Advances in Differential Equations and Control Processes

http://www.pphmj.com
http://dx.doi.org/10.17654/0974324323010
Volume 30, Number 2, 2023, Pages 151-168 **P-ISSN: 0974-3243**

CONVERGENCE AND APPLICATION OF A MODIFIED DOUBLE LAPLACE TRANSFORM (MDLT) IN SOME EQUATIONS OF MATHEMATICAL PHYSICS

Tarig M. Elzaki[1,*], Mourad Chamekh[1,2] and Shams A. Ahmed[3,4]

[1]Mathematics Department
College of Sciences and Arts
Alkamel University of Jeddah
Saudi Arabia
e-mail: tfarah@uj.edu.sa
tarig.alzaki@gmail.com

[2]National Engineering School at Tunis
University of Tunis El Manar
Lamsin, 1002, Tunis
Tunisia
e-mail: chamekhmourad1@gmail.com

Received: December 6, 2022; Revised: February 5, 2023; Accepted: April 3, 2023

2020 Mathematics Subject Classification: 35C05, 35A22.

Keywords and phrases: modified double Laplace transform (MDLT), convergence, pseudo-hyperbolic equation, Benney-Luke equation.

*Corresponding author

How to cite this article: Tarig M. Elzaki, Mourad Chamekh and Shams A. Ahmed, Convergence and application of a modified double Laplace transform (MDLT) in some equations of mathematical physics, Advances in Differential Equations and Control Processes 30(2) (2023), 151-168. http://dx.doi.org/10.17654/0974324323010

Published Online: May 25, 2023

[3]Department of Mathematics
Faculty of Sciences and Arts
Jouf University
Tubarjal, Saudi Arabia
e-mail: cshamsaldeenahmed@gmail.com

[4]Department of Mathematics
University of Gezira
Sudan

Abstract

Dealing with a modified double Laplace transform (MDLT) technique, we have established some convergence results. Then, using this new technique, we propose resolutions of some equations such as pseudo-hyperbolic and the Benney-Luke equations. The advantage of MDLT is that we can obtain exact solutions in one step. The technique aims to provide viable results with respect to particular cases. Finally, some numerical results are presented.

1. Introduction

The nonlinear evolution equation (NLEE) has a lower number of iterations due to its far-reaching decisions. In today's natural sciences, nonlinear phenomena need to take a prominent place in the moss-covered oaks, excellent areas of research, and various extensions of science and technology.

The NLEE-enabled solutions provided are based on data from unpredictable physical phenomena surrounding the structure. These efforts include homotopy checking strategies, gluing strategies, the variational iteration strategy [12-16, 18], the projected differential transform method and Laplace transform (Elzaki [17]), double Laplace transform (Dhunde and Waghmare [15]), and double and modified double Laplace transform [22-26].

Numerous ways to explain and compute nonlinear partial differential equations with fractional derivatives and replicate Laplace transforms have been proposed (Wu et al.), [5-9] to solve the Benney-Luke (Islam et al.), [11] and, solving singular system of hyperbolic equations, (Elzaki et al.) [1]. In this paper, we present how to make the appropriate choice of the initial values for solution using only one step. We also discuss the convergence of one of the problems solved in this paper.

We propose a blend of the MDLT and the present day method to be used. The single modified transform (SMT) is characterized by:

$$\varepsilon[\Phi(v)] = T(\mu) = \mu^2 \int_0^{\infty} \Phi(\mu v) e^{-v} dv.$$

2. The MDLT

In this section, we present the MDLT of a function. Additionally, we discuss its convergence.

Definition. A function $\Phi(w, v)$, $w, v \in R^+$, is said to be a *convergent infinite series* if

$$\varepsilon[\Phi(w, v), \eta, \mu]$$

$$= T(\eta, \mu) = \eta\mu \int_0^{\infty} \int_0^{\infty} \Phi(w, v) e^{-\left(\frac{w}{\eta}+\frac{v}{\mu}\right)} dw dv, \quad w, v > 0, \tag{1}$$

where η and μ are complex numbers.

