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HOPF BIFURCATION OF A DIFFUSIVE PREDATOR-PREY SYSTEM WITH NONLOCAL INTRASPECIFIC COMPETITION

Haoming Wu, Zhaoyan Shi and Ming Liu*

School of Science
Northeast Forestry University
Harbin, 150040, P. R. China
e-mail: liuming_girl@163.com

Abstract

The study of predator-prey models and reaction-diffusion equations helps us to more comprehensively and accurately explain the changes in population density in the natural world, and is an important aspect of biological and mathematical research. The study of Hopf bifurcations is a significant topic of research on reaction-diffusion equations, and it is of great importance for our understanding of population behavior. Firstly, we modify a predator-prey system with local effects studied by Geng et al. [1] and conduct further research based on this system. Secondly, we investigate the existence of

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*Corresponding author

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the positive equilibrium in the system. We find that the positive equilibrium exists only under certain conditions, and we provide criteria through the study of the properties of a cubic function. Thirdly, we present the characteristic equations for two different systems under the scenarios of $n = 0$ and $n \neq 0$. Since the model includes two integral terms, we categorize into two different Hopf bifurcation scenarios based on the magnitude of the value of certain parameters. We provide conditions under which the system undergoes Hopf bifurcation for each case.

1. Introduction

In the past, researchers often assumed that individuals only interact with those nearby in the dynamic processes involving single or multiple species with interactions. However, during the 20th century, studies in population dynamics have shown that the interactions among individuals have a certain range, and assumption that interactions are always nonlocal might not help us understand the behavioral mechanisms of species. Furter and Grinfeld [2] articulated the importance of studying nonlocal interactions in their research. Ermentrout and Cowan [3] demonstrated that nonlocal spatial interactions can lead to the system having more complex bifurcations and spatiotemporal patterns. We refer to [1] for further studied nonlocal competition based on the modified system.

In this paper, we focus on nonlocal interaction on the Hopf bifurcations of the system. We first present the following Holling-Tanner predator-prey model:

$$\begin{cases} u_t = d_1 \Delta u + au\left(1 - \dfrac{1}{k}\displaystyle\int_{\Omega} k(x, y)u(y, t)dy\right) - \dfrac{buv}{u + m}, & x \in \Omega,\ t > 0, \\ v_t = d_2 \Delta v + cv\left(1 - \dfrac{eu}{v}\right), & x \in \Omega,\ t > 0. \end{cases} \tag{1}$$

Ω is a region where organisms live in the Euclidean space, and the integral term in the model can be considered as the nonlocal competition among individuals within the species. In the case where Ω is a bounded region, the analytic expression for $k(x, y)$ is

$$k(x,\ y) = \frac{1}{|\,\Omega\,|}.$$

Here, $|\,\Omega\,|$ characterizes the volume size of the habitat, and the competition strength is the same at any location. We refer to this kind of nonlocal interaction as global interaction. There have been quite a few achievements in the study of this interaction in recent years. For the Holling-Tanner predator-prey model with global interaction, Chen et al. [4] demonstrated that the stable periodic orbits bifurcating from the positive constant steady state can be spatially inhomogeneous. Chen and Yu [5] account for the impact of global interaction on Hopf bifurcations based on the Rosenzweig-MacArthur predator-prey model. Shi et al. [6] proved that global competition can result in spatially inhomogeneous periodic patterns. Geng et al. [1] assume $|\,\Omega\,| = l\pi,$ and write system (1) as

$$\begin{cases} u_t = d_1 u_{xx} + au\left(1 - \dfrac{1}{kl\pi}\displaystyle\int_0^{l\pi} u(y,\ t)\,dy\right) - \dfrac{buv}{u+m}, & x \in (0,\ l\pi),\ t > 0, \\ v_t = d_2 v_{xx} + cv\left(1 - \dfrac{eu}{v}\right), & x \in (0,\ l\pi),\ t > 0, \\ u_x(0,\ t) = v_x(0,\ t) = 0,\ u_x(l\pi,\ t) = v_x(l\pi,\ t) = 0, & t > 0, \\ u(x,\ 0) = u_0(x) > 0, & v(x,\ 0) = v_0(x) > 0. \end{cases} \tag{2}$$

