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ANALYSIS OF GENERALIZED FINITE CONTINUOUS RIDGELET TRANSFORMS WITH SIMPLY SUPPORTED RECTANGULAR KIRCHHOFF PLATES

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Abstract

This work presents the application of generalized finite continuous Ridgelet transform (GFCRT). The solution of Kirchhoff plates with rectangular and simply supported obeying Dirichlet boundary conditions is demonstrated using GFCRT. The inversion formula when applied to the stated problem represents an algebraic solution. In the

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concluding section, the obtained numerical results are discussed with uniformly distributed patch load, general distributed load and point load.

1. Introduction

The continuous Ridgelet transform for a two-variable function $f(x, y)$ was given in [2]. In [6], authors explored Ridgelet transform in the distributions sense. The (CRT) was developed for Schwartz distribution using classical Ridgelet transform on the space of square integrable Boehmians in [4].

Consider a ridge function or ridgelet

$$\left(1 + \sum_{p,q=1}^{\infty} e^{-((pa\pi x/c)+(qa\pi y/d)-b)/a}\right)^{-1}$$

$$= \chi(((pa\pi x/c) + (qa\pi y/d) - b)/a) \tag{1}$$

converging to $\frac{1}{2}[f(-c, -d) + f(c, d)]$ with parameters:

(i) a; the scale of the ridge function,

(ii) b; location of the ridge function,

(iii) $[(pa\pi x/c) + (qa\pi y/d)]$; its orientation,

where $a, b \in \mathbb{R}$.

The classical finite continuous Ridgelet transform was defined in [3] on $[-c, c] \times [-d, d]$ as follows:

$$\Re_{(p,q,a,b)} f(x, y)$$

$$= \frac{a^{-1/2}}{4cd} \int_{-d}^{d} \int_{-c}^{c} f(x, y) \chi\left(\left(\left(\frac{\pi apx}{c} + \frac{\pi aqy}{d}\right) - b\right) \Big/ a\right) dxdy. \tag{2}$$

The inversion formula of (2) is given by

$$\Re^{-1}(\Re_{(p,q,a,b)} f(x, y)) = a^{1/2} \sum_{p,q=1}^{\infty} \Re_{(p,q,a,b)} f(x, y) \chi\left(\left(b - \left(\frac{pa\pi x}{c} + \frac{qa\pi y}{d}\right)\right) \Big/ a\right). \tag{3}$$

The inversion formula due to kernel method in distributional sense is also analyzed. The classical finite continuous Ridgelet transform was extended to generalized functions on certain spaces in [13] with the inversion formula in distributional sense by using kernel method as:

$$\Re_{(p,q,a,b)} f(x, y) = \left\langle f(X), \frac{a^{-1/2}}{4cd} \chi\left(\left(\left(\frac{\pi apx}{c} + \frac{\pi aqy}{d}\right) - b\right) \Big/ a\right)\right\rangle, \tag{4}$$

where $f(x, y) = f(X)$ and $X = (x, y)$.

Using [5], the inversion formula (4) is defined as

$$f(x, y) = \lim_{Q, P \to \infty} \sum_{q=1}^{Q} \sum_{p=1}^{P} a^{-1/2} \Re_{(p,q,a,b)} f(x, y) \times \chi\left(\left(b - \left(\frac{pa\pi x}{c} + \frac{qa\pi y}{d}\right)\right) \Big/ a\right), \tag{5}$$

$\forall P \neq Q.$

The treatment of elastic plates was analyzed in [8]. Classical and shear deformation plate theories are thoroughly discussed with their analytical and numerical solutions for bending, buckling, and natural vibrations. In [9], using the finite Fourier sine transform method, authors solved the boundary value problem for Kirchhoff plates that was supported by simply supported rectangular beams. The numerical results in [10] confirmed that the Radon transform formulation is valid when applied to the vibrations of rectangular thin plates. Bending of fully clamped orthotropic rectangular thin plates solution was presented in [11] using finite continuous Ridgelet transforms subjected to the loadings. Heat conduction in an inverted cone domain was considered by the authors in [12].

The purpose of this paper is to solve several partial differential equations in mathematical physics, such as simply supported rectangular Kirchhoff plates, using the developed GFCRT technique. Also, the methodology is supported by checking validity of the formulation.

