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SUMUDU AND ELZAKI INTEGRAL TRANSFORMS FOR SOLVING SYSTEMS OF INTEGRAL AND ORDINARY DIFFERENTIAL EQUATIONS

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Abstract

We present two integral transforms namely Sumudu (ST) and Elzaki (ET) transforms for solving systems of integral and ordinary differential equations. Also, we study some properties of these transforms. The presented integral transforms are new and simple for solving problems in systems of integral equations and ordinary

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differential equations. Some examples were successfully solved using these integral transforms. The obtained results reveal that the two integral transforms are effective.

1. Introduction

During the last years, many techniques and integral transforms were used to solve different types of problems in integral, ordinary and partial differential equations by researchers in sciences and engineering fields. Finding solutions to these problems of integral, ordinary and partial differential equations is very important nowadays. The integral and differential equations play an important role in a wide variety of applied science fields, including mathematics, geometry, analytical mechanics, biology, physics, chemistry, economics, engineering and others. Therefore, it is still finding new applications.

Factually, many integral transforms like Fourier [1], Laplace [2], Aboodh [3], Mahgoub [4, 5], Sumudu [6] and T. M. Elzaki and S. M. Elzaki [7] are tools which are convenient to solve linear and nonlinear mathematical problems. In our work, we focus on Sumudu and Elzaki transforms with their applications to systems of integral and ordinary differential equations. Also, we describe the basic ideas and properties of these integral transforms.

In recent decades, much work has been conducted by researchers on studding novel methods for solutions of systems of integral and ordinary differential equations. Among these are optimal homotopy asymptotic method [8], homotopy analysis method [9], Adomian decomposition method [10], iterative method [11], differential transform method [12], Laplace transform [13] and Sawi transformation [14].

In this article, we present four sections: Section 2 introduces basic ideas and properties of these integral transforms. In Section 3, some applications of systems of integral and ordinary differential equations are presented. In Section 4, we present conclusions.

2. Preliminaries

In this section, we present some basic relevant concepts [6, 7, 9, 10].

Definition 2.1. The general IVP for a system of ODEs is given by

$$\frac{dU(z)}{ds} = A(z)U(z), \quad U(z_0) = U_0, \tag{1}$$

where $U(z)$ is a column vector and $A(z)$ is a square matrix.

Definition 2.2. The IEs:

$$u(x) = f(x) + \lambda \int_a^{b(x)} k(x, t)u(t)dt, \quad x \in [a, b], \tag{2}$$

where the functions $k(x, t) = (k_{ij}(x, t))$, $f(x) = (f_1(x), f_2(x), ..., f_n(x))$ and $u(x) = (u_1(x), u_2(x), ..., u_n(x))$ are column vectors and $\lambda = \lambda_i$ are constants.

Definition 2.3. Let a function in the set A be defined as

$$A = \{f(t) : \exists M,\ k_1k_2 > 0, |f(t)| < Me^{\frac{|t|}{k_j}};\ t \in (-1)^j \times [0, \infty[\}.$$

Then the ET is stated as

$$E(f(t)) = T(v) = v\int_0^{\infty} f(t)e^{\frac{-t}{v}}dt, \quad t > 0. \tag{3}$$

Definition 2.4. The ST is obtained over the following set:

$$A = \{f(t) : \exists M,\ k_1k_2 > 0, |f(t)| < Me^{\frac{|t|}{k_j}};\ t \in (-1)^j \times [0, \infty[\},$$

as

$$S(f(t)) = F(v) = \frac{1}{v}\int_0^{\infty} f(t)e^{\frac{-t}{v}}dt, \quad t > 0. \tag{4}$$

Theorem 2.1 (Linearity property). *Let* $f(t)$ *and* $g(t)$ *be two functions, and a and b be constants. Then*:

(1) $E(af(t) + bg(t)) = aE(f(t)) \pm bE(g(t)),$ (5)

(2) $S(af(t) + bg(t)) = aS(f(t)) \pm bS(g(t)).$ (6)

