μ).

If $(\mathcal{B}, \|\cdot\|)$ is a Banach space, then $L_2(T, \mu, \mathcal{B})$ denotes the Banach factor-space of all Bochner integrable [14] mappings $f : T \to \mathcal{B}$ with norm $\left(\int_T \| f(\tau)\|^2 \mu(d\tau)\right)^{1/2} < \infty$. Furthermore, for the purpose of comfort, let us agree that

$$\Pi := AC(T, X) \times AC^1(T, X) \times L_2(T, \mu, Y) \times L_2(T, \mu, Z).$$

Next, let $L(T, \mu, R)$ be the factor-space for the classes of μ-equivalence of all real μ-measurable on T functions, and $\leq_L$ be such quasi-ordering in $L(T, \mu, R)$ that $\phi_1 \leq_L \phi_2$ (for $\phi_1, \phi_2 \in L(T, \mu, R)$) holds if and only if $\phi_1(t) \leq \phi_2(t)$ μ-almost everywhere in T. Furthermore, for a fixed subset $W \subset L(T, \mu, R)$, by $\sup_L W$ let us denote the least upper bound (if it exists) of subset W in the structure of quasi-ordering $\leq_L$.

Assume that $\Psi : \Pi \to L(T, \mu, R)$ is a functional Rayleigh-Ritz operator [5, 8]:

$$\Psi(g(t), v(t), w(t), q(t)) := \begin{cases} \| dg(t)/dt \|_X / \| (g(t), v(t), w(t), q(t)) \|_U, \\ \text{when } (g(t), v(t), w(t), q(t)) \neq 0 \in U; \\ 0 \in R, \text{ when } (g(t), v(t), w(t), q(t)) = 0 \in U, \end{cases} \tag{1}$$

where $\| (\cdot, \cdot, \cdot, \cdot) \|_U := (\| \cdot \|_X^2 + \| \cdot \|_X^2 + \| \cdot \|_Y^2 + \| \cdot \|_Z^2)^{1/2}$ is a norm in the Cartesian product $U := X \times X \times Y \times Z$; according to "the parallelogram conditions" [15], space $(U, \| \cdot \|_U)$ is Hilbert. Note incidentally, the motivating arguments for the etymology of operator Ψ are given in [5].

When constructing operator Ψ, we have taken account of the Lemma [16, p. 505] and Lemma 3 [7], according to which the following μ-enclosure holds for all vector-functions $(g, v, w, q) \in \Pi$:

$$\{t \in T : \| (g(t), v(t), w(t), q(t)) \|_U = 0\}$$
$$\subset \{t \in T : \| dg(t)/dt \|_X = 0\} \ (\mathrm{mod}\, \mu).$$

This enclosure provides for functional correctness of construction (1).

The Rayleigh-Ritz operator satisfies the simple (but important) relationships:

$$0 \leq_L \Psi(\phi),\ 0 \in L(T, \mu, R), \quad \phi \in \Pi,$$
$$\Psi(r\phi) = \Psi(\phi), \quad 0 \neq r \in R. \tag{1'}$$

The geometric idea of relationships $(1')$ is explicated below in Theorem 3 and in its Corollaries 3-5, which relate the effect of operator (1) with the methods of projective representations [15].

While fixing the basic terminology, it is possible to identify another property of operator (1).

Definition 1 [5]**.** Rayleigh-Ritz operator is *semiadditive* with weight $p \geq 1$ on the family of dynamic processes $E \subset \Pi$ if for any 2-tuple $(\omega_1, \omega_2) \in E \times E$ the following condition holds:

$$\Psi(\omega_1 + \omega_2) \leq_L p\Psi(\omega_1) + p\Psi(\omega_2).$$

Consider the following second-order controlled (program-positionally controlled) non-stationary differential models (including hyperbolic ones [1]) of the following form on the time interval T:

$$d^2x(t)/dt^2 + A_1(t)dx(t)/dt + A_0(t)x(t) = B(t)u(t)$$
$$+ B^{\#}(t)u^{\#}(dx(t)/dt, x(t)),$$

$$(A_1, A_0, B, B^{\#}) \in L_2(T, \mu, L(X, X)) \times L_2(T, \mu, L(X, X))$$
$$\times L_2(T, \mu, L(Y, X)) \times L_2(T, \mu, L(Z, X)), \tag{2}$$