To obtain the solutions of the Benney-Luke and singular pseudo-hyperbolic equations by the combination of MDLT and the unused strategy, we begin with

$$\varepsilon_2\left[\frac{\partial \Phi}{\partial w}\right] = \frac{1}{\eta} T(\eta, \mu) - \eta T(0, \mu),$$

$$\varepsilon_2\left[\frac{\partial^2 \Phi}{\partial w^2}\right] = \frac{1}{\eta^2} T(\eta, \mu) - T(0, \mu) - \eta \frac{\partial T(0, \mu)}{\partial w},$$

$$\varepsilon_2\left[\frac{\partial \Phi}{\partial v}\right] = \frac{1}{\mu}T(\eta, \mu) - \mu T(\eta, 0),$$

$$\varepsilon_2\left[\frac{\partial^2 \Phi}{\partial v^2}\right] = \frac{1}{\mu^2}T(\eta, \mu) - T(\eta, 0) - \mu\frac{\partial T(\eta, 0)}{\partial v},$$

$$\varepsilon_2\left[\frac{\partial^2 \Phi}{\partial w \partial v}\right] = \frac{1}{\eta\mu}T(\eta, \mu) - \frac{\mu}{\eta}T(\eta, 0) - \frac{\eta}{\mu}T(0, \mu) + \eta\mu T(0, 0). \tag{2}$$

For more details, see [16].

Here we ought to examine a few theorems of convergence of MDLT.

Theorem 1. *If* $\mu\int_0^\infty e^{-\frac{v}{\mu}}\Phi(w, v)dv$ *converges at* $\mu = \mu_0$, *then* $\mu\int_0^\infty e^{-\frac{v}{\mu}}\Phi(w, v)dv$ *converges at* $\mu < \mu_0$.

Proof. Let $p(w, v) = \mu_0\int_0^v e^{-\frac{u}{\mu_0}}\Phi(w, u)du,\ 0 < v < \infty.$ Then

(i) $p(w, 0) = 0$,

(ii) $\lim\limits_{v\to\infty} p(w, v)$ exists, and

(iii) $p_v(w, v) = \mu_0 e^{-\frac{v}{\mu_0}}\Phi(w, v)$.

Choosing $\varepsilon_1,\ R_1,$ such that $0 < \varepsilon_1 < R_1,$ we have

$$\mu\int_{\varepsilon_1}^{R_1} e^{-\frac{v}{\mu}}\Phi(w, v)dt = \mu\int_{\varepsilon_1}^{R_1}\frac{1}{\mu_0}e^{-\frac{v}{\mu}}p_v(w, v)e^{\frac{v}{\mu_0}}dt$$

$$= \frac{\mu}{\mu_0}\int_{\varepsilon_1}^{R_1} e^{-\left(\frac{\mu_0-\mu}{\mu\mu_0}\right)v}p_v(w, v)dv.$$

The last integral becomes

$$\frac{\mu}{\mu_0}\int_{\varepsilon_1}^{R_1} e^{-\left(\frac{\mu_0-\mu}{\mu\mu_0}\right)v} p_v(w, v)\,dv$$

$$= \frac{\mu}{\mu_0}\Bigg\{ e^{-\left(\frac{\mu_0-\mu}{\mu\mu_0}\right)R_1} p(w, R_1) - e^{-\left(\frac{\mu_0-\mu}{\mu\mu_0}\right)\varepsilon_1} p(w, \varepsilon_1)$$

$$+ \left(\frac{\mu_0-\mu}{\mu\mu_0}\right)\int_{\varepsilon_1}^{R_1} e^{-\left(\frac{\mu_0-\mu}{\mu\mu_0}\right)v} p(w, v)\,dv \Bigg\}.$$

Now as, $\varepsilon_1 \to 0,\ R_1 \to \infty,$ and $\mu < \mu_0,$

$$\mu\int_0^\infty e^{-\frac{v}{\mu}}\Phi(w, v)\,dv = \left(\frac{\mu_0-\mu}{\mu\mu_0}\right)\int_0^\infty e^{-\left(\frac{\mu_0-\mu}{\mu\mu_0}\right)v} p(w, v)\,dv, \quad \mu < \mu_0. \tag{3}$$

Theorem 1 is proved if the last integral converges.

Using the limit test for convergence, we obtain

$$\lim_{v\to\infty} v^2 e^{-\left(\frac{\mu_0-\mu}{\mu\mu_0}\right)v} p(w, v) = 0,$$

and hence $\mu\int_0^\infty e^{-\frac{v}{\mu}}\Phi(w, v)\,dv$ converges when $\mu < \mu_0$.