2. Methodology

The terminology and notation used in this article are from [1]. Consider $I = [-c_1, c_1] \times [-d_1, d_1]$. From [3], we get

$$\Omega_{x, y, \theta} = (\sin^2 \theta)\Omega_x - (\cos^2 \theta)\Omega_y, \tag{6}$$

where $\Omega_x = D_x^2$ and $\Omega_y = D_y^2$; c, d are real constants.

The operational formula of GFCRT for every $k = 0, 1, 2, \ldots$ from [13] is given as

$$\Omega^k_{x, y, \theta}\, \chi\left(\left(\left(\frac{\pi apx}{c} + \frac{\pi aqy}{d}\right) - b\right)\Big/ a\right)$$
$$= [(-\eta_p^2)^k \sin^2 \theta \cos^{2k} \theta - (-\eta_q^2)^k \cos^2 \theta \sin^{2k} \theta]$$
$$\times \chi\left(\left(\left(\frac{\pi apx}{c} + \frac{\pi aqy}{d}\right) - b\right)\Big/ a\right). \tag{7}$$

3. GFCRT to Simply Supported Rectangular Kirchhoff Plate

Consider the following equation:

$$\frac{\partial^4 \Psi(x, y)}{\partial x^4} + \frac{\partial^4 \Psi(x, y)}{\partial y^4} + 2\frac{\partial^4 \Psi(x, y)}{\partial x^2 \partial y^2} = \frac{p(x, y)}{D}, \tag{8}$$

which represents the differential equation of Kirchhoff plate, where

(1) $p(x, y)$ - load distributed transversely.

(2) $D = \dfrac{Eh^3}{12(1-\mu^2)}$ - flexural rigidity.

(3) E - Young's modulus.

(4) h - thickness.

(5) μ - Poisson's ratio.

(6) $\Psi(x,\ y)$ - transverse deflection.

For $-c \le x \le c, -d \le y \le d,$ the boundary conditions of (8) are

$$\Psi(x,\ y)|_{x=-c}^{x=c} = \frac{\partial^2 \Psi}{\partial x^2}|_{x=-c}^{x=c} = 0, \tag{9}$$

$$\Psi(x,\ y)|_{y=-d}^{y=d} = \frac{\partial^2 \Psi}{\partial y^2}|_{y=-d}^{y=d} = 0. \tag{10}$$

Transverse distribution load $p(x,\ y)$:

Using GFCRT from (4) to both sides of equation (8), it follows that

$$\left\langle \frac{\partial^4 \Psi}{\partial x^4} + 2\frac{\partial^4 \Psi}{\partial x^2 \partial y^2} + \frac{\partial^4 \Psi}{\partial y^4}, \frac{a^{-1/2}}{4cd}\chi\left(\left(\left(\frac{\pi apx}{c} + \frac{\pi aqy}{d}\right) - b\right)\Big/ a\right)\right\rangle$$

$$= \frac{1}{D}\left\langle p(x,\ y), \frac{a^{-1/2}}{4cd}\chi\left(\left(\left(\frac{\pi apx}{c} + \frac{\pi aqy}{d}\right) - b\right)\Big/ a\right)\right\rangle. \tag{11}$$

Hence

$$\frac{a^{-1/2}}{4cd}\int_{-d}^{d}\int_{-c}^{c}\frac{\partial^4 \Psi}{\partial x^4}\chi\left(\left(\left(\frac{\pi apx}{c} + \frac{\pi aqy}{d}\right) - b\right)\Big/ a\right)dxdy$$

$$= \left(\frac{p\pi}{c}\right)^4 \Re_{(p,q,a,b)}\Psi(x,\ y)$$

$$+ \frac{a^{-1/2}e^{-b/a}}{4cd}\int_{-d}^{d}\left(\frac{p\pi}{c}\right)\left[e^{-\frac{p\pi x}{c}}\frac{\partial^2 \Psi}{\partial x^2}|_{x=c} - e^{-\frac{p\pi x}{c}}\frac{\partial^2 \Psi}{\partial x^2}|_{x=-c}\right]e^{\frac{q\pi y}{d}}dy$$

$$+ \frac{a^{-1/2}e^{-b/a}}{4cd}\int_{-d}^{d}\left(\frac{p\pi}{c}\right)^3\left[e^{-\frac{p\pi x}{c}}\Psi(x,\ y)|_{x=c} - e^{-\frac{p\pi x}{c}}\Psi(x,\ y)|_{x=-c}\right]e^{\frac{q\pi y}{d}}dy.$$