where $x \in AC^1(T, X)$ is the Carathéodory trajectory (C-solution), $u \in L_2(T, \mu, Y)$ is a programmed control, $u^{\#} \in L(X \times X, Z)$ is an operator of linear positional control. We assume that the Cartesian square $X \times X$ has the topology corresponding to norm $(\|\cdot\|_X^2 + \|\cdot\|_X^2)^{1/2}$. While proceeding from the considerations in [7], let us call the vector-function $(dx/dt, x, u, u^{\#}(dx/dt, x))$ also the C-solution of the differential system (2), and the ordered 4-tuple of operator-functions $(A_1, A_0, B, B^{\#})$ - the non-stationary $(A_1, A_0, B, B^{\#})_2$-model of system (2).

From now on, $u^{\#} \in L(X \times X, Z)$ is the given feedback operator in the contour of positional control, N is a fixed family (a bundle) of controlled dynamic processes of the form:

$$N \subset \Pi_{u^{\#}}$$

$$:= \{(dx/dt, x, u, u^{\#}(dx/dt, x)) \in \Pi : x \in AC^1(T, X), u \in L_2(T, \mu, Y)\}, \quad (3)$$

where $\mathrm{Card}\, N \leq \exp \aleph_0$ ($\aleph_0$-aleph-zero, $\exp \aleph_0$-continuum power).

Within the context of [7], the family of processes (3) assigns a continuous *behavioral dynamic system N* (possibly formed a *posteriori*). Let us introduce the following two potential structural properties for system N (below $\subset$ does not exclude $\subseteq$):

Definition 2 [6]**.** The bundle of vector-functions $P \subset N$ possesses:

- the property of *second-order ordinary linear differential compatibility* (*OLD-compatibility*), when either $P = \varnothing$, or there exists a differential system (2) such that the functional bundle P belongs to the class of admissible C-solutions of this system;

- the property of *second-order distributed linear differential compatibility* (*DLD-compatibility*) of degree k (fixed real number), when either $P = \varnothing$ or any subset $P' \subset \text{abs co}(P)$, $\text{Card}\, P' = k$ forms an OLD-compatible second-order set.

Remark 1. Sometimes for the purpose of brevity the term “second-order” (order of the differential system (2)) will be omitted. Furthermore, it is necessary to pay attention to the following facts:

(i) the requirement (geometric condition) in the structure of DLD-compatibility of existence of *absolute* convexity of the functional set P is essential because a situation is possible, when any subset $P' \subset \text{co}(P)$, $\text{Card}\, P' = k$ forms an OLD-compatible set, while set itself P does not possess the property of DLD-compatibility degree k;

(ii) OLD-compatibility of set P does not provide for uniqueness of the $(A_1, A_0, B, B^{\#})_2$-model, which – via the differential equation (2) – realizes the family of dynamic processes P;

(iii) DLD-compatibility of an arbitrary degree k (including the case when $\text{Card}\, P = \aleph_0$) is not equivalent to existence of OLD-compatibility.

We do not plan to consider below the theory of DLD-compatibility of degree k in the broad sense, i.e., when k is a given cardinal number $\geq \aleph_0$. Such a consideration would have led us to undesirable set-theoretic complications (see Conclusion below). On the other hand, in case of the variant, when $\dim \text{Span}\, N < \aleph_0$, such a theory assumes quite natural further geometric generalization, while acquiring the form of analysis on smooth manifolds because the property of DLD-compatibility may be easily reformulated in terms of finite-dimensional projective geometry, say, in terms of the map atlas on real Grassmannian manifolds [15].

Henceforth, we intend to investigate the *relationship* and the *differences* in the structures of OLD- and DLD-compatibility.

2. Elements of Geometrical Analysis of OLD- and DLD-Compatibility

Obviously, second-order OLD- and DLD-compatibility is invariant with respect to the idempotent action of operator Span, what allows us to introduce the following geometric constructions.

Definition 3. If $P^* \subset N$ (similarly $P^\# \subset N$) is a maximum set, which possesses the property of second-order OLD-compatibility (similarly DLD-compatibility of degree k), then $\mathrm{Span}\, P^*$ (respectively, $\mathrm{Span}\, P^\#$) will be called an *ordinary layer* over N (respectively, *distributed layer of degree k* over N) and, if $N \subset \mathrm{Span}\, P^*$ $(N \subset \mathrm{Span}\, P^\#)$, then such a layer will be called *homogeneous*.