Theorem 2. *If* $Q(w, \mu) = \mu\int_0^\infty e^{-\frac{v}{\mu}}\Phi(w, v)\,dv$ *converges for* $\mu < \mu_0$, *and* $\eta\int_0^\infty e^{-\frac{w}{\eta}}Q(w, \mu)\,dw$ *converges at* $\eta = \eta_0$, *then* $\eta\int_0^\infty e^{-\frac{w}{\eta}}Q(w, \eta)\,dw$ *converges for* $\eta < \eta_0$.

Proof. Follows as that of Theorem 1.

Theorem 3. *If* $\Phi(w, v)$ *is continuous and*

$$\eta\mu\int_0^\infty\int_0^\infty e^{-\frac{w}{\eta}-\frac{v}{\mu}}\Phi(w, v)dwdv$$

converges for $\mu = \mu_0$, $\eta = \eta_0$, *then* $\eta\mu\int_0^\infty\int_0^\infty e^{-\frac{w}{\eta}-\frac{v}{\mu}}\Phi(w, v)dwdv$ *converges for* $\eta < \eta_0$, $\mu < \mu_0$.

Proof. We have

$$\eta\mu\int_0^\infty\int_0^\infty e^{-\frac{w}{\eta}-\frac{v}{\mu}}\Phi(w, v)dwdv = \eta\int_0^\infty e^{-\frac{w}{\eta}}\left\{\mu\int_0^\infty e^{-\frac{v}{\mu}}\Phi(w, v)dv\right\}dw$$

$$= \eta\int_0^\infty e^{-\frac{w}{\eta}}Q(w, \mu)dw,$$

where

$$Q(w, \mu) = \mu\int_0^\infty e^{-\frac{v}{\mu}}\Phi(w, v)dv.$$

Using Theorems 1 and 2, we have that

$$\eta\mu\int_0^\infty\int_0^\infty e^{-\frac{w}{\eta}-\frac{v}{\mu}}\Phi(w, v)dwdv \text{ converges for } \eta < \eta_0, \mu < \mu_0.$$

We introduce the general equation that holds in the equations of Benney-Luke and singular pseudo-hyperbolic in the following form:

$$U_{vv} - aU_{ww} + bU_{wwww} - cU_{wwvv} + dU_vU_{ww} + eU_wU_{wv}$$

$$- g(w)\frac{\partial}{\partial w}\left(w\frac{\partial U}{\partial w}\right) - g(w)\frac{\partial^2}{\partial w\partial v}\left(w\frac{\partial U}{\partial w}\right) = f(w, v), \qquad (4)$$

where a, b, c, d and e are constants.

Then

(i) If $g(w) = 0,\ e = 2,\ a = d = 1,$ then equation (4) becomes

$$U_{vv} - U_{ww} + bU_{wwww} - cU_{wwvv} + U_v U_{ww} + 2U_w U_{wv} = f(w, v). \tag{5}$$

Equation (5) is called the *Benney-Luke equation*, where $c = \sigma - \frac{1}{3},$ σ is the Bond number.

(ii) If $a = b = c = d = e = 0,\ g(w) = \frac{1}{w},$ then equation (4) becomes

$$U_{vv} - \frac{1}{w}\frac{\partial}{\partial w}\left(w\frac{\partial U}{\partial w}\right) - \frac{1}{w}\frac{\partial^2}{\partial w \partial v}\left(w\frac{\partial U}{\partial w}\right) = f(w, v). \tag{6}$$

Equation (6) is called the *singular pseudo-hyperbolic equation*.

3. The New Strategy (MDLT)

To clarify our strategy, we consider

$$DU(w, v) + RU(w, v) + GU(w, v) + NU(w, v) = f(w, v), \tag{7}$$

where D may be a second order operator, R a linear operator, N a nonlinear operator, and G a singular operator.