Using (9), it follows that

$$\frac{a^{-1/2}}{4cd}\int_{-d}^{d}\int_{-c}^{c}\frac{\partial^4\Psi}{\partial x^4}\chi\left(\left(\left(\frac{\pi apx}{c}+\frac{\pi aqy}{d}\right)-b\right)\Big/a\right)dxdy$$

$$=\left(\frac{p\pi}{c}\right)^4\Re_{(p,q,a,b)}\Psi(x,\ y), \tag{12}$$

where

$$\Re_{(p,q,a,b)}\Psi(x,\ y)=\left\langle\Psi(x,\ y),\ \frac{a^{-1/2}}{4cd}\chi\left(\left(\left(\frac{\pi apx}{c}+\frac{\pi aqy}{d}\right)-b\right)\Big/a\right)\right\rangle$$

is GFCRT of $\Psi(x,\ y)$.

Thus

$$\frac{a^{-1/2}}{4cd}\int_{-d}^{d}\int_{-c}^{c}\frac{\partial^4\Psi}{\partial y^4}\chi\left(\left(\left(\frac{\pi apx}{c}+\frac{\pi aqy}{d}\right)-b\right)\Big/a\right)dxdy$$

$$=\left(\frac{q\pi}{d}\right)^4\Re_{(p,q,a,b)}\Psi(x,\ y)$$

$$+\frac{a^{-1/2}e^{-b/a}}{4cd}\int_{-c}^{c}\left(\frac{q\pi}{d}\right)\left[e^{-\frac{q\pi y}{d}}\frac{\partial^2\Psi}{\partial y^2}|_{y=d}-e^{-\frac{q\pi y}{d}}\frac{\partial^2\Psi}{\partial y^2}|_{y=-d}\right]e^{\frac{p\pi x}{c}}dx$$

$$+\frac{a^{-1/2}e^{-b/a}}{4cd}\int_{-c}^{c}\left(\frac{q\pi}{d}\right)^3\left[e^{-\frac{q\pi y}{d}}\Psi(x,y)|_{y=d}-e^{-\frac{q\pi y}{d}}\Psi(x,y)|_{y=-d}\right]e^{\frac{p\pi x}{c}}dx.$$

From (10), it follows that

$$\frac{a^{-1/2}}{4cd}\int_{-d}^{d}\int_{-c}^{c}\frac{\partial^4\Psi}{\partial y^4}\chi\left(\left(\left(\frac{\pi apx}{c}+\frac{\pi aqy}{d}\right)-b\right)\Big/a\right)dxdy$$

$$=\left(\frac{q\pi}{d}\right)^4\Re_{(p,q,a,b)}\Psi(x,\ y). \tag{13}$$

Also,

$$\frac{a^{-1/2}}{4cd}\int_{-d}^{d}\int_{-c}^{c}\frac{\partial^4\Psi}{\partial x^2\partial y^2}\chi\left(\left(\left(\frac{\pi apx}{c}+\frac{\pi aqy}{d}\right)-b\right)\Big/a\right)dxdy$$

$$=\left(\frac{p\pi}{c}\right)^2\left(\frac{q\pi}{d}\right)^2\Re_{(p,q,a,b)}\Psi(x,y)$$

$$+\frac{a^{-1/2}e^{b/a}}{4cd}\left(\frac{p\pi}{c}\right)\int_{-d}^{d}\left[e^{-\frac{p\pi x}{c}}\frac{\partial^2\Psi}{\partial y^2}|_{x=c}-e^{-\frac{p\pi x}{c}}\frac{\partial^2\Psi}{\partial y^2}|_{x=-c}\right]e^{\frac{q\pi y}{d}}dy$$

$$+\frac{a^{-1/2}e^{b/a}}{4cd}\left(\frac{p\pi}{c}\right)^2\left(\frac{q\pi}{d}\right)\int_{-c}^{c}\left[e^{-\frac{q\pi y}{d}}\Psi(x,y)|_{y=d}-e^{-\frac{q\pi y}{d}}\Psi(x,y)|_{y=-d}\right]e^{\frac{p\pi x}{c}}dx.$$