Remark 2. It may happen that there exist ordinary layers and a distributed homogeneous layer over N (furthermore, these all do not coincide) or there exists a distributed homogeneous layer of an arbitrary degree k, at the same time, any ordinary layer is absent.

At the first sight, the concept of DLD-compatibility seems rather exquisite in some sense, meanwhile, the considerations and statements of this section shall demonstrate that this concept is analytically productive indeed. For example, the following statement lemma states that second-order DLD-compatibility of any degree is the property of finite character [17]. Consequently, for *each* nonempty DLD-compatible subset of processes $P \subset N$ there exists a *maximum* DLD-compatible subset $P^\#$ such that $P \subset P^\# \subset N$. One can only feel sorry that, in the general case, this property of the dynamical system N does not work with respect to the criterion of OLD-compatibility.

Lemma 1. *The second-order criterion of DLD-compatibility of degree k for the bundle of dynamic processes* $N \subset \Pi_{u^\#}$ *is the property of finite character.*

As noted above, Lemma 1 states the following principal fact: according to the Teichmüller-Tukey Lemma [17], each nonempty family of dynamic processes $N \subset \Pi_{u^\#}$ either does not contain any nonempty subset possessing the property of DLD-compatibility (and hence also of OLD-compatibility) or some distributed layer (possibly a unique one) can obligatorily be found for the bundle N of dynamic processes under scrutiny. Noteworthy, the Teichmüller-Tukey Lemma represents an alternative form of the axiom of choice and, consequently, is independent of the continuum hypothesis, what, in turn, makes the comment expected in the Conclusion obvious.

Let us attribute space

$$H_2 := L_2(T, \mu, X) \times L_2(T, \mu, X) \times L_2(T, \mu, Y) \times L_2(T, \mu, Z)$$

with the topology with the norm

$$\| (g, v, w, q) \|_H$$

$$:= \left(\int_T (\| g(\tau) \|_X^2 + \| v(\tau) \|_X^2 + \| w(\tau) \|_Y^2 + \| q(\tau) \|_Z^2) \mu(d\tau) \right)^{1/2},$$

$$(g, v, w, q) \in H_2.$$

Note, H_2 is a Hilbert space due to the construction of $\| \cdot \|_H$.

Lemma 2. *Let the dynamic bundle* $N \subset \Pi_{u^\#}$ *form a second-order OLD-compatible set. Hence there exists such an ordinary layer E over* $\Pi_{u^\#}$ *closed in space* H_2 *that inclusion* $N \subset E$ *is implementable, furthermore, E is a second-order distributed layer of degree k over the family of processes* $\Pi_{u^\#}$, *where k is a real number.*

Lemma 2 has an important geometric corollary, demonstrating that OLD compatibility is topologically so “good” as it could be wanted.

Corollary 1. *The closure of an OLD-compatible set in the topology of Hilbert space* H_2 *is also an OLD-compatible set.*

According to Corollary 1, any ordinary layer not closed in Hilbert space H_2 may always be topologically extended by the action of the Kuratowski closure operator [17] until its closure with retaining the property of second-order OLD-compatibility. Within the given context, while making the terminology systematic, it is expedient to call such extension *the topological OLD-extension*. This provision is not valid with respect to the distributed layer because topological extension of DLD-compatibility is not always realizable.

The topological construction introduced leads to the question: whether unclosed layers, which assume topological OLD-extension, exist in space H_2? The answer is, no doubt, is positive: due to the Bair theorem on the category [17], these are the layers with a countable Hamel's basis (the algebraic basis [18]; see Remark 3 below). This confirms the relationship between the topological data structure of the layers and their algebraic dimension.

Corollary 2. *If an ordinary or distributed layer over the family of processes* $N \subset \Pi_{u^{\#}}$ *is closed in space* H_2, *then the power of its Hamel's basis cannot be equal to* $\aleph_0$.

Remark 3. When working with Hilbert space, mathematicians, as a rule, use the concept *Hilbert's basis* (its power is equal to $\aleph_0$ for separable spaces), which is different from the concept of *Hamel's basis* in the linear space (including Hilbert's linear space). Note that these bases coincide only in the finite-dimensional case. The difference consists in the fact that, in case of Hilbert's basis, infinite linear combinations are allowed, which are characterized by the topological convergence.