We modify this system, and then take into account the nonlocal effects within the predator population, resulting in the following system:

$$\begin{cases} \dfrac{\partial u}{\partial t} = d_1 \Delta u + ru\left(1 - \dfrac{\displaystyle\int_0^{l\pi} K(x,\ y)u(y,\ t)\,dy}{N}\right) - \dfrac{buv}{a+u}, & x \in (0,\ l\pi),\ t > 0, \\ \dfrac{\partial v}{\partial t} = d_2 \Delta v + \theta\dfrac{buv}{a+u} - dv - \delta v\displaystyle\int_0^{l\pi} K(x,\ y)u(y,\ t)\,dy, & x \in (0,\ l\pi),\ t > 0, \\ u_x(0,\ t) = v_x(0,\ t) = 0,\ u_x(l\pi,\ t) = v_x(l\pi,\ t) = 0, & t > 0, \\ u(x,\ 0) = u_0(x) > 0, & v(x,\ 0) = v_0(x) > 0. \end{cases} \tag{3}$$

Here, $u(x, t)$ and $v(x, t)$ represent the densities of prey and predator species at location x at time t. d_1 and d_2 are the diffusion coefficients for the respective species, r represents the birth rate of the prey, N represents the environmental carrying capacity for the prey, a represents the ability of the prey to evade predators, b represents the competition intensity between prey and predators, θ represents the conversion coefficient of prey turning into nourishment for predators, l can be considered as a measure of the length of the habitat, d represents the death rate of predators, δ represents the intensity of intraspecific competition among predators, and the integral term represents intraspecific competition within the species. d_1, d_2, r, l, N, a, b, θ, b and δ are positive constants.

In the following section of this paper, we focus on system (3), investigating the existence of its positive constant equilibrium. We find that the positive constant equilibrium does not persistently exist, and their analytical expressions are difficult to obtain. Therefore, we further analyze and obtain the criteria for the existence of positive constant equilibrium. Subsequently, by analyzing the Taylor expansion of the system at the positive equilibrium point, we obtain the characteristic equation. Due to the distinct structure of this model, we discuss the distribution of the roots of the characteristic equation in two different situations. We obtain the stability of the positive equilibrium and outline the conditions for the system to undergo a local Hopf bifurcation.

2. The Existence of Positive Equilibrium and Criterion

In this section, we study the existence of positive equilibrium and provide a criterion. Predator-prey model is as follows:

$$\begin{cases} \dfrac{\partial u}{\partial t} = d_1 \Delta u + ru\left(1 - \dfrac{\int_0^{l\pi} K(x, y)u(y, t)dy}{N}\right) - \dfrac{buv}{a+u}, & x \in (0, l\pi),\ t > 0, \\ \dfrac{\partial v}{\partial t} = d_2 \Delta v + \theta \dfrac{buv}{a+u} - dv - \delta v \int_0^{l\pi} K(x, y)u(y, t)dy, & x \in (0, l\pi),\ t > 0. \end{cases}$$

In remaining part of this paper, we set $\alpha = \int_0^{l\pi} K(x, y)u(y, t)dy$, $\beta = \int_0^{l\pi} K(x, y)v(y, t)dy$, $K(x, y) = \frac{1}{l\pi}$, l which is the expression for habitat length. For convenience, we set $\bar{u} = \frac{u}{a}$, $\bar{\delta} = \frac{\delta}{\theta}$, $\bar{d} = \frac{d}{\theta}$, and drop the bar. System (3) can be written as

$$\begin{cases} u_t = d_1 \Delta u + ru\left(1 - \frac{a}{N}\alpha - \frac{bv}{ra(u+1)}\right), \\ v_t = d_2 \Delta v + \theta v\left(\frac{bu}{u+1} - \delta\beta - d\right). \end{cases} \tag{4}$$