From (9) and (10), we have

$$\frac{a^{-1/2}}{4cd}\int_{-d}^{d}\int_{-c}^{c}\frac{\partial^4\Psi}{\partial x^2\partial y^2}\chi\left(\left(\left(\frac{\pi apx}{c}+\frac{\pi aqy}{d}\right)-b\right)\Big/a\right)dxdy$$

$$=\left(\frac{p\pi}{c}\right)^2\left(\frac{q\pi}{d}\right)^2\Re_{(p,q,a,b)}\Psi(x,y). \tag{14}$$

Using (12)-(14), (11) gives

$$\left(\left(\frac{p\pi}{c}\right)^2+\left(\frac{q\pi}{d}\right)^2\right)^2\Re_{(p,q,a,b)}\Psi(x,y)=\frac{1}{D}\Re_{(p,q,a,b)}p(x,y), \tag{15}$$

where

$$\Re_{(p,q,a,b)}p(x,y)=\left\langle p(x,y),\frac{a^{-1/2}}{4cd}\chi\left(\left(\left(\frac{\pi apx}{c}+\frac{\pi aqy}{d}\right)-b\right)\Big/a\right)\right\rangle$$

is GFCRT of $p(x,y)$.

By inverse GFCRT (6), we have

$$\Psi(x,\ y) = \left(\frac{a^{-1/2}}{D\pi^4}\right)\lim_{Q,\,P\to\infty}$$

$$\times \sum_{q=1}^{Q}\sum_{p=1}^{P} \frac{\Re_{(p,q,a,b)} p(x,\ y)\chi\left(\left(b - \left(\frac{pa\pi x}{c} + \frac{qa\pi y}{d}\right)\right)\Big/ a\right)}{\left(\left(\frac{p}{c}\right)^2 + \left(\frac{q}{d}\right)^2\right)^2}. \quad (16)$$

(i) Load $p(x,\ y) = P_0$ **concentrated within the plate at** $x = \xi,\ y = \eta$

Consider

$$p(x,\ y) = P_0\delta(x - \xi)\delta(y - \eta). \quad (17)$$

The Dirac delta function is represented by $\delta(x - \xi)\delta(y - \eta)$ with point load applied on the plate along the coordinates ξ and η and P_0 is a constant.

Considering the plate domain $-c \le \xi \le c;\ -d \le \eta \le d$ and applying GFCRT to (8), we obtain

$$\Re_{(p,q,a,b)}\Psi(x,\ y)$$

$$= \frac{P_0\left\langle \delta(x - \xi)\delta(y - \eta),\ \frac{a^{-1/2}}{4cd}\chi\left(\left(\left(\frac{\pi apx}{c} + \frac{\pi aqy}{d}\right) - b\right)\Big/ a\right)\right\rangle}{D\left(\left(\frac{p\pi}{c}\right)^2 + \left(\frac{q\pi}{d}\right)^2\right)^2}$$

which implies

$$\Re_{(p,q,a,b)}\Psi(x,\ y) = \frac{a^{-1/2}P_0\chi\left(\left(\left(\frac{\pi ap\xi}{c} + \frac{\pi aq\eta}{d}\right) - b\right)\Big/ a\right)}{4cdD\left(\left(\frac{p\pi}{c}\right)^2 + \left(\frac{q\pi}{d}\right)^2\right)^2}. \quad (18)$$

Thus, we obtain (5) as

$$\Psi(x,\ y)$$

$$= \left(\frac{P_0}{4acdD\pi^4}\right)\lim_{Q,\,P\to\infty}$$

$$\times\sum_{q=1}^{Q}\sum_{p=1}^{P}\frac{\chi\left(\left(\left(\frac{\pi ap\xi}{c}+\frac{\pi aq\eta}{d}\right)-b\right)\Big/a\right)\chi\left(\left(b-\left(\frac{pa\pi x}{c}+\frac{qa\pi y}{d}\right)\right)\Big/a\right)}{\left(\left(\frac{p}{c}\right)^2+\left(\frac{q}{d}\right)^2\right)^2}. \tag{19}$$