When following the terminology of "towers" (i.e., ascending sequences) of sets [15] over set $\Pi_{u^{\#}}$, it is possible to state that the structure of OLD-compatibility is *stronger* (in other words, *more exquisite*) than the structure of DLD-compatibility (in other words, DLD-compatibility is *weaker* (*more rough*) than the OLD-compatibility). This means that any OLD-compatible set of dynamic processes from $\Pi_{u^\#}$ is DLD-compatible for any degree *k*. In the general case, the inverse statement is not valid (see item (iii) of Remark 1 above).

Let us recall that G_N is a boundary [17] of set abs $\operatorname{co}(N)$ in the topological space Span *N* with the topology induced from the enveloping space H_2. Hence, the following theorem is valid due to Corollary 2.

Theorem 1. *If* $\operatorname{Card}\dim \operatorname{Span} N = \aleph_0$, *then space* Span *N is not full*.

Analysis of DLD-compatibility of a fixed family of dynamic processes (3) shall start from degree $k = 1$. So, the following statement may occupy not the last place in the set of theoretical aids for such qualitative investigations. The following theorem identifies the necessary conditions of existence of the hyperbolic system to be modeled.

Theorem 2. *A distributed homogeneous layer of degree* $k = 1$ *exists over the family of dynamic processes* $N \subset \Pi_{u^\#}$ *if and only if* $\Psi[G_N] \subset L_2(T, \mu, R)$. *Furthermore*, *the linear manifold Span N forms an ordinary homogeneous layer if and only if set* $\Psi[G_N]$ *is bounded from above in space* $(L_2(T, \mu, R), \leq_L)$, *what is equivalent to the statement that there exists* $\sup_L \Psi[G_N] \in L_2(T, \mu, R)$ *in the structure of quasi-ordering* $\leq_L$.

If each bundle of controlled processes $N_i \subset \Pi_{u^\#}$, $\operatorname{Card} N_i < \aleph_0$, $i = 1, \ldots, n$ *possesses the property of second-order DLD-compatibility of degree* $k = 1$, *then the family of processes* $\bigcup_{i=1,\ldots,n} N_i$ *has some differential realization* (2), *since the Rayleigh-Ritz operator is semiadditive with some weight* $p \geq 1$ *on manifold*

$$\operatorname{Span} N_1 + \operatorname{Span} N_2 + \cdots + \operatorname{Span} N_n.$$

Remark 4. Semiadditivity of the Rayleigh-Ritz operator is under the important geometrical dependence on the Teichmüller-Tukey Lemma [17], to wit, there exist maximum sets in family Π, on each of which operator (1) is semi-additive with some weight $p \geq 0$. So, in case when $p \in (0, 1)$, these sets of dynamic processes cannot be *linear* (the variant when $E = \{0\} \subset \Pi$ is excluded). So, in Theorem 2, the weight of semiadditivity is some constant $p \geq 1$. Within the given context, it is possible demonstrate that semiadditivity of the Relay-Ritz operator with the weight $p \geq 1$ together with the Teichmüller-Tukey Lemma lead to the following fact: if E is an ordinary layer from the statement of Lemma 2, then one can find a maximum linear set in the family of processes E, which is closed in space H_2, on which the Rayleigh-Ritz operator is semi-additive with the weight $p \geq 1$. Finally, this makes the formulation of the second part of Theorem 2 geometrically correct. It is also possible to determine the weight of semiadditivity (likewise in Theorem 2 [19]).

3. Projectivation of the Rayleigh-Ritz Operator

Due to formula $(1')$ it is possible to state that $\Psi \,|\, \operatorname{Span} N = \Psi \,|\, P_N$, where P_N is a real projective space associated with the linear manifold $\operatorname{Span} N$; i.e., P_N is a set of orbits of the multiplicative group $R^* = R\backslash\{0\}$, which acts on $\operatorname{Span} N\backslash\{0\}$. This statement clearly shows the topological properties of space P_N, $\operatorname{Card} N < \aleph_0$, compactness being one of such properties. In particular, if $\dim \operatorname{Span} N = 3$, then 2-manifold P_N represents likewise the Möbius band to which a round is attached along its boundary. Note also that it is possible to introduce a structure of *CW*-complex [20] on space P_N, $\operatorname{Card} N \leq \aleph_0$. This is important in the case of considering the problem of geometrical realization of manifold P_N (see Theorem 9.7 [20]). Having used these refinements, it is possible to write a “projective version” of Theorem 2. In this case, compactness of P_N suggests the idea that continuity of the Rayleigh-Ritz operator may play an important role. Below, when defining the metric structure in $L(T, \mu, R)$, we have used Theorems 15, 16 [18, pp. 65, 67].