We set

$$F(u, v, \alpha) = ru\left(1 - \frac{a}{N}\alpha - \frac{bv}{ra(u+1)}\right),$$

$$G(u, v, \beta) = \theta v\left(\frac{bu}{u+1} - \delta\beta - d\right).$$

To get the positive equilibrium, then we set

$$\begin{cases} F(\tilde{u}, \tilde{v}, \alpha) = 0, \\ G(\tilde{u}, \tilde{v}, \beta) = 0. \end{cases} \tag{5}$$

We obtain

$$f(\tilde{u}) = \delta rap\tilde{u}^3 + \delta ra(2p-1)\tilde{u}^2 + [b^2 - bd + (p-2)\delta ra]\tilde{u} - bd - \delta ra = 0,$$

where

$$p = \frac{a}{N}.$$

Now, we study the property of the cubic function of $\tilde{u}$ to obtain the distribution of the positive equilibrium in system (3). The derivative of $\tilde{u}$ is given by

$$f'(\tilde{u}) = 3\delta rap\tilde{u}^2 + 2\delta ra(2p-1)\tilde{u} + b^2 - bd + (p-2)\delta ra$$

and

$$\Delta = [2\delta ra(2p-1)]^2 - 12\delta rap[b^2 - bd + (p-2)\delta ra].$$

When $\Delta > 0$, the two null points are u_1', u_2', and when $f(\tilde{u})$ has three null points in R, then the null points are u_1, u_2, u_3, where $u_1 < u_2 < u_3$.

Theorem 2.1. *The positive equilibrium* E^* *of the system* (3) *exists if and only if the parameters satisfy one of the following set of conditions*:

(i) $b > d,\ \Delta \le 0,\ f\left(\frac{d}{b-d}\right) < 0,$

(ii) $b > d,\ \Delta > 0,\ u_1' \le 0,\ f\left(\frac{d}{b-d}\right) < 0,$

(iii) $b > d,\ \Delta > 0,\ u_1' > 0,\ f(u_1') < 0,\ f\left(\frac{d}{b-d}\right) < 0,$

(iv) $b > d,\ \Delta > 0,\ u_1' > 0,\ f(u_1') = 0,\ f\left(\frac{d}{b-d}\right) \ge 0,$

(v) $b > d,\ \Delta > 0,\ u_1' > 0,\ f(u_1') > 0,\ f(u_2') > 0,\ f\left(\frac{d}{b-d}\right) < 0,$

(vi) $b > d,\ \Delta > 0,\ u_1' > 0,\ f(u_1') > 0,\ f(u_2') < 0,$ *there exists* $u_i \in \{u_1, u_2, u_3\}$ *such that* $u_i > \frac{d}{b-d},$

(vii) $b > d,\ \Delta > 0,\ u_1' > 0,\ f(u_1') > 0,\ f(u_2') = 0,$ *there exists* $u_i \in \{u_1, u_2, u_3\}$ *such that* $u_i > \frac{d}{b-d}.$

Proof. We only prove (i), (ii), (iii), (v), (vi).

(i) When $\Delta \le 0$, we can obtain that $f(\tilde{u})$ is monotonically increasing in $(0, u_1')$, $f(0) = -bd - \delta ra < 0$, and $\lim_{u' \to +\infty} f(\tilde{u}) = +\infty$. Therefore, there exists $u_1 \in (0, +\infty)$ such that $f(u_1) = 0.$ We obtain $u_1 > \frac{d}{b-d}$ from $\frac{d}{b-d} > 0$ and $f\left(\frac{d}{b-d}\right) < 0.$

(ii) When $b > d,$ $\Delta > 0,$ $u_1' \leq 0,$ we can easily get that there exists $u_1 \in (0, +\infty),$ such that $f(\tilde{u}) < 0$ in $(0, u_1)$ and $f(\tilde{u}) > 0$ in $(u_1, +\infty).$ Thus $u_1 > \frac{d}{b-d}$ from $f\left(\frac{d}{b-d}\right) < 0.$