The initial conditions are

$$U(w, 0) = f_1(w), \quad U_t(w, 0) = f_2(w). \tag{8}$$

To reveal the solution of equations (7), (8), taking modified double Laplace transform (MDLT) of equation (7), and SMT of equation (8), we obtain

$$\frac{1}{\mu^2}\varepsilon_2(U(w, v)) - K_1(\eta) - \mu K_2(\eta)$$

$$= \varepsilon_2[f(w, v) - RU(w, v) - GU(w, v) - NU(w, v)], \tag{9}$$

where $K_1(\eta),\ K_2(\eta)$ are single modified transforms of $f_1(w),\ f_2(w),$ respectively.

We assume that the solution of equation (7) can be expressed in the series form:

$$U(w, v) = \sum_{n=0}^{\infty} U_n(w, v). \tag{10}$$

Taking the inverse MDLT of equation (9), and making use of equation (10), we get

$$\sum_{n=0}^{\infty} U_n(w, v)$$

$$= F(w, v) + \varepsilon_2^{-1}\{\mu^2 \varepsilon_2 [f(w, v) - RU(w, v) - GU(w, v) - NU(w, v)]\}. \tag{11}$$

This strategy depends on how to choose the beginning iteration $U_0(w, v)$, that goes to the exact solution in a few steps for the action in the case we prefer, to choose, $U_0(w, v) = F(w, v)$.

Then the solution, $U(x, t)$, can be recursively decided by utilizing:

$$U_{n+1}(w, v) = \varepsilon_2^{-1}\{\mu^2 \varepsilon_2 [f(w, v) - RU_n(w, v) - GU_n(w, v) - NU_n(w, v)]\},$$

$$U_0(w, v) = F(w, v).$$

From these equations, we obtain

$$U_0(w, v), U_1(w, v), U_2(w, v), \ldots,$$

and after that we get the solution in a series form for equation (10).

4. Application

To demonstrate the productivity of this strategy in solving Benney-Luke hyperbolic and singular pseudo-hyperbolic equations, we consider the following examples:

Example 1. In equation (5), we put $f(w, v) = 2v$ with the initial conditions:

$$U(w, 0) = 1, \quad U_v(w, 0) = w. \tag{12}$$

Utilizing the same steps in area 3,

$$\frac{1}{\mu^2}\varepsilon_2(U(w, v)) - \eta^2 - \mu\eta^3$$

$$= \varepsilon_2[U_{ww} - bU_{wwww} + cU_{wwvv} - U_v U_{ww} - 2U_w U_{wv} + 2v]. \tag{13}$$

Applying the inverse MDLT to equation (13), we obtain

$$U(w, v) = 1 + wv + \varepsilon_2^{-1}\{\mu^2\varepsilon_2[U_{ww} - bU_{wwww} + cU_{wwvv}$$

$$- U_v U_{ww} - 22U_w U_{wv} + 2v]\}.$$

At that point, the recursive relation is as follows:

$$U_{n+1}(w, v) = \varepsilon_2^{-1}\left\{\mu^2\varepsilon_2\begin{bmatrix}(U_n)_{ww} - b(U_n)_{wwww} + c(U_n)_{wwvv} \\ -(U_n)_v(U_n)_{ww} - 2(U_n)_w(U_n)_{wv} + 2v\end{bmatrix}\right\},$$

$$U_0(w, v) = 1 + wv. \tag{14}$$

The primary components are given by

$$U_0(w, v) = 1 + wv, \quad U_1(w, v) = \varepsilon^{-1}\{\mu^2\varepsilon_2[0]\} = 0.$$

Then the solution of equation (5) with $f(w, v) = 2v$ is

$$U(w, v) = \sum_{n=0}^{\infty} U_n(w, v) = 1 + wv.$$

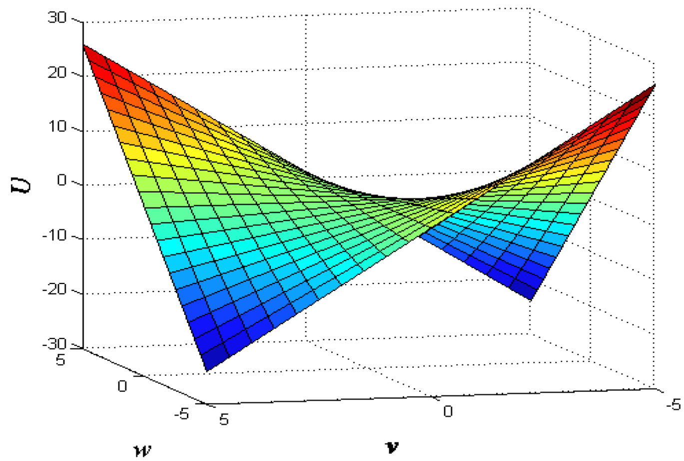

Figure 1. Result obtained in Example 1 of Benney-Luke and singular pseudo- hyperbolic equation.