The bending moment is represented as

$$M_{xx} = -D(\Psi_{xx} + \mu\Psi_{yy}), \tag{20}$$

$$M_{yy} = -D(\Psi_{yy} + \mu\Psi_{xx}). \tag{21}$$

For $x = \xi = a/2$, $y = \eta = b/2$, the change in deflection and loading along the point load p at the centre of the plate becomes:

$$\Psi\left(x=\frac{a}{2},\ y=\frac{b}{2}\right) = \left(\frac{P_0}{4acdD\pi^4}\right)$$

$$\times\lim_{Q,\,P\to\infty}\sum_{q=1}^{Q}\sum_{p=1}^{P}\frac{\left(\chi\left(\left(b-\left(\frac{pa\pi x}{c}+\frac{qa\pi y}{d}\right)\right)\Big/a\right)\right)^2}{\left(\left(\frac{p}{c}\right)^2+\left(\frac{q}{d}\right)^2\right)^2}, \tag{22}$$

where $p,\ q = 1,\ 3,\ 5\ldots.$

From (20) and (21), we obtain

$$M_{xx} = \left(\frac{P_0}{4cd\pi^2}\right)\lim_{Q,\,P\to\infty}$$

$$\times\sum_{q=1}^{Q}\sum_{p=1}^{P}\frac{\left(\left(\frac{p}{c}\right)^2+\mu\left(\frac{q}{d}\right)^2\right)\chi\left(\left(b-\left(\frac{pa\pi x}{c}+\frac{qa\pi y}{d}\right)\right)\Big/a\right)}{pq\left(\left(\frac{p}{c}\right)^2+\left(\frac{q}{d}\right)^2\right)^2} \tag{23}$$

and

$$M_{yy} = \left(\frac{P_0}{4cd\pi^2}\right)\lim_{Q,\,P\to\infty}$$

$$\times\sum_{q=1}^{Q}\sum_{p=1}^{P}\frac{\left(\left(\frac{q}{d}\right)^2+\mu\left(\frac{p}{c}\right)^2\right)\chi\left(\left(b-\left(\frac{pa\pi x}{c}+\frac{qa\pi y}{d}\right)\right)\Big/a\right)}{pq\left(\left(\frac{p}{c}\right)^2+\left(\frac{q}{d}\right)^2\right)^2}. \tag{24}$$

(ii) For the region $c_0 \le x \le c_1,\ d_0 \le y \le d_1$, the uniformly distributed patch load

Due to uniformly distributed patch load, the plate deflection over the region $c_0 \le x \le c_1,\ d_0 \le y \le d_1$ is given by

$$\Psi(x,\ y)$$

$$= \lim_{Q,\,P\to\infty}\sum_{q=1}^{Q}\sum_{p=1}^{P}$$

$$\times\int_{c_0}^{c_1}\int_{d_0}^{d_1}\frac{p_0\chi\left(\left(\left(\frac{\pi ap\xi}{c}+\frac{\pi aq\eta}{d}\right)-b\right)\Big/a\right)\chi\left(\left(b-\left(\frac{pa\pi x}{c}+\frac{qa\pi y}{d}\right)\right)\Big/a\right)}{4acdD\pi^4\left(\left(\frac{p}{c}\right)^2+\left(\frac{q}{d}\right)^2\right)^2}d\xi d\eta$$

$$= \left(\frac{p_0}{aD\pi^6}\right)\lim_{Q,\,P\to\infty}\sum_{q=0}^{Q}\sum_{p=0}^{P}\frac{S_{pq}\chi\left(\left(b-\left(\frac{pa\pi x}{c}+\frac{qa\pi y}{d}\right)\right)\Big/a\right)}{pq\left(\left(\frac{p}{c}\right)^2+\left(\frac{q}{d}\right)^2\right)^2}, \tag{25}$$

where

$$S_{pq} = \chi\left(\left(\left(\frac{\pi apt_1}{c}+\frac{\pi aqt_2}{d}\right)-b\right)\Big/a\right)$$

and

$$t_1 = (c_0 - c_1), \quad t_2 = (d_0 - d_1).$$

And bending moment displacement equations are

$$M_{xx} = \left(\frac{P_0}{cd\pi^4}\right) \lim_{Q, P \to \infty}$$

$$\times \sum_{q=1}^{Q} \sum_{p=1}^{P} \frac{S_{mn}\left(\left(\frac{p}{c}\right)^2 + \mu\left(\frac{q}{d}\right)^2\right) \chi\left(\left(b - \left(\frac{pa\pi x}{c} + \frac{qa\pi y}{d}\right)\right)\Big/ a\right)}{pq\left(\left(\frac{p}{c}\right)^2 + \left(\frac{q}{d}\right)^2\right)^2} \tag{26}$$

and

$$M_{yy} = \left(\frac{P_0}{cd\pi^4}\right) \lim_{Q, P \to \infty}$$

$$\times \sum_{q=1}^{Q} \sum_{p=1}^{P} \frac{S_{mn}\left(\left(\frac{q}{d}\right)^2 + \mu\left(\frac{p}{c}\right)^2\right) \chi\left(\left(b - \left(\frac{pa\pi x}{c} + \frac{qa\pi y}{d}\right)\right)\Big/ a\right)}{pq\left(\left(\frac{p}{c}\right)^2 + \left(\frac{q}{d}\right)^2\right)^2}. \tag{27}$$