Proof. (1)

$$E(af(t) \pm bg(t)) = v\int_0^\infty (af(t) \pm bg(t))e^{\frac{-t}{v}}dt$$

$$= v\int_0^\infty af(t)e^{\frac{-t}{v}}dt \pm v\int_0^\infty bg(t)e^{\frac{-t}{v}}dt$$

$$= av\int_0^\infty f(t)e^{\frac{-t}{v}}dt \pm bv\int_0^\infty g(t)e^{\frac{-t}{v}}dt$$

$$= aE(f(t)) \pm bE(g(t)).$$

(2)

$$S(af(t) \pm bg(t)) = \frac{1}{v}\int_0^\infty (af(t) \pm bg(t))e^{\frac{-t}{v}}dt$$

$$= \frac{1}{v}\int_0^\infty af(t)e^{\frac{-t}{v}}dt \pm \frac{1}{v}\int_0^\infty bg(t)e^{\frac{-t}{v}}dt$$

$$= \frac{a}{v}\int_0^\infty f(t)e^{\frac{-t}{v}}dt \pm \frac{b}{v}\int_0^\infty g(t)e^{\frac{-t}{v}}dt$$

$$= aS(f(t)) \pm bS(g(t)).$$

Theorem 2.2 (Differentiation property). *If* $E(f(t)) = T(v)$ *and* $S(f(t)) = F(v)$, *then*

(1) $E(f^{(n)}(t)) = \frac{1}{v^n} T(v) - \sum_{i=0}^{n-1} v^{2-n+k} f^{(k)}(0), \quad n = 1, 2, \ldots,$ (7)

(2) $S(f^{(n)}(t)) = \frac{1}{v^n} F(v) - \sum_{i=0}^{n-1} v^k f^{(k)}(0), \quad n = 1, 2, \ldots.$ (8)

Proof. We can use mathematical induction to prove this theorem.

Theorem 2.3 (Convolution property)**.** *If* $f(t)$ *and* $g(t)$ *are two functions*, *then*

(1) $E(f(t) * g(t)) = \frac{1}{v} E(f(t)) E(g(t)),$ (9)

(2) $S(f(t) * g(t)) = vS(f(t)) S(g(t)),$ (10)

where

$$(f * g)(t) = \int_0^t f(u) g(t-u) du.$$

Proof. (1) Let $E(f(t)) = \frac{1}{v} \int_0^\infty f(x) e^{\frac{-x}{v}} dx$ and $E(g(t)) = \frac{1}{v} \int_0^\infty g(s) e^{\frac{-s}{v}} ds.$

Now, we have

$$\frac{1}{v} E(f(t)) E(g(t)) = \frac{1}{v} \left(\left(\int_0^\infty f(x) e^{\frac{-x}{v}} dx \right) \left(\int_0^\infty g(s) e^{\frac{-s}{v}} ds \right) \right)$$

$$= \frac{1}{v} \int_0^\infty \left(\int_0^\infty f(x) g(s) e^{\frac{-(x+s)}{v}} ds \right) dx.$$

Let $t = x + s,$ so $dt = ds,$ and

$$\frac{1}{v} \int_0^\infty \left(\int_0^\infty f(x) g(t-x) e^{\frac{-t}{v}} dt \right) dx$$

$$= \frac{1}{v}\int_0^\infty e^{\frac{-t}{v}}\left(\int_0^\infty f(x)g(t-x)dt\right)dx = E((f * g)(t)).$$

(2) Similarly.

Corollary 2.1. *ET and ST of some standard functions are given in the following table*:

Table 1. ET and ST of some standard functions

$f(t)$	$E(f(t)) = T(v)$	$S(f(t)) = F(v)$
1	v^2	1
t^n	$n!v^{n+1}$	$n!v^n$
e^{at}	$\frac{v^2}{1-av}$	$\frac{1}{1-av}$
$\sin at$	$\frac{av^3}{1+a^2v^2}$	$\frac{av}{1+a^2v^2}$
$\cos at$	$\frac{v^2}{1+a^2v^2}$	$\frac{1}{1+av^2}$
$\sinh at$	$\frac{av^3}{1-a^2v^2}$	$\frac{av}{1-a^2v^2}$
$\cosh at$	$\frac{v^2}{1-a^2v^2}$	$\frac{1}{1-a^2v^2}$
$f(at)$	$aT(v)$	$aF(v)$

Proof. We can use Definitions 2.3 and 2.4 to prove the above corollary.