Example 2. In this example, we put

$$f(w, v) = -(w^2 \sin v + 4 \sin v + 4 \cos v)$$

in equation (6), with

$$U(w, 0) = 0, \quad U_v(w, 0) = w^2. \tag{15}$$

Here we use the same steps which we used as before in Example 1:

$$\varepsilon_2(U(w, v))$$

$$= \frac{2\mu^3\eta^4}{1+\mu^2} + \varepsilon_2\left[\frac{1}{w}\frac{\partial}{\partial w}\left(w\frac{\partial U}{\partial w}\right) + \frac{1}{w}\frac{\partial^2}{\partial w \partial v}\left(w\frac{\partial U}{\partial w}\right) - 4\sin v - 4\cos v\right]. \tag{16}$$

Taking the inverse MDLT of equation (16), we obtain

$$U(w, v)$$

$$= w^2 \sin v + \varepsilon_2^{-1}\left\{\mu^2\varepsilon_2\left[\frac{1}{w}\frac{\partial}{\partial w}\left(w\frac{\partial U}{\partial w}\right) + \frac{1}{w}\frac{\partial^2}{\partial w \partial v}\left(w\frac{\partial U}{\partial w}\right) - 4\sin v - 4\cos v\right]\right\}.$$

At that point, the recursive relations is given by

$$U_{n+1}(w, v)$$

$$= \varepsilon_2^{-1}\left\{\mu^2\varepsilon_2\left[\frac{1}{w}\frac{\partial}{\partial w}\left(w\frac{\partial U_n}{\partial w}\right)+\frac{1}{w}\frac{\partial^2}{\partial w\partial v}\left(w\frac{\partial U_n}{\partial w}\right)-4\sin v-4\cos v\right]\right\},$$

$$U_0(w, v) = w^2 \sin v.$$

From above, the primary components take the form:

$$U_0(w, v) = w^2 \sin v, \quad U_1(w, v) = \varepsilon_2^{-1}\{\mu^2\varepsilon_2[0]\} = 0.$$

Then, the solution of equation (6), with

$$f(w, v) = -(w^2 \sin v + 4\sin v + 4\cos v)$$

is

$$U(w, v) = \sum_{n=0}^{\infty} U_n(w, v) = w^2 \sin v.$$

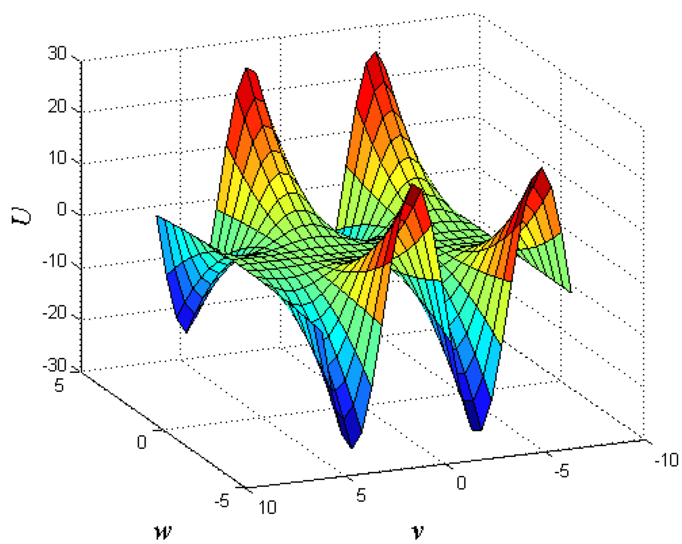

Figure 2. Solutions in Example 2 of Benney-Luke and singular pseudo-hyperbolic equation.

Figures 1 and 2 present the solutions of Benney-Luke and singular pseudo-hyperbolic equations in Examples 1 and 2, respectively.