(iii) $p(x, y) = p_0$ **uniformly distributed over the plate**

The plate deflection is from [7] as follows:

$$\Psi(x, y)$$

$$= \left(\frac{p_0}{4acdD\pi^4}\right) \lim_{Q, P \to \infty} \sum_{q=1}^{Q} \sum_{p=1}^{P}$$

$$\times \frac{\int_{-d}^{d} \int_{-c}^{c} \chi\left(\left(\left(\frac{\pi a p \xi}{c} + \frac{\pi a q \eta}{d}\right) - b\right)\Big/ a\right) \chi\left(\left(b - \left(\frac{pa\pi x}{c} + \frac{qa\pi y}{d}\right)\right)\Big/ a\right) d\xi d\eta}{\left(\left(\frac{p}{c}\right)^2 + \left(\frac{q}{d}\right)^2\right)^2}$$

$$= \left(\frac{p_0}{aD\pi^6}\right) \lim_{Q, P \to \infty} \sum_{q=1}^{Q} \sum_{p=1}^{P} \frac{S'_{pq} \chi\left(\left(b - \left(\frac{pa\pi x}{c} + \frac{qa\pi y}{d}\right)\right) \Big/ a\right)}{pq\left(\left(\frac{p}{c}\right)^2 + \left(\frac{q}{d}\right)^2\right)^2}, \tag{28}$$

where

$$S'_{pq} = e^{\frac{b}{a}} \sinh p\pi \sinh q\pi.$$

The bending moment distribution becomes

$$M_{xx} = \left(\frac{P_0}{cd\pi^4}\right) \lim_{Q, P \to \infty}$$

$$\times \sum_{q=1}^{Q} \sum_{p=1}^{P} \frac{\left(\left(\frac{p}{c}\right)^2 + \mu\left(\frac{q}{d}\right)^2\right) \chi\left(\left(b - \left(\frac{pa\pi x}{c} + \frac{qa\pi y}{d}\right)\right) \Big/ a\right)}{pq\left(\left(\frac{p}{c}\right)^2 + \left(\frac{q}{d}\right)^2\right)^2}, \tag{29}$$

$$M_{yy} = \left(\frac{P_0}{cd\pi^4}\right) \lim_{Q, P \to \infty}$$

$$\times \sum_{q=1}^{Q} \sum_{p=1}^{P} \frac{\left(\left(\frac{q}{d}\right)^2 + \mu\left(\frac{p}{c}\right)^2\right) \chi\left(\left(b - \left(\frac{pa\pi x}{c} + \frac{qa\pi y}{d}\right)\right) \Big/ a\right)}{pq\left(\left(\frac{p}{c}\right)^2 + \left(\frac{q}{d}\right)^2\right)^2}. \tag{30}$$

4. Numerical Results and Discussion

The solution of simply supported rectangular Kirchhoff plates with certain boundary condition has been observed using the newly invented generalized finite continuous Ridgelet transform. The obtained result is discussed under general distributed load $p(x, y)$, uniformly distributed point load P at (ξ, η) over a given area and the entire plate. The deflection functions $\Psi(x, y)$ are shown in (16), (19), (22), (25) and (30) which conclude that $\Psi(x, y)$ is rapidly convergent. The numerical results are compared with [7] as shown in Tables 1, 2 and 3 with comparison graphs 1, 2 and 3, respectively.