(iii) When $b > d,$ $\Delta > 0,$ $u_1' > 0,$ $f(u_1') < 0,$ we have $f(\tilde{u})$ is monotonically increasing in $(0, u_1'),$ monotonically decreasing in $(u_1', u_2'),$ and monotonically increasing in $(u_2', +\infty),$ and hence there exists $u_1 \in (u_2', +\infty),$ such that $f(\tilde{u}) < 0$ in $(0, u_1')$ and $f(\tilde{u}) > 0$ in $(u_1, +\infty).$ Thus $u_1 > \frac{d}{b-d}$ from $f\left(\frac{d}{b-d}\right) < 0.$

(v) When $b > d, \Delta > 0, u_1' > 0, f(u_1') > 0,$ we can easily get that there exists $u_1 \in (0, u_1'),$ such that $f(u_1) = 0,$ $f(\tilde{u})$ is monotonically increasing in $(0, u_1'),$ monotonically decreasing in $(u_1', u_2'),$ and monotonically increasing in $(u_2', +\infty).$ Therefore, $f(\tilde{u}) > 0$ in $(u_1, +\infty).$ We obtain $u_1 > \frac{d}{b-d}$ from $f\left(\frac{d}{b-d}\right) < 0.$

(vi) When $b > d,$ $\Delta > 0,$ $u_1' > 0,$ $f(u_1') > 0,$ $f(u_2') < 0,$ we can easily get that there exist $u_1 \in (0, u_1'),$ $u_2 \in (u_1', u_1')$ and $u_3 \in (u_2', +\infty),$ such that $f(u_1) = f(u_2) = f(u_3) = 0.$ Now, we only need to check whether there exists $u_i \in \{u_1, u_2, u_3\},$ such that $u_i > \frac{d}{b-d}.$

3. Stability of the System and Local Hopf Bifurcation

3.1. Characteristic equation

Based on the system (3), we set $\alpha = \int_0^{l\pi} K(x, y)u(y, t)dy$, $\beta = \int_0^{l\pi} K(x, y)v(y, t)dy$, $K(x, y) = \frac{1}{l\pi}$, $\tilde{\theta} = \theta b$, $\bar{u} = \frac{u}{a}$, and drop the bar of $\bar{u}$. It can be written in the following form:

$$\begin{cases} u_t = d_1 \Delta u + ru\left(1 - \frac{a}{N}\alpha\right) - \frac{bv}{a(u+1)}, \\ v_t = d_2 \Delta v + \frac{\tilde{\theta} uv}{u+1} - \delta v \beta - dv. \end{cases} \tag{6}$$

We set $\begin{pmatrix} u \\ v \end{pmatrix} = \varphi(x) e^{\lambda t} = \sum_{n=0}^{+\infty} \begin{pmatrix} a_n \\ b_n \end{pmatrix} \cos \frac{nx}{l} e^{\lambda t}$ and bring it in system (6).

When $n = 0$, we have

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \int_0^{l\pi} \frac{1}{l\pi} \sum_{n=0}^{+\infty} a_n e^{\lambda t} dx \\ \int_0^{l\pi} \frac{1}{l\pi} \sum_{n=0}^{+\infty} b_n e^{\lambda t} dx \end{pmatrix} = \sum_{n=0}^{+\infty} \begin{pmatrix} a_n \\ b_n \end{pmatrix} e^{\lambda t}.$$

The characteristic equation is

$$\begin{vmatrix} \lambda + (1 - 2c)r + A & \frac{b\tilde{u}}{a(\tilde{u}+1)} \\ \frac{-\tilde{\theta}\tilde{v}}{(\tilde{u}+1)^2} & \lambda + \delta\tilde{v} \end{vmatrix} = 0,$$

where $c = 1 - \frac{a}{N}\tilde{u}$, $A = \frac{b\tilde{v}}{a(\tilde{u}+1)}$. From equation (5), we can easily deduce that c is greater than 0. The characteristic equation can be written in the following form:

$$\lambda^2 + T_0\lambda + D_0 = 0,$$

where

$$T_0 = (1 - 2c)r + A + \delta v,$$

$$D_0 = \delta\tilde{v}[(1 - 2c)r + A] + B,$$

$$B = \frac{\tilde{\theta}b\tilde{u}\tilde{v}}{a(\tilde{u} + 1)^3}.$$

When $n \neq 0$, we have $\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = 0.$