5. Convergence Analysis

Here we study the convergence of MDLT, for the singular pseudo-hyberbolic equation

$$U_{vv} = \frac{1}{w}\frac{\partial}{\partial w}\left(w\frac{\partial U}{\partial w}\right) + \frac{1}{w}\frac{\partial^2}{\partial w \partial v}\left(w\frac{\partial U}{\partial w}\right) + f(w, v). \tag{17}$$

Consider the Hilbert space, $H = L^2((k, h)\times[0, T])$, defined by the set of applications:

$$\left\{\begin{matrix} (U, V) : (k, h)\times[0, T], \text{ with} \\ \varepsilon_{\eta,\mu}^{-1}\left[\mu^2\int_0^p \varepsilon_{w,v}[U(w, v)(k, h)]dp\right], (w, v) < \infty \end{matrix}\right\}. \tag{18}$$

We write the operator in the form:

$$Q(U) = w\frac{\partial^2 U}{\partial v^2} = \frac{\partial U}{\partial w} + w\frac{\partial^2 U}{\partial w^2} + \frac{\partial^2 U}{\partial w \partial v} + w\frac{\partial^3 U}{\partial w^2 \partial v} + wf(w, v). \tag{19}$$

The proposed analytical technique is convergent if we consider the following:

N1. $(Q(U) - Q(V), U - V) \leq q\| U - V \|, \forall U, V \in H, q > 0.$

N2. If $B > 0$ is a positive constant such that $\| U \| \leq B$, $\| V \| \leq B$, then there exists a constant $C(B) > 0$ such that

$$(Q(U) - Q(V), g) \leq C(B)\| U - V \|\| g \|, \quad \forall g, U, V \in H.$$

The next theorem tells us about the sufficient condition for the convergence.

Theorem 4. *The following holds*:

$$Q(U) = \frac{\partial U}{\partial w} + w\frac{\partial^2 U}{\partial w^2} + \frac{\partial^2 U}{\partial w \partial v} + w\frac{\partial^3 U}{\partial w^2 \partial v} + wf(w, v).$$

To prove this theorem, we just verify N1 and N2 for the above equation. We have

$$|Q(U) - Q(V)| = \left(\frac{\partial U}{\partial w} - \frac{\partial V}{\partial w}\right) + \left(w\frac{\partial^2 U}{\partial w^2} - w\frac{\partial^2 V}{\partial w^2}\right) + \left(\frac{\partial^2 U}{\partial w \partial v} - \frac{\partial^2 V}{\partial w \partial v}\right)$$

$$+ \left(w\frac{\partial^3 U}{\partial w^2 \partial v} - w\frac{\partial^3 V}{\partial w^2 \partial v}\right) + w(f(U) - f(V))$$

$$= \frac{\partial}{\partial w}(U - V) + w\frac{\partial^2}{\partial w^2}(U - V) + \frac{\partial^2}{\partial w \partial v}(U - V)$$

$$+ w\frac{\partial^3}{\partial w^2 \partial v}(U - V) + w(f(U) - f(V)).$$

Therefore, the inner product is given by

$$(Q(U) - Q(V), U - V)$$

$$= \left(\frac{\partial}{\partial w}(U - V), U - V\right) + \left(w\frac{\partial^2}{\partial w^2}(U - V), U - V\right)$$

$$+ \left(\frac{\partial^2}{\partial w \partial v}(U - V), U - V\right) + \left(w\frac{\partial^3}{\partial w^2 \partial v}(U - V), U - V\right)$$

$$+ (w(f(U) - f(V)), U - V) \tag{20}$$

if there exists α_1 such that

$$\left(\frac{\partial}{\partial w}(U - V), U - V\right) \geq \| U - V \|^2, \quad \| w \| \leq \alpha_1. \tag{21}$$