Table 1. For simply supported rectangular Kirchhoff plates with loads distributed evenly at $a = 0.0625$, deflection and bending moment coefficients

d/c	Ψ [7]	Ψ	M_{xx} [7]	M_{xx}	M_{yy} [7]	M_{yy}
		Present study		Present study		Present study
1.0	4.07×10^{-3}	4.1×10^{-3}	0.0479	0.05	0.0479	0.05
1.1	4.85×10^{-3}	4.9×10^{-3}	0.0554	0.06	0.493	0.49
1.2	5.64×10^{-3}	5.6×10^{-3}	0.0627	0.06	0.0501	0.05
1.3	6.83×10^{-3}	6.8×10^{-3}	0.0694	0.07	0.0503	0.05
1.4	7.05×10^{-3}	7.1×10^{-3}	0.0755	0.08	0.0502	0.05
1.5	7.724×10^{-3}	7.7×10^{-3}	0.0812	0.08	0.0499	0.05
1.6	8.30×10^{-3}	8.3×10^{-3}	0.0862	0.09	0.0492	0.05
1.7	8.83×10^{-3}	8.8×10^{-3}	0.0908	0.09	0.0486	0.05
1.8	9.31×10^{-3}	9.3×10^{-3}	0.0948	0.09	0.0479	0.05
1.9	9.74×10^{-3}	9.7×10^{-3}	0.0985	0.10	0.0471	0.05
2	10.13×10^{-3}	10.1×10^{-3}	0.1017	0.10	0.0464	0.05
3	12.23×10^{-3}	12.2×10^{-3}	0.1189	0.12	0.0407	0.04
4	12.82×10^{-3}	12.8×10^{-3}	0.1235	0.12	0.0384	0.04
5	12.97×10^{-3}	13.0×10^{-3}	0.1246	0.12	0.0375	0.04
∞	13.021×10^{-3}	13.0×10^{-3}	0.1250	0.12	0.0375	0.04

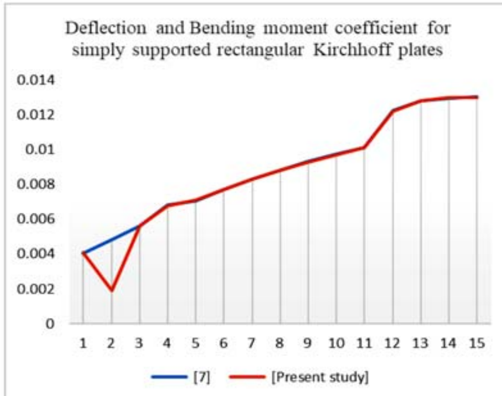

Figure 1. $\Psi = F\dfrac{pa^2}{D}$: comparison between [7] and present work.

Table 2. Convergence analysis for moments of deflection and bending with uniform load at the centre of square Kirchhoff plates having simple support at $a = 0.0625$

No. of terms	Ψ [7]	Ψ	M_{xx} [7]	M_{xx}	M_{yy} [7]	M_{yy}
		Present study		Present study		Present study
1	0.416	0.4	5.34	5.3	5.34	5.3
2	0.405	0.4	4.69	4.4	4.69	4.7
3	0.406	0.4	4.86	4.9	4.94	4.9
4	0.406	0.4	4.81	4.8	4.90	4.9
Exact	0.406	0.4	4.79	4.8	4.79	4.8

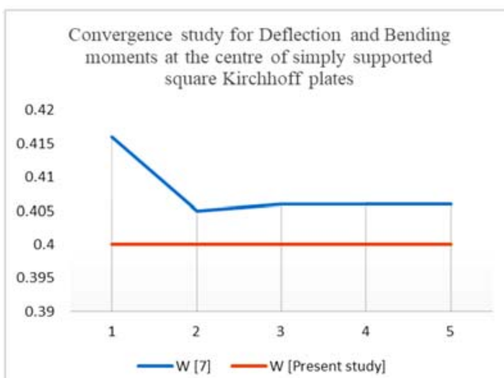

Figure 2. $\Psi = \dfrac{pa^4}{D} \times 10^{-2}$: comparison between [7] and present work.

Table 3. Simply supported rectangular Kirchhoff plates under point load at the centre

d/*c*	Ψ [7]	Ψ present work
1.0	0.01160	0.011
1.2	0.01353	0.013
1.4	0.01464	0.014
1.6	0.01570	0.015
1.8	0.01620	0.016
2	0.01651	0.016

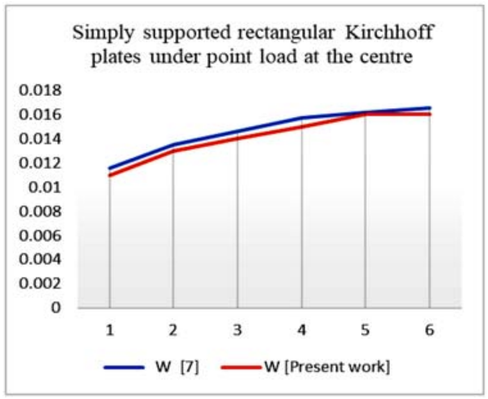

Figure 3. Simply supported rectangular Kirchhoff plates under point load at the centre: comparison between [7] and present work.