Theorem 3. *Consider* $L(T, \mu, R)$ *with the topology induced by convergence with the measure* μ, *or, what is equivalent, consider a complete separable metric space, while introducing the metric for this space*

$$\rho_T(\xi, \zeta) := \int_T |\xi(\tau) - \zeta(\tau)|(1 + |\xi(\tau) - \zeta(\tau)|)^{-1} \mu(d\tau),\ \xi, \zeta \in L(T, \mu, R),$$

and let $N \subset \Pi_{u^\#}$, $\mathrm{Card}\, N < \aleph_0$, *furthermore*, $(g, v, w, q) \in N$ *implies* $w = 0$. *Hence if bundle* N *is DLD-compatible one of degree* 1, *then the Rayleigh-Ritz operator* $\Psi : P_N \to L(T, \mu, R)$ *is continuous*.

Corollary 3. *Operator* $\Psi : P_N \to L(T, \mu, R)$ *is continuous for a finite bundle* $N \subset \Pi_{u^\#}$, *when for an arbitrary choice of* $(g, v, w, q) \in \mathrm{Span}\, N$ *and* $t \in T_g := \{t \in T : g(t) = 0 \in X\}$ *one can find a real number* $\delta_t > 0$ *such that* $\mu((t - \delta_t, t + \delta_t) \cap T_g) = 0$. *In this case*, $\Psi[P_N] = f[F]$, *where* $F \subset [0, 1]$ *is a Cantor set*, $f : F \to L(T, \mu, R)$ *is some continuous mapping*.

(The issue of existence of a mapping similar to *f* has been considered in [21].)

In the process of considering Corollary 3 and the first part of Theorem 2 it is expedient to involve also Corollary 4. In order to use Corollary 4, it is necessary to introduce the following auxiliary construction. Having taken a positive integer *n*, let us denote by W_n some finite n^{-1}-dense subset in $\Psi[P_N]$ (subset W_n may be found due to Corollary 3 and Theorem 2 [18, p. 44]), furthermore, $\{W_n\}_{n=1,2,\dots}$ is an ε-network in the metric space $(\Psi[P_N], \rho_T)$.

Corollary 4. *Let* $f_n := \sup_L \bigcup_{i=1,\ldots,n} W_i$, *hence* $\sup_L \Psi[P_N]$ *exists*, *if and only if* $\{f_n\}$ *is a Cauchy sequence in the metric space* $(L(T, \mu, R), \rho_T)$ *and hence*, *due to Theorem* 2, *when* $\sup_L \Psi[P_N] \in L_2(T, \mu, R)$, *the dynamical bundle N forms an OLD-compatible set*.

Corollary 5. *When one has the bundle* $N \subset \Pi_{u^\#}$, $\operatorname{Card} N < \aleph_0$, *continuity of Rayleigh-Ritz operator*, *which acts onto a CW-complex* P_N, *shall condition the following*:

(i) *set* $\Psi[P_N]$ *is compact*, *what guarantees the existence of point* $\gamma^* \in P_N$, *for which the following estimating relations hold*:

$$\rho_T(\Psi(\gamma^*), 0) = \sup\{\rho_T(\Psi(\gamma), 0) : \gamma \in P_N\} \leq \rho_T(\sup_L \Psi[P_N], 0),$$

$$\Psi(\gamma^*) \notin L_2(T, \mu, R) \Rightarrow \sup_L \Psi[P_N] \notin L_2(T, \mu, R),$$

$$\sup_L \Psi[P_N] = \sup_L \Psi[G_N];$$

(ii) *if the operator* $\Psi : P_N \to L(T, \mu, R)$ *is one-to-one*, *then* $\Psi \mid P_N$ *is a homeomorphism. Furthermore*, *the fundamental group* (*Poincare group*) *of the metric space* $(\Psi[P_N], \rho_T)$ *is a cyclic group* $\mathbb{Z}$ *when* $\dim \operatorname{Span} N = 2$, *and* $\mathbb{Z}_2$ *when* $\dim \operatorname{Span} N \geq 3$.