The characteristic equation is

$$\begin{vmatrix} \lambda + d_1\frac{n^2}{l^2} - cr + A & \frac{\tilde{b}\tilde{u}}{a(\tilde{u} + 1)} \\ \frac{-\tilde{\theta}\tilde{v}}{(\tilde{u} + 1)^2} & \lambda + d_2\frac{n^2}{l^2} \end{vmatrix} = 0,$$

which can be written in the following form

$$\lambda^2 + T_n\lambda + D_n = 0,$$

where

$$T_n = (d_1 + d_2)\frac{n^2}{l^2} + A - Cr,$$

$$D_n = d_1 d_2\frac{n^4}{l^4} + d_2(A - Cr)\frac{n^2}{l^2} + B.$$

3.2. Stability of the positive equilibrium point and Hopf bifurcation value

(1) When $0 < c \leq \frac{1}{2}$, $T_0(r) > 0$, $D_0(r) > 0$. If r_n^H is the Hopf bifurcation value of the system, r_n^H satisfies $T_n(r_n^H) = 0$, $D_n(r_n^H) > 0$, and

we obtain

$$r_n^H = \frac{1}{c}\left[(d_1 + d_2)\frac{n^2}{l^2} + A\right],$$

$$D_n(r_n^H) = B - d_2^2 \frac{n^4}{l^4}.$$

That $D_n(r_n^H) > 0$ if and only if $\frac{n^4}{l^4} < \frac{B}{d_2^2}$. Therefore, the bifurcation value that might cause system (3) to undergo a Hopf bifurcation can be expressed as $\Lambda_1 = \{r_n^H\}_{n=1}^M$, where

$$0 < r_1^H < r_2^H < \cdots < r_M^H < r_{M+1}^H < \cdots.$$

The value of M is determined by the magnitudes of l, B, d_2. We provide a condition and a series of conclusions as follows:

(H_1) $d_2(d_1 + d_2)^2(M + 1)^4 - 4Bd_1l^4 < 0.$

Lemma 3.1. *When the parameter satisfies condition* (H_1), *then the following hold*:

(i) *When* $r \in (0, r_1^H)$, *all the roots of the characteristic equation have negative real parts.*

(ii) *When* $r = r_1^H$, *the characteristic equation has a pair of purely imaginary roots, and all other roots have negative real parts.*

(iii) *When* $r \in (r_j^H, r_{j+1}^H)$, *the characteristic equation has* $2j$ *roots with positive real parts, where* $j = 1, 2, \ldots, M$.

Proof. When (H_1) holds, for any $r \in (0, r_{M+1}^H)$, we have

$$D_n(r) = d_1 d_2 \frac{n^4}{l^4} + d_2(A - Cr)\frac{n^2}{l^2} + B$$

$$> d_1 d_2 \left(\frac{n^2}{l^2}\right)^2 + d_2(A - Cr_{M+1}^H)\frac{n^2}{l^2} + B.$$

We obtain a quadratic function $f\left(\frac{n^2}{l^2}\right)$ with respect to $\frac{n^2}{l^2}$, as follows:

$$L\left(\frac{n^2}{l^2}\right) = d_1 d_2 \left(\frac{n^2}{l^2}\right)^2 + d_2(A - cr_{M+1}^H)\frac{n^2}{l^2} + B.$$

Also, we have

$$d_2^2(A - cr_{M+1}^H)^2 - 4Bd_1d_2 = d_2^2(d_1 + d_2)^2 \frac{(M+1)^4}{l^4} - 4Bd_1d_2$$

$$= \frac{d_2}{l^4}[d_2(d_1 + d_2)^2(M+1)^4 - 4Bd_1 l^4] < 0.$$

Therefore, the equation $L\left(\frac{n^2}{l^2}\right) = 0$ has no real root. Thus $L\left(\frac{n^2}{l^2}\right) > 0.$ Also, $D_n(r) > 0$ for any $r \in (0, r_{M+1}^H)$.