Example 1. With σ as standard deviation, the symmetric Gaussian probability density function is as follows:

$$f(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}. \tag{31}$$

GFCRT of (31), using (4) for $-\pi \leq x \leq \pi$ and $-\pi \leq y \leq \pi$ at $\sigma = 1$, $p = 0$, $q = 1$ and $c = d = \pi$,

$$\left\langle \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}, \frac{a^{-1/2}}{4cd} \chi\left(\left(\left(\frac{\pi apx}{c} + \frac{\pi aqy}{d}\right) - b\right) \Big/ a\right) \right\rangle = 0.0153774. \tag{32}$$

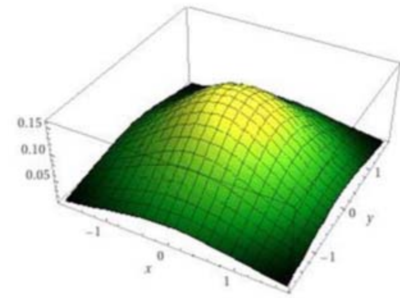

Figure 4. Plot of the symmetric Gaussian probability density function $\frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$ without GFCRT.

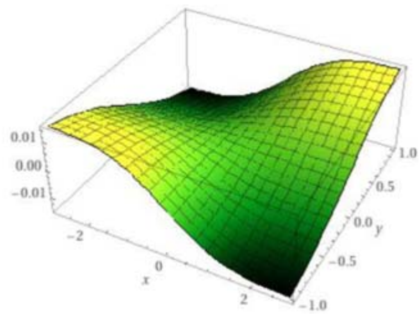

Figure 5. 3D plot of symmetric Gaussian probability density function $\frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$ after application of GFCRT.

Example 2. If nuts and bolts sold per month for hardware manufacturer are represented by x and y, respectively, then the profit function is given by

$$f(x, y) = 16 - (x-3)^2 - (y-2)^2. \tag{33}$$

GFCRT of (33), using (4) for $-\pi \le x \le \pi$ and $-\pi \le y \le \pi$ at $p = 0$, $q = 1$ and $c = d = \pi$,

$$\left\langle (16 - (x-3)^2 - (y-2)^2), \frac{a^{-1/2}}{4cd} \chi\left(\left(\left(\frac{\pi apx}{c} + \frac{\pi aqy}{d}\right) - b\right) \Big/ a\right)\right\rangle = 78.9568. \tag{34}$$

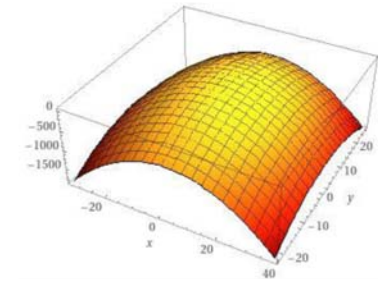

Figure 6. Plot of the profit function for a hardware manufacturer $16-(x-3)^2-(y-2)^2$ without GFCRT.

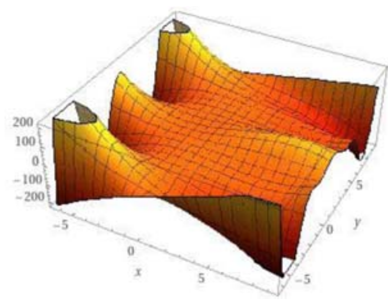

Figure 7. 3D plot of the profit function for a hardware manufacturer $16-(x-3)^2-(y-2)^2$ after application of GFCRT.

5. Conclusion

In this paper, the GFCRT is used to develop analytical flexural solutions for Kirchhoff plates that are simply supported and subjected to various loads. The comparison graph and numerical data demonstrate the validity of the formulation. The GFCRT is also calculated for symmetric Gaussian probability density function and profit function for a hardware manufacturer.

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