4. Parametric Identification of the Realization Model in the Form of Lagrange Equations

Obviously, the procedure of constructing the model of differential realization of the hyperbolic system under scrutiny has to be completed with its parametric identification [11]. So, in the capacity of an illustrative example, let us consider an algorithm intended to identify a differential model of small free damped transversal oscillations of a restrained gravity boom of a gyrostatic satellite [31] in the form of finite-dimensional autonomous Lagrange II kind equations:

$$Md^2 z(t)/dt^2 + Ddz(t)/dt + Sz(t) = 0, \tag{4}$$

where $z(t) \in R^n$, M, D, S are matrices, respectively, of inertia coefficients, damping and rigidity.

Under the conditions of *a posteriori* modeling, equation (4), the problem of identification of matrices $M^{-1}D$, $M^{-1}S$ may be constructed on the basis of measurements of the motions of vector $\check{z}(t) = \mathrm{col}(\check{z}_1(t), ..., \check{z}_n(t)) \in R^n$ on the time interval T taken from the solution of the following problem of parametric optimization

$$\min \int_T \| d^2\ \check{z}(\tau)/d\tau^2 + M^{-1}Dd\ \check{z}(\tau)/d\tau + M^{-1}S\ \check{z}(\tau) \|^2 d\tau, \tag{5}$$

where $\|\cdot\|$ is the Euclidean norm in R^n. This problem statement represents a direct generalization of the statement presuming the application of correlation methods. So, unlike that in case of the approach to identification proposed in [22], this approach shall possess higher noise immunity.

The procedure of solving the optimization problem (5) may be represented by a compact algorithmic scheme bound up with computing matrices $M^{-1}D$, $M^{-1}S$, the result being expressed by the following block-matrix relationship:

$$[M^{-1}D, M^{-1}S] = -\int_T \omega_d(\tau)\omega^*(\tau)d\tau \times \left[\int_T \omega(\tau)\omega^*(\tau)d\tau\right]^{-1},$$

$$\omega_d(t) := \mathrm{col}(d^2\check{z}_1(t)/dt^2, ..., d^2\check{z}_n(t)/dt^2) \in R^n,$$

$$\omega(t) := \mathrm{col}(d\check{z}_1(t)/dt, ..., d\check{z}_n(t)/dt, \check{z}_1(t), ..., \check{z}_n(t)) \in R^{2n}, \tag{6}$$

where $[M^{-1}D, M^{-1}S]$ is a block $n \times 2n$-matrix, "*" is an operation of transposition.

The algorithmic formula (6) allows one, while proceeding from the measurements of vector functions $\omega_d(\cdot)$ and $\omega(\cdot)$, to restore matrices

$M^{-1}D$, $M^{-1}S$. Piezoelectric accelerometers (the operating range being 1-3000 Hz; the range of measured accelerations being 0,1-100 g) were used in the capacity of the sensors [23]. Due to autonomy of system (4), vector-functions $\omega_d : T \to R^n$, $\omega : T \to R^{2n}$ may represent "agglutination" of separate motions on "subintervals" of various durations, while the total time continuity remains T.

Below one can find the result of *a posteriori* modeling (on the time interval $T = 10\,\text{sec}$) system (4) for the package of four standing waves (i.e., $n = 4$) registered in the gravity boom of a gyrostatic satellite, when the number of standing waves is equal to the number of piezoelectric accelerometers. The motion $t \mapsto \check{z}_i(t),\ i = 1, \ldots, 4$ in four fixed points of fixation of these piezoelectric accelerometers and the "disconcordances" (errors) $t \mapsto \Delta z_i(t),\ i = 1, \ldots, 4$ with the dynamics $t \mapsto z_i(t),\ i = 1, \ldots, 4$ of the identified model (4)-(6) are shown in Figures 1 and 2.