When $r \in (0, r_1^H)$, $T_0(r) > 0$, $T_n(r) > T_1(r_1^H) = 0$, the conclusion (i) is established. When $r = r_1^H$, $T_0(r) > 0$, $T_1(r_1^H) = 0$, for any $i \neq 0$, $T_i(r_1^H) > 0$, the conclusion (ii) is proven. When $r \in (r_j^H, r_{j+1}^H)$, $T_0(r) > 0$, $T_i(r) < 0 \ (i = 1, ..., j)$, there are *j* characteristic equations that have a pair of roots with positive real parts, the conclusion (iii) thus holds. Furthermore

$$\frac{d(Re(\lambda))}{dr} = c > 0.$$

Theorem 3.2. *If* $0 < c \leq \frac{1}{2}$ *and condition* (H_1) *holds, then for the system* (3), *the following hold*:

(i) *When* $r \in (0, r_1^H),$ *the system* (3) *is locally asymptotically stable at the positive equilibrium* E^*.

(ii) *When* $r > r_1^H,$ *the system* (3) *is unstable at the positive equilibrium* E^*.

(iii) r_n^H $(n = 1, 2, ..., M)$ *is the Hopf bifurcation value of the system* (3).

Let $c > \frac{1}{2}$. If r_0^H is the Hopf bifurcation value of the system, r_0^H satisfies $T_0(r_0^H) = 0,$ $D_0(r_0^H) > 0,$ and we obtain

$$r_0^H = \frac{1}{2c - 1}(A + \delta\tilde{v}),$$

$$D_0(r_0^H) = B - \delta^2\tilde{v}^2 > 0.$$

As we discussed earlier, we have addressed the expression for r_n^H when it is the bifurcation value of system (3). The bifurcation values that might cause system (3) to undergo Hopf bifurcation can be written as $\Lambda_2 = \{r_i^H\}_{i=0}^M$ $(M \in N)$. The value of M is the same as in the previous context.

Next, we provide two conditions and a series of conclusions:

(H_2) $\delta\overline{v}[(1 - 2c)r_{M+1}^H + A] + B > 0.$

(H_3) $l^2c(A + \delta\overline{v}) - (2c - 1)(Al^2 + d_1 + d_2) < 0.$

Lemma 3.3. *If the parameter satisfies condition* (H_1), (H_2) *and* (H_3), *then the following hold*:

(i) *When* $r \in (0, r_0^H)$, *all the roots of the characteristic equation have negative real parts*.

(ii) *When* $r = r_0^H$, *the characteristic equation has a pair of purely imaginary roots*, *and all other roots have negative real parts*.

(iii) *When* $r \in (r_j^H, r_{j+1}^H)$, *the characteristic equation has* $2(j+1)$ *roots with positive real parts*, *where* $j = 0, 1, 2, \ldots, M$.

Proof. Condition (H_3) is equivalent to

$$\frac{1}{2c-1}(A + \delta\bar{v}) < \frac{1}{c}\left(A + \frac{d_1 + d_2}{l^2}\right),$$

and therefore $\{r_i^H\}_{i=0}^M$ satisfies $0 < r_0^H < r_1^H < \cdots < r_M^H < r_{M+1}^H < \cdots$. When the condition (H_2) is satisfied, $D_0(r) > \delta\bar{v}[(1-2c)r_{M+1}^H + A] + B > 0$ for any $r \in (0, r_{M+1}^H)$. When the condition (H_1) is satisfied, $D_n(r) > 0$ $(n = 1, 2, \ldots)$. The remaining part of the proof is similar to the proof of Lemma 3.1. Furthermore,

$$\frac{d(Re(\lambda))}{dr} = 2c - 1 \text{ or } c > 0.$$

Theorem 3.4. *If* $c > \frac{1}{2}$, (H_1), (H_2) *and* (H_3) *hold*, *then the following hold*:

(i) *When* $r \in (0, r_0^H)$, *the system* (3) *is locally asymptotically stable at the positive equilibrium* E^*.