It follows by the use of Schwartz inequality that

$$-\left(w\frac{\partial^2}{\partial w^2}(U-V),\, U-V\right) \le \|w\| \left\|\frac{\partial^2}{\partial w^2}(U-V)\right\| \|U-V\|$$

$$\le \alpha_1\alpha_2 \|U-V\|^2$$

or

$$\left(w\frac{\partial^2}{\partial w^2}(U-V),\, U-V\right) \ge -\alpha_1\alpha_2 \|U-V\|^2,$$

$$-\left(\frac{\partial^2}{\partial w\partial v}(U-V),\, U-V\right) \le \left\|\frac{\partial^2}{\partial w\partial v}(U-V)\right\| \|U-V\| \le \alpha_3 \|U-V\|^2 \quad (22)$$

or

$$\left(\frac{\partial^2}{\partial w\partial v}(U-V),\, U-V\right) \ge -\alpha_3 \|U-V\|^2,$$

$$-\left(w\frac{\partial^3}{\partial w^2\partial v}(U-V),\, U-V\right) \le \|w\| \left\|\frac{\partial^3}{\partial w^2\partial v}(U-V)\right\| \|U-V\|$$

$$\le \alpha_1\alpha_4 \|U-V\|^2 \quad (23)$$

or

$$\left(w\frac{\partial^3}{\partial w^2\partial v}(U-V),\, U-V\right) \ge -\alpha_1\alpha_4 \|U-V\|^2. \quad (24)$$

If f is a Lipschitzian function and $\varsigma > 0$, then according to Cauchy-Schwartz inequality, we have

$$(-w(f(U)-f(V)),\, U-V)$$

$$\le \|w\| \|f(U)-f(V)\| \|U-V\| \le \alpha_1 \|f(U)-f(V)\| \|U-V\|$$

$$\le \alpha_1\varsigma \|U-V\|^2$$

or

$$(w(f(U)-f(V)),\, U-V) \geq -\alpha_1\varsigma\| U-V \|^2. \tag{25}$$

Substituting (21)-(25) into (20), we obtain

$$(Q(U)-Q(V),\, U-V) \geq (\alpha_1-\alpha_1\alpha_2-\alpha_3-\alpha_1\alpha_4-\alpha_1\varsigma),$$

$$\| U-V \|^2 (Q(U)-Q(V),\, U-V) \geq K\| U-V \|^2,$$

$$K = \alpha_1-\alpha_1\alpha_2-\alpha_3-\alpha_1\alpha_4-\alpha_1\varsigma,$$

where α_1, α_2, α_3 and α_4 are constants.

Thus N1 holds.

Now, we verify N2. We have

$$(Q(U)-Q(V),\ g)$$

$$= \left(\frac{\partial}{\partial w}(U-V),\ g\right) + \left(w\frac{\partial^2}{\partial w^2}(U-V),\ g\right) + \left(\frac{\partial^2}{\partial w\partial v}(U-V),\ g\right)$$

$$+ \left(w\frac{\partial^3}{\partial w^2\partial v}(U-V),\ g\right) + (w(f(U)-f(V)),\ g). \tag{26}$$

Exploiting Schwartz inequality, and the fact that U and V are bounded, there exists a number α_5 such that

$$(Q(U)-Q(V),\ g) \leq \alpha_5\| U-V \|\| g \|.$$

Thus N2 holds.

6. Conclusion

Using a newly developed strategy, we found the exact solutions of the Benney-Luke and singular pseudo-hyperbolic equations. More generally, we observe that this strategy is more time-efficient and less demanding.

Acknowledgment

The authors thank the University of Jeddah for technical and financial supports under grant no. (UJ-21-DR-144).

The authors also thank the anonymous referees for their valuable suggestions and comments which led to the improvement of the presentation of the paper.

References

[1] A. A. Alderremy and Tarig M. Elzaki, On the new double integral transform for solving singular system of hyperbolic equations, J. Nonlinear Sci. Appl. 11 (2018), 1207-1214. Available online at www.isr-publications.com/jnsa.

[2] A. A. Hemeda, New iterative method: an application for solving fractional physical differential equations, Journal of Abstract and Applied Analysis 2013 (2013), Article ID 617010, 9 pp.

[3] Aisha Abdullah Alderremy, Tarig M. Elzaki and Mourad Chamekh, New transform iterative method for solving some Klein-Gordon equations, Results in Physics 10 (2018), 655-659. doi: https://doi.org/10.1016/j.rinp.2018.07.004.

[4] E. Hesameddini and H. Latifizadeh, Reconstruction of variational iteration algorithms using the Laplace transform, Int. J. Nonlinear Sci. Numer. Simul. 10(11-12) (2009), 1377-1382.