3. Application of the ET and ST to Systems of Integral and Ordinary Differential Equations

Some applications are given to explain the procedure of solving the systems of integral and ordinary differential equations using ET and ST.

Example 3.1. Consider a system of two ODEs

$$\frac{dx}{dt} + \alpha y = 0,$$

$$\frac{dy}{dt} - \alpha x = 0, \tag{11}$$

where $x(0) = c_1$ and $y(0) = c_2$.

Solution using ET:

By taking ET to both the sides and using its properties, we have

$$E\left(\frac{dx}{dt}\right) + E(\alpha y) = E(0) \Rightarrow \frac{X(v)}{v} - vx(0) + \alpha Y(v) = 0,$$

$$E\left(\frac{dy}{dt}\right) - E(\alpha x) = E(0) \Rightarrow \frac{Y(v)}{v} - vy(0) - \alpha X(v) = 0.$$

By substituting $x(0) = c_1$ and $y(0) = c_2$ into the above system, we have

$$\frac{X(v)}{v} + \alpha Y(v) = c_1 v,$$

$$\frac{Y(v)}{v} - \alpha X(v) = c_2 v.$$

Now, the solution of the above system is given as

$$X(v) = \frac{c_1 v^2}{\alpha v^2 + 1} - \frac{\alpha c_2 v^3}{\alpha v^2 + 1},$$

$$Y(v) = \frac{c_1 \alpha v^3}{\alpha v^2 + 1} + \frac{c_2 v^2}{\alpha v^2 + 1}.$$

From the inverse ET, we have

$$x(t) = c_1 \cos \alpha t - \alpha c_2 \sin \alpha t \Rightarrow x(t) = c_1 \cos \alpha t - c_3 \sin \alpha t,\ c_3 = \alpha c_2,$$

$$y(t) = \alpha c_1 \sin \alpha t + c_2 \cos \alpha t \Rightarrow y(t) = c_4 \sin \alpha t + c_2 \cos \alpha t,\ c_4 = \alpha c_1.$$

Solution using ST:

By taking ST to both the sides and using its properties, we have

$$S\left(\frac{dx}{dt}\right) + S(\alpha y) = S(0) \Rightarrow \frac{X(v)}{v} - \frac{x(0)}{v} + \alpha Y(v) = 0,$$

$$S\left(\frac{dy}{dt}\right) - S(\alpha x) = S(0) \Rightarrow \frac{Y(v)}{v} - \frac{y(0)}{v} - \alpha X(v) = 0.$$

Substituting $x(0) = c_1$ and $y(0) = c_2$ into the above system, we get

$$\frac{X(v)}{v} + \alpha Y(v) = \frac{c_1}{v},$$

$$\frac{Y(v)}{v} - \alpha X(v) = \frac{c_2}{v}.$$

The solution of the above system is

$$X(v) = \frac{c_1}{\alpha v^2 + 1} - \frac{\alpha c_2 v}{\alpha v^2 + 1},$$

$$Y(v) = \frac{c_1 \alpha v}{\alpha v^2 + 1} + \frac{c_2}{\alpha v^2 + 1}.$$

By taking the inverse ST, we have

$$x(t) = c_1 \cos \alpha t - \alpha c_2 \sin \alpha t \Rightarrow x(t) = c_1 \cos \alpha t - c_3 \sin \alpha t,\ c_3 = \alpha c_2,$$

$$y(t) = \alpha c_1 \sin \alpha t + c_2 \cos \alpha t \Rightarrow y(t) = c_4 \sin \alpha t + c_2 \cos \alpha t,\ c_4 = \alpha c_1.$$

Example 3.2 [15]**.** Consider a system of three ODEs

$$\frac{dx}{dt} - y = e^t,$$

$$\frac{dy}{dt} + z + x = 1,$$

$$\frac{dz}{dt} + x = \sin t, \tag{12}$$

where $x(0) = y(0) = 1$ and $z(0) = 0.$

Solution using ET:

Applying ET to system (12) and using the properties, we get

$$E\left(\frac{dx}{dt}\right) - E(y) = E(e^t) \Rightarrow \frac{X(v)}{v} - vx(0) - Y(v) = \frac{v^2}{1-v},$$

$$E\left(\frac{dy}{dt}\right) + E(z) + E(x) = E(1) \Rightarrow \frac{Y(v)}{v} - vy(0) + Z(v) + X(v) = v^2,$$

$$E\left(\frac{dz}{dt}\right) + E(x) = E(\sin t) \Rightarrow \frac{Z(v)}{v} - vz(0) + X(v) = \frac{v^3}{1+v^2}.$$

Using initial condition, the system becomes

$$\frac{X(v)}{v} - Y(v) = v + \frac{v^2}{1-v},$$

$$\frac{Y(v)}{v} + X(v) + Z(v) = v + v^2,$$

$$\frac{Z(v)}{v} + X(v) = \frac{v^3}{1+v^2}.$$

This in turn gives

$$X(v) = \frac{-v^2 - v^3}{v^3 - v^2 + v - 1} = \frac{v^3}{v^2+1} + \frac{v^2}{1-v},$$

$$Y(v) = \frac{v^2}{v^2+1},$$

$$Z(v) = \frac{v^3}{v-1} = v^2 - \frac{v^2}{1-v}.$$

For solving this system, we take inverse ET to both the sides:

$$x(t) = e^t + \sin t,$$

$$y(t) = \cos t,$$

$$z(t) = 1 - e^t.$$

Solution using ST:

Here, by applying ST and using its properties, we get

$$S\left(\frac{dx}{dt}\right) - S(y) = S(e^t) \Rightarrow \frac{X(v)}{v} - \frac{x(0)}{v} - Y(v) = \frac{1}{1-v},$$

$$S\left(\frac{dy}{dt}\right) + S(z) + S(x) = S(1) \Rightarrow \frac{Y(v)}{v} - \frac{y(0)}{v} + Z(v) + X(v) = 1,$$

$$S\left(\frac{dz}{dt}\right) + S(x) = S(\sin t) \Rightarrow \frac{Z(v)}{v} - \frac{z(0)}{v} + X(v) = \frac{v}{1+v^2}.$$

Using initial condition, the system becomes

$$\frac{X(v)}{v} - Y(v) = \frac{1}{v} + \frac{1}{1-v},$$

$$\frac{Y(v)}{v} + X(v) + Z(v) = 1 + \frac{1}{v},$$

$$\frac{Z(v)}{v} + X(v) = \frac{v}{1+v^2}.$$

The solution of this system is given by

$$X(v) = \frac{-1-v}{v^3 - v^2 + v - 1} = \frac{v}{v^2+1} + \frac{1}{1-v},$$

$$Y(v) = \frac{1}{v^2+1},$$

$$Z(v) = \frac{v}{v-1} = 1 - \frac{v}{1-v}.$$

Now, by taking inverse ET to both the sides, we have

$$x(t) = e^t + \sin t,$$

$$y(t) = \cos t,$$

$$z(t) = 1 - e^t.$$

Example 3.3 [15]**.** Consider a system of second order ODEs

$$\frac{d^2x}{dt^2} + 3x - 2y = 0,$$

$$\frac{d^2y}{dt^2} + \frac{d^2x}{dt^2} - 3x + 5y = 0, \tag{13}$$

with IC: $x(0) = y(0) = 0,\ \frac{dx}{dt}(0) = 3$ and $\frac{dy}{dt}(0) = 2.$

Solution using ET:

Applying ET to both the sides in system (13), we have

$$E\left(\frac{d^2x}{dt^2}\right) + 3E(x) - 2E(y) = E(0)$$

$$\Rightarrow \frac{X(v)}{v^2} - v\frac{dx}{dt}(0) - x(0) + 3X(v) - 2Y(v) = 0,$$

$$E\left(\frac{d^2x}{dt^2}\right) + E\left(\frac{d^2y}{dt^2}\right) - 3E(x) + 5E(y) = E(0)$$

$$\Rightarrow \frac{X(v)}{v^2} - v\frac{dx}{dt}(0) - x(0) + \frac{Y(v)}{v^2} - v\frac{dy}{dt}(0) - y(0) - 3X(v) + 5Y(v) = 0.$$