(ii) *When* $r > r_0^H$, *the system* (3) *is unstable at the positive equilibrium* E^*.

(iii) r_i^H $(n = 0, 1, 2, \ldots, M)$ *is the Hopf bifurcation value of the system* (3).

4. Numerical Simulation

In this section, we assign values to the parameters in system (3) such that they satisfy the conditions stipulated in Theorems 3.2 and 3.4. Utilizing mathematical software, we obtain graphical representations of the numerical solutions for u and v to validate the conclusions drawn in the theorems.

We set $d_1 = 1$, $d_2 = 0.05$, $a = 1$, $r = 0.1$, $l = 5.4$, $N = 4$, $b = 5$, $\theta = 0.4$, $d = 1.86$, and $\delta = 10$. It is straightforward to verify that these parameter values satisfy the conditions in Theorem 3.2(i). When the initial values for u and v are set at $(0.1, 0.1)$, we obtain the graphical representations for u and v, shown in Figure 1. $(u(x, t), v(x, t))$ tends to the positive equilibrium $(0.283, 0.341)$.

For Theorem 3.2(ii), we set $d_1 = 0.97$, $d_2 = 0.14$, $a = 1$, $r = 0.1$, $l = 5.5$, $N = 37.56$, $b = 1.75$, $\theta = 0.4$, $d = 1.86$, and $\delta = 9.27$. When the initial values for u and v are $(0.1, 0.1)$, we obtain Figure 2, which is consistent with our theoretical conclusions.

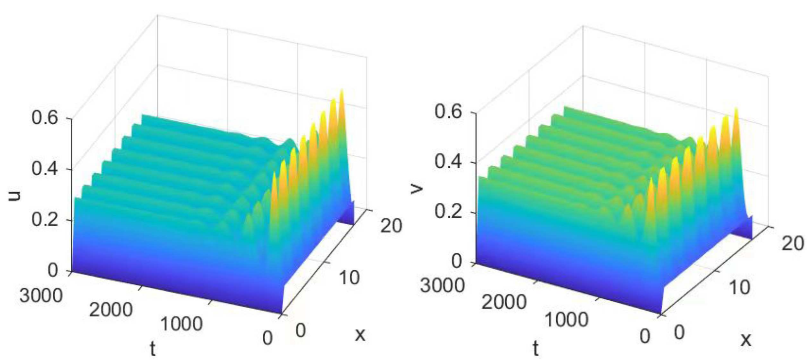

Figure 1. In the case where $d_1 = 1$, $d_2 = 0.05$, $a = 1$, $r = 0.1$, $l = 5.4$, $N = 4$, $b = 5$, $\theta = 0.4$, $d = 1.86$, and $\delta = 10$, $(u(x, t), v(x, t))$ tends to the positive equilibrium $(0.283, 0.341)$.

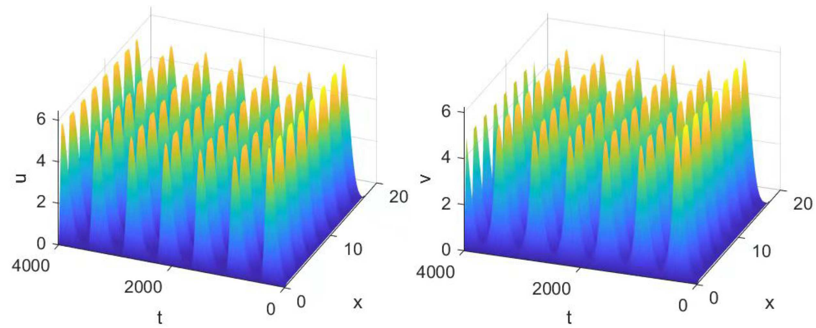

Figure 2. In the case where $d_1 = 0.97$, $d_2 = 0.14$, $a = 1$, $r = 0.1$, $l = 5.5$, $N = 37.56$, $b = 1.75$, $\theta = 0.4$, $d = 1.86$, and $\delta = 9.27$, $(u(x, t), v(x, t))$ is unstable and tends to a spatially inhomogeneous periodic solution, which is consistent with our conclusions.