[5] G. C. Wu and D. Baleanu, Variational iteration method for fractional calculus - a universal approach by Laplace transform, Adv. Difference Equ. 2013 (2013), 18-27.

[6] G. C. Wu, Variational iteration method for solving the time-fractional diffusion equations in porous medium, Chin. Phys. B 21 (2012), 120504.

[7] G. C. Wu and D. Baleanu, Variational iteration method for the Burgers' flow with fractional derivatives-New Lagrange multipliers, Applied Mathematical Modelling 37 (2012), 6183-6190.

[8] G. C. Wu, Challenge in the variational iteration method-a new approach to identification of the Lagrange multipliers, Journal of King Saud University-Science 25 (2013), 175-178.

[9] G. C. Wu, Laplace transform overcoming principle drawbacks in application of the variational iteration method to fractional heat equations, Thermal Science 16(4) (2012), 1257-1261.

[10] Hassan Eltayeb Gadain, Application of double Laplace decomposition method for solving singular one dimensional system of hyperbolic equations, J. Nonlinear Sci. Appl. 10 (2017), 111-121.

[11] Z. Islam, M. M. Hossain and M. A. Sheikh, Exact traveling wave solutions to Benney-Luke Equation, GANIT: Journal of Bangladesh Mathematical Society 37 (2018), 1-14. https://doi.org/10.3329/ganit.v37i0.35721.

[12] J. H. He, Variational iteration method for delay differential equations, Commun. Nonlinear Sci. Numer. Simul. 2(4) (1997), 235-236.

[13] J. H. He, Variational iteration method-a kind of non-linear analytical technique: some examples, Int. J. Nonlinear Mech. 34 (1999), 699-708.

[14] J. H. He, Variational iteration method for autonomous ordinary differential systems, Appl. Math. Comput. 114 (2000), 115-123.

[15] Ranjit R. Dhunde and G. L. Waghmare, On some convergence theorems of double Laplace transform, Journal of Informatics and Mathematical Sciences 6(1) (2014), 45-54.

[16] S. A. Khuri and A. Sayfy, A Laplace variational iteration strategy for the solution of differential equations, Appl. Math. Lett. 25 (2012), 2298-2305.

[17] Tarig M. Elzaki, Application of projected differential transform method on nonlinear partial differential equations with proportional delay in one variable, World Applied Sciences Journal 30(3) (2014), 345-349.
doi: 10.5829/idosi.wasj.2014.30.03.1841.

[18] Tarig M. Elzaki and J. Biazar, Homotopy perturbation method and Elzaki transform for solving system of nonlinear partial differential equations, World Applied Sciences Journal 24(7) (2013), 944-948.
doi: 10.5829/idosi.wasj.2013.24.07.1041.

[19] Tarig M. Elzaki, Salih M. Elzaki and Elsayed A. Elnour, On the new integral transform "ELzaki transform" fundamental properties investigations and applications, Global Journal of Mathematical Sciences: Theory and Practical 4(1) (2012), 1-13.

[20] Tarig M. Elzaki and Salih M. Elzaki, On the connections between Laplace and Elzaki transforms, Advances in Theoretical and Applied Mathematics 6(1) (2011), 1-11.

[21] Tarig M. Elzaki and Eman M. A. Hilal, Solution of telegraph equation by modified of double Sumudu transform "Elzaki transform", Mathematical Theory and Modeling 2(4) (2012), 95-103.

[22] Tarig M. Elzaki, Double Laplace variational iteration method for solution of nonlinear convolution partial differential equations, Archives Des Sciences 65(12) (2012), 588-593.

[23] E. Huntley and A. S. I. Zinober, Applications of numerical double Laplace transform algorithms to the solution of linear partial differential equations, Computing 21(3) (1979), 245-258.

[24] K. Khan, Z. Khan, A. Ali and M. Irfan, Investigation of Hirota equation: modified double Laplace decomposition method, Phys. Scripta 96(10) (2021), 104006.

[25] M. Saif, F. Khan, K. S. Nisar and S. Araci, Modified Laplace transform and its properties, J. Math. Computer Sci. 21 (2020), 137-135.

[26] H. E. Gadain, Solving coupled pseudo-parabolic equation using a modified double Laplace decomposition method, Acta Math. Sci. Ser. B (Engl. Ed.) 38(1) (2018), 333-346.