Using IC, the following system becomes:

$$\frac{X(v)}{v^2} - 3v + 3X(v) - 2Y(v) = 0,$$

$$\frac{X(v)}{v^2} - 3v + \frac{Y(v)}{v^2} - 2v - 3X(v) + 5Y(v) = 0.$$

The solution of the above system is

$$X(v) = \frac{v^3(3 + 25v^2)}{9v^4 + 10v^2 + 1} = \frac{1}{12}\left(\frac{3v^3}{9v^2 + 1}\right) + \frac{11}{4}\left(\frac{v^3}{1 + v^2}\right),$$

$$Y(v) = \frac{2(12v^5 + v^3)}{9v^4 + 10v^2 + 1} = \frac{11}{4}\left(\frac{v^3}{1+v^2}\right) - \frac{1}{4}\left(\frac{3v^3}{1+9v^2}\right).$$

Taking inverse ET to both the sides, we have

$$x(t) = \frac{1}{12}\sin 3t + \frac{11}{4}\sin t,$$

$$y(t) = \frac{11}{4}\sin t - \frac{1}{4}\sin t.$$

Solution using ST:

Applying ST to both the sides in system (13), we have

$$S\left(\frac{d^2x}{dt^2}\right) + 3S(x) - 2S(y) = S(0)$$

$$\Rightarrow \frac{X(v)}{v^2} - \frac{\frac{dx}{dt}(0)}{v} - \frac{x(0)}{v} + 3X(v) - 2Y(v) = 0,$$

$$S\left(\frac{d^2x}{dt^2}\right) + S\left(\frac{d^2y}{dt^2}\right) - 3S(x) + 5S(y) = S(0)$$

$$\Rightarrow \frac{X(v)}{v^2} - \frac{\frac{dx}{dt}(0)}{v} - \frac{x(0)}{v} + \frac{Y(v)}{v^2} - \frac{\frac{dy}{dt}(0)}{v} - \frac{y(0)}{v} - 3X(v) + 5Y(v) = 0.$$

Using IC, the following system becomes:

$$\frac{X(v)}{v^2} - \frac{3}{v} + 3X(v) - 2Y(v) = 0$$

$$\Rightarrow \frac{X(v)}{v^2} + 3X(v) - 2Y(v) = \frac{3}{v},$$

$$\frac{X(v)}{v^2} - \frac{3}{v} + \frac{Y(v)}{v^2} - \frac{2}{v} - 3X(v) + 5Y(v) = 0$$

$$\Rightarrow \frac{X(v)}{v^2} - 3X(v) + \frac{Y(v)}{v^2} + 5Y(v) = \frac{5}{v}.$$

The solution of the above system is

$$X(v) = \frac{v(3 + 25v^2)}{9v^4 + 10v^2 + 1} = \frac{1}{12}\left(\frac{3v}{9v^2 + 1}\right) + \frac{11}{4}\left(\frac{v}{1 + v^2}\right),$$

$$Y(v) = \frac{2(12v^3 + v)}{9v^4 + 10v^2 + 1} = \frac{11}{4}\left(\frac{v}{1 + v^2}\right) - \frac{1}{4}\left(\frac{3v}{1 + 9v^2}\right).$$

By inverse ET to both the sides, the solution is

$$x(t) = \frac{1}{12}\sin 3t + \frac{11}{4}\sin t,$$

$$y(t) = \frac{11}{4}\sin t - \frac{1}{4}\sin t.$$

Example 3.4. Consider a system of second kind IEs

$$u_1(x) = 1 - x^2 + \int_0^x u_2(t)dt,$$

$$u_2(x) = x + \int_0^x u_1(t)dt. \tag{14}$$

Solution using ET:

Applying ET to both the sides in system (13), we have

$$E(u_1(x)) = E(1 - x^2) + E\left(\int_0^x u_2(t)dt\right) \Rightarrow U_1(v) = v^2 - 2v^4 + vU_2(v),$$

$$E(u_2(x)) = E(x) + E\left(\int_0^x u_1(t)dt\right) \Rightarrow U_2(v) = v^3 + vU_1(v).$$