Similarly, for the conditions specified in Theorem 3.4(i), we set $d_1 = 1$, $d_2 = 0.05$, $a = 1.12$, $r = 0.086$, $l = 5.5$, $N = 3.26$, $b = 4$, $\theta = 0.52$, $d = 1.86$, and $\delta = 9.74$. When the initial values for u and v are $(0.07, 0.07)$, we obtain the graphical representations for u and v, shown in Figure 3. Then $(u(x, t), v(x, t))$ tends to the positive equilibrium $(0.340, 0.461)$.

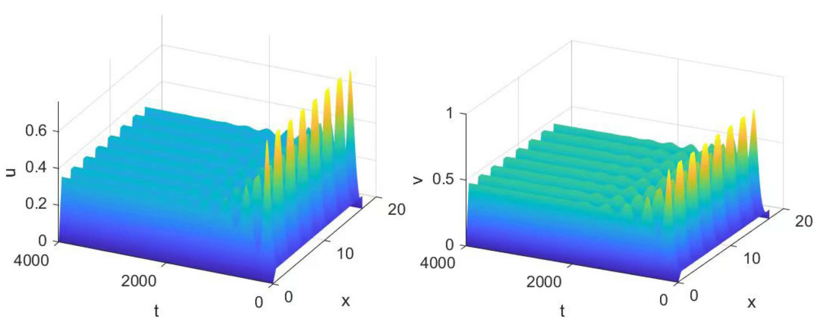

Figure 3. In the case where $d_1 = 1$, $d_2 = 0.05$, $a = 1.12$, $r = 0.086$, $l = 5.5$, $N = 3.26$, $b = 4$, $\theta = 0.52$, $d = 1.86$, and $\delta = 9.74$, $(u(x, t), v(x, t))$ tends to the positive equilibrium $(0.340, 0.461)$.

For Theorem 3.4(ii), we set $d_1 = 1$, $d_2 = 0.14$, $a = 0.75$, $r = 0.11$, $l = 5.5$, $N = 33.1$, $b = 2.0$, $\theta = 0.4$, $d = 2$, and $\delta = 7.25$. When the initial values for u and v are $(0.07, 0.07)$, we obtain Figure 4, which is consistent with our theoretical conclusions.

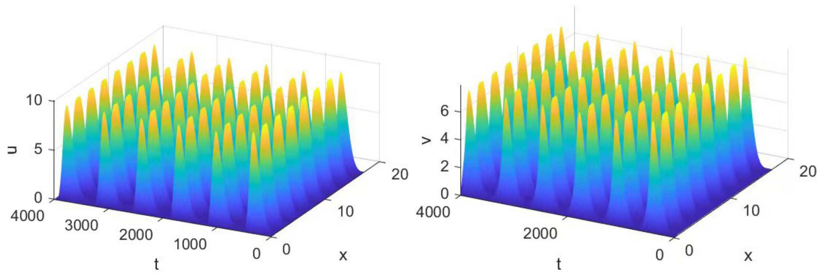

Figure 4. In the case where $d_1 = 1$, $d_2 = 0.14$, $a = 0.75$, $r = 0.11$, $l = 5.5$, $N = 33.1$, $b = 2.0$, $\theta = 0.4$, $d = 2$, and $\delta = 7.25$, $(u(x, t), v(x, t))$ is unstable and tends to a spatially inhomogeneous periodic solution, which is consistent with our conclusions.

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