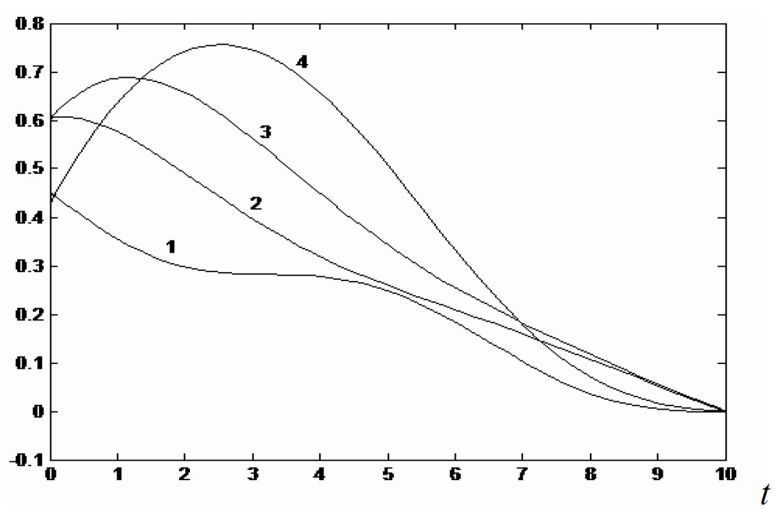

Figure 1. $\check{z}_1(t)$, $\check{z}_2(t)$, $\check{z}_3(t)$, $\check{z}_4(t)$ - indications of sensors of the scrutinized model, $z_1(t)$, $z_2(t)$, $z_3(t)$, $z_4(t)$ - indications of sensors of the identified model (i-graphs of $\check{z}_i(t)$ and $z_i(t)$ coincide because $|\Delta z_i(t)| < 8 \cdot 10^{-5}$, $i = 1, \ldots, 4$; see Figure 2).

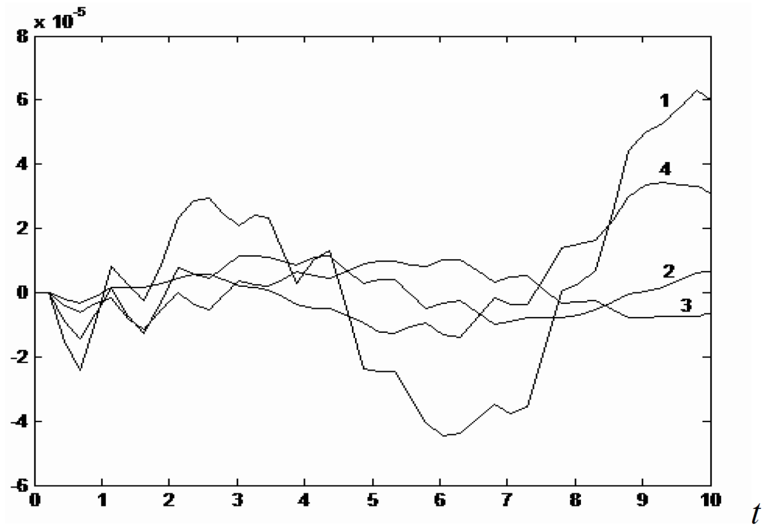

Figure 2. $\Delta z_1(t) := \check{z}_1(t) - z_1(t) < 8 \cdot 10^{-5}$, $\Delta z_2(t) := \check{z}_2(t) - z_2(t) < 2 \cdot 10^{-5}$, $\Delta z_3(t) := \check{z}_3(t) - z_3(t) < 2 \cdot 10^{-5}$, $\Delta z_4(t) := \check{z}_4(t) - z_4(t) < 4 \cdot 10^{-5}$.

5. Conclusion

DLD-compatibility in the variant of extension of Definition 2 up to the position, when degree k of the second-order DLD-compatibility is defined by the *cardinal number*, leads one (outside the frames of continuum hypothesis [24]) either to trivial assumptions, when $k = \operatorname{Card} N$ (for example, if bundle N is countable and DLD-compatible with degree $k = \aleph_0$, then N *a fortiori* has realization (2), similarly with $k = \operatorname{Card} N = \exp \aleph_0$), or to new problem statements for $k < \operatorname{Card} N \leq \exp \aleph_0$. In this case, the general philosophy of theoretical speculations is quite similar to that in the case, when degree k is a real number, but the opportunities are wider (in the context of Corollary 2), unfortunately, these are not always available to the researcher's intuition.

The structure of the tensor product of Fock's space [14] and the functional property of Relay-Ritz operator [25] allow one, who works on the basis of the property of tensor product's universality [15] and the results similar to those of Theorem 2 (on account of Corollary 5), to distribute the methods of projective geometry onto investigations of differential realization of the hyperbolic systems with nonlinear program-positional nonstationary controllers [26], which possess some polylinear structure [27]. It is possible to expect that investigations in this direction will step-by-step outline the contours of a new theory of nonlinear differential realization of higher-order [28-31] infinite-dimensional controlled behavioral systems, the theory, which would presume deeper penetration [32, 33] into the physics content of problems.

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