This in turn gives

$$U_1(v) - vU_2(v) = v^2 - 2v^4,$$

$$U_2(v) - vU_1(v) = v^3.$$

The solution of the above system is

$$U_1(v) = v^2, \quad U_2(v) = 2v^3.$$

Taking inverse ET to both the sides, we have

$$U_1(x) = 1, \quad U_2(x) = 2x.$$

Solution using ST:

$$S(u_1(x)) = S(1 - x^2) + S\left(\int_0^x u_2(t)dt\right) \Rightarrow U_1(v) = 1 - 2v^2 + vU_2(v),$$

$$S(u_2(x)) = S(x) + S\left(\int_0^x u_1(t)dt\right) \Rightarrow U_2(v) = v + vU_1(v).$$

This in turn gives

$$U_1(v) - vU_2(v) = 1 - 2v^2,$$

$$U_2(v) - vU_1(v) = v.$$

The solution of the above system is

$$U_1(v) = 1, \quad U_2(v) = 2v.$$

Taking inverse ST to both the sides, we have

$$U_1(x) = 1, \quad U_2(x) = 2x.$$

Example 3.5. Consider a system of second kind delay IEs

$$u_1(x) = 2 - e^{-x} + \int_0^x (x - t)(u_1(t) + u_2(t))dt,$$

$$u_2(x) = 2x - e^x + 2e^{-x} + \int_0^x (x-t)(u_1(t) - u_2(t))dt. \tag{15}$$

Solution using ET:

By taking ET to both the sides and using its properties, we have

$$E(u_1(x)) = E(2) - E(e^{-x}) + E\left(\int_0^x (x-t)(u_1(t) + u_2(t))dt\right)$$

$$\Rightarrow U_1(v) = 2v^2 - \frac{v^2}{1+v} + v^2 U_1(v) + v^2 U_2(v),$$

$$E(u_2(x)) = E(2x) - E(e^x) + E(2e^{-x}) + E\left(\int_0^x (x-t)(u_1(t) - u_2(t))dt\right)$$

$$\Rightarrow U_2(v) = 2v^3 - \frac{v^2}{1-v} + \frac{v^2}{1+v} + v^2 U_1(v) - v^2 U_2(v).$$

This in turn gives

$$U_1(v)(1 - v^2) - v^2 U_2(v) = 2v^2 - \frac{v^2}{1+v} = \frac{2v^3 + v^2}{1+v},$$

$$U_2(v)(1 + v^2) - v^2 U_1(v) = \frac{-2v^5 - v^3 + v^2}{1 - v^2}.$$

The solution of the above system is

$$U_1(v) = \frac{v^2}{1-v}, \quad U_2(v) = \frac{v^2}{1+v}.$$

Taking inverse ET to both the sides, we have

$$U_1(x) = e^x, \quad U_2(x) = e^{-x}.$$

Solution using ST:

Taking ST to both the sides and using its properties, we get

$$S(u_1(x)) = S(2) - S(e^{-x}) + S\left(\int_0^x (x-t)(u_1(t) + u_2(t))dt\right)$$

$$\Rightarrow U_1(v) = 2 - \frac{1}{1+v} + v^2 U_1(v) + v^2 U_2(v),$$

$$S(u_2(x)) = S(2x) - S(e^{x}) + S(2e^{-x}) + S\left(\int_0^x (x-t)(u_1(t) - u_2(t))dt\right)$$

$$\Rightarrow U_2(v) = 2v - \frac{1}{1-v} + \frac{1}{1+v} + v^2 U_1(v) - v^2 U_2(v).$$

This in turn gives

$$U_1(v)(1-v^2) - v^2 U_2(v) = 2 - \frac{1}{1+v} = \frac{2v+1}{1+v},$$

$$U_2(v)(1+v^2) - v^2 U_1(v) = \frac{-2v^3 - 2v - 2}{1-v^2}.$$

The solution of this system is

$$U_1(v) = \frac{1}{1-v}, \quad U_2(v) = \frac{1}{1+v}.$$

By inverse ET to both the sides, we have

$$U_1(x) = e^{x}, \quad U_2(x) = e^{-x}.$$

4. Conclusion

We have provided two integral transforms, namely ST and ET for solution of systems of integral and ordinary differential equations. We have applied the selected integral transforms for solving some examples of systems of integral and ordinary differential equations. Clearly, these transforms are characterized by their simplicity of use. Also, it is accurate and efficient method for solving such systems of equations.

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